Table 5. Best-fit parameter values versus platoon size.

	Platoon Size Range									
	5-10 Vehicles		11-15 Vehicles		16-20 Vehicles		21-25 Vehicles		26-30 Vehicles	
Site	α	β	α	β	α	β	α	β	α	β
1	0.21	0.99	0.24	0.99	NA	NA	NA	NA	NA	NA
2	0.21	0.92	0.24	0.94	NA	NA	NA	NA	NA	NA
3	0.08	0.99	0.14	0.97	0.12	0.97	0.14	0.98	0.16	0.99
4	0.09	0.98	0.11	0.98	0.13	0.97	NA	NA	NA	NA
5	0.06	0.99	0.10	0.98	0.16	0.96	NA	NA	NA	NA
6	0.13	0.97	0.16	NA	NA	NA	NA	NA	NA	NA

Note: NA = sufficient data not available.

low-friction roadway characteristics by previous research. In addition, the dispersion of passenger-car platoons is less on a four-lane divided arterial street than it is on a two-way two-lane arterial street. Also, larger platoons experience more dispersion than do smaller platoons.

In regard to calibration of the TRANSYT platoon dispersion model, the average results of this research indicate that appropriate values of the dispersion factor α and the travel-time factor β for passenger-car platoons under low-friction traffic flow conditions on urban arterial streets are as follows:

Type of Street	α	β
Two-way two-lane	0.21	0.97
Four-lane divided	0.15	0.97

Thus, in order to more accurately account for the patterns of passenger-car platoon flow for these conditions, the input to the TRANSYT program should be revised to enable the user to specify the travel-time factor β as well as the dispersion factor α.

REFERENCES

1. TRANSYT-7F User's Manual. Office of Traffic Operations, FHWA, Feb. 1981.
2. D.I. Robertson. TRANSYT: A Traffic Network Study Tool. U.K. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, Rept. LR 253, 1969.
3. J.A. Hillier and R. Rothery. The Synchronization of Traffic Signals for Minimum Delay. Transportation Science, Vol. 1, No. 2, May 1967, pp. 81-94.
4. P.A. Sneddon. Another Look at Platoon Dispersion--3: The Recurrence Relationship. Traffic Engineering and Control, Feb. 1972, pp. 442-444.
5. J.F. Collins and P. Gower. Dispersion of Traffic Platoons on A4 in Hounslow. U.K. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, TRRL Rept. SR 29UC, 1974.
6. J.K. Lam. Studies of a Platoon Dispersion Model and Its Practical Applications. Proc., Seventh International Symposium on Transportation and Traffic Theory, Institute of Systems Science Research, Kyoto, Japan, 1977.
7. T.Y. El-Reedy and R. Ashworth. Platoon Dispersion Along a Major Road in Sheffield. Traffic Engineering and Control, April 1978, pp. 186-189.
8. P.J. Tarnoff and P.S. Parsonson. Selecting Traffic Signal Control at Individual Intersections. NCHRP, Rept. 233, June 1981.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.

Evaluation of Dynamic Freeway Flow Model By Using Field Data

N.A. DERZKO, A.J. UGGE, AND E.R. CASE

An attempt to calibrate and validate a dynamic freeway model by using real data from Queen Elizabeth Way in Ontario, Canada, is described. The model used in this research is the one developed by H. Payne; one of the Phillips kinetic models was also applied for comparison purposes. The overall conclusion is that the models exhibit instabilities in their behavior and do not track real road data correctly.

Traffic simulation models are playing an increasingly important role in the development of urban freeway traffic management systems because they provide an economical and safe way to evaluate alternative system designs and control strategies prior to implementation. Freeway models in common use today are adequate for simulating traffic conditions over a period of hours but are not sufficiently realistic for the research and development of new surveillance and control techniques for real-time applications. For such applications, the model must have the ability to realistically represent the shorter-term dynamic phenomena (e.g., shock waves) characteristic of traffic flow. These considerations led to a review a few years ago of the state of the art of traffic flow models and eventually to the conclusion that the Payne model (1-6) seemed to be the most realistic and the most developed of the very few dynamic models available at the time. Unfortunately, although it had been tested to some degree, the model had never undergone a comprehensive validation with actual freeway data. As a result, the decision was made to attempt to calibrate and validate the Payne model by using data from the Queen Elizabeth Way (QEW) Freeway Surveillance and Control System (7). That effort is described in this paper.

Shortly after work started on validation of the Payne model, we became aware of a Boltzmann-type statistical model that was being developed by Phillips (8-10). It is a significant improvement over an earlier model developed by Prigogine (11) and was particularly interesting for two reasons. First, Phillips shows that a family of continuum models of varying levels of refinement can be derived by taking the various moments of the governing stochastic partial-differential equations. One of these continuum models is virtually identical to the Payne model but has some important differences (which affect model performance at high densities) that were later incorporated into the Payne model for comparison purposes. The second reason the Phillips formulation was interesting was because it focused attention on the statistical nature of the calibration and validation process, which in turn provided the basis for developing a realistic methodology.

The results so far have not been encouraging, for reasons that are not yet entirely understood. Nevertheless, it was felt that our experience would possibly be of interest to others working in traffic flow model research and development.

MACK AND FREFLO MODELS

The MACK model was developed by Payne (1-3) as an analytical tool for evaluating ramp control plans and strategies for freeways. It is a macroscopic model that represents traffic flow in terms of aggregate measures such as density, speed, and flow rate.

The FREFLO model (4-6) is a successor to MACK. Both models have the same theoretical foundation.

