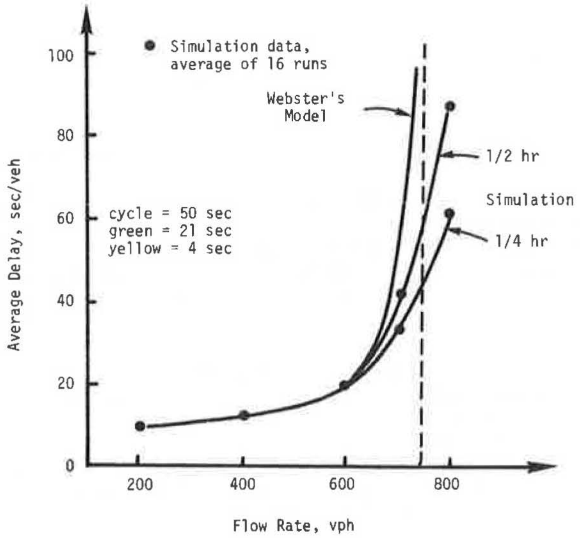
Figure 7. Comparison of simulated delays with Webster's delay under pretimed control.



will not always result in an optimal control under varying traffic-flow conditions. Precisely for this reason, it is desirable to have a reasonably simple yet reliable model to determine the trade-off of timing settings with respect to a typical daily flow pattern. Such a trade-off analysis would enable one to select permanent settings. The use of volume-density control alleviates but does not eliminate this problem.

It would certainly be a blessing if the operation of a signal control could be adequately represented by a primitive and intuitive model. In reality, such intuitive models for analyzing traffic-actuated controls are often misleading and are usually not better than practicing engineers' intuitive judgments. In anticipation of increased use of microcomputers, there is room for developing models that are more reliable than intuitive models but less difficult to use than most existing simulation models.

REFERENCES

1. P.J. Tarnoff and P.S. Parsonson. Selecting Traffic Signal Control at Individual Intersections. NCHRP, Rept. 233, 1981.
2. Interim Materials on Highway Capacity. TRB, Transportation Research Circular 212, Jan. 1980.
3. F.V. Webster. Traffic Signal Settings. U.K. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, TRRL Rept. 39, 1961.
4. F.B. Lin. Predictive Models of Traffic-Actuated Cycle Splits. Transportation Research, Vol. 16, Part B, No. 5, 1982, pp. 361-372.
5. F.B. Lin. Estimation of Average Phase Duration of Full-Actuated Signals. TRB, Transportation Research Record 881, 1982, pp. 65-72.
6. R.W.T. Morris and P.G. Pak-Poy. Intersection Control by Vehicle-Actuated Signals. Traffic Engineering and Control, Oct. 1967, pp. 288-293.
7. K.G. Courage and P.P. Papapanou. Estimation of Delay at Traffic-Actuated Signals. TRB, Transportation Research Record 630, 1977, pp. 17-21.
8. TRANSYT-7F User's Manual. FHWA, Dec. 1981.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.

Another Look at Bandwidth Maximization

KARSTEN G. BAASS

One solution to the problem of fixed-time traffic signal coordination is the provision of a large green band that allows road users to drive at a reasonable speed without stopping. This solution is popular with drivers, although it does not necessarily lead to delay minimization except in special cases. A method for deriving the globally maximal bandwidth together with all possible suboptimal values is described. The programs WAVE1 and WAVE2 can also be used to generate curves that show the continuous relation between uniform progression speed and corresponding maximal bandwidth over a wide range of speeds and cycles. The typical shape of this bandwidth-speed relationship is explained theoretically, and the theory is used in the development of the algorithm. It is shown that bandwidth varies greatly with progression speed and it is suggested that setting bandwidth at the globally optimal value may not always be the best choice. The decision to adopt a progression speed, a bandwidth, and a cycle time should take into account a range of values of speed and cycle. The proposed method was applied to 18 data sets of up to 24 intersections taken from the published literature and the results obtained were compared with those given by the mixed-integer linear-programming approach. Computer execution time is extremely short and the storage space required is negligible, so the method could be of interest in practical applications.

The maximization of bandwidth is one of the two approaches used for determining offsets between fixed-time traffic lights on an artery. There are a number of fairly restrictive hypotheses related to this approach, e.g., the assumptions of a uniform platoon, no platoon dispersion, low volumes, and no or very few cars entering the artery from side streets. Situations corresponding to these assumptions are rare. Nevertheless, the bandwidth-maximizing approach is psychologically attractive to the user, who is unable to distinguish between a nonsynchronized artery and one that is perfectly synchronized for delay and stop minimization but does not allow the user to pass at a reasonable speed through the artery without stopping.

Little and others (1) and Morgan (2) were the first to suggest a mathematical formulation for the bandwidth-maximizing problem, and more recently Little and others (3) published a program called MAXBAND. This program is based on a mixed-integer linear-programming approach and determines the speeds that give the overall maximum bandwidth over a range of acceptable speeds. The linear-programming approach also allows for variations in speed between intersections and enables new constraints to be easily introduced.

This paper describes an algorithm that determines the overall maximum bandwidth together with all suboptimal values, if they exist, for a wide range of speeds. At this time, only two-phase fixed-time traffic light operation is considered and a constant speed on the artery is assumed, but the algorithm could be generalized to changing speeds between intersections.

The solution approach is essentially geometric, and the algorithm is extremely fast for arteries of up to 25 intersections. The limit of 25 is not due to the algorithm nor to computing time but was set because this number is quite high enough for practical synchronization problems. The approach also provides insights into the theoretical relationships and may explain why the bandwidth approach works better in certain cases than in others.

THEORETICAL CONSIDERATIONS

Morgan (2) has shown that for a given speed there is a set of half-integer offsets that gives maximal equal bandwidths. It is shown also that one can derive another optimal solution with unequal speeds and bands from this initial solution. The following discussion will thus pertain only to equal maximal bandwidths.

Consider first one of the possible half-integer sets of offsets, represented for simplicity on a time-space diagram as in Figure 1 (offset scheme 0-1-0-0). For the present, we consider only speeds between a minimum and a maximum speed, as shown in Figure 1 by the dotted and interrupted lines. In the following, when necessary, the constant K = V*C is used instead of a fixed speed and a fixed cycle. This relation is given by a simple scale transformation of the time-space diagram. Figure 1 shows that there are two extremal values of speed possible. The first (V_d) occurs when the speed line is tangential to two of the lower reds and the second (V_u) when the speed line is tangential to two of the upper reds. These two speeds will be called possible optimal speeds. They give rise to two parallel bands of different bandwidths as shown in Figures 2 and 3.

