Multiregional Input-Output Model: A Further Extension

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A multimodal, multioutput, multiregional variable input-output (MMMVIO) model is introduced to evaluate the economic impact of an improvement to a transportation facility. The distinguishing feature of the MMMVIO model over conventional multiregional input-output models is its flexibility. The MMMVIO model is a price and cost-sensitive model, whereas conventional input-output models fail to share these properties. Regional coefficients, trade coefficients, modal split, and the composition of primary and secondary outputs become endogenous variables under the MMMVIO model. The conventional input-output models assume that regional coefficients and trade coefficients are fixed, regardless of changes in either input cost or output price, and each industry produces a single output. The modal split has never been considered explicitly within the context of the conventional input-output model. The MMMVIO model alleviates these unrealistic assumptions.

The conventional multiregional input-output models developed by Isard (1), Moses (2), Leontief and Strout (3), and Polenske (4) are able to describe the industrial transaction, trade flows, industrial output, income, and employment in regional and industrial details. However, the input-output model assumes that:

1. Each industry in each region produces a single output;
2. Regional input-output coefficients are fixed regardless of changes in output prices, input costs, tax structure, or transportation costs;
3. Neither input costs nor output prices nor transportation costs would affect the industry's decision on output, input mix, employment, income, transport modal choice, and trade structure (conventional input-output models fail to respond to cost and price changes);
4. Trade coefficients are independent of transportation costs and input costs; and
5. Transport modal choice has never been fully explained by conventional input-putput models.

To make the input-output model more flexible, we introduce a multimodal, multioutput, multiregional variable input-output (MMMVIO) model, which is not based on such unrealistic assumptions as imposed on conventional input-output models. The MMMVIO model assumes that each industry in each region may produce more than one output. The multioutput and multiinput relation is specified by the production frontiers. Under the MMMVIO model, the regional input-output coefficients become endogenous to the model. A change in output prices, tax structure, or transportation costs affects not only the input-output coefficients but also the trade coefficients. The MMMVIO model assumes that profit maximization guides every business decision on outputs, input mix, employment, income, modal choice of shipment, and trade flows.

The MMMVIO model is derived from the duality between production and price frontiers. The price frontiers are obtained from the dual relations. These price frontiers are expressed in terms of input elasticities, transportation costs, wage rates, service prices of capital, tax rates, and technical progress parameters. The equilibrium prices obtained from the price frontiers determine regional input-output coefficients, trade coefficients, and modal split of commodity shipment. The usual output, income, and employment of each industry in each region are determined by the balance equations.

A derivation of MMMVIO is given in the next section, and a brief description on the usefulness of the model is discussed afterwards.

MMMVIO MODEL

Consider an economy that has m regions and n industries. Each industry produces a primary output and several secondary outputs. Each commodity is shipped to each region by one of g shipping modes.

Industrial outputs in each region are produced by a linear logarithmic production frontier, i.e.,

$$\beta^r_{jj}\ln Y^r_{jj} + \sum_{i\neq j} \beta^r_{ij}\ln Y^r_{ij} - \alpha^r_{oj} - \sum_k \sum_s \sum_i \alpha^{srk}_{ij}\ln X^{srk}_{ij} - \gamma^r_j \ln L^r_j - \delta^r_j \ln K^r_j = 0 \quad (1)$$

$\left(\text{Note, unless otherwise stated, } \sum_k = \sum_{k=1}^{g}, \sum_s = \sum_{s=1}^{m}, \sum_i = \sum_{i=1}^{n}, \text{ and } \sum_{i\neq j} = \sum_{\substack{i=1\\ i\neq j}}^{n}.\right)$

where

Y^r_{jj} = amount of primary output j produced by industry j in region r,

Y^r_{ij} = amount of secondary output i produced by industry j in region r ($i \neq j$),

X^{srk}_{ij} = amount of commodity i produced in region s and delivered to industry j in region r by shipping mode k,

L^r_j = labor employed by industry j located in region r, and

K^r_j = service of capital employed by industry j located in region r.