In both models, the freeway is divided into sections. The time period is divided into uniform time intervals. Each model consists of a set of vehicle equations and a corresponding set of dynamic speed-density equations. Traffic performance data are accumulated for traffic flow in each freeway section defined. The first type of equation expresses the conservation of vehicles:

$$\rho_j^{n+1} = \rho_j^n + (\Delta t/\ell_j \Delta x_j)(\ell_{j-1} q_j^{n+1} - \ell_j q_{j+1}^{n+1} + f_j^{on,n+1} - f_j^{off,n+1}) \quad (1)$$

where

Δt = time interval,
ρ = section density [vehicles/(lane • mile)],
n = time index,
q = flow rate across upstream boundary [vehicles/(h • lane)],
ℓ = number of lanes,
Δx = section length (miles),
f^{on} = on-ramp volume (vehicles/h), and
f^{off} = off-ramp volume (vehicles/h).

The dynamic speed-density equation is

$$u_j^{n+1} = u_j^n - \Delta t \{ u_j^n(u_j^n - u_{j-1}^n)/x_j + (1/T)[u_j^n - u_e(\rho_j^n)] + (\nu/\rho_j^n)(\rho_{j+1}^n - \rho_j^n)/\Delta x_j] \} \quad (2)$$

where

u = section mean speed (miles/h),
T, ν = relaxation and anticipation parameters, and
$u_e(\rho)$ = equilibrium speed-density curve.

The three groups of terms express three physical processes. The first is convection; i.e., vehicles traveling at speed u_{j-1} in the upstream section will tend to continue to travel at that speed as they enter the next section. The second term represents the tendency of drivers to adjust their speeds to the equilibrium speed-density relationship. The third term expresses anticipation of changing travel conditions ahead, i.e., tendency to slow down if the density is perceived to be increasing.

The conservation and dynamic equations are used together to update values from time n to time n + 1. Under conditions of uniform flow,

$$q_{j+1}^{n+1} = \rho_j^n u_j^n \quad (3)$$

and this relation is used when updating densities by means of the conservation equations.

PHILLIPS MODELS AND THEIR COMPARISON WITH PAYNE MODEL

The work of Phillips (8-10) introduces a description of traffic analogous to that used in the kinetic theory of gases.

We begin with a brief review of this description.

Let x denote position along the highway and v denote vehicle speed. At each time t, we define a function (essentially a probability density function) $\varphi(t,x,v)$ such that $\varphi(t,x,v)$ dxdv gives the average number of vehicles found in the interval (x, x + dx) of road with speed in the interval (v, v + dv). We call φ the traffic-distribution function. The description of phase space (x,v) is flexible. We could, for example, talk about a distribution function for each lane of a multilane highway or for the truck component of traffic.

The traffic density K(t,x) is found by integrating:

$$K(t,x) = \int_0^\infty \varphi(t,x,v)dv \quad (4)$$

For many purposes, it is convenient to work with the function f(t,x,v), which gives the probability that a vehicle randomly chosen at point x will have speed in the range (v, v + dv). It is clear that the following relationship holds:

$$\varphi(t,x,v) = K(t,x)f(t,x,v) \quad (5)$$

Phillips (8-10) derives a number of partial-differential equations (or models) satisfied by the moments of the traffic-distribution function. The Phillips model that most closely resembles Payne's is

$$(\partial K/\partial t) + (\partial Ku/\partial x) = 0$$
$$(\partial u/\partial t) + u(\partial u/\partial x) = \lambda(u_e - u) - (1/K)(dP/dK)(\partial K/\partial x) \quad (6)$$

where

u = mean speed,
$u_e(K)$ = equilibrium speed-density relation,
$\lambda(K)$ = delay coefficient, and
$P(K)$ = traffic-pressure function.

There are both strong similarities and very significant differences between the Payne and the Phillips models. Although these have been discussed by Phillips, we present a more quantitative comparison here.

Since the continuity equation is identical in the two models, we begin by rewriting the u-equations in a way that suggests finite difference conversions. Phillips' equation becomes

$$(\partial u/\partial t) = -u(\partial u/\partial x) - [(dP/dK)(\partial K/K\partial x)] - \lambda(K)[u - u_e(K)] \quad (7)$$

Figure 1. Comparison of $\lambda(K)$ and $1/T$.

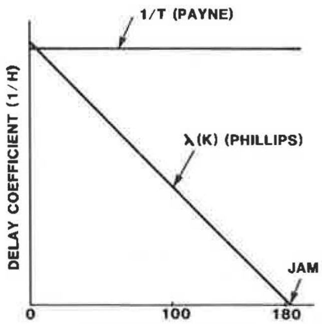


Figure 2. Speed-density curves.

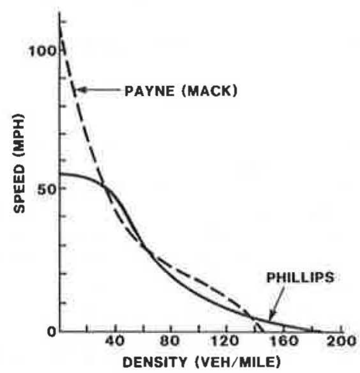
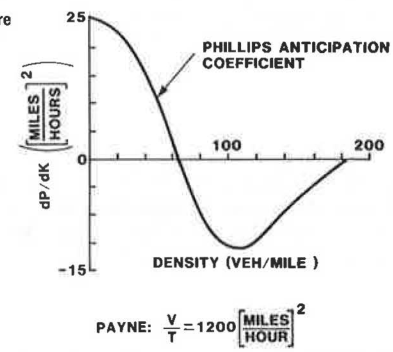


Figure 3. Phillips pressure coefficient.



