Modeling Bus Delays due to Passenger Boardings and Alightings

RICHARD P. GUENTHNER AND KUMARES C. SINHA

Two causes of bus delay are examined: the delay from the stopping and starting at passenger stops and the dwell time as the passengers board and alight from the bus. Evaluation of data on the number of passengers boarding and alighting at stops along a route showed that the negative binomial is a good descriptor of this distribution. Additional data were used to determine dwell time per passenger as a function of the passenger boardings and alightings. By using these intermediate results, a procedure was developed to determine the resulting bus delay and its effect on operating speed, ridership, and ultimately on route performance. This methodology was then tested with data from Milwaukee, Wisconsin.

A significant deterrent to the use of public transportation is the excessive travel time, including both out-of-vehicle and in-vehicle times. The out-of-vehicle or excess travel time includes walking to the bus stop, waiting for the bus, transferring, and walking to the destination. The in-vehicle travel time in bus transportation is also usually longer than that in automobile transportation. The two main elements contributing to this difference are circuitous routing and stopping for passengers and starting again. In the past, not much attention was given to explicitly evaluating the impact on system operation of time spent by the bus in stopping for passengers. By using a performance evaluation model (1,2), we will examine the function for determining how many stops the bus will make along a route, the dwell time required at each stop, and the resulting effect on system operation.

DESCRIPTION OF PERFORMANCE EVALUATION MODEL

The performance evaluation model is used to evaluate changes in operational performance due to a short-range change in service expressed in terms of travel-time changes. After the travel-time changes expected from a service change have been estimated, the ridership change is estimated according to the values of the demand elasticities input into the model, as follows:

$$Q_1 = Q_0[1 + \alpha(\mathrm{DIVTT}) + \beta(\mathrm{DOVTT})]\ r \tag{1}$$

where

- Q_1 = new service demand in passengers per hour over the length of the route,
- Q_0 = old service demand in passengers per hour over the length of the route,
- α = demand elasticity with respect to in-vehicle travel time,
- DIVTT = percentage of change in in-vehicle travel time,
- β = demand elasticity with respect to out-of-vehicle travel time,
- DOVTT = percentage of change in out-of-vehicle travel time, and
- r = ratio of new to old person-trip ends served by the route.

The in-vehicle travel time is a function of the average travel distance and the overall operating speed of the bus after corrections for passenger stops have been made. The bus running speed without stops for passengers is an input into the model. For each passenger stop, a time penalty of delta (δ) is made, averaging 10 to 20 sec. A dwell time of epsilon (ϵ), averaging 3 to 5 sec, is added for each passenger boarding and alighting.

To determine the number of passenger stops, the assumption is made that the passenger demand is uniformly distributed along the route. Consequently, the number of passengers boarding and alighting at each stop would follow a Poisson distribution. The probability of a stop with z passengers boarding and alighting is given as

$$p(z) = \exp(-m)m^z/z! \tag{2}$$

where m is the average number of passengers per stop.

The probability that a stop will actually be made is 1 minus the probability of a zero-passenger stop, as follows:

$$1 - p(0) = 1 - \exp(-m) \tag{3}$$

The total time spent by the bus in the starting and stopping maneuver in seconds per mile is given as

$$D_1 = \delta Y[1 - p(0)] \tag{4}$$

where Y is stops per mile and D_1 is the starting and stopping time per mile.

The dwell time while passengers are boarding and alighting in seconds per mile is given as

$$D_2 = 2Q_1\epsilon\mathrm{HDWY}/L \tag{5}$$

where D_2 is the dwell time per mile and L is the route length in miles.

The operating speed is given as the reciprocal of the total travel time per mile:

$$S = 1/[1/\mathrm{RSPD} + (D_1 + D_2)/3{,}600] \tag{6}$$

where S is the operating speed in miles per hour and RSPD is the running speed in miles per hour without passenger delays.

The in-vehicle travel time is then determined to be as follows:

$$\mathrm{IVTT} = M/S \tag{7}$$

where M is the average passenger-trip length.

During the development of the initial version of the performance evaluation model, two assumptions were made: (a) the passenger boardings and alightings along a route follow a Poisson distribution, and (b) the dwell time per passenger is independent of the number of passengers boarding and alighting at each stop. Field data were used to analyze the validity of these assumptions. A discussion of this analysis is presented below.

