

Development of a Bayesian Acceptance Approach for Bituminous Pavements

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Traditional approaches for estimating the percentage of a lot that is within specification limits (PWL) are based on random samples taken from the lot being evaluated. These approaches suffer from the small sample sizes necessitated by the destructive and time-consuming tests that are usually used in determining the quality of the materials. The development of a Bayesian approach for estimating PWL is presented that incorporates information concerning the contractor's past performance on the project along with the current sample results in determining the estimate for the PWL of the current lot. The procedure assumes that the daily population mean is a random variable that follows a normal distribution, that the production process is also normally distributed, and that the process variance is constant. These assumptions are confirmed by using goodness-of-fit tests on data collected from 13 bituminous runway-paving projects. Computer simulation shows that the Bayesian PWL estimators are slightly biased as compared with the unbiased traditional quality-index method but that the PWL estimators exhibit smaller variances than the traditional method.

To determine the acceptability of and the ultimate payment for bituminous pavements, a procedure is necessary for estimating the quality of those materials. In this paper a method for estimating the quality of construction materials is presented that incorporates empirical Bayes (EB) techniques into the estimate. It is believed that such a procedure will be an improvement over current procedures based solely on classical techniques because it incorporates information concerning the contractor's past production record into the estimate for the current lot of material. The method, which should be applicable for many construction materials, will be developed from data collected on bituminous concrete pavement construction projects.

TYPES OF ACCEPTANCE PLANS

Several different types of acceptance plans have been developed and recommended for bituminous concrete pavement materials. Some acceptance plans (1-4) determine the acceptability of the material from the average, or mean, of the test results from a sample. These plans are based on the assumption that the standard deviation is known (or assumed). This known standard deviation is used to determine the acceptance limits within which the sample means must fall.

Other types of acceptance plans (1,2,5,6) are based on the fact that the standard deviation is not known and must be estimated from the sample results. In one type of plan the sample mean must fall a specified number (k) of standard deviations from the acceptance limit (e.g., $\bar{X} - L \geq k\sigma$). The specified number (k), in essence, determines the percentage of the material that must exceed the acceptance limit before the material is accepted. In an extension of this method (1,2,7-9), the calculated percentage of the material that is within the acceptance limits (percentage within limits, or PWL) is used for acceptance purposes. This method provides a natural measure for the relative quality of the material (presumably 90 PWL is superior to 80 PWL). It can therefore be used for developing a price-adjustment schedule that relates the quality of the material (as measured by PWL) to the payment to be received for the material.

The PWL method has the advantage that it considers both the mean and the variability of the mate-

rial and then incorporates these into one value, PWL, which can then be related to payment level. The PWL approach is a well-accepted method that has been adopted by state (8,9) and federal (10) agencies. The PWL approach to acceptance is the one for which an EB estimator will be developed in this paper.

PROBLEMS WITH EXISTING PLANS

Many acceptance plans currently in use that attempt to account for material variability use the sample range to estimate this variability. When the range is employed in a PWL acceptance plan, it is actually used to estimate the sample standard deviation. It has been pointed out (11) that the range method provides a biased estimate of PWL. The sample standard deviation, which provides an unbiased estimate for PWL, will be used in this paper to provide a better method for estimating PWL.

One major problem common to all types of construction-material acceptance plans is the relatively high costs and destructive nature of many of the tests commonly used to measure the quality and acceptability of the material. The luxury of using a sample size of 100 for each lot may be possible in industrial and manufacturing applications but is totally impractical for most construction situations. Sample sizes in construction-material acceptance plans are typically about four or five samples per lot. The objective in this paper is to develop an acceptance procedure that addresses this problem of a limited sample size.

The method used will be to employ an EB procedure to estimate the quality of a given lot of material. This procedure will incorporate the preceding information from the contractor's production history on the project into the estimate for the quality of the material placed during the day for which the quality is being estimated. In other words, the preceding information about the contractor's process capabilities will be pooled with the test results from the current sample to estimate the quality of the material in question. As pointed out by Martz (12), this pooling of data tends to have the effect of increasing the sample size.

BAYESIAN ESTIMATOR FOR PWL

The case to be considered in the development of the Bayesian estimator is that of basing the acceptance of a lot of material on an estimate of the PWL value for the lot. It is common practice to estimate the PWL value for a given lot of material from the mean (\bar{X}) and standard deviation (s) of a number of tests performed on samples randomly selected from the lot. In this way, each lot is considered totally independent of preceding or subsequent lots. The method to be developed will employ Bayesian concepts to pool information from previous lots with the results from the current lot to estimate the PWL for the current lot.