Payne's formulation is

$$(\partial u/\partial t) = -u(\partial u/\partial x)(\nu/T)(\partial K/K\partial x) - (1/T)(u - u_e) \quad (8)$$

The tendency-to-equilibrium term is the last one in each of the above equations. A graph showing $\lambda(K)$ and $1/T$ is given in Figure 1. The value of $1/T$ shown is the MACK default value. The Phillips coefficient $\lambda(K)$ is calculated by using the following:

$$\lambda(K) = [u_1(K_0 - K)]/\{L_0[C_0K + C_r(K_0 - K)]\} \quad (9)$$

where K_0, L_0, C_0, and C_r are constants given by Phillips (9, p. 12). The equilibrium speed-density curves for Phillips and Payne are compared in Figure 2. Again, for Payne's curve we use the MACK default curve:

$$u_e(K) = 107K^3 - 2.31K^2 + 0.0215K - 7.4 \times 10^{-5}$$

For Phillips, we use

$$u_e(K) = (u_d u_1^7 + u_1^7 u_d^7 \zeta^6)/(u_1^7 - 9\sigma_d^3 u_1^3 u_d^3 \zeta^3 + u_d^7 \zeta^7) \quad (10)$$

where $\zeta = K/(K_0 - K)$ and u_1, u_d, and σ_d are constants given by Phillips (9, p. 27).

The two curves are qualitatively similar, but Payne's curve suffers a bit from the limitations imposed by being cubic. In particular, $u_e = 0$ for $K \geq 145$ is unrealistic.

The second-to-last terms in Equations 7 and 8 also warrant comparison. In Phillips' equation, the term $(dP/dK)(\partial K/K\partial x)$ arises out of traffic-pressure considerations, where (8, p. 64)

$$P(K) = (\sigma_d^2 u_1^2 K)/\{u_1^2 + 14.64\sigma_d^2[K/(K_0 - K)]^2\} \quad (11)$$

In Payne's work, the counterpart $(\nu/T)(\partial K/K\partial x)$ is called the anticipation term. The Phillips pressure coefficient is plotted in Figure 3. We note that $|dP/dK| \leq 25$. Payne's anticipation term is enormous by comparison: $\nu/T = (3600 \times 5)/15 = 1200$.

With Phillips, it must be noted that $dP/dK < 0$ for $K > 62$ vehicles/mile. This property will cause the mean traffic speed to increase for increasing density in this region. This surely cannot be correct.

This completes the quantitative comparison of Payne's and Phillips' first-order mean speed equations. Later in this paper we discuss the results of replacing the subroutine in MACK, which embodies a finite-difference version of Payne's differential equation. The consequences of the differences in the two equations become evident there.

FREEWAY DATA

The QEW Freeway Surveillance and Control System, located west of Toronto, provides continuous traffic-data collection, traffic-responsive control based on both mainline and ramp conditions, incident detection, hardware-status monitoring, a performance evaluation, and reporting capability. The system has been described in detail by Case and Williams (7). Extensive data and information supplied from 10 mainline detector stations and several ramp-metering installations served as an excellent input for our study of a dynamic freeway flow model.

Specifically, the data and information collected during weekday peak hours in October 1978 have been used.

PRELIMINARY STATISTICAL ANALYSIS

The purpose of this section is to understand the substantial statistical fluctuations in real road data. The Phillips description of traffic is the one we adopt as the basis for our analysis.

Flow Rate

The distributions of detector output given here are for stationary conditions. If the expected flow rate is λ vehicles per unit time, then the probability of counting a vehicle in a small time interval T is λT. This type of situation leads to the Poisson distribution. Its properties are well known. The probability of k vehicles in time T is

$$p(k) = (\lambda T)^k \exp(-\lambda T)/k! \quad (12)$$

The mean of this distribution is

$$E(k) = \sum_{k=1}^{\infty} kp(k) = \lambda T \quad (13)$$

and the variance is

$$\mathrm{Var}(k) = \sum_{k=0}^{\infty} (k - \lambda T)^2 p(k) = \lambda T \quad (14)$$

The traffic-distribution function affects the counterdistribution solely through the constant λ.

We proceed to find the relation of λ to the traffic-distribution function $K(x)f(x,v)$. Since the analysis is done at the fixed point x where the detector is located, we shorten the notation to $Kf(v)$. During a time dt, $v\,dtKf(v)dv$ vehicles are counted with speeds in the interval $(v, v + dv)$. The weighting factor v enters because a faster vehicle will be picked up from farther upstream during any given counting interval Δt. Hence, $K\,dt\int_0^\infty vf(v)dv$ vehicles are counted altogether, so that

$$\lambda = K \int_0^\infty vf(v)dv \quad (15)$$

Speed

The reasoning in the previous paragraph produces the speed distribution as a by-product. The speed-distribution function is $\alpha vf(v)$, where α is chosen to make

$$\int_0^\infty \alpha vf(v)dv = 1 \quad (16)$$

The mean speed observed at the detector counter is

$$v_0 = E(v) = \alpha \int_0^\infty v^2 f(v)dv \quad (17)$$

It is important to note that this number may be different from the actual mean speed, defined as

$$\bar{v} = \int_0^\infty vf(v)\,dv \quad (18)$$

We note that

$$v_0 = (\bar{v}^2 + \sigma^2)/\bar{v} = \bar{v} + (\sigma/\bar{v})\sigma \quad (19)$$

where σ is the standard deviation of the random variable v. The variance of the observed mean speed is defined to be

$$\alpha \int_0^\infty (v - v_0)^2 v\phi(v)dv$$

Occupancy

Suppose that $p(\ell)d\ell$ gives the fraction of vehicles on the road with measured lengths in $(\ell, \ell + d\ell)$. Since ℓ is the length as measured by detectors, it must include any component contributed by the detector zone. We make the (not-altogether-justified) assumption that length is independent of speed. Then the probability that a passing vehicle has length in $(\ell, \ell + d\ell)$ and speed in $(v, v + dv)$ is $\alpha v\,f(v)p(\ell)dvd\ell$. Such a vehicle contributes an amount

$$L = \max\,[(\ell/v), \Delta t_{counter}]$$

to the occupancy counter, where $\Delta t_{counter}$ is the counting interval. We make the simplifying assumption that conditions are such that $\ell/v < \Delta t_{counter}$ always; that is,

$$f(v) = 0 \quad \text{for } v \leqslant \ell max/\Delta t_{counter}$$

With this simplifying assumption, the expected occupancy contribution per vehicle becomes

$$E(L) = \alpha \int_0^\infty d\ell \int_0^\infty dv(\ell/v)vf(v)p(\ell) = \alpha\bar{\ell} \int_0^\infty f(v)dv = \alpha\bar{\ell} \quad (20)$$