As progression speed decreases in Figure 2, the bandwidth decreases until a speed is reached where bandwidth is zero. As speed increases in Figure 3 (or the slope of the speed line decreases), the bandwidth will ultimately become zero if the speed does not reach infinity first. These relations can be expressed by the following equations. For the case in which the speed line is tangential to lower reds and i,j are the critical lights, the slope of the speed line is given by

$$s = 360/V_d{*}C = (d_j - d_i)/(x_j - x_i)$$

$$d_m = d_i + s(x_m - x_i) \qquad (1)$$

and the bandwidth at all m intersections is given by

$$b_{md} = (r_m/2) + INT{*}50 + g_m - d_m$$

$$b_{md} = INT{*}50 - [(r_m + r_i)/2] - s(x_m - x_i) \qquad (2)$$

Figure 1. Geometry of a progression.

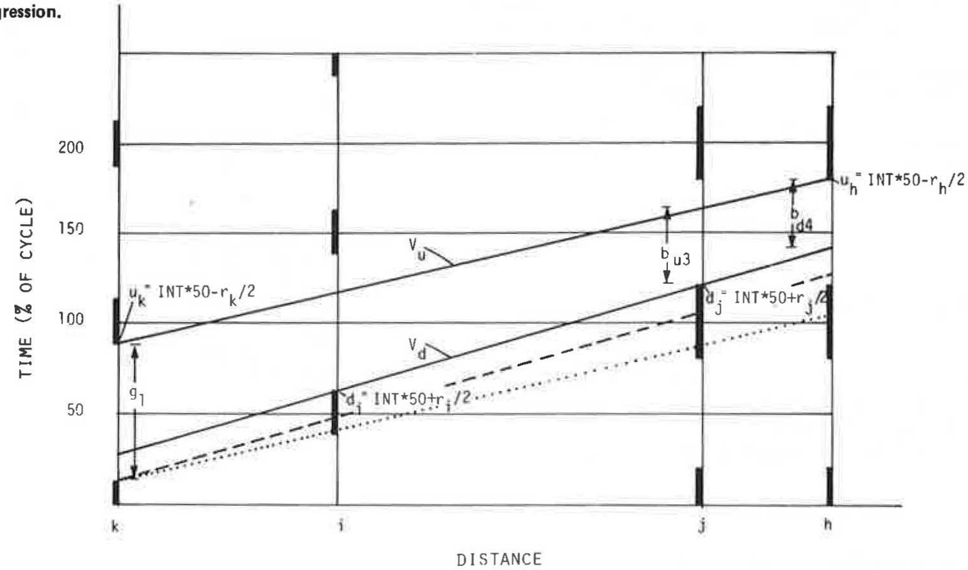


Figure 2. Band corresponding to speed V_d in Figure 1.

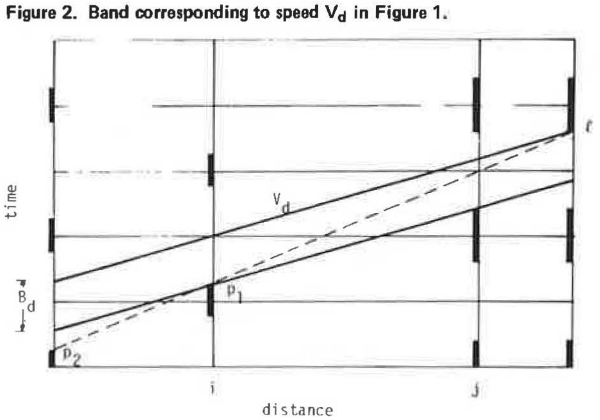
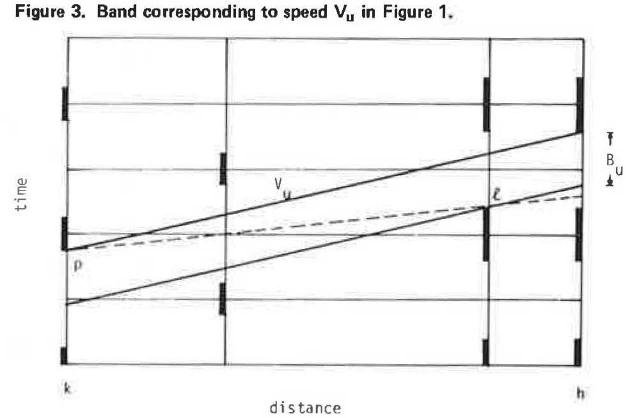


Figure 3. Band corresponding to speed V_u in Figure 1.



where

x_m = cumulative distance from first intersection,
d_m = ordinate of upper edge of lower red,
u_m = ordinate of lower edge of upper red,
r_m = percentage of red at intersection m,
g_m = percentage of green at intersection m, and
C = cycle length.

INT is an integer such that

$0 \leq b_{md} \leq g_m$

where b_{md} is as large as possible within these limits. The maximal bandwidth becomes

$$B_d = \min_m \{b_{md}\}$$

For the case in which the speed line is tangential to the upper reds and h and k are the critical lights,

$$V_u = (360/C)\,[(x_h - x_k)/(u_h - u_k)] \quad (3)$$

$$b_{mu} = \mathrm{INT}*50 - [(r_m + r_k)/2] + s(x_m - x_k) \quad (4)$$

$$B_u = \min_m \{b_{mu}\}$$

The speed line can now be pivoted about p_1 in Figure 2 and speeds and corresponding bandwidths up to a bandwidth of zero can be determined. Clearly, the upper red that limits the bandwidth will have to be found, and it will in certain cases also happen that the speed line touches a red for m < p before B = 0.0 is obtained. In this case, the pivot will have to be changed. The formula that gives the bandwidth for a known pivot point and a known limiting red becomes

$$B_d = \mathrm{INT}*50 - [(r_p + r_\ell)/2] + (360/VC)(x_p - x_\ell) \quad (5)$$

$$B_u = \mathrm{INT}*50 - [(r_p + r_\ell)/2] - (360/VC)(x_p - x_\ell) \quad (6)$$

where p is the number of the pivot intersection and ℓ is the number of the limiting intersection.

We now consider an artery of four intersections in Laval, Quebec, as an example to illustrate these equations. The data are given below and the cycle time is 80 s:

Artery	Distance (m)	Red (%)
1	0.00	25
2	297.18	24
3	803.15	40
4	987.55	40

Figure 5. Possible speed bands from intersection i for speeds between V_{max} and V_{min}.