PASSENGER BOARDING AND ALIGHTING DISTRIBUTION ALONG BUS ROUTE

Two routes in Milwaukee, Wisconsin, were studied to check the validity of the Poisson assumption. Route 27 is a heavily traveled crosstown route traversing primarily high-density residential and commercial areas. Because Milwaukee's streets are in a grid pattern, a large amount of transferring takes place between Route 27 and the downtown-oriented routes with which it intersects. Headways range from 8 min during the peak hours to 17 min late at night. Route 28 is also a crosstown route, but it is located in a primarily suburban area and is lightly used. Neighboring land use is primarily strip commercial, light industrial, and medium-density residential. The bus enters a regional shopping center at the middle of the route. Headways are 30 to 36 min on weekdays. No weekend or evening service is offered.

Data for each route were available from an onboard sample. The passenger boardings and alightings were recorded at each stop once for each scheduled run of the bus. Data from three round trips were selected for each route. One trip occurred during the morning peak hour, one was at midday, and the third was during the evening peak. Northbound and southbound data were analyzed independently for each route.

Table 1 gives the recorded data for each one-way run of the bus for Route 27, and Table 2 gives the same for Route 28. The average number of passengers at a stop, sample variance, chi-square, and test values for the two sets of data are given in Table 3.

By using the original assumptions of the model, the passenger distributions were determined from a Poisson distribution. The Poisson distribution uses

Table 1. Passenger distribution along Route 27.

No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Poisson Prediction	No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Poisson Prediction
Northbound, Morning Peak			Southbound, Morning Peak		
0	41	7.9	0	26	0.5
1	18	20.8	1	7	2.7
2	15	27.4	2	6	6.7
3	11	24.1	3	11	11.3
4	6	15.9	4	6	14.1
5	4	8.3	5	3	14.2
6	2	3.7	6	3	11.9
7	5	1.4	7	3	8.5
8	2	0.5	8	1	5.4
9	1	–	9	1	3.0
11	2	–	10	2	1.5
16	1	–	11	1	0.7
18	1	–	13	3	–
37	1	–	15	1	–
			21	1	–
			22	2	–
			23	1	–
			24	1	–
			31	1	–
			32	1	–
Northbound, Midday			Southbound, Midday		
0	39	4.8	0	44	5.7
1	12	14.3	1	10	16.8
2	11	21.3	2	14	24.7
3	12	21.1	3	13	24.2
4	1	15.6	4	4	17.7
5	3	9.3	5	6	10.4
6	2	4.6	6	2	5.1
7	2	1.9	7	1	2.1
8	0	0.7	8	3	0.8
9	4	–	9	3	–
10	1	–	10	1	–
12	1	–	11	1	–
13	2	–	12	4	–
15	1	–	16	1	–
17	1	–	39	1	–
20	1	–			
28	1	–			
Northbound, Evening Peak			Southbound, Evening Peak		
0	30	3.3	0	19	1.9
1	15	10.9	1	15	7.0
2	10	18.1	2	11	13.2
3	9	20.1	3	10	16.6
4	6	16.6	4	4	15.7
5	5	11.0	5	5	11.9
6	4	6.1	6	5	7.5
7	2	2.9	7	4	4.0
8	1	1.2	11	1	–
10	2	–	12	2	–
12	1	–	13	1	–
13	1	–	16	1	–
14	1	–	23	1	–
15	1	–	25	1	–
21	1	–	28	1	–
22	1	–			
28	1	–			

Table 2. Passenger distribution along Route 28.

No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Poisson Prediction	No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Poisson Prediction
Northbound, Morning Peak			Southbound, Morning Peak		
0	49	44.2	0	48	37.9
1	11	18.4	1	15	23.3
2	5	3.8	2	3	7.1
3	2	0.6	3	1	1.7
			5	2	–
			9	1	–
Northbound, Midday			Southbound, Midday		
0	54	49.7	0	54	39.5
1	9	14.8	1	6	22.6
2	2	2.2	2	4	6.4
3	1	0.2	3	3	1.4
4	1	–	4	1	–
			5	1	–
			8	1	–
Northbound, Evening Peak			Southbound, Evening Peak		
0	44	41.0	0	54	40.1
1	17	20.2	1	8	22.3
2	3	5.0	2	2	6.2
3	2	0.9	3	2	1.3
4	1	–	4	1	–
			5	2	–
			7	1	–

Table 3. Summary of statistics from Tables 1 and 2.

