50

Figure 13. SOAP average delay profiles with NETSÍM derived left-turn saturation-flow rates.



through traffic volumes were studied in three different signal-control variations.

Results were almost identical when a multiphase pretimed traffic signal was simulated. In the case of a two-phase pretimed signal, NETSIM delays were somewhat higher, as expected, and the relative changes in delays corresponding to relative changes in volumes and left turns were similar.

In the case of a two-phase fully actuated signal, the difference between NETSIM and SOAP average delay was higher than in the other two cases but can be explained by too long a unit extension specified in NETSIM. SOAP appeared to overestimate delays at a few points, which corresponded to conditions of high volume/capacity ratios. The overestimated delays for these points were due to conservative SOAP estimates of left-turn saturation-flow rates.

The patterns of NETSIM and SOAP delays were similar enough to indicate that with additional research the correlation could be further improved. In this study no attempt was made to change any of the first set of inputs (unit extension time, minimum green, maximum green, lost time, etc.) in order to increase the correlation between NETSIM and SOAP.

After differences in definitions had been accounted for, NETSIM and SOAP fuel-consumption estimates were found to be identical for all three cases.

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Analysis of Existing Formulas for Delay, Overflow, and Stops

W.B. CRONJÉ

An analysis is made of existing formulas for average delay, average overflow, and average number of stops for undersaturated conditions. The examination of these formulas covers a large variation in flows and cycle lengths, so recommendations are based on a thorough examination. The formulas examined are those developed by Webster, Miller, and Newell. It is concluded that the Newell formulas give the most accurate results.

The delay formulas that are predominant in practice are those developed by Webster ($\underline{1}$), Miller ($\underline{2},\underline{3}$), and Newell ($\underline{4}$). Hutchinson ($\underline{5}$) examined these formulas for accuracy. The standard of comparison is, however, a derived formula. Futhermore, Hutchinson ($\underline{5}$) covered only average delay. In this paper, however, the standard of comparison is computer simulation, and in addition to average delay, average overflow and average number of stops are also examined. The reason for this is that in the optimization of fixed-time signalized intersections, delay as well as number of stops should be used in the optimization process.

Throughout the comparison the value of I, the variance-to-mean ratio of flow per cycle, is taken as 1 because it has been shown ($\underline{6}$) that for the optimization of fixed-time signalized traffic intersections it is immaterial which probability distri-

bution is used for the arriving traffic at a signal. The Poisson distribution, because of its simplicity, is therefore used.

ANALYSIS OF AVERAGE DELAY AND OVERFLOW FORMULAS

The Webster (1) equations are as follows:

$$d = [c(1 - \lambda)^2 / 2(1 - \lambda \cdot x)] + [x^2 / 2 \cdot (1 - x)q] - 0.65(c/q^2)^{1/3} x^{(2+5\cdot\lambda)}$$
(1)

 $Q_0 = q[d - 0.5 \cdot c(1 - \lambda)] \qquad Q_0 < 0$ (2)

The Miller 1 equations (2) are as follows:

$$d = [(1 - \lambda)/2(1 - \lambda \cdot \mathbf{x})] \{c(1 - \lambda) + [(2 \cdot \mathbf{x} - 1)I/q(1 - \mathbf{x})] + [(1 + \lambda \cdot \mathbf{x} - 1)/s] \}$$
(3)

$$Q_0 = 0 \text{ for } x < 0.5$$

= I(2·x - 1)/2(1 - x) for x > 0.5 (4)

The Miller 2 equations (3) are as follows:

$$d = [(1 - \lambda)/2(1 - \lambda \cdot x)] (c(1 - \lambda) + \{exp[-(4/3)][(\lambda \cdot c \cdot s)^{0.5}(1 - x)/x] + q(1 - x)\})$$
(5)

The Newell 1 equations (4) are as follows:

$$d = [c(1 - \lambda)^2 / 2(1 - \lambda \cdot x)] + [I \cdot H(\mu) x / 2 \cdot q(1 - x)] + [I(1 - \lambda) / 2 \cdot s(1 - \lambda \cdot x)^2]$$
(7)

$$Q_0 = I \cdot H(\mu) x/2(1 - x)$$
 (8)

A suggested modification to Newell 1 gives Newell 2:

$$d = [c(1 - \lambda)^2/2(1 - \lambda \cdot x)] + [I \cdot H(\mu)x/2 \cdot q(1 - x)]$$

where

- d = average delay (s/vehicle),
- Q_0 = average overflow at end of cycle,
- y = q/s = ratio of average arrival rate to saturation flow,
- λ = g/c = proportion of cycle that is effectively green,
- g = effective green time (s),
- I = variance-to-mean ratio of flow per cycle,
- x = ratio of average number of arrivals per cycle to maximum number of departures per cycle,
- c = cycle length (s),
- q = average number of arrivals per unit time, and

c

s = saturation flow (vehicles/s).

