Rutting Properties of a Dune-Sand Paving Mixture

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The results of a laboratory evaluation of a dune-sand for use as a base course in a flexible pavement are presented. The principal part of the study was concerned with the development of an equation for relating permanent deformation (rutting) to the combined effects of (a) number of load applications, (b) temperature, (c) surface pressure, and (d) dynamic modulus of elasticity of the supporting subgrade. The mixture evaluated was a dune-sand from Arizona with asphalt of the AR-4000 grade at an optimum content of 14 percent by weight of sand. Permanent deformation specimens were 18 in. in diameter and the height varied from 2 to 6 in. Variation in subgrade support was obtained by using synthetic rubber pads 18 in. in depth and 6 in. in height having estimated moduli of 8,000, 20,000, and 30,000 psi. The constant load system operated at a frequency of 11.5 Hz, and contact area was varied to yield pressures of 38.5, 61.6, and 98.0 psi. It was found that the optimum specimen height was 4 in. if failure was assumed to have occurred when the rut was 0.4 in. in depth. For this constant specimen height, an equation relating load repetitions (N) to temperature (T), stress (o), and subgrade modulus (Eg) is presented.

The structural design of an asphaltic concrete pavement is a complex procedure due to the uncertainties associated with the loads to be carried, the response of the pavement system to the loads, and the effects of the environment on the characteristics of the materials in the pavement system. Structural failure of an asphaltic pavement has been attributed by Hveem (1) to surface appearances of (a) cracks, (b) deformation, and (c) disintegration. The failure type of interest is permanent deformation or rutting, which originates within the layer of concern. (It is possible for an asphaltic concrete surface course to deform or rut after a deformation has occurred in a subsoil layer.)

One of the early pavement design procedures based on elastic theory used repeated vertical strains at the top of the subgrade as the criterion to preclude rutting of the asphaltic surface course (2). However, in certain cases, which are the ones of concern here, a repeated vertical stress or strain on an asphaltic surface course can result in surface ruts or deformations. Until recent years, permanent deformation originating in the surface course had not been used as a factor to be considered in the structural design of asphaltic pavements. It has been shown (3-5) that rutting can originate within an asphaltic concrete surface course that is thicker than about 10 cm (4 in.).

Mechanism of Rutting

Based on the AASHTO Road Test results (3) and AASHTO's own research efforts, investigators of the phenomenon of rutting have generally agreed that it is a longitudinal channel or depression that forms in the wheel path due to compression or lateral movement or both in one or more of the pavement layers as a result of repeated traffic-load applications. Rutting is a manifestation of two different mechanisms: (a) densification (volume change) and (b) shear deformation (plastic flow with no volume change). In the densification process material is forced downward, and shear deformation causes material to flow laterally and upward, as shown in Figure 1 (5). The total rut depth is the difference in elevation between the crest and sag of the surface. As shown in the figure, tensile stresses on the top surface and the outside of the loaded area can cause longitudinal cracking of the asphaltic concrete.

Hofstra and Klop (3) observed in their test track that permanent deformation in the asphaltic concrete layer was greater near the load and gradually decreased with depth below a certain level. Barksdale (7) agreed with this finding because the portion of the rut due to shear would have greater confinement or resistance to plastic flow at depths beneath the applied load. However, McLean and Mone­smith (8) and Morris and others (9), using elastic theory and triaxial laboratory tests, indicated that most rutting should occur in the lower part of the asphaltic concrete layer. This difference in findings makes it important that the test procedure chosen for determining the rutting characteristics of asphaltic concrete simulate field conditions as closely as possible.

Dune-Sand

Dune-sand is found in many parts of the world, and in many cases it is the predominant aggregate available for road construction, especially in desert areas. Asphalt paving mixtures made with dune-sand are generally too weak to be used as a surface course in major highways. This paper is principally concerned with the use of an asphalt-stabilized dune-sand as a base course in a desert environment and under a thin asphaltic surface course. In this situation, it is believed that the principal form of failure would be rutting rather than flexural fatigue cracking.
MATERIALS

Dune-Sand

The dune-sand used in this study came from an area near Yuma, Arizona. The gradation was typical of a windblown deposit; particle sizes ranged from the No. 50 to No. 200 sieves, and the uniformity coefficient (D60/D10) was 1.8. Compaction performed by using ASDPT procedures T99-57 and T180-57 yielded respective maximum dry densities of 102 and 105 pcf, both at an optimum moisture content of 12 percent.

