Rutting Properties of a Dune-Sand Paving Mixture

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The results of a laboratory evaluation of a dune-sand for use as a base course in a flexible pavement are presented. The principal part of the study was concerned with the development of an equation for relating permanent deformation (rutting) to the combined effects of (a) number of load applications, (b) temperature, (c) surface pressure, and (d) dynamic modulus of elasticity of the supporting subgrade. The mixture evaluated was a dune-sand from Arizona with asphalt of the AR-4000 grade at an optimum content of 14 percent by weight of sand. Permanent deformation specimens were 18 in. in diameter and the height varied from 2 to 6 in. Variation in subgrade support was obtained with synthetic rubber pads 18 in. in depth and 6 in. in height having estimated moduli of 8,000, 20,000, and 30, 000 psi. The constant load system operated at a frequency of 11.5 Hz, and contact area was varied to yield pressures of 38.5, 61.6, and 98.0 psi. It was found that the optimum specimen height was 4 in. if failure was assumed to have occurred when the rut was 0.4 in. in depth. For this constant specimen height, an equation relating load repetitions (N) to temperature (T), stress (σ), and subgrade modulus (E_S) is presented.

The structural design of an asphaltic concrete pavement is a complex procedure due to the uncertainties associated with the loads to be carried, the response of the pavement system to the loads, and the effects of the environment on the characteristics of the materials in the pavement system. Structural failure of an asphaltic pavement has been attributed by Hveem (1) to surface appearances of (a) cracks, (b) deformation, and (c) disintegration. The failure type of interest is permanent deformation or rutting, which originates within the layer of concern. (It is possible for an asphaltic concrete surface course to deform or rut after a deformation has occurred in a subsoil layer.)

One of the early pavement design procedures based on elastic theory used repeated vertical strains at the top of the subgrade as the criterion to preclude rutting of the asphaltic surface course (2). However, in certain cases, which are the ones of concern here, a repeated vertical stress or strain on an asphaltic surface course can result in surface ruts or deformations. Until recent years, permanent deformation originating in the surface course had not been used as a factor to be considered in the structural design of asphaltic pavements. It has been shown (3-5) that rutting can originate within an asphaltic concrete surface course that is thicker than about 10 cm (4 in.).

MECHANISM OF RUTTING

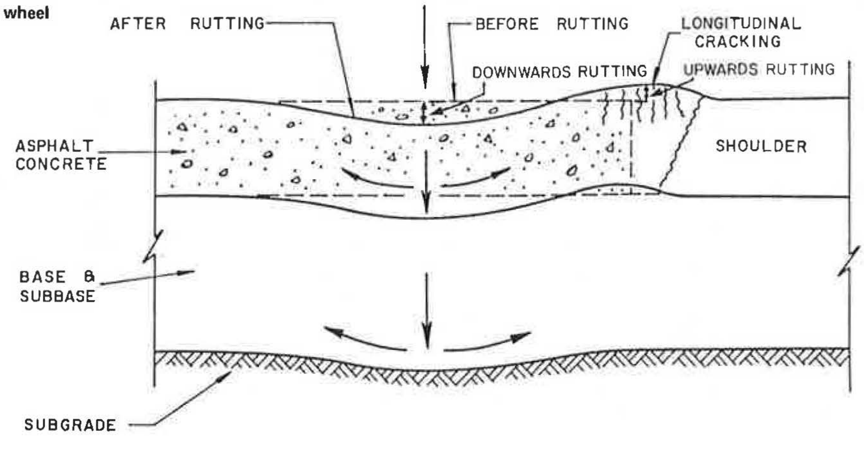
Based on the AASHTO Road Test results (3) and AASHTO's own research efforts, investigators of the phenomenon of rutting have generally agreed that it is a longitudinal channel or depression that forms in the wheel path due to compression or lateral movement or both in one or more of the pavement layers as a result of repeated traffic-load applications. Rutting is a manifestation of two different mechanisms: (a) densification (volume change) and (b) shear deformation (plastic flow with no volume change). In the densification process material is forced downward, and shear deformation causes material to flow laterally and upward, as shown in Figure 1 (6). The total rut depth is the difference in elevation between the crest and sag of the surface. As shown in the figure, tensile stresses on the top surface and the outside of the loaded area can cause longitudinal cracking of the asphaltic concrete.

Hofstra and Klomp (5) observed in their test track that permanent deformation in the asphaltic concrete layer was greater near the load and gradually decreased with depth below a certain level. Barksdale (7) agreed with this finding because the portion of the rut due to shear would have greater confinement or resistance to plastic flow at depths beneath the applied load. However, McLean and Monismith (8) and Morris and others (9), using elastic theory and triaxial laboratory tests, indicated that most rutting should occur in the lower part of the asphaltic concrete layer. This difference in findings makes it important that the test procedure chosen for determining the rutting characteristics of asphaltic concrete simulate field conditions as closely as possible.

DUNE-SAND

Dune-sand is found in many parts of the world, and in many cases it is the predominant aggregate available for road construction, especially in desert areas. Asphalt paving mixtures made with dune-sand are generally too weak to be used as a surface course in major highways. This paper is principally concerned with the use of an asphalt-stabilized dune-sand as a base course in a desert environment and under a thin asphaltic surface course. In this situation, it is believed that the principal form of failure would be rutting rather than flexural fatigue cracking.

Figure 1. Typical rutting mechanism in outer wheel path.



MATERIALS

Dune-Sand

The dune-sand used in this study came from an area near Yuma, Arizona. The gradation was typical of a windblown deposit; particle sizes ranged from the No. 50 to No. 200 sieves, and the uniformity coefficient (D_{60}/D_{10}) was 1.8. Compaction performed by using AASHTO procedures T99-57 and T180-57 yielded respective maximum dry densities of 102 and 105 pcf, both at an optimum moisture content of 12 percent.

Asphalt

The asphalt in the mixtures was a typical 60/70 penetration (AR-4000) grade that met specifications set by the Arizona Department of Transportation.