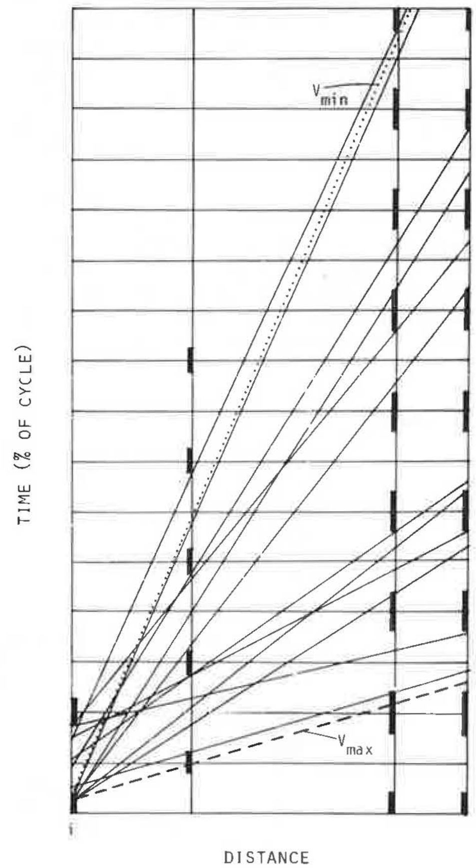
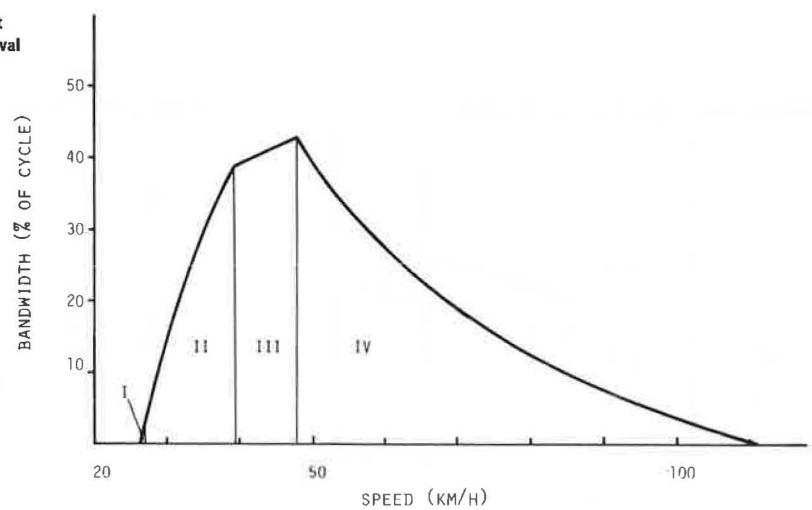


Figure 4. Curve relating speed to bandwidth for offset scheme and speeds V_u and V_d shown in Figure 1 (Laval example).



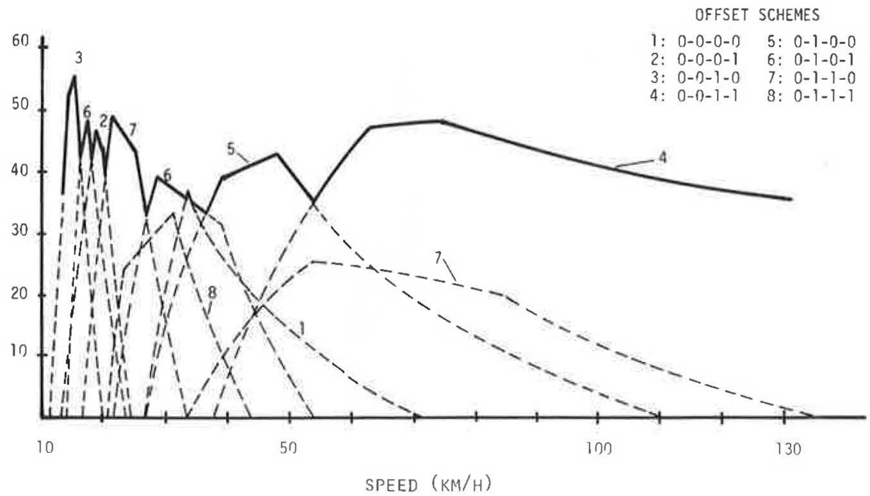


Figure 6. Curves relating speed to bandwidth for all possible offset schemes for Laval example (n = 4).

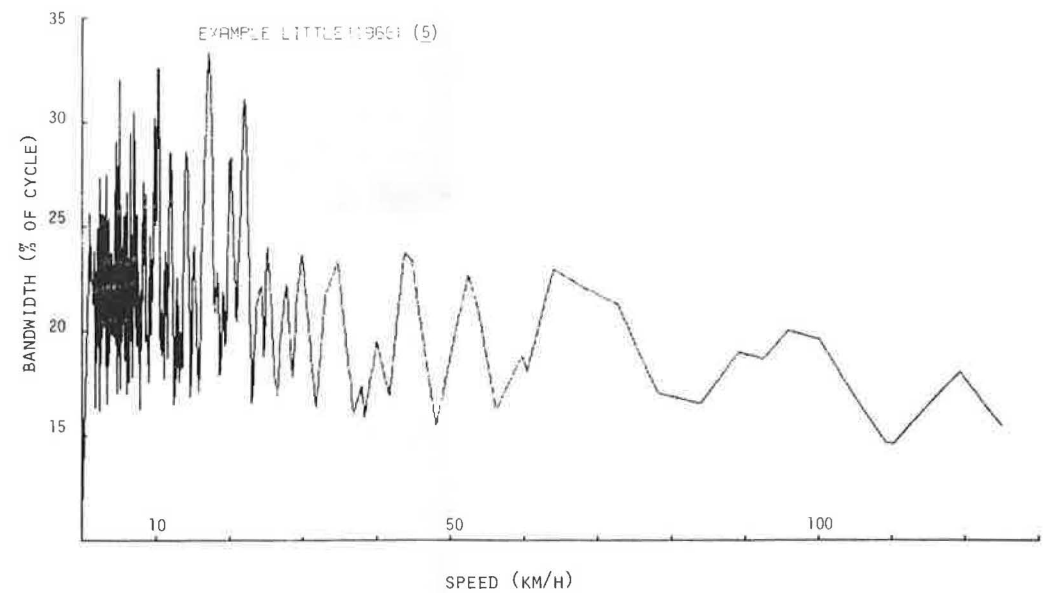


Figure 7. Example of V-B curve with major oscillations at lower speeds (data set 2, 10 intersections, 52 percent minimum green).

Figure 1 shows this artery and from Equations 5 and 6 the relationships tabulated below can be derived:

V_1	V_2	Bandwidth
26.53	27.02	I: B = INT*50 - 32.5 - 4443.975/V
27.02	39.26	II: B = INT*50 - 32.0 - 3106.665/V
39.26	48.04	III: B = INT*50 - 40.0 - 829.800/V
48.04	111.21	IV: B = INT*50 - 32.5 + 3614.175/V

A pivot change from intersection 2 to 1 has to be taken into account at a speed of 27.02 km/h. The equations above give the relation between B and V (or K = V*C and B) as shown in Figure 4. This curve is typical; it has a concave part up to the lower optimal speed and a convex section for speeds greater than the highest optimal speed. There is only one optimal speed in the case in which the critical lights for V_d and V_u are the same and have the same amount of green.