Data Set	Mean No. of Passengers	Variance	Chi-Square	Test Value
Route 27				
Northbound				
Morning peak	2.636	21.142	221.[a]	18.47 (7 df)
Midday	2.968	23.414	310.[a]	16.81 (6 df)
Evening peak	3.318	25.316	253.[a]	18.47 (7 df)
Southbound				
Morning peak	5.024	51.950	323.[a]	20.09 (8 df)
Midday	2.935	24.040	300.[a]	16.81 (6 df)
Evening peak	3.778	28.770	105.[a]	16.81 (6 df)
Route 28				
Northbound				
Morning peak	0.403	0.569	3.83[b]	9.21 (2 df)
Midday	0.298	0.538	3.64[b]	9.21 (2 df)
Evening peak	0.493	0.698	1.49[b]	9.21 (2 df)
Southbound				
Morning peak	0.614	2.008	6.02[b]	9.21 (2 df)
Midday	0.571	1.873	18.4[a]	9.21 (2 df)
Evening peak	0.557	1.818	16.9[a]	9.21 (2 df)

[a]Significantly different from the Poisson distribution at the 0.01 level.
[b]Evidence is not present that the distribution is not Poisson.

a single parameter (m), which is equal to both the mean and the variance of the distribution. The resulting distributions for separate data sets are shown in Tables 1 and 2. A chi-square (χ^2) goodness-of-fit test was performed for each distribution (Table 3). A significance level of 1 percent was used. This means that if the hypothesis that the passenger distribution is Poisson is rejected, there is a 1 percent chance that the data may in fact be Poisson distributed. The observed χ^2-value along with the test value are shown for each data set in Table 3. It can be noted that because of the low demand for Route 28, very few heavily used stops were recorded. In the northbound direction, no stop was observed with more than four passengers boarding and alighting. The same results were obtained for the southbound morning-peak data. It can therefore be concluded that the Poisson distribution cannot be rejected only in a situation where the passenger boardings and alightings are relatively low. Route 27 has a higher demand and the distribution of passenger boardings and alightings was found to be more significantly different from the Poisson distribution than that observed in the case of the southbound midday and afternoon peak-hour data for Route 28.

A review of the Route 27 data in terms of the Poisson distribution, as shown in Table 1, reveals two major discrepancies. First, many more stops at which no boardings or alightings occurred were observed in the field than the Poisson distribution predicts. The result of using the Poisson distribution in this situation would be a much lower level of service because the model would simulate that the bus stopped far too many times.

The Poisson distribution also projects too few stops serving a large number of passengers. For example, the northbound data for Route 27 during the morning peak included six stops involving more than eight passengers, whereas the Poisson distribution indicated none. Although this is only 5.4 percent of the total stops, 102 boardings and alightings or 35.2 percent of the total is involved.

It therefore appears that the Poisson distribution is not a good descriptor of the distribution of passengers along a route. An improved procedure is necessary to estimate the number of stops with a given number of passengers. This was accomplished as discussed in the following section.

DEVELOPMENT OF IMPROVED PROCEDURE

A comparison of the variances with the corresponding means, as given in Table 3, revealed that the variance is higher in all cases presented here. The Poisson distribution did not fit these data because it requires a variance equal to the mean. Note that these two parameters are most nearly equal for the data from northbound Route 28, where passenger boardings and alightings are low and the assumption of a Poisson distribution could not be rejected.

Consequently, the applicability of the negative binomial distribution was explored. The negative binomial distribution is characterized as having a variance higher than the mean. Three parameters of the distribution are defined as follows:

$$p = m/s^2 \quad (8a)$$

$$q = 1 - p \quad (8b)$$

$$k = pm/q = m^2/(s^2 - m) \quad (8c)$$

where

m = sample mean,
S^2 = sample variance, and
p,q,k = parameters of negative binomial distribution.

The probability of a zero occurrence is then given as follows:

$$P(0) = p^k \quad (9)$$

The probability of z is given as follows:

$$P(z) = [(z + k - 1)/z] \times q \times P(z - 1) \quad (10)$$

The negative binomial distribution was found to be an acceptable descriptor of passenger boardings and alightings under all volume conditions observed in the sample.