Rubber

The purpose of the rubber in this study was to serve as a subgrade to support the asphaltic dune-sand specimens. This artificial subgrade had a diameter of 18 in. and a thickness of 6 in. In order to cover a range of subgrade strength, especially of compacted dune-sand, the rubber pads were classified as hard, medium, or soft, which corresponded to durometer values of 50, 40, and 30, respectively. Table 1 gives physical properties of the rubber pads. The selection for values of hardness for the rubber pads was influenced by the following:

1. Bowles (11) has presented the following density relation for sands:

Condition	Density (psi)
Loose	17.3-57.8
Medium	34.7-289.4
Dense	231.5-463

2. Al-Salloum (12) has reported that in situ California bearing ratio (CBR) measurements on Arabian compacted dune-sand subgrades showed values ranging from 15 to 20.
3. Heukelom and Klomp (13) suggest the following relation between CBR and dynamic modulus of elasticity (E_D): E_D(psi) = 1500 CBR.

Compacted Dune-Sand and Asphalt Mixtures

The effects of asphalt content on the physical properties of compacted mixtures were determined by using the Marshall method of design and also a double-punch procedure for strength and dynamic modulus of elasticity.

Marshall Design

The Marshall design procedure described in Asphalt Institute Manual MS-2 (14) was followed for evaluation of mixture characteristics; however, by using a portion of the Hveem procedure [centrifuge kerosene equivalent (CKE)], it was found that the dune-sand had a surface-area value of 74 ft^2/lb and an optimum asphalt content of 4.8 percent by weight. The fineness of the sand, its uniformity in size, and its high surface-area value suggested discounting the CKE asphalt content as an optimum value. Test specimens for the Marshall evaluation were prepared with asphalt contents ranging from 4.5 to 16 percent by weight of sand.

Table 2 gives the data obtained from the Marshall evaluation along with the calculated asphalt film thickness on the sand particles.

Figures 2 and 3 show plots of the Marshall data and indicate that an optimum asphalt content for stability was reached at about 14 percent. It is noted that Marshall stabilities are low and do not meet general requirements for a surface course. As expected, the values for voids in mineral aggregate (VMA) were high (±38 percent) and remained fairly constant as the asphalt content varied; as a consequence, asphalt content caused a linear variation in density and air void content. At the optimum asphalt content, Marshall stability was 175 lbf, flow was 13, air void content was 11.8 percent, and asphalt film thickness was 9 μm.

Table 1. Physical properties of rubber subgrade.

Durometer Value	Condition	Tensile Strength[a] (psi)	Modulus of Subgrade Reaction[b] (pci)	Dynamic Modulus of Elasticity[c] (psi)
30	Soft	1,950	57	8,000
40	Medium	2,140	200	20,000
50	Hard	2,600	370	30,000

[a] From Adamac Rubber Manufacturers, Inc., Tucson, Arizona.
[b] From loadings with 6-in.-diameter plate.
[c] Assumed from relations suggested by Bowles (11), Al-Salloum (12), and Heukelom and Klomp (13).

Table 2. Marshall stability test data.

Asphalt Content (%)	Specimen No.	Specimen Density (g/cm³)	Specimen Height (in.)	Air Voids (%)	VMA (%)	Load Dial	Marshall Stability (lb)	Marshall Flow (0.01 in.)	Film Thickness[a] (μm)
4.5	1	1.712	2.50	30.7	38.3	2	40	7	2.93
	2	1.725	2.50	29.5	37.8	2	40	8	
6	1	1.746	2.51	27.5	38.1	2	40	10	3.91
	2	1.754	2.50	27.1	37.8	3	50	10	
8	1	1.826	2.50	23.0	36.6	4	70	9	5.21
	2	1.801	2.40	23.9	37.5	3.5	64	9	
10	1	1.836	2.47	19.7	37.7	4	71	12	6.51
	2	1.827	2.48	20.0	38.0	5	92	11	
12	1	1.866	2.50	16.1	38.0	5	90	10	7.81
	2	1.862	2.52	16.2	38.2	5.5	97	11	
14	1	1.910	2.47	11.8	38.0	7.5	153	14	9.10
	2	1.909	2.48	11.8	38.1	10	194	13	
16	1	1.928	2.53	8.6	38.9	7.5	134	16	10.42
	2	1.928	2.50	8.6	38.9	8	150	18	

[a] Film thickness is not usually included in the Marshall mix design method. It is added here to show how it varies with asphalt content. It is calculated as the ratio of the volume of asphalt to the surface area of aggregates (18).

Figure 2. Marshall stability test data: stability and flow versus asphalt content at 140° F wet.

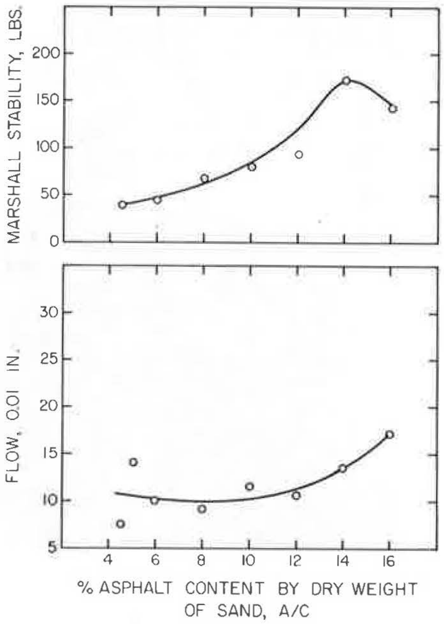
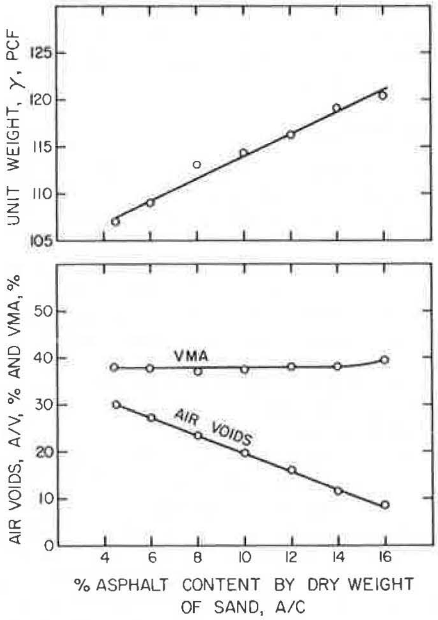


Figure 3. Marshall stability test data: unit weight and air voids or VMA versus asphalt content.