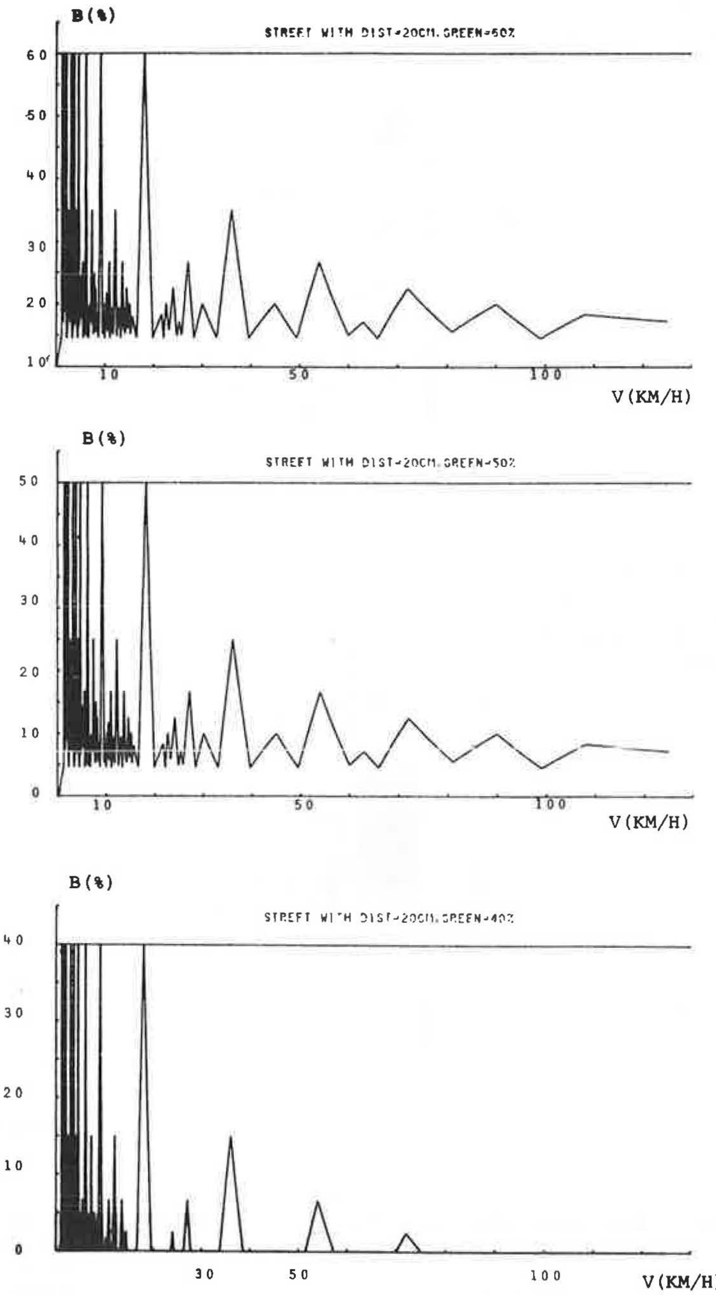
If speeds are allowed to vary to a larger extent, other speed bands will become possible for the same set of offsets shown in Figure 1. This is depicted in Figure 5. Each of these speed bands will produce a V-B curve similar to the one in Figure 4. There are 2**(n - 1) possible half-integer sets of offsets to be considered in this way, and the resulting curves of speed against bandwidth can be drawn together in one graph as shown in Figure 6. The envelope of these curves will be the curve of maximal bandwidth for each and every speed for a given cycle length. And since V*C is a constant, one can also derive all possible combinations of V and C that have the same bandwidth by using this envelope curve.

The extremal point of this curve between a minimum and a maximum speed is the same as the one obtained by the linear-programming approach of MAXBAND for the special case of uniform speeds. Some interesting remarks can be made by analyzing different V-B envelope curves.

1. There are many optimal values at low speeds and less at higher speeds as in Figure 7. This will be explained later by a simple formula, but it is also intuitively clear from Figure 5.

2. For the special case of equal distances between intersections and equal green times, maximum bandwidths corresponding to the minimum green may not be obtained at reasonable speeds. As the green time is decreased (as in Figure 8) from 60 percent to 50 percent and to 40 percent, the same envelope curve is displaced on the ordinate by 10 percent.

Figure 8. Comparison of speed-bandwidth curves for different minimum greens and equal distances between intersections.



Increasing the distance between lights (four times, for example) does not change the envelope curve. The range of speeds in Figure 9 (top) between 0 and 30 km/h is merely stretched four times to speeds between 0 and 120 km/h. This entails an increase in oscillations of the V-B curve.

3. As average distance between intersections increases, the envelope curve becomes more unstable. The stability of the band with respect to speed decreases. Figure 6 gives an example of a relatively stable situation at least between speeds of 60-90 km/h with an 80-s cycle. Stability should also be taken into account in the choice of a band and progression speed. If, for example, in Figure 9 (bottom) an optimal speed of 36 km/h and a band of 50 percent is chosen and speed increases or decreases by only 2 km/h, the bandwidth will fall to only 5 percent. This may explain why certain bands are apparently very inefficient as volume increases slightly; a slight reduction in speed is produced, but there is a major decrease in bandwidth. It may not always be good to adopt the extremal value of the V-B curve if this value is on a steep part of the envelope curve.

4. As the number of intersections increases, the envelope curve becomes more and more unstable, as is shown in Figure 10. In this case, oscillations increase and attainable bandwidth is small.

Clearly, the discussion up to now is purely theoretical and certainly impractical, since it is impossible to enumerate all possible combinations of half-integer offsets in order to produce the envelope curve that gives the extremal values. In the case of 24 intersections there would be 8 388 608 offset schemes to be investigated and for each offset scheme there would be a certain number of possible speeds, depending on the range of speeds to be studied.

THE ALGORITHM

The aim is to determine all possible extremal points on the V-B envelope curve for up to 25 intersections and also for a reasonable range of speeds and cycles.

It appears impractical to enumerate all possible half-integer offset combinations. But this enumeration can be avoided. It is obvious that the speeds giving rise to the extremal points on the V-B curve must be straight lines in the time-space diagram, i.e., lines that correspond to V_d and V_u. These possible optimal speeds can be calculated as

Figure 9. Arteries with equal distance between intersections: top, 100 m; bottom, 400 m.

