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Tandem Toll Booths for the Golden Gate Bridge

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Many toll plazas are constrained in width by buildings or other physical barriers. These barriers may make the cost of adding toll lanes prohibitive. One method for increasing the capacity of a toll facility without increasing its width is to use tandem toll booths. A tandem toll booth consists of two toll takers in a single toll lane both serving alternating sets of vehicles simultaneously. The capacity of tandem toll booths is calculated with time-space diagrams and the cumulative headway distributions of vehicles at a conventional toll booth. The capacity depends on the maximum of two random variables, which correspond to the service times at the two booths, and is found by taking the product of their cumulative headway distributions. Adjacent tandem toll booths were found to increase the capacity of the Golden Gate Bridge toll plaza by about 15 percent, and batch tandem toll booths increase capacity by 25 percent or more. Thus, tandem toll booths would eliminate the queueing that now exists during the morning commute period without the cost of expanding the toll plaza's width.

The Golden Gate Bridge is the primary transportation link between Marin County, California, and the City of San Francisco. Each day 100 000 vehicles traverse the six-lane 2-mile span, 20 000 of which travel in each commute period. The bridge is operated by the Golden Gate Bridge, Highway, and Transportation District and is financed by user fees. Tolls are collected in the southbound direction at the south (San Francisco) end of the bridge.

In February 1981 the bridge board of directors approved an increase in the automobile toll from \$1.00 to \$1.25. Soon afterward, it became evident that the toll plaza could no longer accommodate peak-period traffic. Queues extended as far back as 4 miles; occasionally delays were up to 30 min. Predictably, many motorists were upset and the new toll became the subject of media scrutiny.

The ability of a toll facility to accommodate large traffic flow depends on two factors: the number of servers (toll takers) and the service time per vehicle. The Golden Gate solution to the queuing problem was to reduce the service time per vehicle. Delay had increased with the \$1.25 toll because more motorists needed change and because some motorists folded their dollar bill around their quarter (which caused the toll taker to spend extra time sorting money). The added delay caused the \$1.25 toll to be rescinded in July 1981. The toll was eventually replaced by a split toll of \$1.00 from Sunday through Thursday and \$2.00 on Friday and Saturday. Since that time the number of complaints has dropped considerably, but the equity of the split toll has been questioned. Clearly, the split toll was not motivated by traditional pricing considerations but simply by the need to reduce service time while maintaining revenue.

The alternative approach of increasing the number of servers was perceived to be infeasible in the short run. The peculiar geography of the facility meant that adding toll lanes would require relocation of the entire toll plaza at a cost of \$16 million (1). Other ideas, such as building separate toll facilities for the different bridge lanes, would also be capital intensive.

The purpose of this paper is to demonstrate the consequences of adding toll booths to existing lanes in series rather than adding toll lanes in parallel. The unit of study will be a pair of toll takers serving a single traffic lane. Such a unit, which we call a tandem toll booth, can potentially increase the per-lane throughput, or capacity, and may obviate the need for increasing the width of the toll facility. When the width of a toll facility is physically constrained (by buildings or other barriers), as is the case with the Golden Gate Bridge, tandem toll booths may be cost efficient. Because the length of the toll plaza is also restricted, this paper focuses on tandem-toll-booth strategies that are not greatly affected by the distance between the upstream and downstream toll takers. Although other strategies (such as staggered toll plazas for different lanes or the alternating-toll-booth strategy, described by Rubenstein in another paper in this Record) might increase capacity more, queuing can be nearly eliminated at the Golden Gate Bridge without resorting to these capital-intensive alternatives.

We next describe how to calculate the capacity of tandem toll booths and then report on a case study at the Golden Gate Bridge.

PER-LANE CAPACITY

Single Toll Booths

Before the operation of a tandem toll booth is explained, a single toll booth will be considered. Let the service position be the location of a vehicle when it pays its toll and let the waiting position be the location of the following vehicle in line (Figure 1). Furthermore, let vehicles be numbered 0, 1, 2, ... beginning from the vehicle in the service position.

At the time vehicle 1 leaves the service position, the headway between it and vehicle 0 (H) equals the sum of (a) the headway at the time vehicle 0 left service (H_0) and (b) the service time for vehicle 1 (S), which is the time needed to pay the toll. For the vehicle trajectories on the time-space diagram of Figure 2, H_0 is the sum of a reaction time (R) and a move-up time (M). The reaction time equals the elapsed time between the moment vehicle 0 leaves the service position and the moment vehicle 1 begins to move into the service position. The move-up time equals the time needed to drive from the waiting position to the service position. However, some drivers do not actually stop before entering the service position (Figure 3). Rather, many coast into service at a slow speed, coming to a halt only if vehicle 0 takes particularly long to pay its toll. For these drivers, the reaction time

Figure 1. Conventional toll booth.

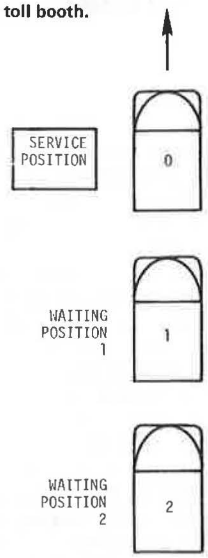


Figure 2. Vehicle trajectories at conventional booth.