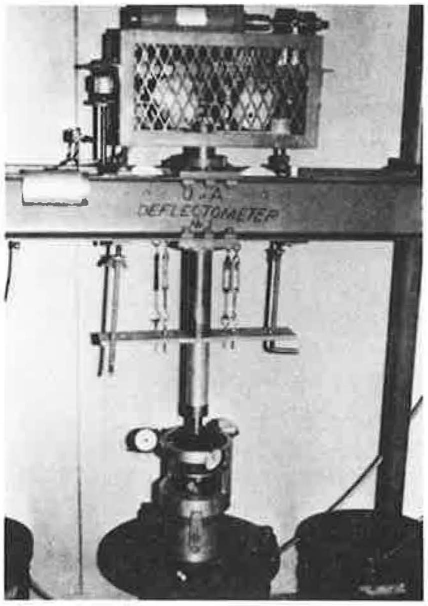


Double-Punch Tests

In 1970 Chen (15) presented a method for determining the tensile strength of concrete by loading a cylindrical specimen with two steel punches centered on the flat surfaces. In 1975 Jimenez (16) extended the double-punch test to obtain a measure of the dynamic modulus of elasticity under repeated rapid loading. Figure 4 shows the tester used in the procedure.

The graphs shown in Figures 5 and 6 present some of the results obtained in this evaluation. The complete testing procedure and the results are fully described elsewhere by Al-Juraiban (17). A review of Al-Juraiban's data will show that the densities of the 4-in.-diameter specimens were equal to those of the 18-in.-diameter specimen of comparable composition, formed by vibratory kneading compaction. Jimenez (18) has described vibratory kneading compaction and shown comparisons of physical properties obtained for 4- and 18-in.-diameter specimens.

Figure 4. Setup for dynamic modulus of elasticity by double-punch procedure.



The curves shown in Figures 5 and 6 confirm the finding that 14 percent was the optimum asphalt content.

PREPARATION AND TESTING OF SPECIMENS FOR RUTTING PROPERTIES

Preparation of Large Test Specimens

The large test specimens were to have a diameter of 18 in. and heights varying from 2 to 6 in.; the weight of a specimen thus ranged from 31 to 104 lb.

Mixing

Enough sand and asphalt were weighed to make enough mixture to produce a 2-in.-high specimen. Sand (30 lb) in covered metal containers was heated in a forced-draft oven to a temperature of 300° ± 5°F. Sufficient asphalt for the number of specimens to be compacted was heated in a sealed metal container to a temperature of 250° ± 5°F.

The mixing bowl and beater of a large-capacity Blakeslee mixer were heated before mixing. After the sand and asphalt were combined in the desired proportion, it was found that 3 min of mixing was sufficient to produce a uniform mixture.

After mixing of a sufficient amount of the asphaltic mixture to produce a 2-in. specimen, the specimen was placed in a lidded metal container and stored in a 250° ± 5°F oven before compaction. Specimens were compacted in increments of 2-in. layer thickness.

Compaction

The 18-in.-diameter specimens were made with a vibratory kneading compactor developed by Jimenez

Figure 5. Tensile strength (by double-punch) versus asphalt content for different testing temperatures.

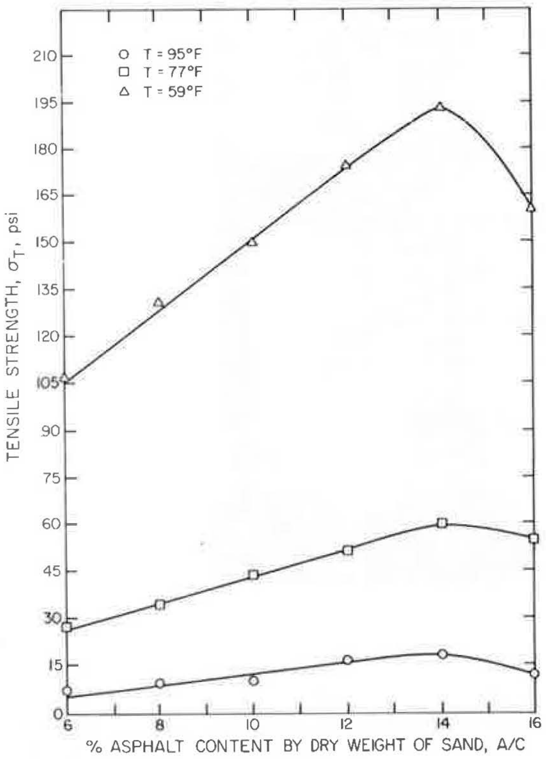
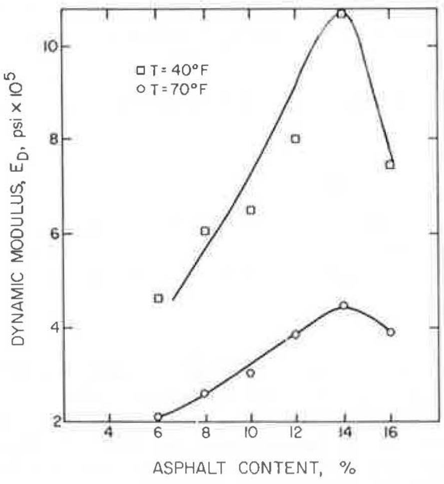
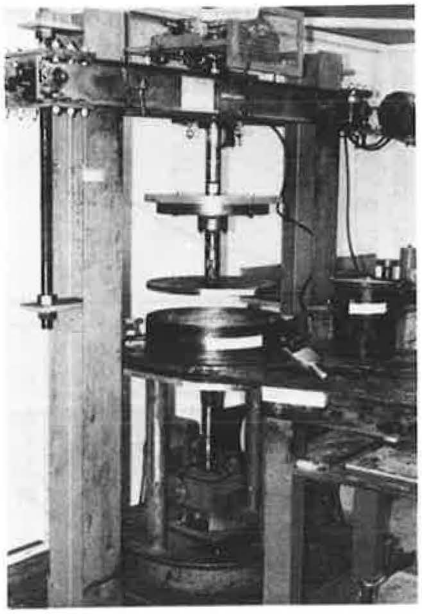


Figure 6. Dynamic modulus versus asphalt content for dune-sand asphalt.