Asphalt

The asphalt in the mixtures was a typical 60/70 penetration (AR-4000) grade that met specifications set by the Arizona Department of Transportation.

Rubber

The purpose of the rubber in this study was to serve as a subgrade to support the asphaltic dune-sand compacted dune-sand, the rubber pads were classified by durometer values of 1.928, which corresponded to hardness, medium, or soft, which corresponded to durometer values of 50, 40, and 30, respectively. Table 1 gives physical properties of the rubber pads. The selection for values of hardness for the rubber pads was influenced by the following:

1. Bowles has presented the following density relation for sands:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Density (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose</td>
<td>17.3-57.8</td>
</tr>
<tr>
<td>Medium</td>
<td>34.7-289.4</td>
</tr>
<tr>
<td>Dense</td>
<td>231.5-463</td>
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</table>

2. Al-Salloum has reported that in situ California bearing ratio (CBR) measurements on Arabian compacted dune-sand subgrades showed values ranging from 15 to 20.

3. Heukelom and Klopp suggest the following relation between CBR and dynamic modulus of elasticity (Eg): Eg (psi) = 1500 CBR.

Compacted Dune-Sand and Asphalt Mixtures

The effects of asphalt content on the physical properties of compacted mixtures were determined by using the Marshall method of design and also a double-punch procedure for strength and dynamic modulus of elasticity.

Marshall Design

The Marshall design procedure described in Asphalt Institute Manual MS-2 (14) was followed for evaluation of mixture characteristics; however, by using a portion of the Hvem procedure (centrifuge kerosene equivalent (CKE)), it was found that the dune-sand had a surface-area value of 74 ft²/lb and an optimum asphalt content of 4.8 percent by weight. The fineness of the sand, its uniformity in size, and its high surface-area value suggested discounting the CKE asphalt content as an optimum value. Test specimens for the Marshall evaluation were prepared with asphalt contents ranging from 4.5 to 16 percent by weight of sand.

Table 2 gives the data obtained from the Marshall evaluation along with the calculated asphalt film thickness on the sand particles.

Figures 2 and 3 show plots of the Marshall data and indicate that an optimum asphalt content for stability was reached at about 14 percent. It is noted that Marshall stabilities are low and do not meet general requirements for a surface course. As expected, the values for voids in mineral aggregate (VMA) were high (±8 percent) and remained fairly constant as the asphalt content varied; as a consequence, asphalt content caused a linear variation in density and air void content. At the optimum asphalt content, Marshall stability was 175 lbf, flow was 13, air void content was 11.8 percent, and asphalt film thickness was 9 μm.

Table 1. Physical properties of rubber subgrade.

<table>
<thead>
<tr>
<th>Durometer Value</th>
<th>Condition</th>
<th>Tensile Strengtha (psi)</th>
<th>Modulus of Subgrade Reactionb (psi)</th>
<th>Dynamic Modulus of Elasticityc (psi)</th>
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<tr>
<td>30</td>
<td>Soft</td>
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<td>Medium</td>
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<td>Hard</td>
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<td>370</td>
<td>30,000</td>
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</table>

bFrom loadings with 6-in.-diameter plate.
cAssumed from relations suggested by Bowles (11), Al-Salloum (12), and Heukelom and Klopp (13).

Table 2. Marshall stability test data.

<table>
<thead>
<tr>
<th>Asphalt Content (%)</th>
<th>Specimen No.</th>
<th>Specimen Density (g/cm³)</th>
<th>Specimen Height (in.)</th>
<th>Air Voids (%)</th>
<th>VMA (%)</th>
<th>Load Dial</th>
<th>Marshall Stability (lb)</th>
<th>Marshall Flow (0.01 in.)</th>
<th>Film Thicknessa (μm)</th>
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<td>1.712</td>
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<td>30.7</td>
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<td>70</td>
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<td>16.1</td>
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<td>2.50</td>
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<td>38.2</td>
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<td>1.928</td>
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<td>38.9</td>
<td>8</td>
<td>150</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

aFilm thickness is not usually included in the Marshall mix design method. It is added here to show how it varies with asphalt content. It is calculated as the ratio of the volume of asphalt to the surface area of aggregates (18).
Double-Punch Tests

In 1970 Chen (15) presented a method for determining the tensile strength of concrete by loading a cylindrical specimen with two steel punches centered on the flat surfaces. In 1975 Jimenez (16) extended the double-punch test to obtain a measure of the dynamic modulus of elasticity under repeated rapid loading. Figure 4 shows the tester used in the procedure.