In the development of the Bayesian estimator it will be assumed that the sampling is from a normal process, i.e., that the daily test results are normally distributed. It will also be assumed that the

process has a constant variance. The final assumption to be made is that the daily population means are also normally distributed. The Bayesian approach considers this daily population mean as a random variable and assigns some distribution to it. The appropriateness of these assumptions will be verified against data collected from 13 field construction projects, and this analysis will be presented in a later section.

The normal-normal assumption is quite convenient and is frequently used when the data are only approximately normally distributed because it forms what is known as a conjugate family. Conjugate families make updating of the preceding distribution with current data to form the subsequent distribution relatively easy. That is, if one samples from a normal process with a fixed variance (σ^2) and the preceding distribution for the mean is normal, the subsequent distribution for the mean will also be normally distributed. This allows for a fairly convenient estimator for the mean of the subsequent distribution.

Bayes Estimator for Daily Population Mean

Parameters Known

The underlying assumptions on which the estimator will be developed are presented in Figure 1 and may be summarized as follows:

$$X_{ij} \sim N(\mu_i, \sigma^2) \quad (1)$$

$$\mu_i \sim N(\mu_p, \sigma_p^2) \quad (2)$$

where

- X_{ij} = j daily test results for day i ;
- μ_i = population mean for day i ;
- σ^2 = process variance (assumed equal for all days, i.e., constant variance);
- μ_p = preceding mean of the distribution of daily population means (μ_i); i.e., μ_i is a random variable with mean equal to μ_p ;
- σ_p^2 = preceding variance of the distribution of daily population means (μ_i); i.e., μ_i is a random variable with variance equal to σ_p^2 ;
- $i = 1, 2, 3, \dots, N$ = number of days; and
- $j = 1, 2, \dots, n$ = number of tests per day.

If σ^2 , μ_p , and σ_p^2 were known, then with a squared error loss function, the Bayes estimator for daily population mean (call it $\hat{\mu}$) can be shown to be given by

$$\hat{\mu}_i = [\sigma_p^2 \bar{X}_i + \mu_p(\sigma^2/n)] / [\sigma_p^2 + (\sigma^2/n)] \quad (3)$$

The derivation of this equation has been discussed by Burati (13).

Parameters Unknown

The estimator in Equation 3 was based on the assumption that the process variance and preceding distribution were known. Because σ^2 , μ_p , and σ_p^2 are not known in the typical construction situation, their values must be estimated from the sample results. Based on the results of N previous days, natural estimators for μ_p and σ^2 are

$$\hat{\mu}_p = (1/N) \sum_{i=1}^N \bar{X}_i \quad (4)$$

$$\hat{\sigma}^2 = (1/N) \sum_{i=1}^N s_i^2 \quad (5)$$

where

- N = number of previous days,
- \bar{X}_i = daily mean for day i , and
- s_i^2 = daily variance for day i .

Equation 5 is the pooled estimate for variance for the case of an equal number of tests each day. This is typically the case for density test results in asphalt pavement construction. If the number of tests per day is not constant, which may be the case for certain test results, such as the Marshall and extraction tests, the following formula must be used to determine the pooled estimate for process variance:

$$\hat{\sigma}^2 = [(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \dots + (n_k - 1)s_k^2] / (n_1 + n_2 + \dots + n_k - k) \quad (6)$$

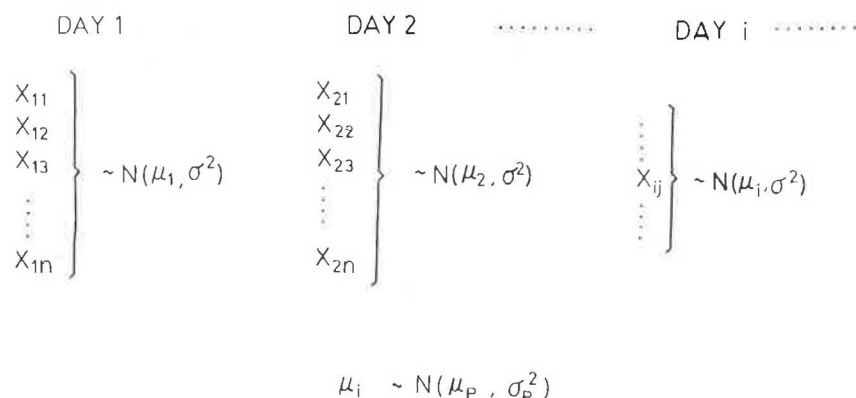
where $s_1^2, s_2^2, \dots, s_k^2$ are the daily sample variances for day 1, 2, \dots , k and n_1, n_2, \dots, n_k are the number of tests per day for day 1, 2, \dots , k .