(18,19). The compactor can be used to make specimens with diameters from 2 to 18 in. and heights from 1 to 6 in. During compaction, the mixture is held in a circular mold on a tilted turntable and is subjected to an impacting cyclic vertical load while the mold is rotating. After a predetermined period of compaction time, the loading is stopped to take the tilt out of the turntable and the specimen is squared with an additional short period of vibratory loading. The kneading effect of the compactor comes from the combination of a vertical force on a rotating slanted surface. The compactor is shown in Figure 7.

Figure 7. Jimenez vibratory kneading compactor ready to compact a large specimen.



The standard compaction procedure was modified by eliminating the tilt in the process. The principal reason for this was concern that the compactor might be damaged in the attempt to compact the thickest specimens, which with the steel mold would involve sliding a weight in excess of 125 lb. As a consequence, even though equal densities were obtained between the 4-in.-diameter and 18-in.-diameter specimens, the strength of the 18-in. specimens was lower due to the lack of kneading effect caused by a tilt of zero on the turntable.

Testing for Rutting Properties

The apparatus used for the rutting study was essentially the one developed by Jimenez (18) for flexure fatigue testing. The main difference between the two was the support given a specimen; in the original device, the specimen was given a hydraulic support--that is, uniform pressure over the bottom of the specimen; in the present device, the specimen was supported with a rubber pad. The test apparatus is shown in Figure 8.

The loading system consisted of counter-rotating balanced eccentric masses placed in mirrored positions so that the effective centrifugal force was a vertical one. In order to eliminate impact (collision), the dead load of the system was greater than the live load caused by the rotating eccentric masses. The loading standard for this research was a dead load of 178 lb, a sinusoidal live load with a maximum value of 130 lbf, and a load frequency of 11.5 Hz. The contact pressure on the specimen was varied by changing the area of the load disc to yield maximum contact pressures of 38.5, 61.6, and 98.0 psi.

Specimens were mounted on the testing device as shown in Figure 8. Surface displacement measurements were made periodically as a function of load repetitions and with extensometers graduated to 0.001 in. Two extensometers were used to determine the permanent deformation at the center of the specimen and also at a distance of 1.0 in. outside the loaded area. These measurements were made 4 min after stopping and raising the loading system. Another extensometer was mounted on the load shaft to determine the repeated deflection and to check the permanent deflection at the center of the specimen.

Loading and recordings were continued until the rut depth (permanent deformation) was equal to 0.4

Figure 8. Permanent deformation test apparatus.

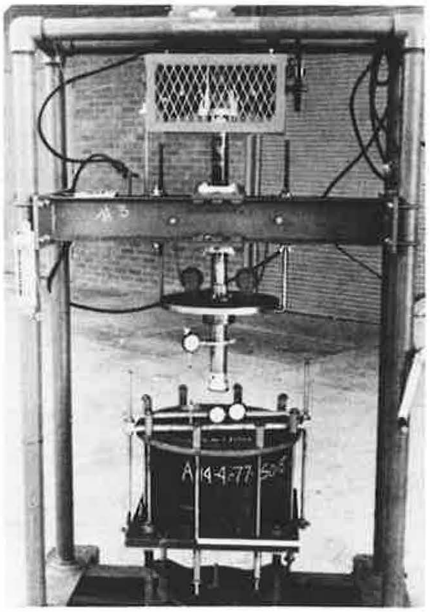
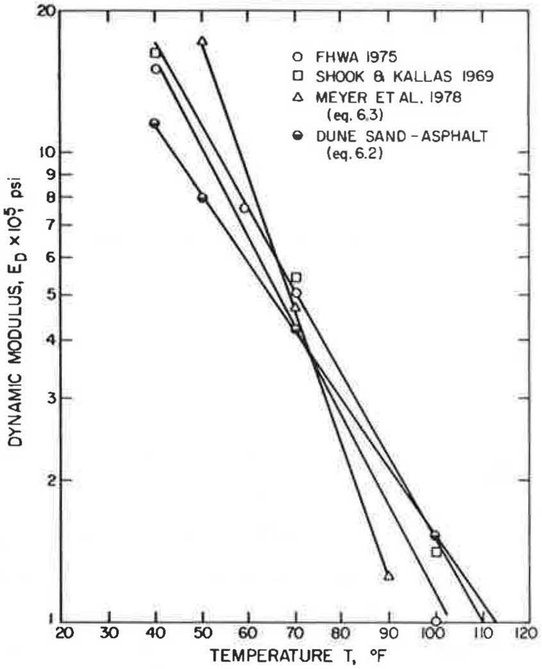


Figure 9. Dynamic modulus versus temperature for dune-sand asphalt and asphalt concrete.



in. as obtained by the algebraic addition of the deflection values on the surface of the specimen. This value of rut depth as the failure criterion was established after a review of the literature (17) indicated that, in consideration of hydroplaning, the rut depth should be limited to 0.375 in. and that, in the United Kingdom, pavements are considered to have failed when the rut depth is 0.75 in.

TEST RESULTS AND DISCUSSIONS

Dynamic Modulus of Elasticity

The graphs shown in Figures 5 and 6 were developed from data not presented here for the sake of brevity; however, a statistical analysis and comparisons of values obtained by other researchers are presented.

In our test for determining a dynamic modulus of elasticity, the specimen was compacted by using a vibratory kneading procedure; the repeated sinusoidal stresses varied from a low of 3 psi to a high of 12 psi at a load frequency of 11.5 Hz.