The graphs shown in Figures 5 and 6 present some of the results obtained in this evaluation. The complete testing procedure and the results are fully described elsewhere by Al-Juraiban (17). A review of Al-Juraiban's data will show that the densities of the 4-in.-diameter specimens were equal to those of the 18-in.-diameter specimen of comparable composition, formed by vibratory kneading compaction. Jimenez (18) has described vibratory kneading compaction and shown comparisons of physical properties obtained for 4- and 18-in.-diameter specimens.

The curves shown in Figures 5 and 6 confirm the finding that 14 percent was the optimum asphalt content.

PREPARATION AND TESTING OF SPECIMENS FOR RUTTING PROPERTIES

Preparation of Large Test Specimens

The large test specimens were to have a diameter of 18 in. and heights varying from 2 to 6 in.; the weight of a specimen thus ranged from 31 to 104 lb.

Mixing

Enough sand and asphalt were weighed to make enough mixture to produce a 2-in.-high specimen. Sand (30 lb) in covered metal containers was heated in a forced-draft oven to a temperature of 300° ± 5°F. Sufficient asphalt for the number of specimens to be compacted was heated in a sealed metal container to a temperature of 250° ± 5°F.

The mixing bowl and beater of a large-capacity Blakeslee mixer were heated before mixing. After the sand and asphalt were combined in the desired proportion, it was found that 3 min of mixing was sufficient to produce a uniform mixture.

After mixing of a sufficient amount of the asphaltic mixture to produce a 2-in. specimen, the specimen was placed in a lidded metal container and stored in a 250° ± 5°F oven before compaction. Specimens were compacted in increments of 2-in. layer thickness.

Compaction

The 18-in.-diameter specimens were made with a vibratory kneading compactor developed by Jimenez.
Figure 5. Tensile strength (by double-punch) versus asphalt content for different testing temperatures.

Figure 6. Dynamic modulus versus asphalt content for dune-sand asphalt.

Figure 7. Jimenez vibratory kneading compactor ready to compact a large specimen.

The standard compaction procedure was modified by eliminating the tilt in the process. The principal reason for this was concern that the compactor might be damaged in the attempt to compact the thickest specimens, which with the steel mold would involve sliding a weight in excess of 125 lb. As a consequence, even though equal densities were obtained between the 4-in.-diameter and 18-in.-diameter specimens, the strength of the 18-in. specimens was lower due to the lack of kneading effect caused by a tilt of zero on the turntable.

Testing for Rutting Properties

The apparatus used for the rutting study was essentially the one developed by Jimenez (18) for flexure fatigue testing. The main difference between the two was the support given a specimen; in the original device, the specimen was given a hydraulic support—that is, uniform pressure over the bottom of the specimen; in the present device, the specimen was supported with a rubber pad. The test apparatus is shown in Figure 8.

The loading system consisted of counter-rotating balanced eccentric masses placed in mirrored positions so that the effective centrifugal force was a vertical one. In order to eliminate impact (collision), the dead load of the system was greater than the live load caused by the rotating eccentric masses. The loading standard for this research was a dead load of 178 lb, a sinusoidal live load with a maximum value of 130 lbf, and a load frequency of 11.5 Hz. The contact pressure on the specimen was varied by changing the area of the load disc to yield maximum contact pressures of 38.5, 61.6, and 98.0 psi.

Specimens were mounted on the testing device as shown in Figure 8. Surface displacement measurements were made periodically as a function of load repetitions and with extensometers graduated to 0.001 in. Two extensometers were used to determine the permanent deformation at the center of the specimen and also at a distance of 1.0 in. outside the loaded area. These measurements were made 4 min after stopping and raising the loading system. Another extensometer was mounted on the load shaft to determine the repeated deflection and to check the permanent deflection at the center of the specimen.