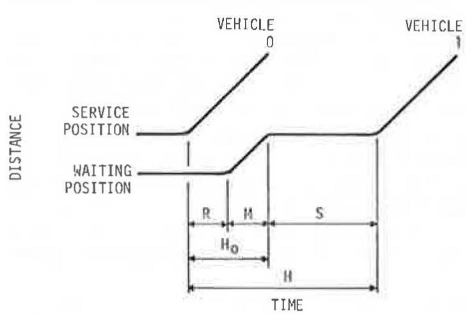
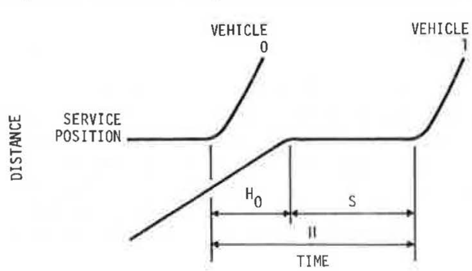


Figure 3. Vehicle coasting into service at conventional booth.



R is virtually zero. (In general, reaction times are very small because drivers anticipate the departure of vehicles from the booth by watching the toll takers.)

If the headway varies randomly from vehicle to vehicle, its expectation is given by

$$E(H) = E(H_0) + E(S) \qquad E(H) = E(R) + E(M) + E(S) \tag{1a}$$

One vehicle is processed per headway, so the capacity per toll lane (C) is the inverse of E(H):

$$C = 1/E(H) \tag{1b}$$

<u>Tandem Toll Booth: Adjacent Servers</u>

Adjacent tandem toll booths have two service positions (position 1 in the front and position 2 in the rear) and no waiting room between (Figure 4). The waiting positions for the following two vehicles are identically numbered. Vehicles that wait at position 1 are served at position 1, and vehicles that wait at position 2 are served at position 2. Let vehicles now be numbered 0, 1, 2, ... beginning from the vehicle initially at service position 2.

In Figure 5, the elapsed time from the moment vehicle 0 leaves service until the moment vehicle 1 completes and leaves service (T_1') is the sum of a reaction, move-up, and service time:

$$T_1' = R_1' + M_1' + S_1' \tag{2}$$

where R_1', M_1', and S_1' are, respectively, the reaction time, move-up time, and service time of vehicle 1.

Vehicle 2 cannot enter its service position until vehicle 1 does the same. Thus, the elapsed time from the moment vehicle 0 leaves service until vehicle 2 completes service (T_2') contains an additional reaction time:

$$T_2' = R_1' + (R_2' + M_2' + S_2') \tag{3}$$

Figure 4. Tandem toll booth.

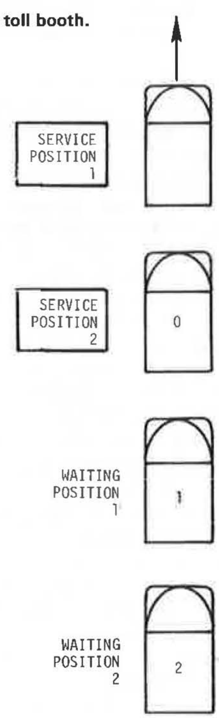
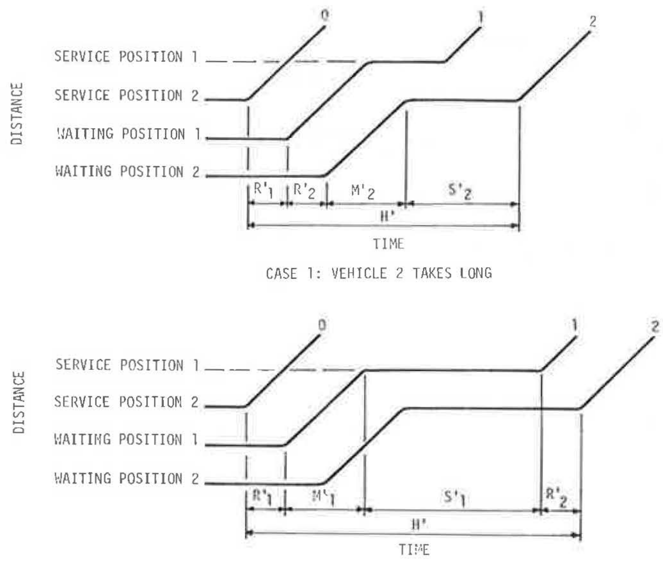


Figure 5. Vehicle trajectories at adjacent tandem toll booths.