The Statistical Package for the Social Sciences (SPSS) (20) computer program was used to establish functional relations between dynamic modulus of elasticity (E_D) and asphalt content (AC), test temperature (T_F), and air void content (AV). An exponential model yielded the following constants:

$$\text{Log } E_D = 5.89741(AC)^{0.09828}(T_F)^{-0.09736}(AV)^{0.04207} \qquad (1)$$

$$R^2 = 0.877 \quad SEE = 0.0053 \quad df = 11$$

For the optimum asphalt content of 14 percent and the corresponding air voids and test load rate, Equation 1 reduced to

$$\text{Log } E_D = 6.62294 - 0.01436(T_F) \quad R^2 = 0.98 \quad SEE = 0.041 \qquad (2)$$

Figure 9 shows relations between temperature and dynamic modulus of elasticity (21-23). The semi-log plots of Figure 9 show similarities among the four curves; however, it should be recognized that there were great variations in the mixture types, specimen compaction procedures, test methods, and stress levels used.

Rutting or Permanent Deformation

The chart shown in Figure 10 shows the variables and their level used to investigate the rutting properties of asphaltic dune-sand mixtures. At least two specimens were used in each test block. As shown in the chart, the variables were temperature, subgrade strength expressed as rubber durometer, specimen thickness, asphalt content, and contact pressure expressed as loaded area.

Response of Permanent Deformation to Repeated Loads

As indicated earlier, values of permanent deformation for corresponding number of repeated loads were recorded until a rut depth of 0.4 in. was reached. Table 3 gives values of permanent deformation as affected by number of load repetitions and several test variables for a particular mixture. For the sake of brevity, comprehensive test results are not presented in this paper, but a statistical analysis of the data is presented that gives a measure of reliability of the relations to be shown.

The effect of load repetitions on permanent deformation is shown in Figure 11; also shown is a curve representing the findings of Hofstra and Klomp (5) that shows the similarity of shape for the two materials and tests. For the dune-sand mixture shown and a general model that relates permanent deformation (P_d) to number of repetitions (N) in the form $P_d = aN^b$, the SPSS program yielded the following equation:

$$\text{Log } P_d = 0.1159 + 0.4690 \log N \quad R^2 = 0.979 \quad SEE = 0.098 \quad df = 11 \qquad (3)$$

Equation 3 was developed for a test temperature of 77°F, a subgrade dynamic modulus of 20,000 psi, an applied stress of 61.6 psi, an asphalt content of 14 percent, and a specimen height of 4 in. A more general equation including several variables is presented later in this paper.

Effect of Asphalt Content

Figure 12 shows the effects of asphalt content on the number of load repetitions to failure. The curves show that the optimum asphalt content was 14 percent, the same value obtained from the Marshall and double-punch test data. Because 14 percent was the optimum asphalt content for the dune-sand, this percentage is taken as a typical asphalt content and the one on which most of the analyses are concentrated.

Effect of Specimen Thickness

The curves presented in Figure 13 show that an optimum value of 4 in. was obtained with reference to maximum number of load repetitions to failure (a rut depth of 0.4 in.).

Examination of the 2-in.-thick specimens showed radial cracks at the top of the specimen at failure. These cracks were associated with a tensile fatigue failure. When the specimen cracked, there

Figure 10. Testing program chart.

Temperature, °F		59									77									95								
Rubber Durometer (Subgrade Strength)		30 (Soft)			40 (Medium)			50 (Hard)			30 (Soft)			40 (Medium)			50 (Hard)			30 (Soft)			40 (Medium)			50 (Hard)		
Specimen Thickness, in		2	4	6	2	4	6	2	4	6	2	4	6	2	4	6	2	4	6	2	4	6	2	4	6	2	4	6
Loaded Area, in²	Asphalt Content, %																											
5	4.5																x	x										
	6																x	x										
	8										x	x	x				x	x	x									
	10										x	x	x				x	x	x									
	12	x		x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x		x				x	
	14	x	x		x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x		x	x		x	x	
	16	x	x	x	x		x	x	x		x	x	x	x	x	x	x	x	x	x			x	x		x		
π	14		x			x			x			x			x			x			x			x		x	x	
8	14		x			x			x			x			x			x			x			x			x	

Table 3. Typical magnitudes of permanent deformation as affected by various test variables (asphalt content = 14 percent and specimen height = 4 in.).

Permanent Deformation[a] ($P_d \times 10^{-3}$ in.)	Applied Stress (psi)	Subgrade Dynamic Modulus (psi)	Test Temperature (°F)	No. of Load Repetitions (000s)
95	38.5	8,000	59	100
222	38.5	8,000	59	400
285	38.5	8,000	59	638
325	38.5	8,000	59	728
400	38.5	8,000	59	1,150
105	61.6	8,000	59	100
160	61.6	8,000	59	200
240	61.6	8,000	59	400
320	61.6	8,000	59	600
400	61.6	8,000	59	825
107	98.0	8,000	59	68
130	98.0	8,000	59	100
255	98.0	8,000	59	212
325	98.0	8,000	59	258
400	98.0	8,000	59	303
90	38.5	20,000	59	200
150	38.5	20,000	59	400
195	38.5	20,000	59	1,000
275	38.5	20,000	59	2,000
400	38.5	20,000	59	3,900
69	38.5	30,000	77	10
132	38.5	30,000	77	100
225	38.5	30,000	77	400
330	38.5	30,000	77	800
400	38.5	30,000	77	1,100
80	61.6	30,000	77	10
113	61.6	30,000	77	20
161	61.6	30,000	77	40
258	61.6	30,000	77	80
400	61.6	30,000	77	140
80	98.0	30,000	77	4
120	98.0	30,000	77	10
210	98.0	30,000	77	20
280	98.0	30,000	77	30
400	98.0	30,000	77	48
72	38.5	8,000	77	10
140	38.5	8,000	77	40
210	38.5	8,000	77	80
270	38.5	8,000	77	110
360	38.5	8,000	77	150
48	61.6	8,000	77	2
80	61.6	8,000	77	5
120	61.6	8,000	77	10
220	61.6	8,000	77	20
400	61.6	8,000	77	40
55	98.0	8,000	77	2
96	98.0	8,000	77	5
180	98.0	8,000	77	10
305	98.0	8,000	77	17
440	98.0	8,000	77	24
160	61.6	30,000	95	1
240	61.6	30,000	95	3
275	61.6	30,000	95	3
300	61.6	30,000	95	4
360	61.6	30,000	95	5

Note: A/C = 14 percent and h = 4 in.