The development of an estimate of σ_p^2 is not so obvious as the case of μ_p and σ^2 , because there is no natural estimator that is readily apparent. The daily means can be thought of as being equal to the daily population mean (μ_i) plus some average error term ($\bar{\epsilon}$); i.e.,

$$\bar{X}_i = \mu_i + \bar{\epsilon} \quad (7)$$

This can be illustrated as follows:

Figure 1. Assumptions made in development of Bayesian estimator for daily population mean.



$$X_{i1} = \mu_i + e_1$$

$$X_{i2} = \mu_i + e_2$$

$$X_{i3} = \mu_i + e_3$$

$$X_{in} = \mu_i + e_n$$

and

$$\bar{X}_i = (1/n) \sum_{j=1}^n X_{ij} \quad (8)$$

but this can also be written as follows:

$$\bar{X}_i = (1/n) \left(n\mu_i + \sum_{j=1}^n e_j \right) \quad (9)$$

or

$$\bar{X}_i = \mu_i + (1/n) \sum_{j=1}^n e_j \quad (10)$$

but

$$(1/n) \sum_{j=1}^n e_j$$

is merely the average error (\bar{e}), so

$$\bar{X}_i = \mu_i + \bar{e} \quad (11)$$

If it is assumed that the daily population mean (μ_i) and the average error term for the day (\bar{e}) are independent of each other, because

$$\bar{X}_i = \mu_i + \bar{e} \quad (12)$$

then

$$\text{Var}(\bar{X}_i) = \text{Var}(\mu_i) + \text{Var}(\bar{e}) \quad (13)$$

It may be recalled that $\mu_i \sim N(\mu_p, \sigma_p^2)$; then $\text{Var}(\mu_i) = \sigma_p^2$. Also, it may be noted that

$$X_{ij} \sim N(\mu_i, \sigma^2) \quad (14)$$

Then

$$e_{ij} \sim N(0, \sigma^2) \quad (15)$$

so

$$\bar{e} \sim N(0, \sigma^2/n) \quad (16)$$

Therefore,

$$\text{Var}(\bar{e}) = \sigma^2/n \quad (17)$$

If this information is combined, it can be stated

$$\text{Var}(\bar{X}_i) = \sigma_p^2 + (\sigma^2/n) \quad (18)$$

The variance of the daily means can also be stated as follows:

$$\text{Var}(\bar{X}_i) = [1/(N-1)] \sum_{i=1}^N (\bar{X}_i - \bar{\bar{X}})^2 \quad (19)$$

where

N = number of previous days;

\bar{X}_i = daily mean for day i ; and

$\bar{\bar{X}}$ = grand mean, i.e., mean of daily means for N previous days.

If the right side of Equation 19 is referred to as S_N^2 , then the estimate for σ_p^2 is therefore as follows:

$$\hat{\sigma}_p^2 = S_N^2 - (\sigma^2/n) \quad (20)$$

Because all these values are estimates, it may be possible on some occasions for $\hat{\sigma}_p^2/n$ to exceed S_N^2 . Because it is not possible to have a variance less than zero, the estimate of σ_p^2 will be as follows:

$$\hat{\sigma}_p^2 = S_N^2 - (\sigma^2/n) \quad \text{if positive,} \\ = 0 \quad \text{if } S_N^2 - (\sigma^2/n) \text{ is negative}$$

To summarize, an EB estimate for the daily population mean ($\hat{\mu}_i$) may be calculated from the sample results as follows:

$$\hat{\mu}_i = [\hat{\sigma}_p^2 \bar{X}_i + \hat{\mu}_p (\hat{\sigma}^2/n)] / [\hat{\sigma}_p^2 + (\hat{\sigma}^2/n)]$$

where $\hat{\sigma}_p^2$, $\hat{\mu}_p$, and $\hat{\sigma}^2$ are as defined above.