One advantage of the Poisson distribution is that only one parameter is required (the mean), whereas the negative binomial distribution requires both a mean and a variance. Nevertheless, a careful examination of the sample means and variances in Table 3 indicates that they may be correlated with each other. Consequently, a linear regression analysis was performed to develop the following second-order equation:

$$s^2 = -1.305 + 4.870m + 1.085m^2 \quad (11)$$

R^2 for this equation was the high value of 0.991. This equation was found to be an accurate predictor except in the situation of an extremely small mean. The predicted variance actually equals the mean at a value of 0.31 and drops below the mean at lower values. Because the negative binomial distribution dictates a variance higher than the mean, the equation was modified to assign a variance of 1.1 times the mean at values below 0.32. This modification produced significant results. One must be careful not to use this equation on other data, however, without careful examination of the results from those data.

By using the predicted value of the variance, negative binomial distributions were fitted to the passenger data as shown in Tables 4 and 5. A goodness-of-fit test was performed for each distribution at a level of significance of 1 percent. The chi-square, test values, and predicted and actual variances for Route 27 and Route 28 data are given in Table 6.

DWELL TIME

The assumption that dwell time per passenger is independent of the number of passengers boarding and alighting may be erroneous. For example, Kraft (3) found that this function follows an Erlang distribution. To further examine this distribution, dwell-time data from a survey of one of the more heavily used routes in Lafayette, Indiana, were obtained (4), as shown in Table 7. Data were tabulated during six 30-min runs of the route. There were two runs from each of the morning-peak, off-peak, and evening-peak periods of the day. The dwell time was recorded along with the number of persons boarding and alighting at each stop. It is interesting to note that although the total dwell time increases, the time per passenger decreases as the number of passengers at a stop increases.

For the data in Table 7, the total number of passengers boarding and alighting is 357, total number of stops is 113, overall average dwell time is 9.54 sec, SD is 7.46, and average dwell time per passenger weighted by passenger is 3.02 and that weighted by stop is 4.07.

By using these data, a regression analysis was performed to relate the natural logarithm of the

Table 4. Test of negative binomial model by using predicted variances for Route 27.

No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Negative Binomial Model Prediction	No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Negative Binomial Model Prediction
Northbound, Morning Peak			Southbound, Morning Peak		
0	41	47.64	0	26	22.52
1	18	17.36	1	7	11.25
2	15	10.64	2	6	7.87
3	11	7.41	3	11	6.04
4	6	5.46	4	6	4.83
5	4	4.16	5	3	3.96
6	2	3.24	6	3	3.30
7+	13	14.07	7+	19	21.33
Northbound, Midday			Southbound, Midday		
0	39	37.91	0	44	43.85
1	12	14.71	1	10	16.92
2	11	9.25	2	14	10.61
3	12	6.55	3	13	7.51
4	1	4.91	4	4	5.61
5	3	3.79	5	6	4.33
6+	16	16.88	6	2	3.42
			7+	15	15.75
Northbound, Evening Peak			Southbound, Evening Peak		
0	30	34.16	0	19	27.85
1	15	14.04	1	15	12.20
2	10	9.03	2	11	8.07
3	9	6.51	3	10	5.93
4	6	4.95	4	4	4.58
5	5	3.88	5	5	3.64
6	4	3.10	6+	17	18.73
7+	12	15.32			

Table 5. Test of negative binomial model by using predicted variances for Route 28.

No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Negative Binomial Model Prediction	No. of Boardings and Alightings per Stop	Observed Data (no. of stops)	Negative Binomial Model Prediction
Northbound, Morning Peak			Southbound, Morning Peak		
0	49	50.93	0	48	51.21
1	11	9.93	1	15	9.22
2	5	3.53	2	3	4.09
3+	2	2.61	3+	4	5.48
Northbound, Midday			Southbound, Midday		
0	54	50.41	0	54	51.76
1	9	13.68	1	6	9.23
2+	4	2.91	2	4	4.00
			3+	6	5.01
Northbound, Evening Peak			Southbound, Evening Peak		
0	44	50.41	0	54	51.94
1	17	9.02	1	8	9.24
2	3	3.68	2	2	3.97
3+	3	3.90	3+	6	4.85

dwell time per person to the number of boardings and alightings at each stop. The equation was found to be

$$\epsilon = 5.0 - 1.2[\ln(z)] \qquad (12)$$

where ϵ is the dwell time per passenger and z is the number of boardings and alightings at a stop.