Table 1. Average delay for macroscopic simulation and standard formulas.

 $H(\mu) = \exp[-\mu - (\mu^2/2)]$

where

(9)

0.90

0.95

1026

1083

26.60

42.21

28.69

44.79

31.21

48.34

24.78

40.13

27.79

44.69

25.81

42.38

 $\mu = (1 - x) \cdot (s \cdot g)^{0.5}$

The values obtained for average delay and average overlow and the simulation values are indicated in Tables 1 and 2.

If we take the simulation values as the standard and calculate the standard deviation for the values in Tables 1 and 2, the values given below are obtained:

	Standard	Formula			
		Miller	Miller	Newell	Newell
Variable	Webster	1	2	1	2
Avg de- lay	2.061	3.820	2.122	1.445	1.471
Avg over-	0.293	1.063	0.193	0.204	

ANALYSIS OF AVERAGE NUMBER OF STOPS

Figure 1 is a queue-length diagram for a signal cycle with overflow at the end of the cycle in which the overflow of vehicles at the end of previous cycle (Q_B) , the overflow of vehicles at the end of

					Avg Delay (d	1)					
c						Standard Formula					
	g	g 5 x	x	q	Simulation	Webster	Miller 1	Miller 2	Newell 1	Newell 2	
40	12	0.5	0.50	270	13.07	13.76	11.65	11.95	13.42	12.45	
			0.70	378	17.12	17.32	18.22	15.89	17.59	16.47	
			0.90	486	38.42	37.60	42.10	38.15	40.06	38.75	
			0.95	513	78.90	70.25	75.82	71.58	73.62	72.25	
40	24	0.5	0.50	540	5.20	5.99	4.74	4.61	5.52	4.70	
	2.	0.0	0.70	756	7 34	8 35	8 00	6.28	7.85	6.66	
			0.90	972	18 37	18.82	20.31	16.61	10.05	18.06	
			0.95	1026	36.68	35.14	37 35	33.05	37 22	35.06	
			0.75	1020	50.00	55.14	51.55	55.05	51.22	33.00	
60	18	0.5	0.50	270	18.47	19.37	17.42	17.50	18.75	17.78	
			0.70	378	22.32	23.11	24.42	21.15	22.74	21.62	
			0.90	486	43.71	43.47	48.81	42.93	45.06	43.74	
			0.95	513	81.17	76.12	82.67	76.25	78.67	77.30	
60	36	0.5	0.50	540	7.40	8.24	7.03	6.87	7.72	6.90	
			0.70	756	9.76	10.96	10.75	8.76	10.16	8.97	
			0.90	972	20.76	21.85	23.79	19.04	22.29	20.40	
			0.95	1026	37.63	38.29	41.07	35.42	39.69	37.52	
80	24	0.5	0.50	270	24.08	25.00	23.18	23 17	24 29	23 32	
00	2.	010	0.70	378	27.92	28.00	30.62	26.76	28 22	27 10	
			0.90	486	49.22	49.51	55 52	18 14	50.36	40.05	
			0.95	512	95.50	92.10	20.52	91 22	94.01	97.64	
80	10	0.5	0.55	515	05.55	10.50	09.32	01.52	0.07	014	
00	40	0.5	0.30	756	12.22	12.50	9.51	9.13	9.91	9.10	
			0.70	/30	12.52	13.39	13.51	11.57	12.00	11.47	
			0.90	972	23.40	24.96	21.21	21.72	24.86	22.97	
			0.95	1026	40.36	41.56	44.79	38.05	42.37	40.20	
100	32	0.5	0.50	288	28.41	29.31	27.65	27.57	28.60	27.64	
			0.70	403	32.23	33.49	35.19	31.12	32.44	31.31	
			0.90	518	51.73	52.89	59.05	50.62	52.93	51.59	
			0.95	547	84.23	83.38	90.93	81.37	84.34	82.94	
100	64	0.5	0.50	576	9.98	10.81	9.70	9.53	10.31	9.53	
			0.70	806	12.79	14.10	13.96	11.93	13.14	11.96	
			0.90	1037	23.56	25.38	27.60	21.70	24.88	22.87	
			0.95	1094	38.70	40.90	44.05	36.65	41.28	38.94	
120	38	0.5	0.50	285	34.14	35.00	33.42	33.32	34.32	33.35	
			0.70	399	38,18	39.51	41.47	37.10	38.33	37.21	
			0.90	513	57.82	59.57	66.28	56.79	59.09	57.76	
			0.95	542	90.74	91 78	99 99	89.08	92.20	90.81	
120	76	0.5	0.50	570	12.26	13.09	11.07	11.91	12.50	11.01	
				1/1/		1.1.1.0			1 / 17		