Loading and recordings were continued until the rut depth (permanent deformation) was equal to 0.4
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Figure 8. Permanent deformation test apparatus.

Figure 9. Dynamic modulus versus temperature for dune-sand asphalt and asphalt concrete.

in. as obtained by the algebraic addition of the deflection values on the surface of the specimen. This value of rut depth as the failure criterion was established after a review of the literature (17) indicated that, in consideration of hydroplaning, the rut depth should be limited to 0.375 in. and that, in the United Kingdom, pavements are considered to have failed when the rut depth is 0.75 in.

TEST RESULTS AND DISCUSSIONS

Dynamic Modulus of Elasticity

The graphs shown in Figures 5 and 6 were developed from data not presented here for the sake of brevity; however, a statistical analysis and comparisons of values obtained by other researchers are presented.

In our test for determining a dynamic modulus of elasticity, the specimen was compacted by using a vibratory kneading procedure; the repeated sinusoidal stresses varied from a low of 3 psi to a high of 12 psi at a load frequency of 11.5 Hz.

The Statistical Package for the Social Sciences (SPSS) (20) computer program was used to establish functional relations between dynamic modulus of elasticity \( E_0 \) and asphalt content (AC), test temperature \( T_f \), and air void content (AV). An exponential model yielded the following constants:

\[
\log E_0 = 5.89741(AC)^{0.09936}(T_f)^{0.09736}(AV)^{0.04207}
\]

\( R^2 = 0.877 \quad \text{SEE} = 0.0053 \quad df = 11 \) \( \text{(1)} \)

For the optimum asphalt content of 14 percent and the corresponding air voids and test load rate, Equation 1 reduced to

\[
\log E_0 = 6.62294 - 0.01436(T_f) \quad R^2 = 0.98 \quad \text{SEE} = 0.041
\]

Equation 3 was developed for a test temperature of 77°F, a subgrade dynamic modulus of 20,000 psi, an applied stress of 61.6 psi, an asphalt content of 14 percent, and a test load rate of 12 psi at a load frequency of 11.5 Hz.

Rutting or Permanent Deformation

The chart shown in Figure 10 shows the variables and their level used to investigate the rutting properties of asphaltic dune-sand mixtures. At least two specimens were used in each test block. As shown in the chart, the variables were temperature, subgrade strength expressed as rubber durometer, specimen thickness, asphalt content, and contact pressure expressed as loaded area.

Response of Permanent Deformation to Repeated Loads

As indicated earlier, values of permanent deformation for corresponding number of repeated loads were recorded until a rut depth of 0.4 in. was reached. Table 3 gives values of permanent deformation as affected by number of load repetitions and several test variables for a particular mixture. For the sake of brevity, comprehensive test results are not presented in this paper, but a statistical analysis of the data is presented that gives a measure of reliability of the relations to be shown.

The effect of load repetitions on permanent deformation is shown in Figure 11; also shown is a curve representing the findings of Hofstra and Klomp (5) that shows the similarity of shape for the two materials and tests. For the dune-sand mixture shown and a general model that relates permanent deformation \( P_d \) to number of repetitions \( N \) in the form \( P_d = aN^b \), the SPSS program yielded the following equation:

\[
\log P_d = 0.1159 + 0.4690 \log N \quad R^2 = 0.979 \quad \text{SEE} = 0.098 \quad df = 11
\]

Equation 3 was developed for a test temperature of 77°F, a subgrade dynamic modulus of 20,000 psi, an applied stress of 61.6 psi, an asphalt content of 14 percent, and a test load rate of 12 psi at a load frequency of 11.5 Hz.

compaction procedures, test methods, and stress levels used.
percent, and a specimen height of 4 in. A more general equation including several variables is presented later in this paper.

**Effect of Asphalt Content**

Figure 12 shows the effects of asphalt content on the number of load repetitions to failure. The curves show that the optimum asphalt content was 14 percent, the same value obtained from the Marshall double-punch test data. Because 14 percent was the optimum asphalt content for the dune-sand, this percentage is taken as a typical asphalt content and the one on which most of the analyses are concentrated.