The variance of ℓ is

$$V(L) = \int_0^\infty d\ell \int_0^\infty dv[(\ell/v) - \alpha\bar{\ell}]^2 vf(v)p(\ell) \quad (21)$$

The calculations are simplified if we note that the assumed independence of v and ℓ together with $\ell/v = (1/v)\ell$ lead to the formulas

$$E(\ell/v) = E(\ell)E(1/v) \quad (22)$$

$$E[(\ell/v)^2] = (E(\ell^2)E(1/v^2) \quad (23)$$

from which

$$\begin{aligned} Var(L) &= E(\ell^2)E(1/v^2) - [E(\ell)E(1/v)]^2 \\ &= Var(\ell)Var(1/v) + E(\ell)^2Var(1/v) + E(1/v)^2Var(\ell) \quad (24) \end{aligned}$$

It can be checked that the standard deviation of a single length measurement is small relative to the mean of such a measurement.

We shall see shortly that the dominant cause of variation in the output of the occupancy counter is the fact that the number of vehicles counted tends to follow the Poisson distribution, except at very high density.

The occupancy counter records the sum S of a random number N (having Poisson distribution) of identically distributed length measurements L_i. This situation leads to the following formulas:

$$\begin{aligned} E(S) &= E(L)E(N) \\ Var(S) &= Var(L)E(N) + E(L)^2Var(N) \\ &= Var(L) + E(L)^2E(N) \quad (25) \end{aligned}$$

when N has a Poisson distribution. The mean occupancy reading, which uses Equations 13 and 15, is

$$E(S)/T = \lambda E(L) \quad (26)$$

Density/Occupancy Ratio

In using road data to check speed times density = flow rate, we first calculate speed times occupancy. If we substitute mean values for each factor on the left, we should obtain from Equations 19, 20, and 26

$$[\int_0^\infty v^2f(v)dv/\int_0^\infty vf(v)dv]\,(\lambda\alpha\bar{\ell})$$

The estimate of mean flow rate is

$$\lambda = K \int_0^\infty vf(v)dv \quad (27)$$

from Equation 15. Now, consider the following estimate:

$$\text{Flow rate/(speed x occupancy)} = [\int_0^\infty vf(v)dv]^2/\int_0^\infty v^2f(v)\,d\bar{v\ell} \quad (28)$$

which is a useful estimate for the density/occupancy (d/o) ratio $1/\ell$ only for very narrow distributions $f(v)$ (which is often not the case). The degree of bias in Equation 28 can be better judged from the following formula:

$$\begin{aligned} \text{Density/occupancy} &= \text{(speed x density)/(speed x occupancy)} \\ &= (1/\ell)[\mu^2/(\mu^2 + \sigma^2)] \quad (29) \end{aligned}$$

where μ and σ are the mean and standard deviation of v with respect to the density function $f(v)$. This means that the d/o estimates based on Equation 28 are too low. The correction factor is

$$(\mu^2 + \sigma^2)\mu^2 = 1 + (\sigma/\mu)^2 \quad (30)$$

Calculations of this correction for road data we have been using yield correction factors < 1.05 in most cases and < 1.10 for very broad distributions.

Figure 4. Evaluation procedure.

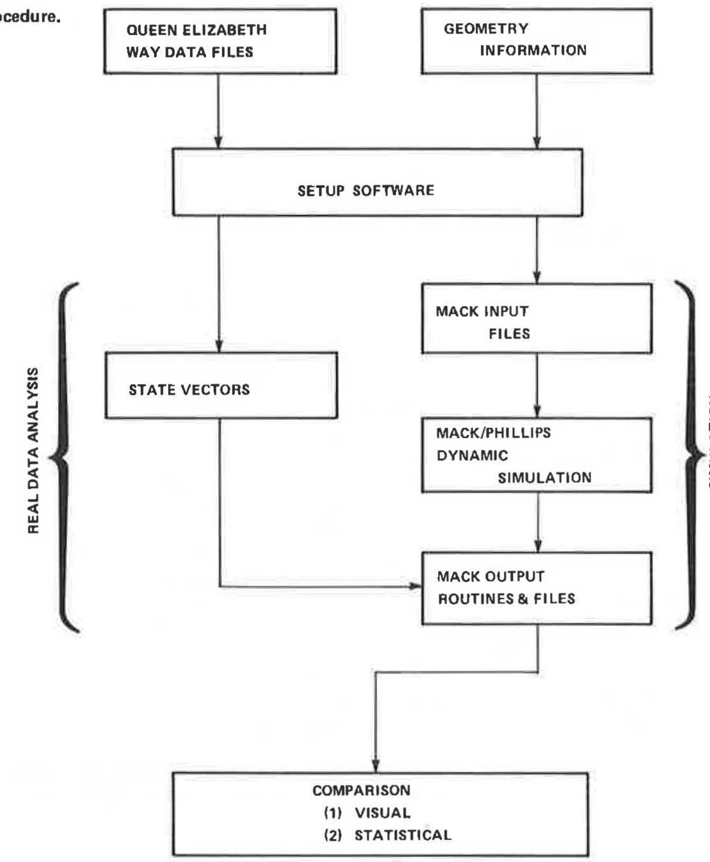
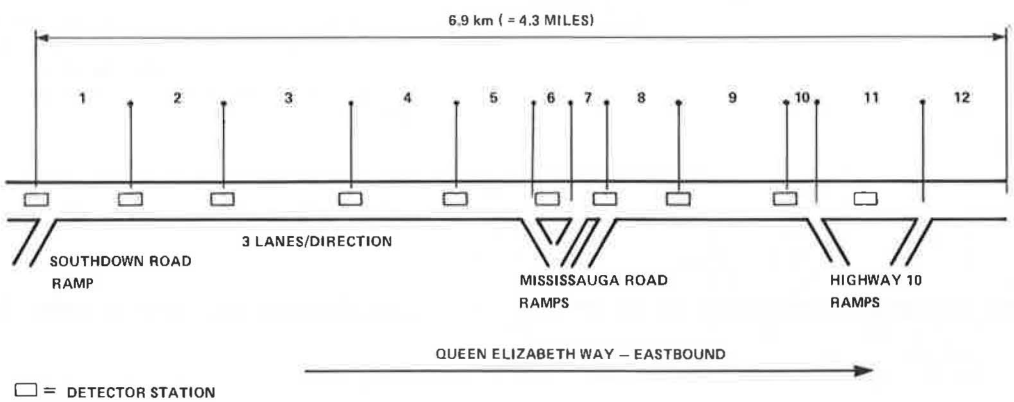