This equation was significant at the 99 percent level by using the F-test. Nevertheless, the relatively low value of 0.36 for R^2 indicates that the variation in dwell time depends not only on the number of passengers but also on other factors. For example, a lower value for dwell time would be expected if

1. Many people board and alight from the bus at only a few stops, such as in an express bus service, or
2. Monthly passes or tokens are in effect, reducing the time needed to pay fares.

The average dwell time might increase if

1. Many elderly and handicapped persons are present,
2. A complex fare structure is used,
3. The basic fare requires a large number of coins (e.g., 45 cents requires at least three coins), or
4. Small buses that have only one door are used.

Further analysis of Equation 12 reveals that the maximum dwell time per stop occurs when there are approximately 24 passengers. The value of ϵ is 1.2 sec at this point. It can be expected that the total dwell time will continue to increase as the number of boardings and alightings increases. Consequently, the model assigns the value of 1.2 sec to

Table 6. Summary of statistics from Tables 4 and 5.

Data Set	Chi-Square[a]	Test Value	Variance Predicted	Variance Actual
Route 27				
Northbound				
Morning peak	5.09	18.47 (7 df)	19.069	21.142
Midday	8.56	16.81 (6 df)	22.705	23.414
Evening peak	3.16	18.47 (7 df)	26.803	25.316
Southbound				
Morning peak	5.83	18.47 (7 df)	50.541	51.950
Midday	9.66	18.47 (7 df)	22.334	24.042
Evening peak	8.05	16.81 (6 df)	32.573	28.765
Route 28				
Northbound				
Morning peak	0.94	11.34 (3 df)	0.833	0.569
Midday	2.26	9.21 (2 df)	0.328[b]	0.5378
Evening peak	8.20	11.34 (3 df)	1.356	0.698
Southbound				
Morning peak	4.51	11.34 (3 df)	2.096	2.008
Midday	1.42	11.34 (3 df)	1.831	1.873
Evening peak	1.50	11.34 (3 df)	1.744	1.818

[a] Evidence against the distribution's being negative binomial by using predicted variances is not present.
[b] Mean = 1.1.

Table 7. Lafayette dwell-time survey results.

No. of Passengers Boarding and Alighting	No. of Stops	Avg Dwell Time (sec)	Standard Deviation of Dwell Time (sec)	Avg Dwell Time per Passenger (sec)
1	41	4.83	2.14	4.83
2	29	9.45	8.21	4.73
3	18	10.82	3.68	3.61
4	10	11.53	6.31	2.88
5	2	6.85	1.63	1.37
6	3	12.00	5.19	2.00
7	1	21.00	–	3.00
8	2	30.50	9.19	3.81
9	1	28.00	–	3.11
14	2	24.00	0.00	1.71
15	2	19.00	0.00	1.27
22	1	26.00	–	1.18
24	1	24.00	–	1.00

ε whenever 24 or more passengers are using a given stop. The final dwell time as a function of the number of passengers at a stop is then given as

$$\mathrm{TIME}(z) = z\,[5.0 - 1.2\ln(z)] \quad z \leq 23 \tag{13a}$$

$$\mathrm{TIME}(z) = 1.2z \quad z \geq 24 \tag{13b}$$

METHODOLOGY FOR ESTIMATING BUS DELAY

The findings from the passenger distribution and dwell-time analyses were used to develop a procedure that can estimate the bus delay time as a function of the number of passengers along a route as discussed below.

By using the approach of Sinha and Bhandari (1), the average demand at a stop is obtained:

$$m = 2Q_1(\mathrm{HDWY})/(Y \times L) \tag{14}$$

The variance of passenger demand is computed by using Equation 11. The parameters for the negative binomial distribution can then be determined from Equations 8a, 8b, and 8c on the basis of the values for the mean and variance. The number of nonzero stops per mile can be found from Equation 9 as follows:

$$\mathrm{SPM} = Y \times [1 - P(0)] \tag{15}$$

where SPM is the number of nonzero stops per mile.