[a] 400 = failure.

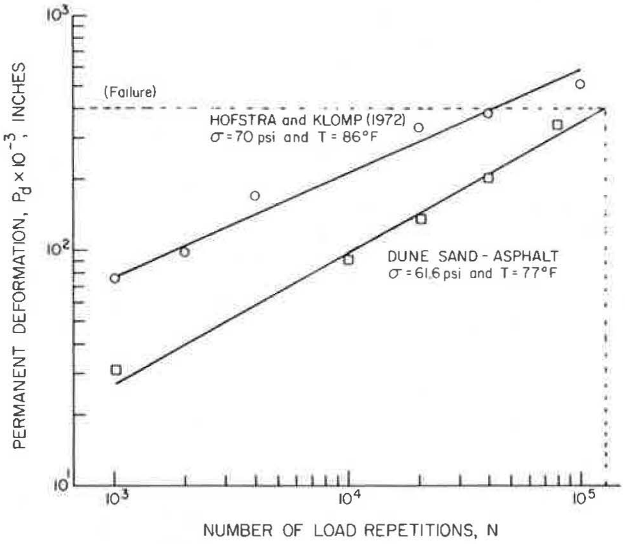


Figure 11. Comparison between measured permanent deformations for dune-sand asphalt and Hofstra and Klomp (5) test track.

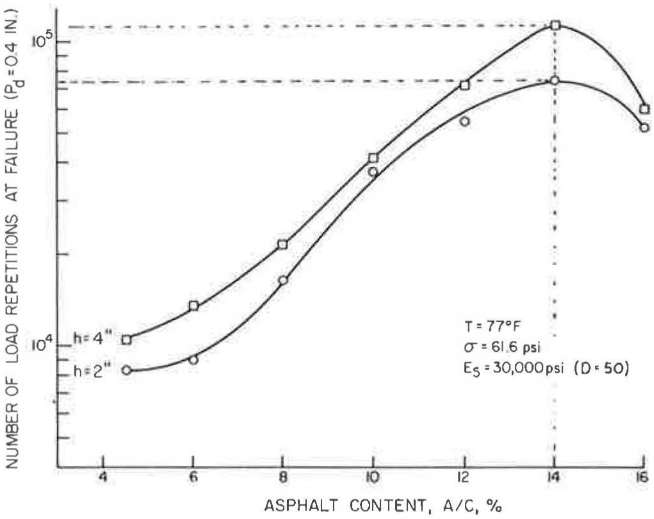


Figure 12. Number of load repetitions at failure versus asphalt content for specimen heights of 2 and 4 in.

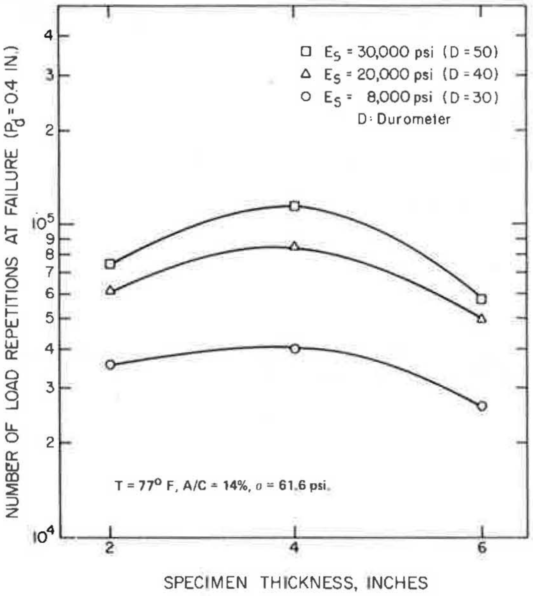


Figure 13. Typical relations between specimen thickness and number of load repetitions at failure for different subgrade (rubber) strengths.

was open space for the material to flow into and thus the resistance to rutting was decreased.

The 6-in.-thick specimens failed before the 4-in. ones, most likely because of densification. The 6-in.-thick specimens had a longer column under the load and so had more voids in an absolute sense to decrease the resistance to rutting.

The fact that permanent deformation increases with layer thickness was observed by Hofstra and Klomp (5) in their test track. This behavior was also found by Van de Loo (24) from measurements in a laboratory test track.

The study data thus far indicated that, to optimize the resistance to rutting of the dune-sand and asphalt specimens, they should have an asphalt content of 14 percent and a thickness of 4 in. As a consequence, the effects of subgrade modulus, temperature, and contact pressure on resistance to rutting were determined on the optimum specimen.

Effects of Subgrade Strength

Three rubber pads with durometer values of 30, 40, and 50 were used to represent sand subgrades of loose, medium, and dense conditions. It has been shown that these durometer values corresponded to dynamic modulus of elasticity values of 8,000, 20,000, and 30,000 psi, respectively.

As anticipated, the number of load repetitions to rutting failure increased as the subgrade modulus increased. For a test temperature of 77°F and a contact pressure of 61.6 psi, the following equation was obtained for relating rut depth (P_d) to number of load repetitions (N) and subgrade dynamic modulus (E_S):

$$\text{Log } P_d = 1.4108 + 0.5892 \log N - 0.4176 \log E_s \qquad R^2 = 0.97 \quad \text{SEE} = 0.057 \quad \text{df} = 11 \tag{4}$$

Effects of Applied Pressure

The curves presented in Figure 14 show the influence of applied pressure on the number of load repetitions to failure. An exponential equation relating number of repetitions (N) and applied stress (pressure σ) to the values of permanent deformation is given below:

$$\text{Log } P_d = 1.9472 + 0.4638 \log N + 1.1422 \log \sigma \qquad R^2 = 0.92 \quad \text{SEE} = 0.10 \quad \text{df} = 11 \tag{5}$$

Equation 5 is valid for a test temperature of 77°F and a subgrade value of 20,000 psi.