However, since service position 1 is just one car space from service position 2, vehicle 2 cannot leave service until its driver perceives that vehicle 1 has left service. Thus, the elapsed cycle time from the moment vehicle 0 leaves service until vehicle 2 leaves service (H') must be greater than or equal to $T_1' + R_2'$:

$$H' \geq T_1' + R_2' \tag{4}$$

In fact, vehicle 2 will depart after both T_2' and $(T_1' + R_2')$ have elapsed. Thus, the cycle time H' between vehicles 0 and 2 is

$$\begin{aligned} H' &= \max(T_2', T_1' + R_2') \\ &= \max[R_1' + (R_2' + M_2' + S_2'), R_2' + (R_1' + M_1' + S_1')] \end{aligned} \tag{5}$$

Suppose that vehicles 1 and 2 were observed as they passed through a conventional toll booth. R_1' and R_2' would then be their approximate reaction times at this single booth (R_1 and R_2), and S_1' and S_2' would be their service times (S_1 and S_2). However, move-up time for the single booth would be less than that for a tandem booth. Move-up time for vehicle 1 (M_1') would equal the M_1 observed at the single booth plus a value ΔM equal to the time needed to traverse the extra distance between the two service positions (Figures 2 and 5). The extra move-up time for vehicle 2 would equal the time needed to traverse an extra car position in the queue. Since the spacing between service positions is approximately the same as the distance between cars in the queue, the move-up time for vehicle 2 would also be $M_2 + \Delta M$. Thus, additional move-up time (ΔM) can be estimated by dividing the distance between vehicles queued at a conventional booth by their peak velocity as they drive into the service position.

If we substitute the service, move-up, and reaction times observed at a single booth for the corresponding variables in Equation 5 and assume (in agreement with observation) that reaction time and ΔM do not vary greatly among vehicles, the following simplified cycle-time equation is obtained:

$$\begin{aligned} H' &\approx R + \Delta M + \max(R_2 + M_2 + S_2, R_1 + M_1 + S_1) \\ &\approx R + \Delta M + \max(H_2, H_1) \end{aligned} \tag{6}$$

where H_1 and H_2 are the headways observed at a conventional booth. As noted earlier, not all drivers stop before entering service at a single toll booth, and most have very short reaction times. Thus, R was not measured precisely. Instead, the sum $R + \Delta M$ was approximated by the headway between cars discharging from traffic signals. According to the well-known Greenshields paper (2), this headway is approximately 3.2 s. However, headways at tandem toll booths might be somewhat larger than those at traffic signals because drivers handle money while driving to the booth and because drivers might be delayed by vehicles at the front booth. But reaction times have decreased since Greenshields' paper [due to automatic transmissions (3)], so the net effect makes 3.2 s a reasonable estimate for $R + \Delta M$. Fortunately, $R + \Delta M$ is easily bounded. It must be between ΔM and H_0 $(R + M)$. Thus, the next section gives capacity estimates for the following reaction times:

1. Low, $R + \Delta M = \Delta M$;
2. Medium, $R + \Delta M = 3.2$ s; and
3. High, $R + \Delta M = H_0$.

The expectation of H' also depends on the expectation of $\max(H_1, H_2)$. The probability that $\max(H_1, H_2)$ is less than any time t equals the probability both that H_1 is less than t and that H_2 is less than t. Thus, the cumulative distribution function for $\max(H_1, H_2)$ is the product of the distribution functions for H_1 and H_2 (Figure 6). The expectation equals the area above the cumulative distribution for $\max(H_1, H_2)$ between t = 0 and t = ∞ and is calculated by numerical integration:

$$E[\max(H_1, H_2)] = \int_0^\infty 1 - P(H_1 < t)P(H_2 < t)dt \tag{7}$$

The capacity of the tandem toll booth in vehicles per unit time (C') is twice the inverse of the cycle time:

$$C' = 2/E(H') \tag{8}$$

because two vehicles are served in each cycle.

The percentage of increase in capacity with adjacent tandem toll booths is derived from Equations 1 and 8:

$$\begin{aligned} \text{Percent capacity increase} &= 100\,\{[2/E(H')]/[1/E(H)] - 1\} \\ &= 100\left(2E(H)/\{R + \Delta M + E[\max(H_1, H_2)]\} - 1\right) \end{aligned}$$

Figure 6. Expectation of max(H_1, H_2).

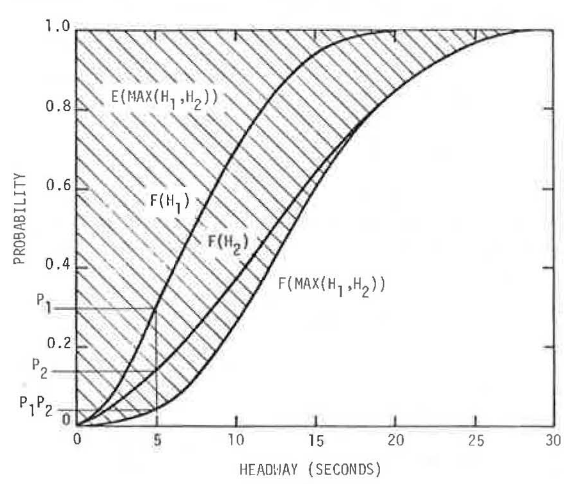


Figure 7. Vehicle trajectories: long service time for first n vehicles.