ESTIMATING PERCENTAGE WITHIN LIMITS

Because the acceptance procedure under consideration is based on PWL, it is necessary to develop a method for estimating PWL by using the EB estimator for daily population mean (EB mean estimator) that was developed above. A number of possible estimators for PWL that are based on the EB mean estimator can be developed.

Traditional Quality-Index Approach

The traditional approach uses the daily sample mean and standard deviation results to calculate a quality index (Q_L or Q_U). Once the quality index has been calculated for a given lot, the estimated PWL can be determined from tabled values. The quality index can be calculated as follows:

$$Q_L = (\bar{X} - L)/s \quad (21)$$

or

$$Q_U = (U - \bar{X})/s \quad (22)$$

where

Q_L = lower quality index,
 Q_U = upper quality index,
 L = lower specification limit,
 U = upper specification limit,
 \bar{X} = sample mean, and
 s = sample standard deviation.

Table 1 is used to estimate PWL values based on Q_L and Q_U values. The procedure for deriving this table was developed by Willenbrock and Kopac (11).

An example will help to describe this approach, referred to as method 1, for estimating PWL. The following values are known for the case of acceptance of a lot based on mat density: $\bar{X} = 97.6$ percent, $s = 1.05$ percent, $L = 96.7$ percent, and $n = 4$. The quality index can then be calculated as follows:

$$Q_L = (\bar{X} - L)/s = (97.6 - 96.7)/1.05 = +0.857.$$

From Table 1, the estimated PWL for the lot is 78.6 percent.

Bayes Quality-Index Approach

Logically, the first step in incorporating the EB

Table 1. Standard deviation method for estimating percentage of lot within limits.