Figure 5. QEW geometry.



EVALUATION PROCEDURE

The evaluation was done by using MACK, a program that incorporates a finite-difference version of Payne's model. Currently, MACK has been superseded by FREFLO. However, the underlying finite-difference scheme describing the traffic dynamics is identical, as can be seen by a comparison of the UPDATE subroutines in each. Consequently, we would expect similar results for FREFLO.

MACK calculates the traffic-state functions (density and mean speed) along the road at each time interval by using as input the initial state and the upstream on-ramp and off-ramp flow rates as functions of time. This corresponds to an initial-boundary-value problem for the underlying partial-differential equation.

MACK is linked to the QEW road data files by means of a program that calculates the MACK input information and produces a MACK input data set. The input of road geometry is also done in MACK input format to minimize modifications to MACK software.

The evaluation procedure is summarized in Figure 4, and Figure 5 shows the sectioning geometry and ramp configurations.

STATISTICAL AND VISUAL COMPARISONS

If we assume that the section deviations between MACK output and road data have approximately normal distributions and are independent for different road sections, then the normalized sum of squares of section deviations has a chi-square distribution and standard statistical theory can be used.

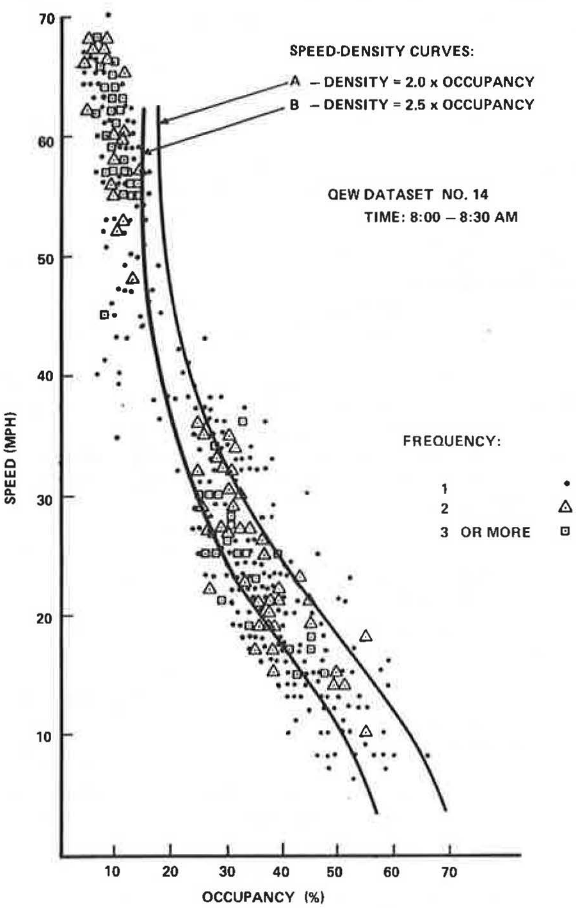
As an example, we apply the test to evaluate how closely the density calculated by MACK agrees with the density derived from occupancy observations for one of our runs. The relevant figures are contained

Table 1. Calculated versus observed densities.

Road Section	Density Observed (vehicles/mile)	Density Calculated (vehicles/mile)	Estimated Standard Observed Density
1	14.0	15.7	4.0
2	18.0	16.3	4.0
3	25.0	17.3	4.1
4	53.0	18.1	4.2
5	79.3	18.6	4.2
6	86.4	18.7	4.3
7	85.0	20.0	4.5
8	76.9	21.2	4.6
9	67.0	21.3	4.6
10	62.6	21.4	4.6
11	60.5	19.4	4.4
12	60.0	23.1	4.8

Note: 2-min data, fixed time; $\chi^2 = \Sigma[(\text{observed} - \text{expected})/\text{standard}]^2 = 5353$.

Figure 6. Speed-occupancy data.



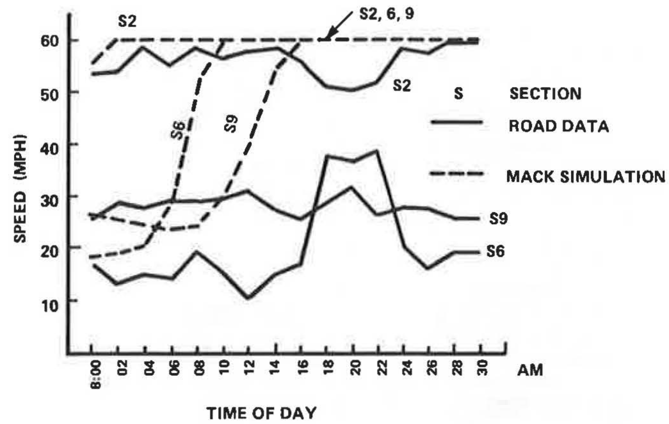
in Table 1. A value of χ^2 in this range has extremely small probability (<< 0.005), which can be seen from a standard chi-square table with f = 12. The hypothesis is that the values calculated by MACK are the correct section density values. Under the circumstances, a reasonable conclusion is that MACK is not correct.