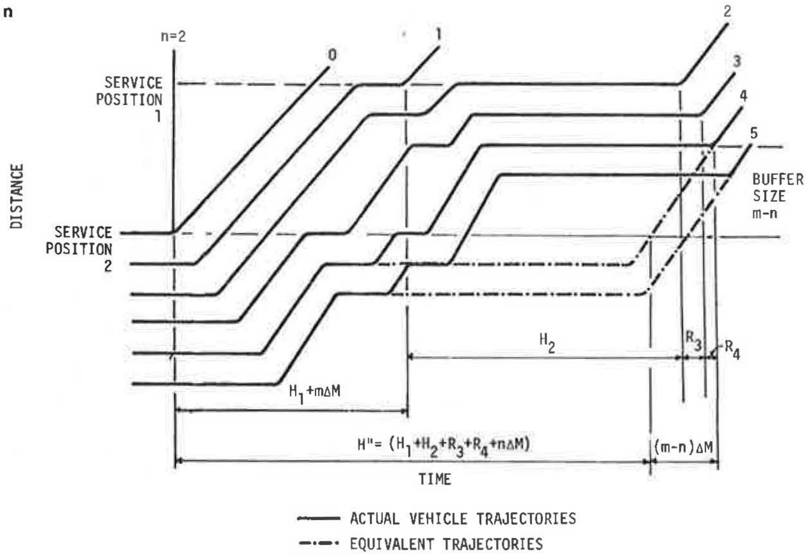
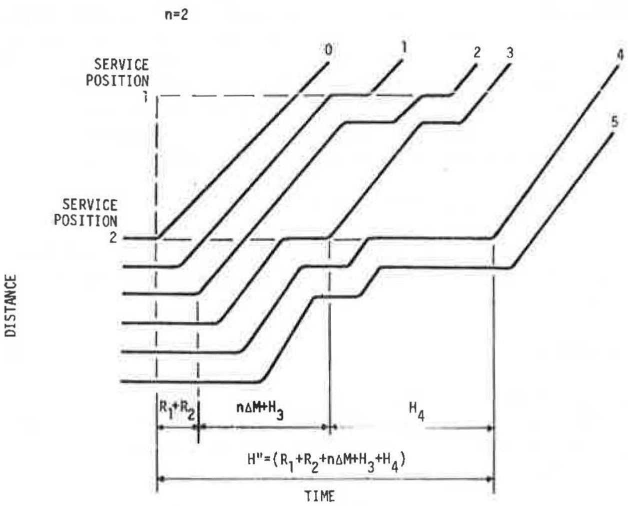


Figure 8. Vehicle trajectories: short service time for first n vehicles.



Tandem toll booths increase capacity if their cycle time is less than twice the headway of conventional booths. Because the reaction time R and move-up time M tend to be small compared with the service time, capacity increase should be substantial. Also note that capacity increases most substantially if headways do not vary greatly among vehicles. Otherwise, $E[\max(H_1, H_2)]$ will be much larger than $E(H)$, and the capacity ratio will be small.

Tandem Toll Booths: Batch Servers

The capacity of tandem toll booths is increased with batch processing. Rather than have the servers collect tolls one vehicle at a time, each server would serve a batch of vehicles in succession. The advantage of this strategy is that random variations in service times are moderated and idle time is reduced.

Suppose that each toll booth processes n vehicles at a time. Then, the last vehicle in a batch served at position 2 would be followed by 2n vehicles. The first n would stop at service position 1 and be processed in succession (each leaving as soon as it pays its toll). The second n vehicles would pay their tolls at service position 2. However, they must queue up behind the vehicles at service position 1 until they all finish paying their tolls. The time-space diagrams in Figures 7 and 8 depict batch tandem booths of size n = 2 for two cases: one in which the service time for the first n vehicles is large and the other in which the service time for these vehicles is small. Again, vehicles are numbered 0, 1, 2, ... beginning from the vehicle initially at service position 2.

To ensure that service is not blocked at position 2 by vehicles waiting for service at position 1, the distance between servers should be somewhat greater than n vehicle position lengths. In fact, from standard results of point processes (renewal theorem), blocking would be rare if the number of positions (m) is at least as follows:

$$m \geq n + [2\sigma_H/E(H)]\,(n-1)^{1/2}$$

where σ_H is the standard deviation of H. The ratio $2\sigma_H/E(H)$ is generally close to 1 (see the next section) and should not change much with different tolls. Therefore, the rounded-up value of $n + (n-1)^{1/2}$ gives the minimum number of positions between servers needed to prevent blocking:

n	1	2	3	4	5	6	...	10	...
m	1	3	5	6	7	9	...	13	...

Values of m smaller than these could result in occasional blockage, and larger values could result in unnecessary distance between servers.

The cycle time between the moment vehicle 0 leaves service and vehicle 2n leaves service (H") equals the sum of n reaction and move-up times and n headways (Figures 7 and 8):

$$H'' = n(R + \Delta M) + \max\left(\sum_{i=1}^{n} H_i, \sum_{i=n+1}^{2n} H_i\right) \quad (9)$$

If $\bar{H}_1^{(n)}$ and $\bar{H}_2^{(n)}$ are the average of two sets of n independent headways (which represent the n vehicles at servers 1 and 2), Equation 9 reduces to the following:

$$E(H'') = nE\,\{R + \Delta M + \max[\bar{H}_1^{(n)}, \bar{H}_2^{(n)}]\} \quad (10)$$