Percentage Within Limits	Negative Values of Q_U or Q_L					Percentage Within Limits	Positive Values of Q_U or Q_L				
	n = 3	n = 4	n = 5	n = 6	n = 7		n = 3	n = 4	n = 5	n = 6	n = 7
50	0.0000	0.0000	0.0000	0.0000	0.0000	99	1.1510	1.4700	1.6719	1.8016	1.8893
45	0.1806	0.1500	0.1406	0.1364	0.1338	98	1.1476	1.4400	1.6018	1.6990	1.7615
40	0.3568	0.3000	0.2823	0.2740	0.2689	97	1.1439	1.4100	1.5428	1.6190	1.6662
39	0.3912	0.3300	0.3106	0.3018	0.2966	96	1.1402	1.3800	1.4898	1.5500	1.5868
38	0.4252	0.3600	0.3392	0.3295	0.3238	95	1.1367	1.3500	1.4408	1.4892	1.5184
37	0.4587	0.3900	0.3678	0.3577	0.3515	94	1.1330	1.3200	1.3946	1.4332	1.4562
36	0.4917	0.4200	0.3968	0.3859	0.3791	93	1.1263	1.2900	1.3510	1.3813	1.3990
35	0.5242	0.4500	0.4254	0.4140	0.4073	92	1.1170	1.2600	1.3091	1.3328	1.3465
34	0.5564	0.4800	0.4544	0.4426	0.4354	91	1.1087	1.2300	1.2683	1.2866	1.2966
33	0.5878	0.5100	0.4837	0.4712	0.4639	90	1.0977	1.2000	1.2293	1.2421	1.2494
32	0.6187	0.5400	0.5131	0.5002	0.4952	89	1.0864	1.1700	1.1911	1.2001	1.2045
31	0.6490	0.5700	0.5424	0.5292	0.5211	88	1.0732	1.1400	1.1538	1.1592	1.1615
30	0.6788	0.6000	0.5717	0.5586	0.5506	87	1.0596	1.1100	1.1174	1.1196	1.1202
29	0.7076	0.6300	0.6018	0.5880	0.5846	86	1.0446	1.0800	1.0819	1.0813	1.0793
28	0.7360	0.6600	0.6315	0.6178	0.6095	85	1.0286	1.0500	1.0469	1.0437	1.0413
27	0.7635	0.6900	0.6619	0.6480	0.6395	84	1.0118	1.0200	1.0125	1.0073	1.0032
26	0.7905	0.7200	0.6919	0.6782	0.6703	83	0.9940	0.9900	0.9782	0.9718	0.9673
25	0.8164	0.7500	0.7227	0.7093	0.7011	82	0.9748	0.9600	0.9453	0.9367	0.9315
24	0.8416	0.7800	0.7535	0.7403	0.7320	81	0.9555	0.9000	0.9123	0.9028	0.8966
23	0.8661	0.8100	0.7846	0.7717	0.7642	80	0.9342	0.9000	0.8798	0.8693	0.8626
22	0.8896	0.8400	0.8161	0.8040	0.7964	79	0.9122	0.8700	0.8479	0.8363	0.8290
21	0.9122	0.8700	0.8479	0.8363	0.8290	78	0.8896	0.8400	0.8161	0.8040	0.7964
20	0.9342	0.9000	0.8798	0.8693	0.8626	77	0.8661	0.8100	0.7846	0.7717	0.7642
19	0.9555	0.9300	0.9123	0.9028	0.8966	76	0.8416	0.7800	0.7535	0.7403	0.7320
18	0.9748	0.9600	0.9453	0.9367	0.9315	75	0.8164	0.7500	0.7227	0.7093	0.7011
17	0.9940	0.9900	0.9782	0.9718	0.9673	74	0.7905	0.7200	0.6919	0.6782	0.6703
16	1.0118	1.0200	1.0125	1.0073	1.0032	73	0.7635	0.6900	0.6619	0.6480	0.6395
15	1.0286	1.0500	1.0469	1.0437	1.0413	72	0.7360	0.6600	0.6315	0.6178	0.6095
14	1.0446	1.0800	1.0819	1.0813	1.0793	71	0.7076	0.6300	0.6018	0.5880	0.5846
13	1.0597	1.1100	1.1174	1.1196	1.1202	70	0.6788	0.6000	0.5717	0.5586	0.5506
12	1.0732	1.1400	1.1538	1.1592	1.1615	69	0.6490	0.5700	0.5424	0.5292	0.5211
11	1.0864	1.1700	1.1911	1.2001	1.2045	68	0.6187	0.5400	0.5131	0.5002	0.4925
10	1.0977	1.2000	1.2293	1.2421	1.2494	67	0.5878	0.5100	0.4837	0.4712	0.4639
9	1.1087	1.2300	1.2683	1.2866	1.2966	66	0.5564	0.4800	0.4544	0.4426	0.4354
8	1.1170	1.2600	1.3091	1.3328	1.3465	65	0.5242	0.4500	0.4254	0.4140	0.4073
7	1.1263	1.2900	1.3510	1.3813	1.3990	64	0.4917	0.4200	0.3968	0.3859	0.3791
6	1.1330	1.3200	1.3946	1.4332	1.4562	63	0.4587	0.3900	0.3678	0.3577	0.3515
5	1.1367	1.3500	1.4408	1.4892	1.5184	62	0.4252	0.3600	0.3392	0.3295	0.3238
4	1.1402	1.3800	1.4898	1.5500	1.5868	61	0.3912	0.3300	0.3106	0.3018	0.2966
3	1.1439	1.4100	1.5428	1.6190	1.6662	60	0.3568	0.3000	0.2823	0.2740	0.2689
2	1.1476	1.4400	1.6018	1.6990	1.7615	55	0.1806	0.1500	0.1406	0.1364	0.1338
1	1.1510	1.4700	1.6719	1.8016	1.8893	50	0.0000	0.0000	0.0000	0.0000	0.0000

mean estimator into the PWL estimate is simply to substitute it for the sample mean in the calculation of the quality index. This approach, referred to as method 2, can be written as follows:

$$Q_L = (\hat{\mu}_{EB} - L)/s \quad (23)$$

or

$$Q_U = (U - \hat{\mu}_{EB})/s \quad (24)$$

where $\hat{\mu}_{EB}$ is the EB estimator for daily population mean and Q_L , Q_U , L , U , and s are as described before.