A visual comparison is possible because MACK output includes a number of graphs and a density plot. Since our procedure generated road data output in the same format, a great deal can be learned by simply comparing plots.

MODIFIED MACK

When difficulties were experienced with MACK, it was

Figure 7. Speeds.



decided to try a modified version prepared by replacing the original finite-difference scheme with one based on Equation 7. The relevant formulas are as follows:

$$\rho_j^{n+1} = \rho_j^n + [(\ell_{j-1} q_j^{n+1} - f_j^{off,n+1} - \ell_j q_{j+1}^{n+1} + f_j^{on,n+1})\Delta t/\ell_j \Delta x_j] \quad (31)$$

$$q_j^{n+1} = \rho_j^n u_j^n \quad (32)$$

$$u_j^{n+1} = u_j^n - \Big[u_j^n \{(u_j^n - u_{j-1}^n)/[(\Delta x_j + \Delta x_{j-1})/2]\} + \Big(\lambda(\rho_j)u_j^n - u_e(\rho_j^n)$$
$$+ (dP/dK)(\rho_j^n)\{(\rho_{j+1}^n - \rho_j^n)/[\rho_j^n(\Delta x_j + \Delta x_{j-1})/2]\}\Big)\Delta t\Big] \quad (33)$$

where λ, u_e, and (dP/dK) are as defined by Phillips.

RESULTS

We rely on visual comparisons in this presentation because they suffice for indicating the magnitude of the deviation between MACK output and road data. We describe selected output from a typical sequence of runs. The d/o ratio used is 2.0. This value of the d/o ratio has been standardized in all our runs in order to come as close as possible to preserving the continuity relationships: flow = speed times density. It can be seen from Figure 6 that this value of the d/o ratio leads to a speed-density curve that is too far from the origin. This means, of course, that the equilibrium speed at any given density is too high.

Figure 7 shows selected mean speed graphs for real road data. The corresponding output for MACK that uses the default equilibrium speed-density curve gives the same information from MACK output. The key feature to be noted is that the mean speeds increase and the road empties as time progresses in the MACK output, whereas the road data exhibit fairly steady conditions. The same pattern in terms of densities can be seen in Figure 8a and b.

In an attempt to calibrate MACK, the speed-density curve was shifted by scaling the density-occupancy axis. The effects of scaling are shown in Figure 6. A larger scaling constant produces a curve that is closer to the origin, that is, one that yields a lower speed at fixed density. The best fit of speed-density curve to our observed scatter diagrams was obtained with a scaling constant of 1.25 (i.e., density = 2.5 times occupancy). Use of the best-fit speed-density curve did not correct the road-clearing effects exhibited by MACK output (see Figure 8c).

Figure 8. Densities.

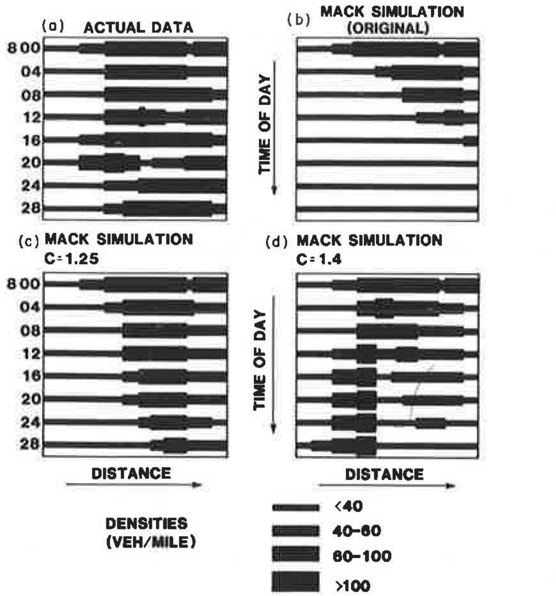
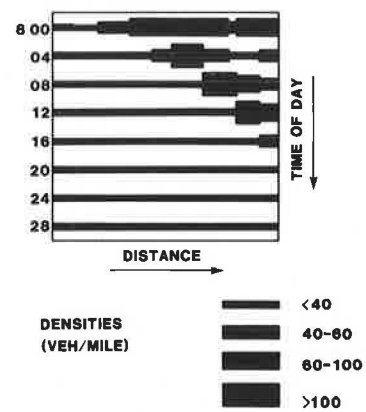


Figure 9. Densities: Phillips simulation.



Next, an attempt was made to see whether MACK would track road data correctly for some speed-density curve, even if it did not fit the usual definition of such a curve. Figure 8d shows a density plot for a scaling constant of 1.4 (i.e., density = 2.8 times occupancy). Downstream and upstream clearing is still evident. In addition, however, density concentrations as in a traffic jam have appeared at section 4. When the scaling constant is increased to 1.4, the density concentration continues to coexist with upstream and downstream road clearing except that now an actual shock wave forms that moves upstream.

Finally, the performance of our modification of MACK based on Phillips' differential equation was tested to check whether the differences in terms already discussed could account for the poor MACK tracking properties. Figure 9 shows a density plot for the same run as the one presented in Figure 8. Once more we see that abnormally low and high densities coexist even for the speed-density relation provided by Phillips.

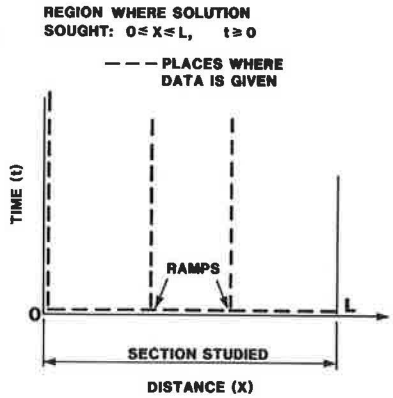
DISCUSSION AND CONCLUSIONS

The behavior we have witnessed in this series of runs is suggestive of an instability in the simulation method. There are several areas where the simulation method, which is actually a solution algorithm for a system of first-order partial-differential equations, needs a more detailed study. The handling of the downstream boundary condition is responsible for the fact that density decrease and associated increases in speed move from the downstream to the upstream end in all cases. It is also conceivable that the handling of the merging process has a significant effect on the relationship between speed and density. This effect does not seem to be large enough to cause the simulation error to concentrate downstream of on ramps. These occur in the present geometry in sections 1, 7, and 12.