Note that Equations 9 and 10 do not depend on the spacing between servers. Although increasing spacing increases the travel time between servers, vehicles can leave service at booth 2 earlier (because they can queue up behind vehicles at booth 1). These two factors cancel each other. Capacity (C") equals 2n (the number of vehicles per cycle) divided by the cycle time:

$$C'' = 2n/E(H'') \tag{11}$$

If toll takers are similar, $\bar{H}_1^{(n)}$ and $\bar{H}_2^{(n)}$ will have the same expectation [E(H)] and standard deviation $[\sigma_H/(n)^{1/2}]$. If n is greater than 2 or 3, $\bar{H}_1^{(n)}$ and $\bar{H}_2^{(n)}$ will also be approximately normally distributed. Under these conditions, the expectation of the maximum is (4)

$$E\{\max[\bar{H}_1^{(n)}, \bar{H}_2^{(n)}]\} \approx E(H) + [0.4\sigma_H/(n)^{1/2}] \tag{12}$$

which yields the capacity

$$C'' \approx 2\ \{[E(R) + E(\Delta M) + E(H)] + [0.4\sigma_H/(n)^{1/2}]\}^{-1} \tag{13}$$

Even though H is not exactly normally distributed, Equation 12 is fairly accurate, even for small n. Thus, Equation 13 with n = 1 approximates C' (adjacent booths) and eliminates the need for numerical integration.

The capacity improvement with batch processing exhibits decreasing marginal returns with n, as illustrated for the following hypothetical values:

$E(R) + E(\Delta M) = 2.0$ s, $E(H) = 5$ s, and $\sigma_H = 2.5$ s.

From Equation 1b we obtain

C = 0.200 vehicle/s (720 vehicles/h),

and from Equation 13 we obtain

C' $\approx$ 0.250 vehicle/s (900 vehicles/h),
C" $\approx$ 0.267 vehicle/s (960 vehicles/h) if n = 4,
C" $\approx$ 0.276 vehicle/s (1000 vehicles/h) if n = 16,
C" $\approx$ 0.286 vehicle/s (1028 vehicles/h) if n = ∞.

TANDEM TOLL BOOTHS AT GOLDEN GATE BRIDGE

The Golden Gate Bridge toll plaza consists of 13 bidirectional traffic lanes and 12 conventional toll booths. Typically 11 or 12 booths serve southbound (toll-paying) vehicles during the morning commute, whereas just one lane serves northbound traffic. These lanes are reassigned throughout the day to match prevailing traffic patterns.

Following the procedures outlined earlier, data were collected to determine the impact of tandem toll booths on the plaza's vehicle capacity. The morning commute period was chosen for study because the flow of toll-paying vehicles (approximately 7000 vehicles/h) is largest during these hours. As mentioned previously, the bridge collects a split toll; $2.00 is charged two days a week and $1.00 five days a week. Data were collected in good weather for both tolls and then used to estimate the change in capacity and delay. Capacity with a $1.25 toll was also estimated.

Capacity

Vehicles were individually timed as they passed through the toll plaza on $1.00 days and $2.00 days. Consecutive departure times (the times when vehicles began to accelerate from the service position) were recorded for approximately 100 vehicles in each of four toll lanes. Table 1 gives the average and standard deviation of H for the different lanes. The averages closely match the prevailing traffic volumes recorded by the bridge authority during peak periods.

Table 1. Average and SD of headways.

Item	No. of Observations	$\bar{H}$ (s)	S(H) (s)
$1.00 Toll Day			
Lane			
A	95	5.47	2.23
B	86	5.26	2.42
C	78	5.81	1.93
D	96	5.56	2.31
Total	355		
Avg		5.52	2.24[a]
Bridge avg[b]		5.56	
$2.00 Toll Day			
Lane			
A	118	7.43	3.35
B	116	6.39	3.79
C	113	6.30	3.44
D	108	7.26	4.00
Total	455		
Avg		6.84	3.65[a]
Bridge avg[b]		6.53	

[a] Root mean square.
[b] Derived from average traffic volume, 7:30-8:00 a.m.

Reaction and move-up times were later recorded by timing vehicles as they approached the toll plaza. The variable ΔM was estimated by timing vehicles as they traversed a measured distance of 25 ft. Only those vehicles that maintained a constant speed over the entire interval were recorded. The average value of ΔM was 2.7 s; the standard deviation was 0.41 s, which translates to a speed of 6.3 mph.

The expectation of H_0 was determined by recording the elapsed time from the moment a vehicle in the service position began to accelerate away from the toll plaza until the moment the outstretched arm of the following driver reached the toll taker. (Due to time constraints, these observations were recorded at the San Francisco--Oakland Bay Bridge. However, the observed value of H_0 should not differ greatly from that at the Golden Gate Bridge since H_0 is not influenced by the toll charged.) The average of H_0 was 4.22 s; the standard deviation was 0.67 s.

The cumulative headway distributions for the four surveyed lanes were paired into all possible combinations and by following the method described earlier, they were multiplied and integrated to estimate the maximum headways (Table 2). The capacity change was then determined by adding $E(\Delta M)$ (optimistic estimate), 3.2 s (medium estimate), or $E(H_0)$ (pessimistic estimate) to the expectation of the maximum and performing necessary calculations (Equations 7 and 8).