The next logical step in developing a Bayes quality-index approach is to extend the concept of pooling preceding information with current sample results to the estimate of the variability of the material. Because in the development of the EB mean estimator it was assumed that the process had a constant variance, the estimate for the process variance ($\hat{\sigma}^2$), as defined in Equation 5, should be an improvement on the use of the sample standard deviation (s) in the quality-index calculation. This approach, method 3, can be written as follows:

$$Q_L = (\hat{\mu}_{EB} - L)/(\hat{\sigma}^2)^{1/2} \quad (25)$$

or

$$Q_U = (U - \hat{\mu}_{EB})/(\hat{\sigma}^2)^{1/2} \quad (26)$$

where $(\hat{\sigma}^2)^{1/2}$ is the estimated process variance as defined in Equation 5 and Q_L , Q_U , $\hat{\mu}_{EB}$, L , and U are as described before. Although the use of $\hat{\sigma}^2$ may not be strictly Bayesian, it is based on the same principle of pooling preceding information because it is defined as the average of all daily sample variances on the project. As long as the constant-process variance assumption is appropriate, this third method should provide a good estimate for PWL.

Normal-Distribution Approach

Another approach to estimating the PWL for the lot of material is simply to consider the process to be normally distributed and to use the daily estimates for mean and standard deviation as the parameters for the normal distribution that describes the population for the day. In this way, the PWL can be estimated by using a standardized statistic (z) and tables of the standard normal distribution to estimate the proportion of the daily population that is within the specification limits.

Two Bayes approaches can be developed for calculating the z -value to be used for estimating PWL. These approaches, methods 4 and 5, differ in the estimator that is used for the daily population standard deviation. These two methods parallel the

way that variability was estimated in methods 2 and 3, which have been described previously.

Method 4 uses the daily sample standard deviation to estimate the population standard deviation along with the EB mean estimator. The calculation for z can then be described by the following:

$$Z_L = (L - \hat{\mu}_{EB})/s \quad (27)$$

or

$$Z_U = (U - \hat{\mu}_{EB})/s \quad (28)$$

where Z_L is the standardized variate for the lower specification limit; Z_U is the standardized variate for the upper specification limit; and L , U , $\hat{\mu}_{EB}$, and s are as described before.

Method 5, on the other hand, uses the pooled estimate of process variance as the estimate for the daily population variability. This can be described as follows:

$$Z_L = (L - \hat{\mu}_{EB})/(\hat{\sigma}^2)^{1/2} \quad (29)$$

or

$$Z_U = (U - \hat{\mu}_{EB})/(\hat{\sigma}^2)^{1/2} \quad (30)$$

where all terms are as described before.

VERIFICATION OF ASSUMPTIONS

In the previous section, several semiempirical Bayes estimators for PWL (EB PWL estimators) were developed. These estimators were based on three assumptions concerning the production process and the distribution of the daily population means. It was assumed that the process was normally distributed with a constant variance from day to day and that the daily population means were normally distributed. Before the EB PWL estimators can be evaluated, it is necessary to determine whether the assumptions on which they were developed are appropriate for the case of a bituminous concrete surface course. In order to verify or refute these assumptions, a number of goodness-of-fit (GOF) tests were conducted on density, asphalt content, and aggregate gradation data collected on actual paving projects. Although GOF tests were conducted on all of these properties, only density is discussed in this paper.

Data were collected from 13 bituminous concrete runway-paving projects in four states. The projects, designated A through M, and their locations and approximate tonnages placed are given in Table 2. More than 200,000 tons of bituminous concrete were placed on these projects.

Daily Population Means

The first assumption to be considered is that the daily population means are normally distributed. This assumption can readily be checked by applying a GOF test to the daily sample means from the 13 projects for which data are available to see whether they can be assumed to follow a normal distribution. Three commonly employed GOF tests that were considered for the analysis included the chi-square (χ^2), Kolmogorov-Smirnov (K-S), and Cramer-von Mises (CvM) tests. The χ^2 -test requires a relatively large sample size and was therefore not appropriate in this instance because the largest number of days on any project was only 26. The K-S test is a widely accepted general-purpose GOF test. The CvM test is particularly appropriate for small sample sizes, which was the case in this analysis.

The K-S and CvM tests were implemented by means

Table 2. Projects from which data were collected.

Project	State	Concrete Placed (tons)
A	New York	8,210
B	Pennsylvania	2,500
C	Virginia	25,300
D	New York	10,660
E	New York	60,000
F	Pennsylvania	6,850
G	New York	3,250
H	New Jersey	6,850
I	Virginia	2,230
J	Virginia	3,050
K	Virginia	8,000
L	Virginia	33,000
M	New York	36,000
Total		205,900

Table 3. K-S GOF results for assumption of normal distribution for daily population density.