Effects of Test Temperature

As would be expected, when the test temperature decreased the resistance to rutting increased. By using the prior exponential models to relate load repetitions and temperature to permanent deformation, the following equation was obtained:

$$\text{Log } P_d = -13.4998 + 0.5695 \log N + 7.0329 \log T \quad R^2 = 0.96 \quad \text{SEE} = 0.067 \quad \text{df} = 11 \tag{6}$$

Again, the equation is valid for σ = 61.6 psi, E_S = 20,000 psi, AC = 14 percent, and h = 4 in. From Equation 6 it is noted that temperature had a tremendous influence on rutting; this is shown by the coefficient value of 7.03.

Figure 15 shows the linear logarithmic effect of test temperature on number of load repetitions to failure. It is seen that both reduction of temperature and increase of subgrade modulus increase the rutting resistance of the paving mixture. Note that at temperatures below 50°F the number of load repetitions to failure would be in excess of 10 million, which suggests that rutting failure would not occur at those temperatures.

General Model for Permanent Deformation of Asphaltic Dune-Sand Mixture

The stepwise SPSS computer program was used to obtain a general model for predicting permanent deformation of the dune-sand mixture. All significant variables were included in the model to represent a mixture containing an asphalt content of 14 percent and a layer thickness of 4 in. The model will allow the prediction of the magnitude of permanent deformation at any number of load repetitions, any temperature, any applied stress, or any subgrade modulus. The general model chosen follows the form of those used previously:

$$P_d = aN^bT^c\sigma^dE_S^e \tag{7}$$

The equation constants were evaluated to yield

$$\text{Log } P_d = -10.7019 + 0.4820 \log N + 5.7627 \log T + 0.9756 \log \sigma - 0.4043 \log E_S \quad R^2 = 0.76 \quad \text{SEE} = 0.133 \quad \text{df} = 130 \tag{8}$$

where

P_d = permanent deformation (10^{-3} in.),
N = number of load repetitions,
T = test temperature (°F),
σ = applied compressive stress (psi), and
E_S = subgrade dynamic modulus of elasticity (psi).

The effect of the independent variables on the permanent deformation model (Equation 8) is shown by the data given in Table 4. The improvements in R^2 values and reduction in SEE values as the number of variables increased are evident.

If one agrees that a rut depth of 1 cm (0.4 in.) corresponds to a failure condition, then the model can be modified to express the number of load repetitions to failure. This equation then takes the following form:

$$\text{Log } N = 27.8133 - 12.0000 \log T - 2.4614 \log \sigma + 0.9664 \log E_S \quad R^2 = 0.97 \quad \text{SEE} = 0.223 \quad \text{df} = 23 \tag{9}$$

Even though Equation 9 has a higher R^2 than Equation 8, it has a higher SEE and a lower df. We suggest that both models are acceptable for characterization of the paving mixture.

Figure 14. Typical relations between number of load repetitions at failure and applied stress for different supporting strengths.

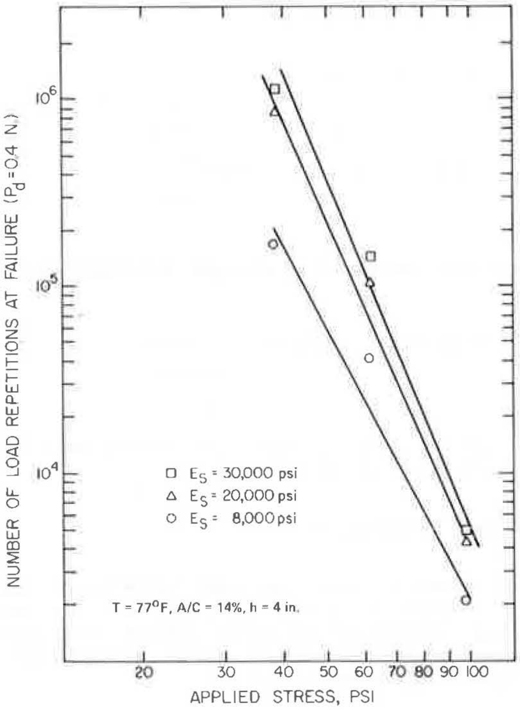


Figure 15. Typical relations between number of load repetitions at failure and test temperature.

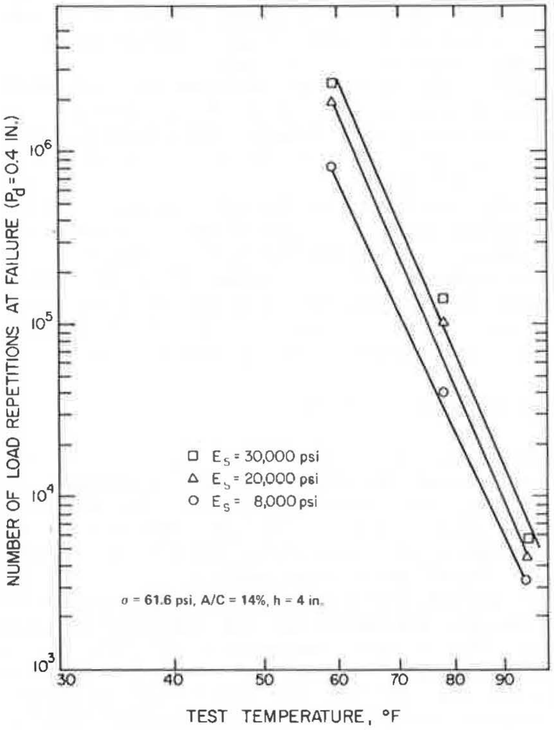


Table 4. Improvement in permanent deformation model due to considering T, σ, and E_S for n = 135.

Terms Considered in Equation 7	R^2	Standard Error of Estimate	F[a]
Only N	0.05	0.262	6.88
N and T	0.44	0.201	52.32
N, T, and σ	0.64	0.162	77.61
N, T, σ, and E_s	0.76	0.133	102.47

[a] All highly significant at 0.5 percent level.