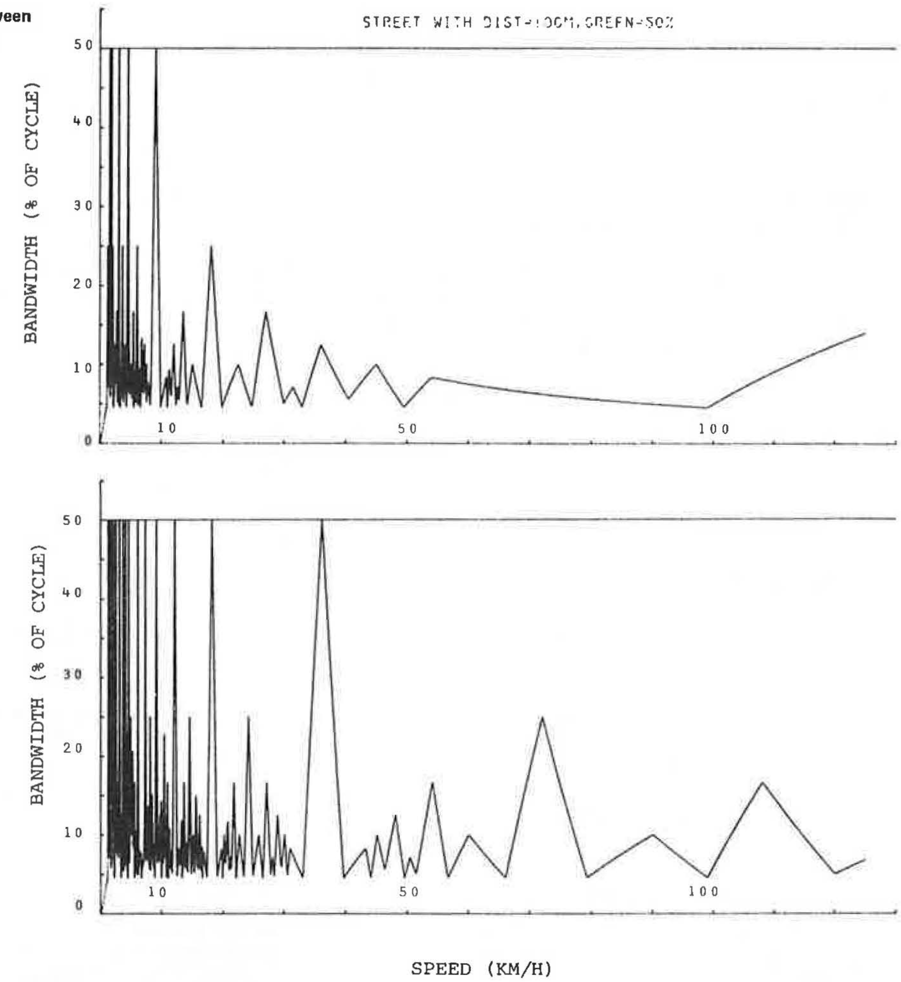
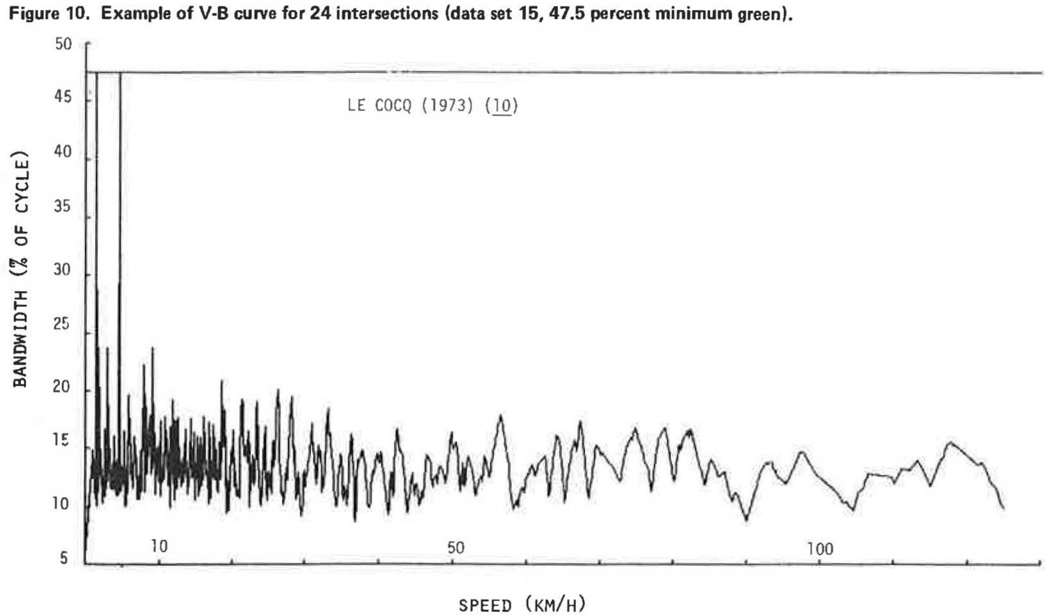


Figure 10. Example of V-B curve for 24 intersections (data set 15, 47.5 percent minimum green).



straight lines between any two critical lights i and j for all i = 1...n and all j = (i + 1)...n. Furthermore, as Figure 11 shows, there may be several possible optimal speeds between each pair of i and j.

For V_d (referring to Figure 11), we have

$$V_d = (720/C)\,[(x_j - x_i)/(r_j - r_i + 100\ell)] \tag{7}$$

For V_u (referring to Figure 12), we have

$$V_u = (720/C)\,[(x_j - x_i)/(r_j - r_i + 100k)] \tag{8}$$

where $\ell, k = 0 \ldots m$ such that

$$V_{min} \leq V \leq V_{max}$$

The resulting speeds are speeds that are extremal points on the subset of V-B curves and that may be extremal points of the envelope curve.

We now show that the number (N) of speeds to be calculated in this way is small or at least computationally feasible for all cases in which the number of intersections (n) is less than 25. With V_{min} and V_{max}, the limits of the speed range to be investigated, fixed, we have from Equation 7 for V_d

$$\ell_{i1} \leq (720/100CV_{min})(x_j - x_i) + (r_i - r_j)/100 \tag{9}$$

$$\ell_{i2} \geq (720/100CV_{max})(x_j - x_i) + (r_i - r_j)/100 \tag{10}$$

Figure 11. Geometric relationships for V_d.

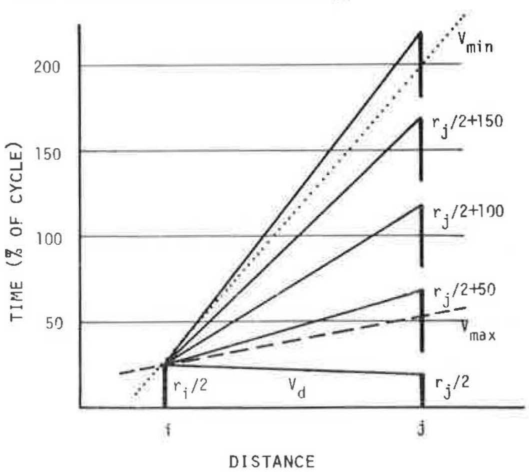
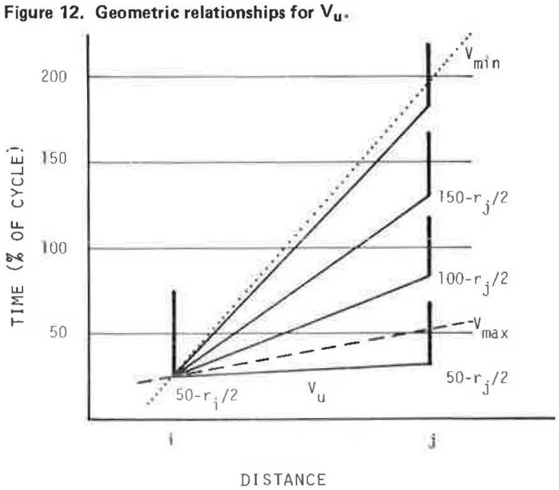


Figure 12. Geometric relationships for V_u.