The delay per mile for the stopping and starting maneuver of a bus is given by

$$D_1 = \delta \times \mathrm{SPM} \tag{16}$$

The dwell time per mile for stops with 23 or less boardings and alightings can be found by combining Equations 10 and 13 as follows:

$$D_2' = Y \sum_{z}^{23} \mathrm{TIME}(z) \times P(z) \tag{17}$$

The dwell time for those stops with 24 or more boardings and alightings is simply set as 1.2 sec times the number of passengers involved, as follows:

$$D_2'' = 1.2Y \times \left\{m - \sum_{z=1}^{23} [P(z) \times z]\right\} \tag{18}$$

The total dwell time per mile is then

$$D_2 = D_2' + D_2'' \tag{19}$$

The total delay in hours per mile caused by the bus's stopping for passengers is then

$$\mathrm{Delay} = (D_1 + D_2)/3{,}600 \tag{20}$$

This value can be directly substituted into Equation 6 to compute the bus operating speed.

ANALYSIS OF RESULTS

The performance evaluation model was appropriately modified to include a negative binomial distribution for passenger distributions at stops as well as the revised procedure to compute vehicle dwell time. The computation of the operating speed was thus also modified by incorporating the logic for bus delay. The model was then applied to analyze the operation of the same routes in Milwaukee where the passenger data had been obtained. Some key input data are shown in Table 8. The average ridership and trip length were obtained from the data supplied by the Milwaukee County Transit System. The route lengths and numbers of posted stops per mile were determined from measuring the route from an automobile. The running speeds were determined by recording times while riding in the bus.

In Table 9 a comparison is shown of the model results with the actual recorded values from the two routes. In general, the results proved to be within about 10 percent. The notable exceptions were the results of the analysis of Route 27 during the morning peak hour. This difference can be accounted for by exceptionally low ridership during the sampling. As a means of examining how the model performs under different inputs, the ridership and the posted stops per mile were tested at the following levels:

1. Ridership: level 1, as observed in the field (Table 8); level 2, 20 percent increase.
2. Stops per mile: level 1, 12.5; level 2, existing as shown in Table 8; level 3, 2.5.

Some of the results of the analysis of the northbound parts of the routes are shown in Table 10. The ridership results were determined from Equation 1 by using elasticity values of -0.35 for in-vehicle travel time (α) and -0.70 for out-of-vehicle travel time (β). The conclusions obtained from the southbound data were the same and therefore are not discussed here.

An evaluation of the Route 27 results reveals that the decrease in posted stops per mile to 2.5 produced about an 11 to 16 percent increase in oper-

Table 8. Input data for application of performance evaluation on Milwaukee routes.

Route	Direction	Time Period	Avg Ridership per Hour	Avg Trip Length (miles)	Route Length (miles)	Avg Headway (min)	Posted Stops per Mile	Fare (cents)	Estimated Running Speed (mph)
27	Northbound	Morning peak	603.7	2.20	13.22	11.25	6.9	75	17.5
		Midday	509.8	2.20	12.18	10.59	6.9	75	18.6
		Evening peak	1,067.2	2.20	12.44	8.18	6.9	75	16.1
		Evening	274.0	2.20	12.58	11.66	6.9	75	17.0
		Night	137.3	2.20	9.03	17.60	6.9	75	17.0
27	Southbound	Morning peak	1,176.2	2.20	13.01	7.50	6.9	75	16.9
		Midday	565.3	2.20	12.57	9.47	6.9	75	16.4
		Evening peak	705.3	2.20	13.14	12.85	6.9	75	16.8
		Evening	289.4	2.20	12.97	11.67	6.9	75	17.0
		Night	129.6	2.20	13.08	14.28	6.9	75	17.0
28	Northbound	Morning peak	12.6	2.50	12.1	30.00	5.6	75	22.3
		Midday	10.8	2.50	12.1	36.00	5.6	75	24.0
		Evening peak	28.8	2.50	12.1	30.00	5.6	75	25.1
28	Southbound	Morning peak	28.5	2.50	12.0	30.00	5.6	75	24.3
		Midday	21.0	2.50	12.0	36.00	5.6	75	22.2
		Evening peak	30.2	2.50	12.0	30.00	5.6	75	19.7

Table 9. Comparison of model output with actual recorded values.