Project	Degrees of Freedom	K-S Test Statistic	Critical Value ($\alpha = 0.05$)	Decision
A	15	0.1375	0.220	@
B	6	0.2062	0.319	@
C	19	0.1127	0.195	@
D	17	0.1530	0.206	@
E	12	0.1809	0.242	@
F	10	0.1972	0.258	@
G	23	0.1060	0.184	@
H	10	0.2092	0.258	@
I	13	0.1003	0.234	@
J	9	0.1690	0.271	@
K	21	0.0959	0.188	@
L	26	0.1146	0.176	@
M	18	0.1572	0.200	@

Note: @ = do not reject normality assumption.

of a computer program, GOF, written by Don T. Phillips of Purdue University (14). The program, which had to be modified to run on the compiler that was being used, is capable of conducting χ^2 , K-S, and CvM GOF tests on up to 500 sample observations.

The results of the K-S tests on daily population density are given in Table 3. The critical values for a 5 percent level of significance ($\alpha = 0.05$) given in the table are taken from a paper by Lilliefors (15). These values, which were determined by Monte Carlo methods, are appropriate for the case where the mean and standard deviation for the theoretical distribution to be tested are determined from the sample observations. As can be seen from an examination of Table 3, there are no instances in which the normality assumption can be rejected at the $\alpha = 0.05$ level. This provides a solid argument in favor of the validity of the normal distribution assumption for daily population means.

The results of the CvM tests on daily population density are given in Table 4. The critical values for $\alpha = 0.05$ shown in the table are taken from the monograph by Phillips (14). Once again, there are no instances in which the normality assumption can be rejected.

Distribution of Production Process

The next assumption to be considered is that the process from which the daily samples are drawn is normally distributed. The logical way to test this assumption is to conduct GOF tests on the individual daily test results to determine whether they can be assumed to follow a normal distribution. This approach is not feasible because of the small number

Table 4. CvM GOF results for assumption of normal distribution for daily population density.

Project	Project Days	CvM Test Statistic	Critical Value ($\alpha = 0.05$)	Decision
A	15	0.1009	0.461	@
B	6	0.0602	0.461	@
C	19	0.0782	0.461	@
D	17	0.1003	0.461	@
E	12	0.0620	0.461	@
F	10	0.0792	0.461	@
G	23	0.2029	0.461	@
H	10	0.2700	0.461	@
I	13	0.0698	0.461	@
J	9	0.0491	0.461	@
K	21	0.2974	0.461	@
L	26	0.0674	0.461	@
M	18	0.1306	0.461	@

Note: @ = do not reject normality assumption.

Table 5. K-S GOF results for assumption of normal distribution for density residuals.

Project	Degrees of Freedom	K-S Test Statistic	Critical Value ($\alpha = 0.05$)	Decision
A	60	0.0996	0.114	@
B	24	0.0746	0.182	@
C	73	0.0843	0.104	@
D	65	0.0690	0.110	@
E	36	0.0623	0.148	@
F	30	0.1093	0.161	@
G	69	0.1072	0.1067	X
H	40	0.0807	0.140	@
I	91	0.0850	0.093	@
J	34	0.0760	0.152	@
K	84	0.0894	0.097	@
L	91	0.0473	0.093	@
M	68	0.0823	0.107	@

Note: @ = do not reject normality assumption; X = reject normality assumption.

of tests (usually four) per day. This sample size is too small for even the CvM test to provide meaningful results. To conduct a GOF test it was therefore necessary to pool, or group, the individual daily test results for all days on the project.

When the data were pooled from day to day, it was not possible simply to combine the daily tests, because each day had a different sample mean. To eliminate the day-to-day variation in sample means, instead of pooling the actual test values, the daily residuals were combined to form one large sample for each project. The residuals were defined as the difference between the individual test results for each day and the daily sample mean for the respective day. The procedure for determining the residuals can be illustrated in equation form as follows:

$$r_{ij} = (x_{ij} - \bar{X}_i) \quad (31)$$

where

- x_{ij} = test result number j for day i ,
- \bar{X}_i = daily sample mean for day i , and
- r_{ij} = residual number j for day i .

If the production process from which the individual daily test samples were drawn is normal, these $i \times j$ residuals should be normally distributed with mean equal to zero.