If we take the same number for V_u, the number of speeds N_i to be calculated for each pair i,j would be

$$N_i = 2[\mathrm{IFIX}(\ell_{i1}) - \mathrm{IFIX}(\ell_{i2})]$$

where IFIX(ℓ) denotes the integer part of ℓ. An upper bound for N_i can be obtained by using

$$N_i \approx 2(\ell_{i1} - \ell_{i2}) \approx (14.4/C)(x_j - x_i)\,[(1/V_{min}) - (1/V_{max})]$$

$$N_i \approx F(x_j - x_i)$$

$$F = (14.4/C)\,[(1/V_{min}) - (1/V_{max})] \tag{11}$$

This result is not surprising. If the range of speeds to be considered between V_{min} and V_{max} is small, very few possible speeds will fall between these two extremes. Also, as V_{min} becomes small many speeds are possibly optimal, which explains the many oscillations in the V-B curve at low speeds or low K-values. From the tabulation below, the importance of the lower speed range can be seen. In fact, 50 percent of possible speeds between 10 and infinity and 20 and infinity lie between 10 and 20 km/h:

Range of Speed			
V_{min}	V_{max}	F	Percentage
10	∞	0.1	100.00
10	100	0.09	90.00
15	∞	0.067	67.00
20	∞	0.05	50.00
30	∞	0.033	33.33
40	∞	0.025	25.00
30	60	0.016 7	16.66
20	80	0.037 5	37.50
15	125	0.058 67	58.67

N_i depends approximately only on the distances between intersections and one can develop a summation formula for N over all intersections i, which constitutes an upper bound for the number N:

$$N \leq F\,[\sum_{i=1}^{n} (2i - n - 1)\,x_i] \tag{12}$$

Eighteen data sets were analyzed. These data sets were taken from the published literature, and the second column in Table 1 gives the references. An approximate lower bound for N was obtained by considering that distances between intersections are, on average,

$$L = x_n/n$$

In this case formula 12 becomes

$$N \geq FL(n^3 - n)/6 \tag{13}$$

The usefulness of Equations 12 and 13 can be verified from the data in Table 1. In fact, the formulas correspond fairly well to the number (N) of speeds actually calculated. Figure 13 shows the relation between N and n. It may also be seen in Table 1 that the mean distance between intersections alone explains more than 80 percent of the number N. The remaining percentage is due to the differences in reds and the variation of distances about the mean. The rapidity of the algorithm (called WAVE1) is further increased because in many cases the configuration of the reds is such that the same speeds V_d and V_u are generated repeatedly and the bandwidth corresponding to these speeds does not have to be determined several times. Equations 7 and 8 show also that $V_d = V_u$ when $r_i = r_j$. This further reduces the number N.

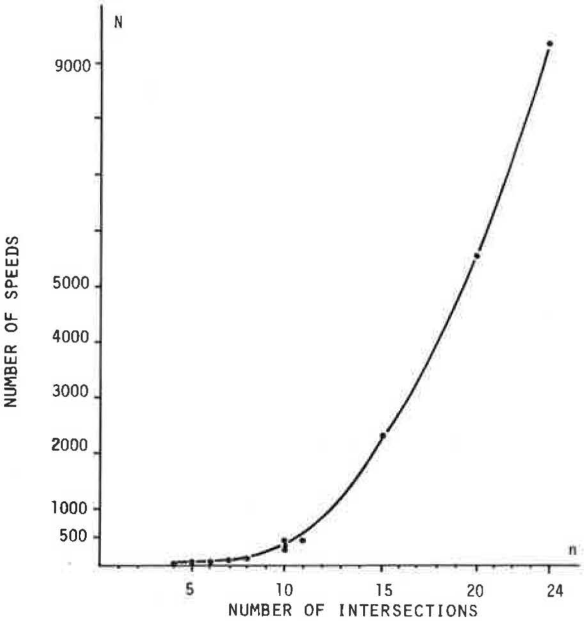
The main steps in the algorithm for finding the extremal points of the envelope curve and the cor-

Table 1. Number N for 18 data sets.

Data Set	Reference	No. of Intersections	N Exact	Lower Bound	Upper Bound	Avg Distance (m)
1	MAGTOP (1975) (4)	5	82	64	88	301.8
2	Little (1966) (5)	10	367	321	373	184.4
3	Le Cocq (1971) (6)	10	468	427	472	245.0
4	Institute of Traffic Engineers (40 percent) (1950) (7, pp. 229-239)	8	136	132	142	148.6
5	Institute of Traffic Engineers (1950) (7, pp. 229-239)	8	139	132	142	148.6
6	Davidson (1960) (8)	7	138	116	138	195.9
7	Purdy (1967) (9)	6	88	77	88	208.3
8	LeCocq (1973) (10)	4	61	42	61	401.3
9	Kell (1956) (11)	10	340	303	344	173.7
10	Laval	4	34	26	36	246.9
11	Pignataro (1973) (12, pp. 372-381)	6	38	38	46	104.1
12	Woods (1960) (13, pp. 7-40 to 7-43)	8	164	157	168	177.1
13	Morgan (1964) (2)	9	169	170	193	135.5
14	Kelson (1980) (14)	5	58	53	63	253.0
15	Le Cocq (1973) (10)	24	9384	9057	9394	364.6
16	Le Cocq (1973) (10)	20	5513	5021	5522	357.5
17	Le Cocq (1973) (10)	15	2387	2148	2396	363.3
18	200 m, 50 percent green	11	450	464	464	200.0

Note: All data sets in metric system; C = 80 s; V_{min} = 15 km/h and V_{max} = 125 km/h.

Figure 13. Number of possible optimal speeds N versus number of intersections n.



responding offsets, if they are required, over all ranges of K are described below. Note that certain steps and tests that are crucial for rapid execution and efficient storage are omitted here for conciseness. However, these steps are not essential for an understanding of the proposed procedure.

1. Do for i = 1...n; if i = n, go to step 6.
2. Do for j = (i + 1)...n; if j = n, go to step 1.

$C1 = r_j - r_i$

$C2 = (720/C)(x_j - x_i)$

3. Do for all ℓ = 0...m so that

$V_{min} \leqslant V_{dij} \leqslant V_{max}$

If no more ℓ satisfy the condition, go to step 2.