Route	Direction	Time Period	Nonzero Stops per Mile		Total Bus Delay (sec/mile)		Operating Speed (mph)	
			Model	Actual	Model	Actual	Model	Actual
27	Northbound	Morning peak	3.8	3.0	82.3	53.5	12.5	13.9
		Midday	3.6	3.3	75.0	71.9	13.4	13.5
		Evening peak	4.3	4.8	99.3	95.0	11.1	11.4
27	Southbound	Morning peak	4.3	3.5	97.3	61.3	11.6	13.1
		Midday	3.5	3.3	73.1	70.4	12.3	12.4
		Evening peak	4.3	4.4	98.3	92.7	11.5	11.7
28	Northbound	Morning peak	0.9	0.9	13.5	13.5	20.5	20.5
		Midday	0.9	0.9	13.8	17.1	22.0	21.5
		Evening peak	1.4	1.3	22.7	22.9	21.7	21.4
28	Southbound	Morning peak	1.3	1.4	22.7	21.4	17.5	21.2
		Midday	1.3	1.2	21.5	20.7	19.6	19.7
		Evening peak	1.3	1.2	22.2	21.4	21.2	17.6

Table 10. Model results on delay from passenger boardings.

Route	Direction	Time Period	Stops per Mile	Ridership Level	Nonzero Stops per Mile	Total Bus Delay per Mile (sec)	User Cost per Passenger ($)	Operating Speed (mph)	Annual Ridership
27	Northbound	Morning peak	2.5	1	1.9	50.1	1.12	14.07	406,439
				2	2.0	55.1	1.13	13.80	490,068
			6.9	1	3.8	82.3	0.99	12.49	461,831
				2	4.1	92.0	1.00	12.09	554,197
			12.5	1	5.2	102.8	0.97	11.66	473,419
				2	5.7	115.9	0.99	11.12	566,399
		Midday	2.5	1	1.8	46.2	1.08	14.99	681,379
				2	1.9	50.9	1.09	14.70	821,636
			6.9	1	3.6	75.0	0.95	13.39	779,994
				2	3.9	84.1	0.96	12.95	935,993
			12.5	1	4.8	93.2	0.92	12.54	801,904
				2	5.3	105.2	0.94	12.03	959,659
		Evening peak	2.5	1	2.0	58.2	1.05	12.78	702,310
				2	2.1	64.0	1.06	12.52	846,560
			6.9	1	4.3	99.3	0.93	11.15	816,408
				2	4.6	110.2	0.94	10.79	979,690
			12.5	1	6.1	126.8	0.92	10.28	841,132
				2	6.7	142.3	0.94	9.84	1,005,822
28	Northbound	Morning peak	2.5	1	0.6	9.8	1.90	21.03	9,057
				2	0.6	10.4	1.84	20.95	10,907
			5.6	1	0.9	13.5	1.69	20.58	9,639
				2	1.1	16.0	1.70	20.29	11,567
			12.5	1	1.0	14.4	1.63	20.47	9,923
				2	1.2	17.2	1.63	20.16	11,902
		Midday	2.5	1	0.6	9.9	1.92	22.52	15,607
				2	0.6	10.5	1.92	22.42	18,799
			5.6	1	0.9	13.8	1.77	21.97	16,524
				2	1.1	16.4	1.77	21.63	19,829
			12.5	1	1.0	14.8	1.71	21.85	16,977
				2	1.2	17.6	1.71	21.48	20,361
		Evening peak	2.5	1	0.8	15.4	1.76	22.67	20,874
				2	0.9	17.3	1.82	22.40	25,029
			5.6	1	1.4	22.7	1.68	21.67	22,032
				2	2.1	31.2	1.63	20.61	22,329
			12.5	1	2.5	36.7	1.64	19.98	26,600
				2	2.8	43.5	1.67	17.13	26,496

ating speed. However, an increase to 12.5 stops per mile caused a somewhat smaller decrease. For Route 28, the effect of a change in posted stops on the operating speed is similar but much less. If the number of stops is reduced to 2.5, the operating speed increases only 2 to 5 percent. On the other hand, if the number of stops is increased to 12.5, the change in operating speed is much less. The implication here is that there is very little demand for posted stops beyond eight per mile along this route with low ridership levels.