β^r_{ij}, α^r_{oj}, α^{srk}_{ij}, γ^r, and δ^r_j are parameters of the production frontier, and it is assumed to be a linear homogeneous function, i.e.,

$$\beta^r_{jj} + \sum_{i\neq j} \beta^r_{ij} - \sum_k \sum_s \sum_i \alpha^{srk}_{ij} - \gamma^r_j - \delta^r_j = 0 \quad (j = 1, \ldots n; r = 1, \ldots m) \quad (2)$$

The commodity i produced by all industries in region s is demanded by industries and final users of all regions, and the shipment of the commodity is made by transportation mode k:

$$\sum_k \sum_r \sum_j X^{srk}_{ij} + \sum_k \sum_r F^{srk}_i = \sum_j Y^s_{ij} \quad (3)$$

The profit maximization with Equations 1-3 yields the following solutions. (Note: the full mathematical derivation is available from the authors.)

$$Y^r_{ij} = \beta^r_{ij}(1 - t^r_j)P^r_j Y^r_{jj}/[\beta^r_{jj}(1 - t^r_i)P^r_i] \quad (\text{for } i \neq j) \quad (4)$$

$$X^{srk}_{ij} = \alpha^{srk}_{ij}(1 - t^r_j)P^r_j Y^r_{jj}/(\beta^r_{jj}C^{srk}_i P^s_i) \quad (5)$$

$$L^r_j = \gamma^r_j(1 - t^r_j)P^r_j Y^r_{jj}/(\beta^r_{jj}w^r_j) \quad (6)$$

$$K^r_j = \delta^r_j(1 - t^r_j)P^r_j Y^r_{jj}/(\beta^r_{jj}v^r_j) \quad (7)$$

P^r_j is the equilibrium price of commodity j produced in region r, and C^{srk}_i is one plus the unit cost of delivering commodity i from region s to region r by shipping mode k. C_i^{srk} is called the transportation cost factor in this study.

From Equations 1, 2, 4-7, we obtain the price frontiers that can be conveniently presented as a matrix form:

$$(B - S)\ln p = \hat{\gamma}\ln w + \hat{\delta}\ln v - B\ln(1 - t) + W\ln c + A_o \tag{8}$$

where

$$\underset{(nm,1)}{\ln p} = \begin{pmatrix} \ln p^1 \\ \vdots \\ \ln p^m \end{pmatrix} \quad \text{and} \quad \underset{(n,1)}{\ln p^r} = \begin{pmatrix} \ln p_1^r \\ \vdots \\ \ln p_n^r \end{pmatrix}$$

$$\underset{(nm,nm)}{B} = \begin{bmatrix} B^1 & & 0 \\ & B^2 & \\ 0 & & B^m \end{bmatrix} \quad \text{and} \quad \underset{(n,n)}{B^r} = \begin{bmatrix} B_{11}^r & \cdots & B_{n1}^r \\ \vdots & \ddots & \vdots \\ B_{1n}^r & \cdots & B_{nn}^r \end{bmatrix}$$

$$S = A^1 + \ldots + A^g$$

$$\underset{(nm,nm)}{A^k} = \begin{bmatrix} \alpha^{11k} & \cdots & \alpha^{m1k} \\ \vdots & \ddots & \vdots \\ \alpha^{1mk} & \cdots & \alpha^{mmk} \end{bmatrix} \quad \text{and} \quad \underset{(n,n)}{\alpha^{srk}} = \begin{bmatrix} \alpha_{11}^{srk} & \cdots & \alpha_{n1}^{srk} \\ \vdots & \ddots & \vdots \\ \alpha_{1n}^{srk} & \cdots & \alpha_{nn}^{srk} \end{bmatrix}$$

$\underset{(mn,mn)}{\hat{\gamma}}$ = diagonal matrix of γ_j^r;

$\underset{(mn,mn)}{\hat{\delta}}$ = diagonal matrix of δ_j^r;

$\ln w$ = mn component vector of $\ln w_j^r$;

$\ln v$ = mn component vector of $\ln v_j^r$;

$\ln(1-t)$ = mn component vector of $\ln(1-t_j^r)$;

$\underset{(mn,nmm^g)}{W} = (W^1, W^2, \ldots W^g)$

$$\underset{(nm,nmm)}{W^k} = \begin{bmatrix} S^{1k} & 0 & \cdots & 0 \\ 0 & S^{2k} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & & S^{mk} \end{bmatrix} \quad \text{and} \quad \underset{(n,nm)}{S^{rk}} = \begin{bmatrix} \alpha_{11}^{1rk} \cdots \alpha_{n1}^{1rk} & \cdots & \alpha_{11}^{mrk} \cdots \alpha_{n1}^{mrk} \\ \vdots \quad \ddots \quad \vdots & & \vdots \quad \ddots \quad \vdots \\ \alpha_{1n}^{1rk} \cdots \alpha_{nn}^{1rk} & \cdots & \alpha_{1n}^{mrk} \cdots \alpha_{nn}^{mrk} \end{bmatrix}$$

$(k = 1, \ldots g)$

$$\underset{(nmmg,1)}{\ln c} = \begin{pmatrix} \ln c^1 \\ \vdots \\ \ln c^g \end{pmatrix} \quad \text{and} \quad \underset{(nmm,1)}{\ln c^k} = \begin{pmatrix} \ln c_1^{11k} \\ \vdots \\ \ln c_n^{11k} \\ \vdots \\ \ln c_1^{mmk} \\ \vdots \\ \ln c_n^{mmk} \end{pmatrix}$$

A_o = nm component vector of A_{oj}^r.