$V_{dij} = C2/(C1 + \ell 100)$

$V_{uij} = C2/(\ell 100 - C1)$

Do not retain speed if it hits a red light for all k $\neq$ i,j.

4. Calculate for ℓ and do for k = 1...n; if k = n, go to step 5.

$C3 = 360\,(x_j - x_k)/V_{dij}C$ or V_{uij}

$C4 = (r_j + r_k)/2$

$b_{kd} = C3 - C4 \pm INT*50$ if V_d

$b_{ku} = -C3 - C4 \pm INT*50$ if V_u

INT is such that

$0 \leqslant b_{kd} \leqslant g_k$ or $0 \leqslant b_{ku} \leqslant g_k$

Calculate offset, if required. If not, go to step 4.

$TET = C3 - b_{kd} - C4$ if V_d

$TET = C3 + b_{ku} + C4$ if V_u

If TET is negative, change the sign.

5.

$THETA_k = AMOD\,(TET, 100)$

$B_d = \underset{k}{MIN}\,(b_{kd})$

$B_u = \underset{k}{MIN}\,(b_{ku})$

Go to step 3.

6. The extremal points are found by a simple search algorithm that eliminates extremal points not on the envelope curve.

7. Choose speed and cycle over all acceptable extremal points by using the relationship K = VC.

Another algorithm based on a modified Brooks (15) algorithm, which gives the envelope curve and the extremal points, is called WAVE2. This algorithm produces the same extremal points as WAVE1 but to a lesser degree of accuracy. All data sets except 15 to 18 were also tested by Couture (16), who used the MAXBAND program of Kelson (14). The accuracy of these approaches is compared in Table 2. For the purposes of comparison only, the extremal point found by MAXBAND is given. WAVE1 determines all extremal points and WAVE2 produces also the continuous V-B envelope curve. All data sets were tested over a range of K_{min} = 640 to K_{max} = 10 000, which represents the practical limits of V and C.

Table 3 compares execution times in seconds for WAVE1 and WAVE2 on an IBM 4341 computer over all ranges of K and n and for up to 24 intersections. Execution time for WAVE1 is highly dependent on the average distance beteen intersections. Figure 14 shows this comparison and it can be seen that the 20-intersection case would be the cutoff point between the two programs.

Figure 15 gives the graphic output of program WAVE2 and Figure 16 is an output listing of the program WAVE1.

CONCLUSION

Maximum bandwidth varies with speed, especially at lower speeds. The extent of this problem depends mostly on the distances between intersections. In certain cases (when the average distance is great), maximum bandwidth changes so rapidly with speed that no stability can be expected, which may partly explain the unreliable functioning of certain progressions. It would seem important not only to consider the extremal points of bandwidth but also to take into account their stability with regard to changing speeds. The best choice of speed and bandwidth may in fact be lower than the overall maximum. The proposed method generates the entire relationship be-

Table 2. Comparison of precision for MAXBAND, WAVE1, and WAVE2.

Data Set	MAXBAND K	MAXBAND B	WAVE1 K	WAVE1 B	WAVE2 K	WAVE2 B
1	2156.00	0.3000	2156.00	0.3000	2152.00	0.3000
2	1364.80	0.3364	1364.80	0.3363	1368.00	0.3333
3	745.89	0.4698	745.89	0.4698	744.00	0.4597
4	1410.21	0.2724	1410.21	0.2724	1416.00	0.2712
5	1410.21	0.3724	1410.21	0.3724	1416.00	0.3712
6	704.20	0.4297	704.20	0.4297	704.00	0.4295
7	1060.80	0.3775	1060.80	0.3775	1064.00	0.3750
8	3549.70	0.4444	3549.70	0.4444	3552.00	0.4444
9	1563.41	0.2369	1563.62	0.2368	1560.00	0.2293
10	1215.52	0.5539	1215.20	0.5538	1216.00	0.5527
11	1426.41	0.3393	1426.46	0.3392	1424.00	0.3388
12	2506.67	0.3500	2506.50	0.3500	2512.00	0.3500
13	1096.95	0.6000	1096.93	0.6000	1096.00	0.5953
14	3017.50	0.4000	3017.60	0.4000	3024.00	0.4000
15			1488.00	0.2100	1488.00	0.2093
18			1440.00	0.5000	1440.00	0.5000

Table 3. Central-processing-unit time for programs WAVE1 and WAVE2.

Data Set	WAVE1	WAVE2	n	Data Set	WAVE1	WAVE2	n
1	0.49	3.82	5	10	0.40	3.49	4
2	0.79	5.61	10	11	0.39	3.72	6
3	0.94	5.39	10	12	0.60	4.46	8
4	0.61	4.58	8	13	0.56	4.76	9
5	0.56	4.71	8	14	0.41	3.40	5
6	0.54	4.24	7	15	33.9	21.5	24
7	0.44	3.52	6	16	16.6	12.1	20
8	0.44	3.47	4	17	5.72	7.78	15
9	0.98	5.57	10	18	1.54	4.93	11

Figure 14. Comparison of central-processing-unit time for programs WAVE1 and WAVE2.

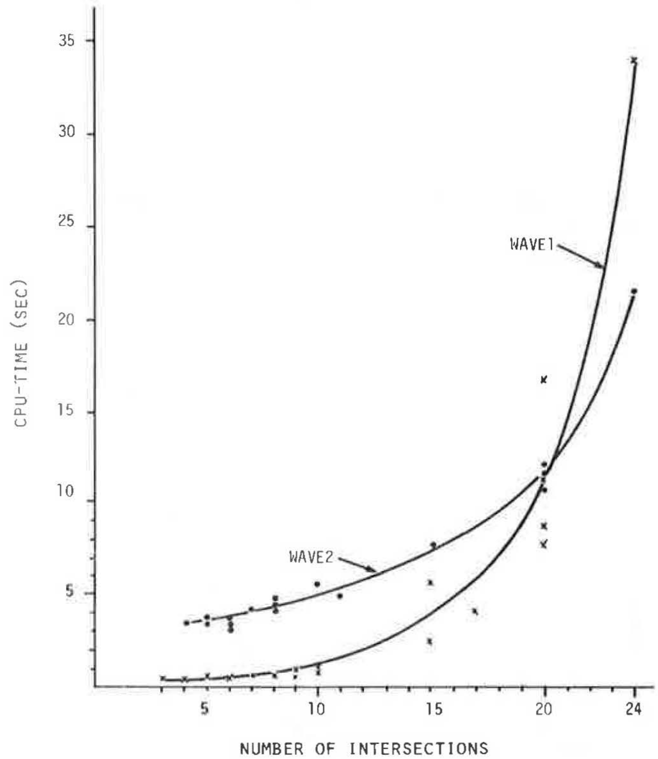


Figure 15. Graphic output of WAVE2.