As shown in Table 2, the capacity change for adjacent tandem booths does not vary greatly among the lane combinations. The optimistic estimates for $1.00 days range from a 16 percent increase to a 22.2 percent increase; the average is 18.5 percent. The capacity increase is greater on $2.00 days because service time is longer relative to reaction and move-up times. The increase ranges from 15.2 to 29.0 percent; the average is 21.1 percent. However, the optimistic estimate differs considerably from the pessimistic estimate: 18.5 percent versus 1.8 percent on $1.00 days, 21.5 percent versus 7.0 percent on $2.00 days. Although it is impossible to predict exactly how drivers would behave with tandem toll booths, the actual capacity would likely be close to the medium estimate. This assertion is based on the short reaction times of commuters at the bridge toll plaza.

The capacity estimates for batch tandem toll booths are given in Table 3. As discussed earlier, capacity increases with batch size, although the amount of increase decreases with batch size. As batch size becomes large, lane capacity increases by as much as 10-20 percent over adjacent booths. Even with a batch of size 5, the additional increase falls in the range of 7-13 percent. According to the pessimistic prediction, batch tandem toll booths of size n = 5 increase capacity by at least 8.8 percent on $1.00 days and 17.0 percent on $2.00 days over conventional booths. More realistically, the increase on $1.00 days should be more than 20 percent and on $2.00 days close to 30 percent.

Although the $1.25 toll was discontinued prior to this study, the effect of tandem toll booths on capacity can be inferred. Traffic flow per toll lane was approximately 9.5 vehicles/min with the $1.25 toll, which is nearly the same as the 9.2-vehicle/min rate with the $2.00 toll. Thus, it is likely that a similar number of drivers needed change and that the headway distributions were comparable. Therefore, the percentage capacity increase with tandem booths would likely be similar to that of the $2.00 toll; it would fall in the range of 7-21 percent for adjacent booths and most likely be more than 15 percent. The capacity increase with a batch of size 5 would fall in the range of 17-24 percent; most likely it would be close to 30 percent.

Delay

A precise estimate could not be obtained for the distribution of vehicle arrival times within the constraints of this study. Therefore, attention was given to the capability of tandem toll booths to handle the maximum traffic volume entering the plaza. During one morning commute period, vehicles were counted at the north end of the bridge 2.2 min from the plaza in uninterrupted flow conditions. The maximum flow rate was slightly less than 7800 vehicles/h, which is close to saturation for the Golden Gate Bridge roadway (four lanes with no shoulder), and was sustained for about 15 min. One can deduce that the arrival rate of vehicles at the plaza would never greatly exceed 7800 vehicles/h over any reasonably long time interval (1 min or greater). Thus, a toll plaza capacity in excess of 7800 vehicles/h would ensure that transient queues and delays would always be short (30 s or less).

Even during the busiest periods, the average arrival rate is considerably less than 7800 vehicles/h. Toll plaza queues on $1.00 days tend to be small or nonexistent. Therefore, vehicle counts recorded at the plaza on these days are representative of both arrivals and departures. The average 15-min vehicle count on $1.00 days between 7:30 and 8:00 a.m. on 97 workdays in 1981 was 1782 vehicles, or a rate of 7127 vehicles/h. If the capacity of the toll plaza were approximately 7200 vehicles/h, transient queues would be common but they would not become increasingly long. Even if the maximum flow rate were sustained for 30 min (which is unlikely), delay would reach just 2.5 min. Average delay would likely be a minute or less.

Table 2. Effect of adjacent tandem booths on lane capacity.

Item	$E[\max(\bar{H}_1, \bar{H}_2)]$ (s)	C′ [vehicles/(lane·min)] High Estimate	Medium Estimate	Low Estimate
$1.00 Toll Day				
Lane combination				
A, A	6.57	12.94	12.28	11.12
A, B	6.46	13.09	12.42	11.23
A, C	6.70	12.77	12.12	10.99
A, D	6.66	12.82	12.17	11.03
B, B	6.33	13.29	12.59	11.38
B, C	6.62	12.88	12.22	11.07
B, D	6.55	13.02	12.31	11.14
C, C	6.80	12.63	12.00	10.89
C, D	6.78	12.65	12.02	10.91
D, D	6.74	12.71	12.07	10.95
Avg	6.62	12.88	12.22	11.07
Single toll booth			10.87	
$2.00 Toll Day				
Lane combination				
A, A	9.09	10.18	9.76	9.02
A, B	8.66	10.56	10.12	9.31
A, C	8.56	10.66	10.20	9.40
A, D	9.15	10.12	9.72	8.97
B, B	8.05	11.16	10.67	9.78
B, C	7.96	11.31	10.75	9.85
B, D	8.64	10.59	10.14	9.34
C, C	7.87	11.35	10.84	9.92
C, D	8.56	10.66	10.20	9.39
D, D	9.17	10.10	9.70	8.95
Avg	8.57	10.66	10.21	9.39
Single toll booth			8.77	

Table 3. Effect of batch tandem toll booths on lane capacity.