(Note that the figures inside the parentheses indicate the size of the matrix.)

The price frontier (Equation 8) is expressed in terms of local wage rates (w_j^r), regional service price of capital (v_j^r), effective tax rates (t_j^r), transportation cost factor by each mode (C_i^{srk}), input-output elasticities (β_j^r, α_{ij}^{srk}, γ_j^r, δ_j^r), and technical progress parameters (α_{oj}^r).

The equilibrium prices (P_j^r) of the model are determined by the price frontier equation (Equation 8). The above equilibrium prices solve the output coefficients (d_{ij}^s) and the primary input coefficients (a_{ij}^{srk}); i.e.,

$$d_{ij}^s = Y_{ij}^s/Y_{jj}^s = \beta_{ij}^s(1 - t_j^s)P_j^s/[\beta_{jj}^s(1 - t_i^s)P_i^s] \tag{9}$$

$$a_{ij}^{srk} = X_{ij}^{srk}/Y_{jj}^r = \alpha_{ij}^{srk}(1 - t_j^r)P_j^r/(\beta_{jj}^r C_i^{srk} \cdot P_i^s) \tag{10}$$

Dividing Equation 3 by the primary output (Y_{jj}^r), the following balance equation is obtained, i.e.,

$$\sum_j d_{ij}^s \cdot Y_{jj}^s - \sum_r \sum_j A_{ij}^{sr} \cdot Y_{jj}^r = F_i^s \tag{11}$$

where

$A_{ij}^{sr} = \sum_k a_{ij}^{srk}$ and $F_i^s = \sum_k \sum_r F_i^{srk}$.

A matrix form of Equation 11 is as follows:

$$(D - A^*)Y = F \tag{12}$$

where

$$\underset{(nm,nm)}{D} = \begin{pmatrix} D^1 & & 0 \\ & D^2 & \\ 0 & & D^m \end{pmatrix} \quad \text{and} \quad D^s = \begin{pmatrix} d_{11}^s & \cdots & d_{1n}^s \\ \vdots & \ddots & \vdots \\ d_{n1}^s & \cdots & d_{nn}^s \end{pmatrix}$$

$$\underset{(nm,nm)}{A^*} = \begin{bmatrix} A^{11} & \cdots & A^{1m} \\ \vdots & \ddots & \vdots \\ A^{m1} & \cdots & A^{mm} \end{bmatrix} \quad \text{and} \quad A^{sr} = \begin{bmatrix} A_{11}^{sr} & \cdots & A_{1n}^{sr} \\ \vdots & \ddots & \vdots \\ A_{n1}^{sr} & \cdots & A_{nn}^{sr} \end{bmatrix}$$

$$\underset{(nm,1)}{Y} = \begin{bmatrix} Y^1 \\ \vdots \\ Y^m \end{bmatrix} \quad \text{and} \quad Y^r = \begin{bmatrix} Y_{11}^r \\ \vdots \\ Y_{nn}^r \end{bmatrix} \qquad \underset{(nm,1)}{F} = \begin{bmatrix} F^1 \\ \vdots \\ F^m \end{bmatrix} \quad \text{and} \quad F^r = \begin{bmatrix} F_1^r \\ \vdots \\ F_n^r \end{bmatrix}$$

The balance equation (Equation 12) determines the primary outputs of each industry in each region (Y).

Once the primary outputs (Y) and the equilibrium prices (P) are determined, the secondary products (Y_{ij}^r for $i \neq j$), intermediate purchases (X_{ij}^{srk}), labor demands (L_j^r), and capital demands (K_j^r) are determined by Equations 4-7.