For Route 28, the user cost per passenger always decreases with an increased number of stops because of the shorter walking distance, but for Route 27, the user cost per passenger decreases only negligibly with added stops. The reason is that the increase in user cost due to an increase in in-vehicle travel time as the bus makes more stops offsets the associated decreased walking time.

It should be explained that the user costs are larger for Route 28 because of the longer waiting times due to longer headways. In addition, the configuration of Route 28 is such that the walking time to the bus stop is longer.

The ridership decreased by about 5 to 6 percent for Route 27 and by about 11 to 14 percent for Route 28; there was a decrease in posted stops per mile to 2.5. The implication here is that the out-of-vehicle travel time becomes longer with fewer stops, which causes a decrease in ridership. Nevertheless, because of the higher operating speed due to less stops, the in-vehicle travel time decreases, which attracts additional riders. The net result was, however, a reduction in ridership. With an increase to 12.5 posted stops per mile, the ridership increased by much less than the magnitude of the decrease caused by fewer stops per mile. This is a reasonable result because the percentage of change in walking distance is also about one-half of the magnitude, as it was with the decrease in stops per mile.

For both routes, the model indicated that a 20 percent increase in ridership generally caused a decrease in operating speed by about 1 to 4 percent. Also, the number of stops with some passengers boarding or alighting increased by about 5 to 15 percent for a 20 percent increase in ridership. The total delay increased proportionately. For Route 28, the user cost per passenger increased only slightly. Due to the already crowded conditions on Route 27, however, the user costs per passenger were affected to a greater extent. Still, the average increase was only about 1 percent.

CONCLUSIONS

In determining a transportation mode choice, the overall travel time is a very important element. Although the adverse effect of out-of-vehicle travel time is most severe, it is also important to reduce the in-vehicle travel time as much as possible. A major disadvantage of the bus is that it has to stop continually to allow passengers to board and alight. Not much attention has been given to determining explicitly the impact that this stopping has on the overall operating speed of the route.

By using data from Milwaukee, Wisconsin, the distribution of passengers boarding and alighting at stops along a route was analyzed. It was found that a Poisson distribution could be used only on routes with low ridership. Nevertheless, the negative binomial distribution was found to be a good descriptor of passenger boardings and alightings over a range of ridership levels. Data from Lafayette, Indiana, were used to analyze the bus dwell time. It was found that the bus dwell time per passenger decreases with the natural logarithm of the boardings and alightings at the stop. From these findings, a procedure was developed to determine the resulting bus delay and its effect on operating speed.

The methodology was then tested by using data from Milwaukee, Wisconsin, and assuming different numbers of stops per mile. Analysis of the output revealed two major findings. First, a change in posted stops along a low-demand route will have only a minor effect on bus operating speed but will reduce the user's walking distance. Second, because additional posted stops along a high-demand route will save walking distance at the cost of greater in-vehicle travel time, an optimum number of posted stops per mile should be sought.

This methodology can be applied to all operating-policy changes that have an effect on the operating speed. Appropriate performance measures can then be used to examine the impact of the various policy options.

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Economics of Commuter Express Bus Operations

BRAD WILLIAMS AND BILL WELLS

With the recent cuts in federal subsidies for transit operations, planners are looking for ways to reduce their operating costs. One way of doing this is to allow the private sector to provide commuter express bus service at little or no subsidy. A study of commuter express bus operations is summarized in which it is concluded that the operating cost for a private carrier is only about half that of the public carriers in Southern California. After 22 public bus lines had been evaluated, the conclusion was that more than $5 million per year in subsidy could be eliminated if the 22 bus lines were operated by private carriers. The cost savings are attributed to more favorable work rules and the ability to use less costly equipment. One other factor is that private operators will continue operation of a bus only if it is nearly full. The analysis was based on operating budgets for the two transit districts in Los Angeles and Orange Counties and on a survey of private agencies in the region.

This paper is the product of a 10-month study that has focused on the respective roles of the public and private sectors in providing commuter express bus services. The study has examined two critical, interrelated issues affecting public policy decisions in this area. The issues are (a) the comparative economics of public and private agencies and (b) the institutional and regulatory framework within which services are currently provided and that constrain policy changes.

In this paper we concentrate on the economic analysis that was performed during the course of the