the cycle $({\rm Q}_{\rm E})\,,$ and the effective red time in seconds (r) are shown. The total delay per cycle is the area under the queue-length curve such that

$$D = [(2 \cdot Q_{B} + q \cdot r)r/2] + [(q \cdot r + Q_{B} + Q_{E})g/2]$$
(10)

where D is the total delay per cycle in seconds.

The number of stops per cycle is the number of arrivals while there is a queue plus the overflow at the beginning of the cycle such that

$$N = c \cdot q + Q_B \tag{11}$$

where N is the number of stops per cycle.

Figure 2 is a queue-length diagram for a signal

cycle without overflow at the end of the cycle. This case also has to be considered.

$$D = [(2 \cdot Q_{B} + q \cdot r)r/2] + [(q \cdot r + Q_{B})^{2}/2(s - q)]$$
(12)

$$N = \{r + [(q + Q_B)q/(s - q)]\} + Q_B$$
(13)

The average delay is

$$d = D/(q \cdot c) \tag{14}$$

and the average number of stops is

$$n = N/(q \cdot c)$$

(15)

Table 2. Average overflow for macroscopic simulation and standard formulas.							Avg Overflow (Q ₀)					
							-	Standard Formula				
			c g	g	S	x	q	Simulation	Webster	Miller 1	Miller 2	Newell 1
			40	12	0.5	0.50	270	0.06	0.00	0.00	0.04	0.07
						0.70	378	0.43	0.35	0.67	0.41	0.43
						0.90	480	3.34	3.19	4.00	3.48	3.42
			40	24	0.5	0.95	515	9.28	8.02	9.00	0.42	0.02
			40	24	0.5	0.30	340	0.02	0.00	0.00	0.01	0.02
						0.70	/30	0.25	0.07	0.07	2.00	2.00
						0.90	972	2.97	2.92	4.00	3.00	3.00
						0.93	1020	0.27	1.74	9.00	1.05	1.07
			60	18	0.5	0.50	270	0.03	0.00	0.00	0.02	0.04
						0.70	378	0.32	0.22	0.67	0.30	0.32
						0.90	486	3.14	3.03	4.00	3.21	3.19
						0.95	513	8.62	7.85	9.00	8.11	8.09
			60	36	0.5	0.50	540	0.01	0.00	0.00	0.00	0.01
						0.70	756	0.16	0.00	0.67	0.15	0.15
						0.90	972	2.64	2.66	4.00	2.67	2.69
						0.95	1026	7.45	7.49	9.00	7.43	7.51
			80	24	0.5	0.50	270	0.02	0.00	0.00	0.01	0.02
			00	2.	0.0	0.70	378	0.25	0.10	0.67	0.23	0.24
						0.90	486	2.97	2.90	4.00	3.00	3.00
						0.95	513	8 27	7 72	9.00	7.85	7 87
			80	48	0.5	0.50	540	0.00	0.00	0.00	0.00	0.00
			00	10	015	0.70	756	0.10	0.00	0.67	0.10	0.09
						0.90	972	2 41	2 42	4 00	2 42	2 4 5
						0.95	1026	7.15	7.28	9.00	7.10	7.22
			100	20	0.5	0.50	200	0.01	0.00	0.00	0.00	0.01
			100	32	0.5	0.50	288	0.01	0.00	0.00	0.00	0.01
						0.70	403	0.18	0.00	0.00	0.17	0.17
						0.90	518	2.71	2.12	3.97	2.74	2.75
			100	64	0.5	0.95	547	1.12	7.50	8.93	7.49	7.56
			100	04	0.5	0.50	5/6	0.00	0.00	0.00	0.00	0.00
						0.70	806	0.06	0.00	0.00	0.07	0.05
						0.90	1037	2.17	2.13	4.01	2.17	2.19
						0.95	1094	0.60	0.90	8.93	0.00	0.81
			120	38	0.5	0.50	285	0.00	0.00	0.00	0.00	0.01
						0.70	399	0.15	0.00	0.67	0.14	0.13
						0.90	513	2.59	2.65	4.00	2.63	2.65
						0.95	542	7.59	7.65	9.18	7.54	7.64
			120	76	0.5	0.50	570	0.00	0.00	0.00	0.00	0.00
						0.70	798	0.04	0.00	0.67	0.05	0.03
						0.90	1026	2.00	1.91	4.00	2.01	2.01
						0.95	1083	6.45	6.85	9.00	6.50	6.66







Figure 2. Queue-length diagram for cycle without overflow at end of cycle.