The output produced by industry j in region r ($Y_{\cdot j}^r$) is computed as

$$Y_{\cdot j}^r = \sum_i Y_{ij}^r \tag{13}$$

The regional input-output coefficients (a_{ij}^{sr}) are identified by the following equation:

$$a_{ij}^{sr} = \sum_k (X_{ij}^{srk})/Y_{\cdot j}^r \tag{14}$$

The regional technical coefficient (a_{ij}^{r}) is the sum of the regional input-output coefficients over regions, i.e.,

$$a_{ij}^{r} = \sum_{s} a_{ij}^{sr} \tag{15}$$

The trade coefficients by each mode (t_{ij}^{srk}) are computed as

$$t_{ij}^{srk} = X_{ij}^{srk}/(\sum_{s} X_{ij}^{srk}) \tag{16}$$

Note that this definition of trade coefficients coincides with that of Moses (2), except that Moses did not break down the transport modal split, i.e.,

$$t_{ij}^{sr} = a_{ij}^{sr}/(\sum_{s} a_{ij}^{sr}) = X_{ij}^{sr}/(\sum_{s} X_{ij}^{sr}).$$

Following Moses (2), it is assumed that each industry in region r consumes some fraction of the import of commodity i from region s so that the trade coefficients of the transportation mode k (t_{ij}^{srk}) are the same regardless of the final users, i.e.,

$$t_{ij}^{srk} = t_{i}^{srk} \tag{17}$$

We impose this property by averaging the trade coefficients over industries, i.e.,

$$t_{i}^{srk} = (1/n)(\sum_{j} t_{ij}^{srk}) \tag{18}$$

An improvement of a transportation mode reduces the transportation cost factor (C_{i}^{srk}), which changes regional industrial outputs (Y_{ij}^{r}), trade coefficients (t_{i}^{srk}), regional coefficients (A_{ij}^{sr}), and modal choice ($t_{i}^{k} = \sum_{sr}\sum t_{i}^{srk}$).

POTENTIAL USEFULNESS OF MMMVIO MODEL

The MMMVIO model is capable of determining the feasibility of constructing new transportation systems such as highways, waterways, bridges, or railways. The model can be employed to evaluate the existing transportation system, measure the economic impact of an energy crisis, appraise the development impact of rail abandonment, and predict the economic conditions of a region that has a sustained shortage of essential resources.

The MMMVIO model is an extension of the multiregional variable input-output (MRVIO) model that has been in operation since 1979. The basic input data of the MMMVIO are the same as those of MRVIO. MRVIO was employed to evaluate an existing waterway (5,6), to appraise the feasibility of a new waterway (7), to measure the development impact of a water shortage (8), to evaluate the pollution impact of the relocation of an industry (9), and to assess the growth impact of an energy crisis (10). The sources of the data and the computer programs for the MRVIO model are described in the reports cited.

The MMMVIO model requires additional data besides those employed for MRVIO. The Make of Commodities by Industry (Survey of Current Business, April 1979) can be used to identify the primary and secondary products. The modal-split information may require a sample survey of commodity shipment. A rough estimation on the modal split can be made by using the 1972 Transportation Margin Tape (from the U.S. Department of Commerce), which identifies the transportation margin of goods delivered by each mode.

REFERENCES

1. W. Isard. Interregional and Regional Input-Output Analysis: A Model of a Space Economy. Review of Economics and Statistics, Vol. 33, Nov. 1951.
2. L. Moses. The Stability of Interregional Trading Patterns and Input-Output Analysis. American Economic Review, Vol. 45, No. 5, 1955.
3. W. Leontief and A. Strout. Multiregional Input-Output Analysis. In Structural Interdependence and Economic Development (T. Barna, ed.), St. Martin Press, Inc., New York, 1963.
4. K. Polenske. A Multiregional Input-Output Model for the United States. Harvard Economic Research Project, Economic Development Administration, Cambridge, MA, 1970.
5. C.K. Liew and C.J. Liew. Use of a Multiregional Variable Input-Output Model to Analyze Economic Impacts of Transportation Cost. TRB, Transportation Research Record 747, 1980, pp. 5-12.
6. C.K. Liew and C.J. Liew. The McClellan-Kerr Waterway and Regional Economic Development. Institute of Water Research, U.S. Army Corps of Engineers, Vicksburg, MS, 1981.
7. C.K. Liew and C.J. Liew. The Development Impact of the Coosa River Navigation Project on the Regional and National Economies. Institute of Water Research, U.S. Army Corps of Engineers, Vicksburg, MS, 1982.
8. C.K. Liew and C.J. Liew. The Development Impact of Water Shortage on Regional Economy. Proc., Business Economic Section, American Statistical Association Meeting, 1982.
9. C.J. Liew. Pollution-Related Variable Input-Output Model: The Tulsa SMSA as a Case Study. Journal of Urban Economics (in preparation).
10. C.J. Liew. The Impact of Higher Energy Prices on Growth and Inflation in an Industrializing Economy: The Korean Case. Journal of Policy Modeling, 1980.

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