**Effect of Specimen Thickness**

The curves presented in Figure 13 show that an optimum value of 4 in. was obtained with reference to maximum number of load repetitions to failure (a rut depth of 0.4 in.). Examination of the 2-in.-thick specimens showed radial cracks at the top of the specimen at failure. These cracks were associated with a tensile fatigue failure. When the specimen cracked, there

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**Table 3. Typical magnitudes of permanent deformation as affected by various test variables (asphalt content = 14 percent and specimen height = 4 in.).**

<table>
<thead>
<tr>
<th>Permanent Deformation (Pd x 10^3 in.)</th>
<th>Applied Stress (psi)</th>
<th>Subgrade Dynamic Modulus (psi)</th>
<th>Test Temperature (°F)</th>
<th>No. of Load Repetitions (000s)</th>
<th>Permanent Deformation (Pd x 10^3 in.)</th>
<th>Applied Stress (psi)</th>
<th>Subgrade Dynamic Modulus (psi)</th>
<th>Test Temperature (°F)</th>
<th>No. of Load Repetitions (000s)</th>
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<tbody>
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<td>30,000</td>
<td>77</td>
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<td>360</td>
<td>61.6</td>
<td>30,000</td>
<td>95</td>
<td>5</td>
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</table>

Note: A/C = 14 percent and h = 4 in.

400 = failure.
Figure 11. Comparison between measured permanent deformations for dune-sand asphalt and Hofstra and Klomp test track.

was open space for the material to flow into and thus the resistance to rutting was decreased.

The 6-in.-thick specimens failed before the 4-in. ones, most likely because of densification. The 6-in.-thick specimens had a longer column under the load and so had more voids in an absolute sense to decrease the resistance to rutting.

The fact that permanent deformation increases with layer thickness was observed by Hofstra and Klomp in their test track. This behavior was also found by Van de Loo from measurements in a laboratory test track.

The study data thus far indicated that, to optimize the resistance to rutting of the dune-sand and asphalt specimens, they should have an asphalt content of 14 percent and a thickness of 4 in. As a consequence, the effects of subgrade modulus, temperature, and contact pressure on resistance to rutting were determined on the optimum specimen.

Effects of Subgrade Strength

Three rubber pads with durometer values of 30, 40, and 50 were used to represent sand subgrades of loose, medium, and dense conditions. It has been shown that these durometer values corresponded to dynamic modulus of elasticity values of 8,000, 20,000, and 30,000 psi, respectively.

As anticipated, the number of load repetitions to rutting failure increased as the subgrade modulus increased. For a test temperature of 77°F and a contact pressure of 61.6 psi, the following equation was obtained for relating rut depth (Pd) to number of load repetitions (N) and subgrade dynamic modulus (Eg):

\[ \log P_d = 1.4108 + 0.5892 \log N - 0.4176 \log E_g \]

\[ R^2 = 0.97 \]

\[ \text{SEE} = 0.057 \]

\[ \text{df} = 11 \]

Equation 4 is valid for a test temperature of 77°F and a subgrade value of 20,000 psi.

Effects of Applied Pressure

The curves presented in Figure 14 show the influence of applied pressure on the number of load repetitions to failure. An exponential equation relating number of repetitions (N) and applied stress (pressure \( \sigma \)) to the values of permanent deformation is given below:

\[ \log P_d = 1.9472 + 0.4638 \log N + 1.1422 \log \sigma \]

\[ R^2 = 0.92 \]

\[ \text{SEE} = 0.10 \]

Equation 5 is valid for a test temperature of 77°F and a subgrade value of 20,000 psi.

Effects of Test Temperature

As would be expected, when the test temperature decreased the resistance to rutting increased. By using the prior exponential models to relate load repetitions and temperature to permanent deformation, the following equation was obtained:
Log $P_d = -13.4998 + 0.5695 \log N + 7.0329 \log T$ \hspace{1cm} R$^2 = 0.96$ \hspace{1cm} (6)  

SEE = 0.067  

$df = 11$  

Again, the equation is valid for $\sigma = 61.6$ psi, $E_s = 20,000$ psi, $AC = 14$ percent, and $h = 4$ in. From Equation 6 it is noted that temperature had a tremendous influence on rutting; this is shown by the coefficient value of 7.03.