Table 3. Average number of stops for macroscopic simulation and standard formulas.

					Avg No. of Stops (n)					
с					-	Standard Formula				
	g	\$	x	q	Simulation	Webster	Miller 1	Miller 2	Newell 1	
40	12	0.5	0.50 0.70 0.90	270 378 486	0.90 1.04 1.60 2.62	0.82 0.99 1.59	0.82 1.09 1.74	0.84 1.01 1.64	0.85 1.01 1.63 2.46	
40	24	0.5	0.95 0.50 0.70 0.90 0.95	540 756 972 1026	2.62 0.64 0.81 1.21 1.69	0.57 0.70 1.27 1.68	0.57 0.83 1.37 1.79	0.57 0.74 1.28 1.69	0.58 0.74 1.28 1.69	
60	18	0.5	0.50 0.70 0.90	270 378 486 513	0.87 0.97 1.36 2.00	0.82 0.93 1.37 1.92	0.82 1.02 1.49 2.05	0.83 0.95 1.40 1.95	0.83 0.95 1.39 1.95	
60	36	0.5	0.50 0.70 0.90 0.95	540 756 972 1026	0.62 0.77 1.08 1.40	0.57 0.69 1.16 1.44	0.57 0.78 1.25 1.53	0.57 0.71 1.16 1.43	0.57 0.71 1.17 1.44	
80	24	0.5	0.50 0.70 0.90 0.95	270 378 486 513	0.86 0.94 1.25 1.71	0.82 0.90 1.27 1.68	0.82 0.99 1.37 1.79	0.83 0.92 1.28 1.69	0.83 0.92 1.28 1.69	
80	48	0.5	0.50 0.70 0.90 0.95	540 756 972 1026	0.60 0.75 1.03 1.27	0.57 0.69 1.11 1.32	0.57 0.76 1.19 1.39	0.57 0.70 1.11 1.31	0.57 0.70 1.11 1.32	
100	32	0.5	0.50 0.70 0.90 0.95	288 403 518 547	0.83 0.92 1.16 1.49	0.81 0.88 1.19 1.49	0.81 0.95 1.28 1.59	0.81 0.90 1.19 1.49	0.81 0.90 1.19 1.50	
100	64	0.5	0.50 0.70 0.90 0.95	576 806 1037 1094	0.55 0.70 0.97 1.17	0.53 0.65 1.02 1.23	0.53 0.71 1.14 1.29	0.53 0.66 1.03 1.22	0.53 0.66 1.03 1.22	
120	38	0.5	0.50 0.70 0.90	285 399 513 542	0.83 0.91 1.12 1.41	0.81 0.88 1.15 1.42	0.81 0.94 1.23 1.51	0.81 0.89 1.15 1.42	0.81 0.89 1.15 1.42	
120	76	0.5	0.50 0.70 0.90 0.95	570 798 1026 1083	0.56 0.70 0.95 1.13	0.54 0.66 0.98 1.19	0.54 0.70 1.12 1.25	0.54 0.66 0.99 1.18	0.54 0.66 0.99 1.18	

By substituting the average-overflow equations (Equations 2, 4, 6, and 8) into Equations 10 through 15, the values for average number of stops are obtained, which are indicated in Table 3.

If we take the simulation values as the standard and calculate the standard deviation for the values in Table 3, the values below are obtained:

	Standard	Formula			
Variable	Webster	Miller 1	Miller 2	Newell 1	
Avg no. of	0.060	0.094	0.049	0.050	
stops					

CONCLUSIONS AND RECOMMENDATIONS

From an inspection of the values for average delay given above, it is clear that the Newell formulas for average delay approximate the simulated values much more closely than the other formulas. Newell 1 gives a slightly better approximation than Newell 2. For practical purposes, this difference is negligible and the fact that Newell 2 contains one term less than Newell 1 makes the former equation, namely, Equation 9, preferable for calculating average delay at all signalized intersections for undersaturated stationary conditions.

From an inspection of the values for average overflow, it is seen that Miller 2 approximates the simulated values more closely than the other formulas. However, since Newell is, for practical purposes, as accurate and since Equation 9, which has already been suggested for calculating delay, also contains $H(\mu)$, the Newell equation, Equation 8, is

recommended for calculating average overflow at all signalized intersections for undersaturated stationary conditions.