It should be noted that Equation 8 was developed for a mixture containing 14 percent asphalt and a layer thickness of 4 in. because these conditions yielded the most resistance to rutting. It is possible to establish other constants for other thicknesses or asphalt contents or both for other values of rut depth as long as higher values than 0.4 in. satisfy Equation 3.

CONCLUSIONS

The aims of this study were to characterize an asphaltic dune-sand mixture by using the Marshall procedure and also its resistance to rutting (permanent deformation) when subjected to repeated loads. Within the bounds of the experiment, the following conclusions are warranted:

1. The Yuma dune-sand had an optimum asphalt content of 14 percent by weight of sand. This value was obtained from results of the following tests: Marshall stability, double-punch tensile strength, dynamic modulus of elasticity, and permanent deformation.
2. The dune-sand had a large surface area and compacted mixtures had large values of VMA; therefore, standard asphaltic concrete criteria do not apply to these dune-sand mixtures.
3. The large (18-in.-diameter) specimens were compacted with a vibratory procedure (no tilt on the vibratory kneading compactor) and were equal in density to the 4-in.-diameter specimens compacted by the Marshall procedure and those compacted by the standard vibratory kneading (with tilt) procedure. However, the structural characteristics of these specimens may be different.
4. The relation between amount of permanent deformation and number of load repetitions was a linear logarithmic one that compared favorably with a similar relation presented by Hofstra and Klomp (5).
5. An optimum specimen thickness (4 in.) existed for maximum resistance to rutting. The thinner one (2 in.) failed initially because of surface cracking, and the thicker one (6 in.) failed because of excessive densification under the load disc.
6. There were linear logarithmic relations between values of rutting and each of subgrade modulus of elasticity, temperature, and applied surface pressure. The combination of all of the above variables yielded the equation presented as Equation 8 in this paper:

$$\begin{aligned}\text{Log } P_d = -10.7019 + 0.4820 \log N + 5.7627 \log T + 0.9756 \log \sigma \quad & R^2 = 0.76\\ - 0.4043 \log E_S \quad & \text{SEE} = 0.133\\ & \text{df} = 130\end{aligned}$$

This equation was developed for an optimum asphalt content of 14 percent by weight of sand and a layer thickness of 4 in.

7. If it is assumed that a rut depth of 0.4 in. constitutes failure, then the general rutting equation can be transformed as follows to yield the number of repetitions to failure (Equation 9):

$$\text{Log } N = 27.8133 - 12.0000 \log T - 2.4614 \log \sigma + 0.9664 \log E_s$$

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Discussion

David G. Tunnicliff

The authors have presented an interesting and stimulating approach to the use of sand-asphalt base courses. The data in the paper are conclusive and show that the methodology is satisfactory; however, the mixture of Yuma sand and 14 percent asphalt (12.3 percent by weight of mixture) raises questions. Such a high asphalt content in any practical paving mixture is unheard of. If the best asphalt content for this sand-asphalt base mixture is 12.3 percent, the mixture will almost certainly rut excessively and should not be used. Because all of the data are obtained from what appears to be a useless mixture, it is suggested that a practical sand-asphalt mixture be included in future research before it is concluded that the methodology is satisfactory.

Authors' Closure

In response to Tunnicliff's discussion, we believe the methodology is applicable to any asphaltic mixture in view of the similarity between the results we obtained and those obtained by Hofstra and Klomp (5) from a test road and results obtained by Morris (6).

The paper is concerned principally with the properties of a dune-sand paving mixture. We do not believe the material described to be entirely useless. We believe it could be used as a base course for a low-volume road with load restrictions during hot periods, especially in locales that do not have good paving aggregates.

Publication of this paper sponsored by Committee on Characteristics of Bituminous Paving Mixtures to Meet Structural Requirements.

Visco-Elasto-Plastic Constitutive Law for a Bituminous Mixture Under Repeated Loading

MORDECHAI PERL, JACOB UZAN, AND ARIEH SIDES

A constitutive law for a bituminous mixture subjected to repeated loading is presented. The elastic, plastic, viscoelastic, and viscoplastic strain components are incorporated into the model as they are simultaneously present in the loading process. The model parameters are extracted from a series of repeated uniaxial creep and creep recovery experiments conducted under constant compression stress. The experiments were performed at constant temperature for various stress levels, time periods, and numbers of cycles. The elastic strain is found to depend solely and linearly on the stress. The plastic strain is linearly proportional to stress and exhibits a power-law dependence on the number of loading cycles. The viscoelastic strain is nonlinear with respect to stress and is governed by a power law of time. The viscoplastic strain component is nonlinear with respect to stress and thus can be represented by the product of a second-order polynomial of stress and two power laws of time and number of cycles, respectively. The reliability of this constitutive equation was evaluated by means of two verification tests. Good agreement was found between the predicted and measured strains.

In recent years, efforts have been made to develop rational methods for the design of flexible pavements that make it possible to predict pavement performance in terms of fatigue crack growth and rutting. Because the bituminous mixture plays a major role in pavement design, it is imperative to characterize it by a constitutive law that will account for the various effects occurring in in situ service. This constitutive law can be expressed as follows:

$$\epsilon_{ij} = \epsilon_{ij}(\sigma_{ij}, t N, T) \qquad (1)$$

where

ϵ_{ij} and σ_{ij} = strain and stress tensor components, respectively;
t = time;
N = number of loading cycles; and
T = temperature.

N is included in Equation 1 to represent the repetitive loading mode.

In the various approaches to analyzing failure of flexible pavements due to fatigue, the bituminous mixture is assumed to be either a linear (1-3) or a viscoelastic material (4). These models, however, do not fully account for the actual behavior of the bituminous mixture. The effects of factors such as rest period (5,6), healing time (7), and crack retardation, which might increase pavement fatigue life by more than one order of magnitude, cannot be anticipated. Thus, the development of a more rational model for predicting pavement fatigue life under cyclic loading requires a more comprehensive and more realistic material law.

It is the purpose of this paper to present an improved rheological model for the bituminous mixture that accounts for the elastic, plastic, viscoelastic, and viscoplastic responses of the material.

MATERIAL AND SPECIMENS

A single sand-asphalt mixture was used. It con-