Toll ($)	Conventional Booth [vehicles/(lane·min)]	n = 1 Vehicles per Lane per Minute	n = 1 Increase (%)	n = 2[a] Vehicles per Lane per Minute	n = 2[a] Increase (%)	n = 5[a] Vehicles per Lane per Minute	n = 5[a] Increase (%)	n = 10[a] Vehicles per Lane per Minute	n = 10[a] Increase (%)	n = ∞ Vehicles per Lane per Minute	n = ∞ Increase (%)
High Estimate[b]											
1.00	10.87	12.88	18.5	13.52	24.4	13.89	27.1	14.08	29.5	14.58	34.1
2.00	8.77	10.66	21.5	11.36	29.5	11.79	34.4	12.01	36.9	12.60	43.7
Medium Estimate[b]											
1.00	10.87	12.22	12.40	12.83	18.0	13.17	21.0	13.33	22.6	13.76	26.6
2.00	8.77	10.21	16.4	10.84	23.6	11.22	28.0	11.43	30.3	11.95	36.3
Low Estimate[b]											
1.00	10.87	11.07	1.8	11.57	6.4	11.83	8.8	11.97	10.1	12.32	13.3
2.00	8.77	9.39	7.0	9.93	13.2	10.26	17.0	10.42	18.8	10.86	23.8

[a] Clark approximation (4).
[b] Vehicles per lane per minute averaged over all lane combinations.

Table 4. Vehicle capacity of toll plaza.

Toll ($)	Capacity (vehicles/h) Current[a]	Adjacent Tandem Booths High	Adjacent Tandem Booths Medium	Adjacent Tandem Booths Low	Batch Tandem (n = 2) High	Batch Tandem (n = 2) Medium	Batch Tandem (n = 2) Low	Batch Tandem (n = 5) High	Batch Tandem (n = 5) Medium	Batch Tandem (n = 5) Low
1.00	7127	8116	7790	7223	8431	8089	7469	8576	8250	7597
2.00	6616	7683	7430	6963	8080	7787	7271	8323	8005	7460
1.25	6824	7924	7663	7187	8274	8032	7499	8584	8257	7694

[a]Capacity estimate based on average traffic flow from 7:30 to 8:00 a.m. on 97 $1.00 days, 86 $1.25 days, and 14 $2.00 days in 1981.

The layout of the toll plaza prevents installing tandem toll booths on all traffic lanes, particularly the far-right lane (which serves many trucks and buses) and the two far-left lanes (which already operate below capacity when other lanes have long queues). Tandem toll booths could possibly be installed on the other nine lanes.

Table 4 gives the current capacity of the toll plaza under the $1.00, $1.25, and $2.00 tolls and capacity estimates for adjacent and batch tandem toll booths. These capacities are based on the average traffic flow between 7:30 and 8:00 a.m. on workdays, a period when the plaza is usually congested. This estimate is slightly conservative since the arrival rate sometimes falls below capacity (such as when there is an accident on the bridge). The tandem estimates account for nine tandem and three conventional booths and are based on the percentages given in Table 3. Furthermore, it was assumed that all lanes carry equal amounts of traffic (the three lanes without tandem booths actually carry slightly less than average, so this underestimates tandem capacity).

Tandem capacity on $1.25 days is between 7187 and 7924 vehicles/h. Thus, even under the most pessimistic estimate, capacity would still exceed the average traffic volume and queues would never become excessively long. Under the optimistic estimate, capacity would exceed the maximum arrival rate at the toll plaza (7800 vehicles/h), and queues would never become more than a few vehicles long. The medium estimate yields a capacity of more than 7600 vehicles/h. Therefore, we conclude that nine tandem booths would accommodate all traffic during the morning commute without significant delay.

The capacity estimates for $2.00 days are also encouraging. It is very likely that adjacent tandem booths would handle the average traffic volume entering the toll plaza (see medium estimate). However, the toll plaza would not handle the maximum arrival rate. Also, the pessimistic capacity estimate falls below the average traffic volume recorded on $1.00 days. The capacity of batch tandem toll booths (size n = 2) does exceed the average traffic volume and would likely be very close to the maximum arrival rate. Because part of the delay incurred on $2.00 days results from confusion regarding the split toll, capacity would surely be greater than these estimates if a $2.00 toll were charged every day of the week and likely be close to that for $1.25 days.

DISCUSSION

It was not feasible to collect arrival and service time data for all time periods, but it is still possible to make a good estimate of the impact of tandem toll booths throughout the day and week. Of particular interest are Sunday afternoons, when many vacationers return to San Francisco, and Friday evenings, when many Marin residents travel into the city. Queues have been particularly long at these times because fewer lanes are allocated to the toll-paying direction (the northbound traffic is greater than in the morning) and because service times are considerably longer than during the morning commute (drivers are less familiar with the toll-taking system). This longer service time makes tandem booths more desirable. Thus, the capacity should increase by a greater percentage than the estimates provided above and would likely be 20 percent or greater on $2.00 days for adjacent booths.

The estimates provided above apply specifically to days when the weather is fair. Tandem toll booths may not be as effective on rainy or very foggy days. Drivers would likely be more cautious moving into the service position. Thus, the move-up time and reaction time should be larger relative to the service time, and the capacity increase should be less than that predicted earlier. However, the capacity on the bridge itself may be sufficiently small on poor weather days that queues never appear at the toll plaza anyway.