From an inspection of the values for average number of stops, it follows that for the same reasons as in the case of average overflow, the Newell equation for average overflow is suggested for calculating the average number of stops for undersaturated stationary conditions.

Hutchinson (5) indicates that within the practical limits in which flow and saturation flows can be measured, any formula can be used for calculating average delay. The Newell 2 delay equation, because of its simplicity and accuracy, however, is recommended for the calculation of average delay at fixed-time signalized traffic intersections.

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Derivation of Equations for Queue Length, Stops, and Delay for Fixed-Time Traffic Signals

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The existing methods for the calculation of queue length, number of stops, and delay for isolated traffic intersections are applicable either to undersaturated stationary conditions or to oversaturated conditions. As far as is known, no model exists that is applicable to all conditions. Equations are derived for the calculation of queue length, number of stops, and delay for isolated fixed-time signalized intersections that are applicable to undersaturated as well as to oversaturated conditions. In the derivation the macroscopic approach to traffic flow is used. This approach has been shown to be sufficiently accurate for practical purposes. Traffic flow at a signalized intersection is considered a Markov process. Equations are derived for expected queue lengths, expected number of stops, and expected total delay. These equations can also be used for the optimization of isolated fixed-time signalized intersections.

Traffic flow at a signalized intersection is a Markov process. The states being considered are the queue lengths at the beginning and the end of the signal cycle, and the time interval over which changes in these states take place is the length of the signal cycle.

The equation governing the states is as follows:

$$Q_E = Q_B + q \cdot c - s \cdot g \qquad Q_E < 0 \tag{1}$$

where

- Q_E = overflow of vehicles at end of cycle,
- Q_B^- = overflow of vehicles from previous cycle,
- q = average arrivals per unit time interval,
- c = cycle length (s),
- g = effective green time (s),
- s = saturated flow (vehicles/s),
- q c = number of arriving vehicles per cycle, and s'g = maximum number of departing vehicles per
 - cycle.

Equation 1 can be represented by the transition probability matrix shown in Figure 1, in which $\mathtt{P}\left(\mathtt{Q}_{B},\mathtt{Q}_{\overline{E}}\right)$ is the probability of transition from state Q_B to state Q_E , P(s•g) is the probability distribution of departing vehicles, and P(q*c) is the probability distribution of arriving vehicles.

Equation 1 is illustrated by Figure 2, in which r is effective red time in seconds.

DERIVATION

Consider one approach to an intersection controlled by a fixed-time signal. Consider one cycle on the approach in which vehicles are expected to arrive according to a distribution P(q.c). The saturation-flow vehicles are distributed according to P(s•g).

Let $P(Q_B,Q_E)$ be the probability of an overflow of Q_E vehicles at the end of the cycle, given an overflow $\boldsymbol{Q}_{\mathrm{B}}$ at the start of the cycle. The form of this probability is shown as a matrix in Figure 3. The expected overflow at the end of the cycle is

given by

$$\begin{split} E(Q_E) &= \sum_{Q_E=0}^{\infty} Q_E \cdot P(Q_E) \\ &= \sum_{s \cdot g} P(s \cdot g) \left[\sum_{q \cdot c = 0}^{\infty} P(q \cdot c) \sum_{Q_B=0}^{\infty} (Q_B + q \cdot c - s \cdot g) P(Q_B) \right. \\ &\left. - \sum_{q \cdot c = 0}^{s \cdot g - 1} P(q \cdot c) \sum_{Q_B=0}^{s \cdot g - q \cdot c - 1} (Q_B + q \cdot c - s \cdot g) P(Q_B) \right] \\ &= E(Q_B) + E(q \cdot c) - E(s \cdot g) \\ &\left. - \sum_{s \cdot g} P(s \cdot g) \sum_{q \cdot c = 0}^{s \cdot g - 1} P(q \cdot c) \sum_{Q_B=0}^{s \cdot g - q \cdot c - 1} (Q_B + q \cdot c - s \cdot g) P(Q_B) \right] \end{split}$$

A queue-length diagram with overflow at the end of the cycle is indicated in Figure 4.

The total number of stops per cycle is the number of vehicle arrivals while there is a queue. Overflow vehicles stop twice.

If there is overflow at the end of the cycle, the total number of stops per cycle is as follows:

$$N = Q_{\rm H} + q \cdot c \tag{3}$$

A queue-length diagram without overflow at the end of the cycle is indicated in Figure 5.

If there is no overflow at the end of the cycle, the total number of stops per cycle is as follows:

$$N = Q_{B} + r \cdot q + [(Q_{B} + r \cdot q)/(s - q)] q$$
(4)

Figure 1. Transition probability matrix for Equation 1.