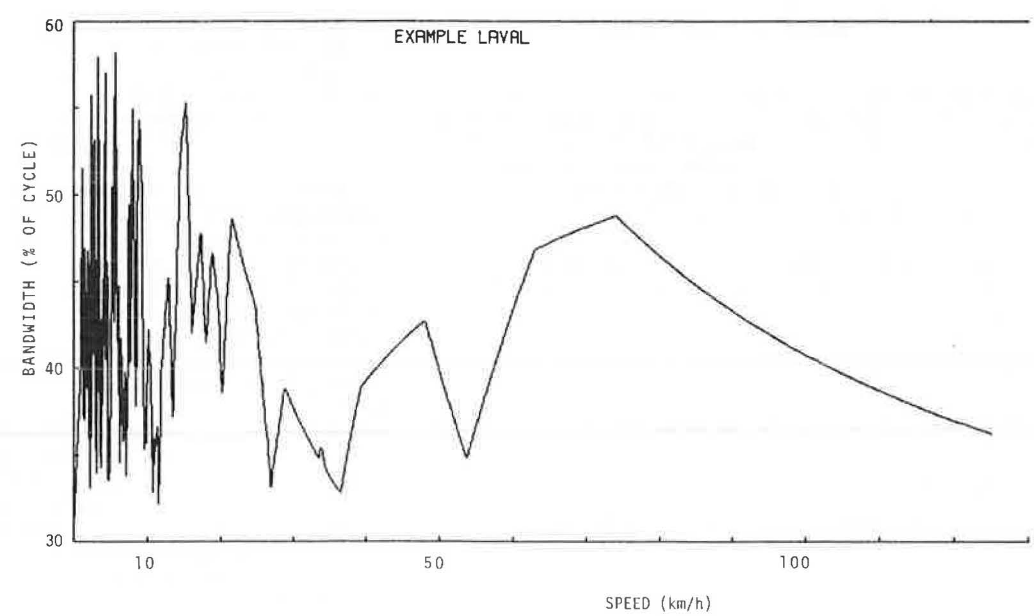


Figure 16. Output of WAVE1.

EXEMPLE LAVAL

CARREFOUR	ABSCISSE	ROUGE
1	0.0 MEIRES	25.00 POURCENI
2	297.18 MEIRES	24.00 POURCENI
3	803.15 MEIRES	40.00 POURCENI
4	987.55 MEIRES	40.00 POURCENI

GMIN= 60.00 SEC DUREE DU CYCLE= 80.00 SEC VMIN= 15 KM/H VMAX= 125 KM/H

EXEMPLE LAVAL

VIIESSES OPIIMALES POSSIBLES 34

VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE	VII.	BANDE
15.19	55.38	16.03	11.75	16.18	42.85	16.60	35.27	17.26	48.01	17.42	12.36	18.33	14.72	18.77	46.73	19.66	43.51	21.08	20.64
21.42	48.75	21.88	22.07	22.95	23.84	23.09	24.06	24.75	43.53	25.36	28.23	26.48	35.32	27.02	33.72	28.22	28.10	28.77	38.85
31.19	33.39	33.62	35.32	33.77	35.43	39.07	31.24	39.26	38.86	41.34	30.07	48.04	42.73	53.56	25.49	54.21	25.31	62.86	46.80
73.97	48.78	77.29	20.74	85.04	19.76	104.56	38.29												

EXEMPLE LAVAL

VIIESSES OPIIMALES SUR LA COURBE D'ENVELOPPE

VII.	BANDE	PCENI	VII.	BANDE	PCENI	VII.	BANDE	PCENI	VII.	BANDE	PCENI	VII.	BANDE	PCENI	VII.	BANDE	PCENI
15.19	55.38	*****	16.18	42.85	77.37	17.26	48.01	86.69	18.77	46.73	84.37	21.42	48.75	88.02	24.75	43.53	78.60
26.48	35.32	63.77	28.77	38.85	70.14	33.77	35.43	63.97	39.26	38.86	70.17	48.04	42.73	77.15	62.86	46.80	84.50
73.97	48.78	88.08	104.56	38.29	69.14												

tween speed and bandwidth in the form of an envelope curve, or it may generate all extremal points of this curve in extremely short execution times for arteries with up to 25 intersections. The proposed procedure may be practically useful since it provides more than just a single point solution and may contribute to a better understanding of the basic relationships.

REFERENCES

1. J.D.C. Little and others. Synchronizing Traffic Signals for Maximal Bandwidth. Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA, Rept. R 64-08, March 1964, 54 pp.
2. J.T. Morgan. Synchronizing Traffic Signals for Maximal Bandwidth. Operations Research, Vol. 12, 1964, pp. 896-912.
3. J.D.C. Little and others. MAXBAND: A Program for Setting Signals on Arteries and Triangular Networks. TRB, Transportation Research Record 795, 1981, pp. 40-46.
4. MAGTOP: User's Manual Sample Input and Output. FHWA, 1975, 291 pp.
5. J.D.C. Little. Synchronization of Traffic Signals by Mixed Integer Linear Programming. Operations Research, Vol. 14, 1966, pp. 568-594.
6. J.P. Le Cocq. Coordination des feux sur un itinéraire: ACHILLE II--Maximisation de l'onde verte. Services d'Etudes Techniques des Routes and Autoroutes (SETRA), Ministère de l'Equipement et du Logement, Paris, France, 1971, 57 pp.
7. H.K. Evans, ed. Traffic Engineering Handbook. Institute of Traffic Engineers, New York, 1950.
8. B. Davidson. Design of Signal Systems by Graphical Solutions. Traffic Engineering, Vol. 30, Nov. 1960, pp. 32-38.
9. F.G. Purdy. The Arithmetic of a Balanced Two-Way Signal Progression. Traffic Engineering, Vol. 37, No. 4, Jan. 1967, pp. 48-49.
10. J.P. Le Cocq. Coordination des feux sur un itinéraire: ACHILLE. SETRA, Paris, France, 1973, 64 pp.
11. J.H. Kell. Coordination of Fixed Time Traffic Signals. Institute of Transportation and Traffic Engineering, Univ. of California, Berkeley, Course Notes, Aug. 1956, 19 pp.
12. L.J. Pignataro. Traffic Engineering: Theory and Practice. Prentice-Hall, New York, 1973.
13. K.B. Woods and others. Highway Engineering Handbook. McGraw-Hill, New York, 1960.
14. M.D. Kelson. Optimal Signal Timing for Arterial Signal Systems: Vol. 2--MAXBAND User's Manual. FHWA, Rept. FHWA/RD-80/083, Dec. 1980, 230 pp.
15. W.D. Brooks. Vehicular Traffic Control--Designing Arterial Progression Using a Digital Computer. IBM Data Processing Division, Kingston, NY, 1964.
16. L. Couture. La coordination des feux de circulation sur une artère pour optimiser la bande verte en considérant les retards. Ecole Polytechnique, Université de Montréal, Montreal, Canada, master's thesis, 1982.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.