Figure 15 shows the linear logarithmic effect of test temperature on number of load repetitions to failure. It is seen that both reduction of temperature and increase of subgrade modulus increase the rutting resistance of the paving mixture. Note that at temperatures below 50°F the number of load repetitions to failure would be in excess of 10 million, which suggests that rutting failure would not occur at those temperatures.

General Model for Permanent Deformation of Asphaltic Dune-Sand Mixture

The stepwise SPSS computer program was used to obtain a general model for predicting permanent deformation of the dune-sand mixture. All significant variables were included in the model to represent a mixture containing an asphalt content of 14 percent and a layer thickness of 4 in. The model will allow the prediction of the magnitude of permanent deformation at any number of load repetitions, any temperature, any applied stress, or any subgrade modulus. The general model chosen follows the form of those used previously:

$$P_d = aN^{0.4820}T^{5.7627}E_s^{-0.9664}$$ \hspace{1cm} (7)  

The equation constants were evaluated to yield

$$Log P_d = -10.7019 + 0.4820 \log N + 5.7627 \log T + 0.9664 \log E_s$$ \hspace{1cm} $R^2 = 0.76$ \hspace{1cm} (8)  

$df = 130$

Figure 14. Typical relations between number of load repetitions at failure and applied stress for different supporting strengths.

where

$P_d$ = permanent deformation (10$^{-4}$ in.),  
$N$ = number of load repetitions,  
$T$ = test temperature ($^\circ$F),  
$\sigma$ = applied compressive stress (psi), and  
$E_s$ = subgrade dynamic modulus of elasticity (psi).

The effect of the independent variables on the permanent deformation model (Equation 8) is shown by the data given in Table 4. The improvements in $R^2$ values and reduction in SEE values as the number of variables increased are evident.

If one agrees that a rut depth of 1 cm (0.4 in.) corresponds to a failure condition, then the model can be modified to express the number of load repetitions to failure. This equation then takes the following form:

$$Log N = 27.8133 - 12.0000 \log T - 2.4614 \log \sigma + 0.9664 \log E_s$$ \hspace{1cm} $R^2 = 0.97$ \hspace{1cm} (9)  

$SEE = 0.223$  

$df = 23$

Even though Equation 9 has a higher $R^2$ than Equation 8, it has a higher SEE and a lower df. We suggest that both models are acceptable for characterization of the paving mixture.

Figure 15. Typical relations between number of load repetitions at failure and test temperature.

Table 4. Improvement in permanent deformation model due to considering $T$, $\sigma$, and $E_s$ for $n = 135$.  

<table>
<thead>
<tr>
<th>Terms Considered in Equation 7</th>
<th>$R^2$</th>
<th>Standard Error of Estimate</th>
<th>$P^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only $N$</td>
<td>0.05</td>
<td>0.262</td>
<td>6.88</td>
</tr>
<tr>
<td>$N$ and $T$</td>
<td>0.44</td>
<td>0.201</td>
<td>32.32</td>
</tr>
<tr>
<td>$N$, $T$, and $\sigma$</td>
<td>0.64</td>
<td>0.162</td>
<td>77.61</td>
</tr>
<tr>
<td>$N$, $T$, $\sigma$, and $E_s$</td>
<td>0.76</td>
<td>0.133</td>
<td>102.47</td>
</tr>
</tbody>
</table>

$^*All$ highly significant at 0.5 percent level.
It should be noted that Equation 8 was developed for a mixture containing 14 percent asphalt and a layer thickness of 4 in. because these conditions yielded the most resistance to rutting. It is possible to establish other constants for other thicknesses or asphalt contents or both for other values of rut depth as long as higher values than 0.4 in. satisfy Equation 3.

**CONCLUSIONS**

The aims of this study were to characterize an asphaltic dune-sand mixture by using the Marshall procedure and also its resistance to rutting (permanent deformation) when subjected to repeated loads. Within the bounds of the experiment, the following conclusions are warranted:

1. The Yuma dune-sand had an optimum asphalt content of 14 percent by weight of sand. This value was obtained from results of the following tests: Marshall stability, double-punch tensile strength, dynamic modulus of elasticity, and permanent deformation.