The capacity of tandem booths is most accurately estimated with a simple experiment. An additional toll taker could temporarily collect tolls in tandem behind each of the existing toll booths. Car counts could then be compared with the current averages to obtain an accurate estimate of the capacity change. The experiment should be repeated over several time periods so that capacity throughout the day and week can be determined.

CONCLUSIONS

It is clear from Equation 12 that the capacity of tandem toll booths is greatest when service times have a small variance and the move-up and reaction times are small relative to the service time. Although the Golden Gate Bridge is an ideal location for tandem booths from the aspect of small service time variance, its small service time average keeps the capacity increase relatively low. Because traffic only slightly exceeds capacity, this increase would eliminate most of the existing delay at the toll plaza. A capacity increase of about 15 percent is expected for adjacent booths and about 25 percent for batch booths of size 5. In both cases the actual capacity is influenced by the size of the toll, familiarity of drivers with the toll-taking system, and weather conditions.

Although tandem toll booths use personnel less efficiently, they do increase land use efficiency; they offer increased capacity without increased plaza width. Since the capacity of the toll plaza at the Golden Gate Bridge is insufficient during only a few hours a day, the labor cost would be small compared with the cost of constructing new lanes. Tandem toll booths would cost $1.5 million to install and $150 000 per year to staff, which appears attractive compared with the $16 million cost of installing a new toll plaza (1).

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Abridgment

Tandem Toll Collection Systems

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By using two or more collection stations in the same traffic lane, tandem toll or parking-fee collection increases lane capacity and reduces the need for plaza widening. Data are presented relating processing rates to toll fee; e.g., the rate for a $1.25 fee is 30 percent slower than that for a $1.00 fee. Toll agencies that have implemented $1.25 tolls have encountered extreme congestion, especially with the weekend recreational traveler. Several operational configurations of tandem tolls are described. A coordination device is described to automate the control of motorist traffic signal and payment signal to distinguish between axle registrations of successive vehicles, even under dense conditions. Slow collection devices such as paper-money acceptors or flexible-ticket readers that are impractical at a conventional active lane are feasible in tandem. The expected capacity increase depends on the conventional cycle time, its standard deviation, and the distance between the toll stations. When the distance is several vehicle lengths, the stations are buffered, which results in better performance and independence of capacity increase on cycle time variance. The slower the existing collection time, the greater the capacity increase, e.g., 6 s/vehicle yields a 34 percent increase, 20 s/vehicle, 1.75 percent increase, when buffered.

There is a growing need for measures such as tandem toll booths to rapidly increase the traffic capacity of existing toll plazas. The experience at the Golden Gate Bridge and the Triborough Bridge and Tunnel Authority with long queues when toll rates were raised to more than $1 can be expected to be repeated at other tollway facilities. The high inflation rate of the last several years, one-way toll collection, and the use of toll surpluses to subsidize mass transit operations are pushing many toll fees to above the $1 level.

Stop-watch surveys that I have conducted indicate the relative effect of the toll fee on the vehicle-processing rate; they are summarized in Table 1.

Many existing toll plazas were designed when traffic volumes were lower and vehicle-processing rates higher and are not equipped to accommodate fees of more than $1. As toll fees rise, the problem will become more widespread.

This approaching problem will remain for years. Efforts by the U.S. Treasury Department to popularize the use of a $1 coin have not been successful. Similarly, efforts by toll operators to promote use of high-value tokens have met public resistance and are not very effective with the weekend recreational traveler. Busy motorists are not willing to accept the inconvenience and advance payment requirements of token prepurchase without a substantial discount. Even a 10 percent discount for tokens will reduce the revenue of many facilities a greater amount than the total cost of the existing toll-collection system. Token discounts also increase opportunities for employee fraud.

New technologies such as automatic vehicle identification had offered potential for speeding toll processing, but after years of development they have still not overcome the operational, cost, and privacy obstacles to their widespread implementation. Toll-collection computerization programs have been directed at improved auditing capabilities and not improved traffic flow.

The patronage of toll booths in the outer roadway lanes is lower than that in the central lanes, even under congested conditions. The approach to a toll plaza must be widened gradually over a long distance, which increases construction and maintenance costs, particularly on elevated plazas. If there are heavy weaving movements due to the location of particular entrance/exit ramps, even long tapers may not be effective. Tandem lanes would also lessen air-pollution levels in the toll plaza.

Table 1. Effect of toll fee on processing rate.

Passenger-Car Fee ($)	Bridge Surveyed	Manual Lane-Processing Rate[a] (s/car)	
		Sample Avg	Best Avg
1.20	San Diego-Coronado, California	10.0	9.1
1.25	Throggs Neck, New York City	9.8	9.0
2.00	Golden Gate, San Francisco, California	8.8	6.5
0.75	San Francisco-Oakland Bay, San Francisco, California	6.9	6.6
1.00	Golden Gate	6.4	6.1
0.40	Carniquez, I-80, Vallejo, California	6.3	5.9
0.25	Vincent Thomas, Long Beach, California	5.9	5.5

[a]Observations are based on 120 observations per bridge, under moderate traffic; survey was conducted in spring 1982 during hours when commuter carpool free-passage rates were not in effect. Plaza grades were zero to slight. Best averages exclude patrons with exceedingly long service times, apparently unrelated to the toll fee.