It is also conceivable that a careful adjustment of the speed-density curves on a section-by-section basis may improve the fit between the simulation and road data for one particular data set. We must remember, however, that the qualitative character of the simulation has shown itself remarkably sensitive to the simulator speed-density relation. The work simulator is emphasized because the curve that must be used to retard the road-clearing phenomena is far below the picture of speed versus density that one obtains in plots of real road data. Then, within a tiny range of curves, the shock phenomena set in.

Each of these phenomena must be carefully examined with regard to its statistical properties in designing a simulator that will properly track road data. They fall into the realm of fine tuning. We do not believe that they contain the seeds of the explanation and correction of the performance of MACK or for that matter any simulator based on a finite-difference method for solving a system of first-order differential equations governing mean density and speed. We hasten to note that for the MACK modification by using a differential equation based on Phillips' theory, the results were quantitatively worse than those for MACK.

Figure 10. Initial-boundary value outline.



Well-Posedness of Partial-Differential Equation

"Well-posedness" means that the solution must be unique and depend on the given data in a continuous fashion.

We observe that either of the first-order models is a 2-x-2 system of first-order partial-differential equations. The intuitively reasonable initial-boundary value problem (IBVP) is outlined in Figure 10. That is, $u(x,t)$ and $K(x,t)$ are to be found, given initial states on $t = 0$, $0 < x < L$, given upstream traffic data (state) for $x = 0$, $t > 0$, and given on/off-ramp data for appropriate values of x and $t > 0$.

Strictly speaking, the well-posedness of the IBVP needs proof. The method of characteristics is available to handle such questions for first-order equations, and although our intuition about traffic flow strongly backs a conclusion that the IBVP is well posed, a study along these lines should be undertaken at some point in the light of the difficulties we have experienced.

Finite-Difference Scheme

In examining this aspect of a simulator, we must remember that we are in fact trying to solve a stochastic problem in partial-differential equations.

Several key questions emerge in looking at a finite-difference solution to the first-order IBVP:

1. Given steady boundary conditions, does there exist a steady-state solution to the IBVP?
2. Does the time-dependent state approach the steady state from any initial value?
3. Do the effects of a momentary perturbation in a boundary condition die out as they would in a real traffic situation? (This is a key stability question.) If they do not, then the statistical fluctuations inherent in real road data make them unusable as initial and boundary data.

The use of MACK or FREFLO to track real road data in fact requires deeper knowledge than the above. We must, in fact, know something about the distribution of state variables generated from distribution of input data. These distributions should reflect stability of the finite-difference scheme as already mentioned and they are needed to devise meaningful tests to determine whether the simulator is tracking faithfully. Techniques are available for tackling these questions, and at this point, it seems that they are well worth trying.

Concluding Remarks

The overall conclusion must be that MACK and FREFLO, by virtue of their identical underlying differential equation, exhibit instabilities in their behavior that make them unsuitable for use as simulators tracking real road data in the sense of solving an IBVP. Furthermore, it appears that the difficulties cannot be corrected by using the first-order differential equation derived by Phillips.

Numerous reasons can be advanced for the observations we have made, but two are dominant in importance. First, road traffic has a very strong and complex stochastic aspect. Fluctuations on a moderate time scale of 5-10 min are very high relative to the mean size of quantities being measured. These fluctuations of necessity find their way into any scheme designed to simulate road behavior and wreak havoc if there are any instabilities present. In fact, for meaningful results we must ask not just for the absence of instabilities--that is, continuous dependence on data and parameters--but for the presence of strong stability properties to cause decay of the influences of earlier fluctuations.

The second reason is that the concepts involved in obtaining a differential equation and an IBVP for the mean speed function lead to a very sensitive dependence on the speed-density relation and possibly other parameters. Once the upstream flow rate and the on/off ramp flow rates are known, a time-averaged flow rate for each road position is a consequence. Suppose that at some point the speed-density relationship is too slow to move a sufficient number of vehicles over the longer time scale of 5-10 min. Then densities will increase upstream of the point in question to conserve vehicles. This effect will depress the road speed even further, ultimately starting the typical shock wave moving upstream. Since such effects are cumulative, it is clear that they will be observed even if the speed-density relation is mismatched by a tiny amount. Conversely, it is clear that if the speed-density relation used by the simulator is a bit too large, we will find the scenario of less density and more speed, which moves ultimately to clear the road of traffic.

Discussion

Harold J. Payne

This paper contributes to the examination of the FREFLO model necessary to reach a judgment concerning its ability to represent freeway traffic flow and to indicate steps necessary to make use of the model.

In this discussion, three points are addressed:

1. The calibration/validation process necessary before using FREFLO,
2. Applicability and restrictions in the use of FREFLO, and
3. Recent model improvements.

The authors describe an effort to calibrate FREFLO for application to the QEW freeway in Toronto. This entailed collection of data, rectification of measurements to model variables (outputs), and then a cycle of execution of the FREFLO model, assessment of match of model results to data, and adjustments to FREFLO. Similar efforts have been successfully undertaken in connection with NCHRP Project 3-22A, Guidelines for Design and Operation of Ramp Control Systems (12). In that study, FREFLO was calibrated for freeways in Los Angeles and Dallas. The calibrated model subsequently played a major role in that study.