2. The dune-sand had a large surface area and compacted mixtures had large values of VMA; therefore, standard asphaltic concrete criteria do not apply to these dune-sand mixtures.

3. The large (18-in.-diameter) specimens were compacted with a vibratory procedure (no tilt on the vibratory kneading compactor) and were equal in density to the 4-in.-diameter specimens compacted by the Marshall procedure and those compacted by the standard vibratory kneading (with tilt) procedure. However, the structural characteristics of these specimens may be different.

4. The relation between amount of permanent deformation and number of load repetitions was a linear logarithmic one that compared favorably with a similar relation presented by Hofstra and Klomp (5).

5. An optimum specimen thickness (4 in.) existed for maximum resistance to rutting. The thinner one (2 in.) failed initially because of surface cracking, and the thicker one (6 in.) failed because of excessive densification under the load disc.

6. There were linear logarithmic relations between values of rutting and each of subgrade modulus of elasticity, temperature, and applied surface pressure. The combination of all of the above variables yielded the equation presented as Equation 8 in this paper:

\[
\log P_d = -10.7019 + 0.4820 \log N + 5.7627 \log T + 0.9756 \log \sigma^2 = 0.76 \\
-0.4043 \log E_k \]

\[
SEF = 0.133 \\
df = 130
\]

This equation was developed for an optimum asphalt content of 14 percent by weight of sand and a layer thickness of 4 in. If it is assumed that a rut depth of 0.4 in. constitutes failure, then the general rutting equation can be transformed as follows to yield the number of repetitions to failure (Equation 9):

\[
\log N = 27.3133 - 12.0000 \log T - 2.4614 \log \sigma + 0.9664 \log E_k
\]

**REFERENCES**


Visco-Elasto-Plastic Constitutive Law for a Bituminous Mixture Under Repeated Loading

MORDECHAI PERL, JACOB UZAN, AND ARIEH SIDES

A constitutive law for a bituminous mixture subjected to repeated loading is presented. The elastic, plastic, viscoelastic, and viscoplastic strain components are incorporated into the model as they are simultaneously present in the loading process. The model parameters are extracted from a series of repeated uniaxial creep and creep recovery experiments conducted under constant compression stress. The experiments were performed at constant temperature for various stress levels, time periods, and numbers of cycles. The elastic strain is found to depend solely and linearly on the stress. The plastic strain is linearly proportional to stress and exhibits a power-law dependence on the number of loading cycles. The viscoelastic strain is nonlinear with respect to stress and is governed by a power law of time. The viscoplastic strain component is nonlinear with respect to stress and thus can be represented by the product of a second-order polynomial of stress and two power laws of time and number of cycles, respectively. The reliability of this constitutive equation was evaluated by means of two verification tests. Good agreement was found between the predicted and measured strains.

In recent years, efforts have been made to develop rational methods for the design of flexible pavements that make it possible to predict pavement performance in terms of fatigue crack growth and rutting. Because the bituminous mixture plays a major role in pavement design, it is imperative to characterize it by a constitutive law that will account for the various effects occurring in situ service. This constitutive law can be expressed as follows:

\[ e_{ij} = \epsilon_{ij}(\sigma_{ij}, N, T) \]  

where \( e_{ij} \) and \( \sigma_{ij} \) = strain and stress tensor components, respectively; 
\( t = \) time; 
\( N = \) number of loading cycles; and 
\( T = \) temperature.

\( N \) is included in Equation 1 to represent the repetitive loading mode.

In the various approaches to analyzing failure of flexible pavements due to fatigue, the bituminous mixture is assumed to be either a linear (1-3) or a viscoelastic material (4). These models, however, do not fully account for the actual behavior of the bituminous mixture. The effects of factors such as rest period (5, 6), healing time (7), and crack retardation, which might increase pavement fatigue life by more than one order of magnitude, cannot be anticipated. Thus, the development of a more rational model for predicting pavement fatigue life under cyclic loading requires a more comprehensive and more realistic material law.

It is the purpose of this paper to present an improved rheological model for the bituminous mixture that accounts for the elastic, plastic, viscoelastic, and viscoplastic responses of the material.

MATERIAL AND SPECIMENS

A single sand-asphalt mixture was used. It con-