A key element of that calibration, and one that was unfortunately and unnecessarily lacking in the Toronto work, was the involvement of the FREFLO model builders in the calibration effort. At this time, the special requirements for effective use of FREFLO are not widely known or fully documented, so this type of involvement is very important.

In the instance of the work reported in this paper, a critical model restriction was not properly observed. The situation is depicted in Figure 11. The downstream extent of the congestion modeled by FREFLO must be completely interior to the freeway segment modeled; some uncongested zone must exist at the downstream end. (There are adequate techniques to deal with congestion at the upstream end, however.) Such was not the case in the Toronto work. As a consequence, all calibration efforts were doomed to failure.

As a final point, some recent work (13) has revealed that a discontinuous speed-density relationship, of the sort depicted in Figure 12, more

Figure 11. Critical restriction on use of FREFLO.

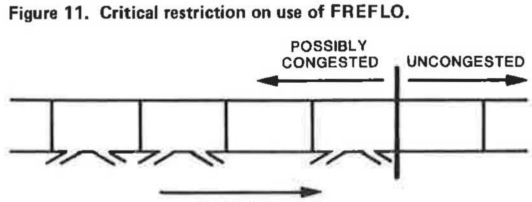
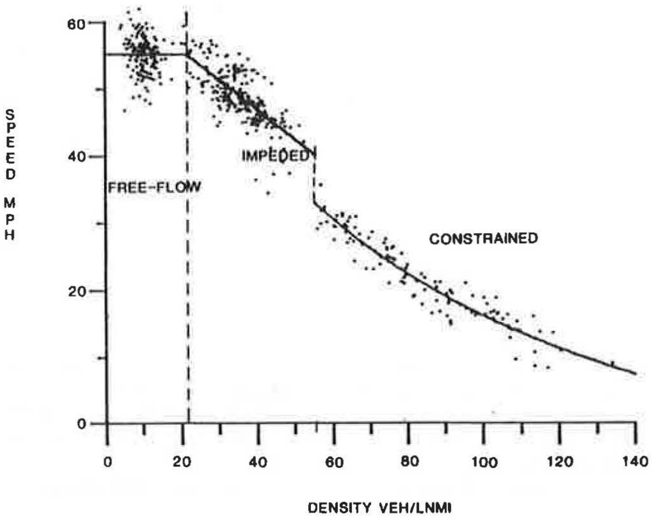


Figure 12. Recent improvements to FREFLO.



accurately reflects the equilibrium behavior of freeway traffic. The discontinuity has significant impacts on the ability of FREFLO to model geometric discontinuities. It may also be very important to the study of ramp metering (13).

Authors' Closure

We would like to thank Payne for his comments.

During the study, and especially in its initial stage, the FREFLO model builder was involved in our work. We had considerable documentation and even the benefit of his personal instructions.

We would like to point out that the behavior exhibited by the model was very similar to that observed in an earlier FREFLO simulation (14) of an idealized freeway with a bottleneck in the middle. The situation with uncongested downstream sections was simulated by using simplified hypothetical test cases, but the results exhibited incorrect patterns similar to those shown by Payne (6). Many different "cures" to the problem were tried, including those suggested by Payne, but to no avail. Finally, it was concluded that the hypothetical speed-density curve used was unrealistic and the decision was made to proceed with an evaluation based on data obtained from an actual system, the subject of this paper.

For a model to be of any practical value, the model builder must strive to develop ones that do not require any additional involvement by the builder during implementation and application stages.

REFERENCES

1. H.J. Payne. Models of Freeway Traffic and Control. Simulation Council Proc., Mathematical Models of Public Systems, Vol. 1, No. 1, 1971, pp. 51-61.
2. H.J. Payne, W.A. Thompson, and L. Isaksen. Design of a Traffic-Responsive Control System for a Los Angeles Freeway. IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC-3, No. 3, May 1973, pp. 213-224.
3. D.N. Goodwin, S.D. Miller, and H.J. Payne. MACK: A Macroscopic Simulation Model of Freeway Traffic. Technology Service Corporation, Santa Monica, CA, July 1974.
4. H.J. Payne. FREFLO: A Macroscopic Simulation Model of Freeway Traffic--Version I: User's Guide. ESSCOR, San Diego, CA, July 1978.
5. H.J. Payne. FREFLO: A Macroscopic Simulation Model of Freeway Traffic--Version I: Program Documentation. ESSCOR, San Diego, CA, July 1978.
6. H.J. Payne. FREFLO: A Macroscopic Simulation Model of Freeway Traffic. TRB, Transportation Research Record 722, 1979, pp. 68-75.
7. E.R. Case and K.M. Williams. Queen Elizabeth Way Freeway Surveillance and Control System Demonstration Project. TRB, Transportation Research Record 682, 1978, pp. 84-93.
8. W.F. Phillips. Kinetic Model for Traffic Flow. U.S. Department of Transportation, Rept. DOT/RSPD/DPB/50-77/17, 1977.
9. W.F. Phillips. A New Continuum Model for Traffic Flow. Utah State Univ., Logan, June 1979.
10. W.F. Phillips. A Kinetic Model for Traffic Flow with Continuum Implications. Transportation Planning and Technology, Vol. 5, 1979, pp. 131-138.
11. I. Prigogine. A Boltzmann-Like Approach to the Statistical Theory of Traffic Flow. In Theory of Traffic Flow (R. Herman, ed.), Elsevier, Amsterdam, Netherlands, 1961, pp. 158-164.
12. Guidelines for Selection of Ramp Control Systems. NCHRP, Rept. 232, 1981.
13. H.J. Payne. The Discontinuity in Equilibrium Freeway Traffic Flow: Empirical Findings and Implications for Control. Presented at National Meeting of the Operations Research Society of America, San Diego, CA, 1982.
14. E. Hauer and V.F. Hurdle. Discussion of paper by Payne--FREFLO: A Macroscopic Simulation Model of Freeway Traffic. TRB, Transportation Research Record 722, 1979, pp. 75-76.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.