K-S tests were conducted on the residuals for each project to determine whether they could be assumed to follow a normal distribution. Because of the larger sample sizes for the residuals, CvM tests were not conducted. The results of the K-S test for

normality on the individual test residuals for density are shown in Table 5. Once again, the critical values for $\alpha = 0.05$ are from Lilliefors (15). With the exception of one project, the normality assumption cannot be rejected at the $\alpha = 0.05$ level. This is convincing evidence of the appropriateness of the assumption that the production process is normally distributed.

Assumption of Constant Process Variance

The final assumption to be addressed is that of a constant process variance from day to day throughout the project. To determine the appropriateness of this assumption it is necessary to test the equality of the sample variances from each of the project days. Bartlett's test is most often used (16) to test homogeneity of variances for random samples drawn from several populations. This test is based on a statistic whose sampling distribution approximates a chi-square when the random samples are drawn from independent normal populations. The normality assumption has already been addressed in the preceding paragraphs.

The form of the Bartlett test used to evaluate the assumption of constant process variance is that presented by Neter and Wasserman (17). The procedure consists of determining a pooled estimate for process variance (s_p^2) by using the following:

$$s_p^2 = [1/(n_T - k)] \sum_{i=1}^k (n_i - 1) s_i^2 \quad (32)$$

where

- s_p^2 = pooled estimate for process variance,
- s_i^2 = k -sample variances,
- n_i = sample size for k -project days,
- k = number of sample variances, and

$$n_T = \sum_{i=1}^k n_i$$

Once the pooled variance estimate has been determined, the test statistic (B) can be determined from the following:

$$B = (2.302585/C) \left[(n_T - k) \log s_p^2 - \sum_{i=1}^k (n_i - 1) \log s_i^2 \right] \quad (33)$$

where

$$C = 1 + [1/3(k-1)] \left(\left\{ \sum_{i=1}^k [1/(n_i - 1)] \right\} - [1/(n_T - k)] \right) \quad (34)$$

The test statistic is a value of a random variable that approximately follows a chi-square distribution with $k - 1$ degrees of freedom.

A simple FORTRAN program was written to calculate the values of the Bartlett test statistics given in Table 6 for density. The critical values given in the table were determined from the appropriate chi-square distribution for the 5 percent level of significance ($\alpha = 0.05$).

The constant-variance assumption was rejected on 4 of 13 projects for density. Although the constant-variance assumption did not fare as well as the two normality assumptions tested by the K-S and CvM procedures, based on the results given in Table 6 it still appears to be a reasonable assumption.

COMPUTER SIMULATION

The performance of the EB estimator for PWL can be evaluated against the traditional method by means of computer simulation. An extensive computer simula-

Table 6. Results of Bartlett test for constant variance on density.

Project	Pooled Variance	Test Statistic	Degrees of Freedom	Critical Value ($\alpha = 0.05$)	Decision
A	0.8278	18.8862	14	23.68	@
B	2.2600	3.1491	5	11.07	@
C	2.4903	25.3461	18	28.87	@
D	1.3883	11.0024	16	26.30	@
E	0.333	9.9718	11	19.68	@
F	0.3107	20.3424	9	16.92	X
G	0.5200	34.0504	22	33.92	X
H	0.7799	6.3637	9	16.92	@
I	0.1255	34.3288	12	21.03	X
J	0.5570	13.3109	8	15.51	@
K	1.4526	37.9786	20	31.41	X
L	0.5676	19.9066	25	37.64	@
M	0.4250	12.2958	17	27.59	@

Note: @ = do not reject normality assumption; X = reject normality assumption.

tion analysis was conducted with the five methods for estimating PWL that were described previously. A detailed description and discussion of this simulation analysis have been made (13), and a paper detailing this analysis is also currently in preparation. A brief description of the basic simulation procedure used is presented in this paper along with some of the results of the analysis.

Simulation Design

Each simulation run consisted of 3,000 project days. Because paving projects are typically of relatively short duration, the 3,000 project days were made up of 100 projects of 30 days' duration each. In the simulation the values for the daily population means (μ_i) were generated from a (μ_p , σ_p^2) normal distribution. Values of σ^2 were held constant for a given project but allowed to vary among projects. For each simulation, however, the ratio of the variance of the daily sample means (σ^2/n) to the preceding variance (σ_p^2) was held constant for all projects. The ratio $[(\sigma^2/n)/\sigma_p^2]$ was chosen to be 2.0, 1.0, or 0.5. A value of 2.0 would favor the EB estimator because it meant that the preceding distribution had a smaller variance than did the daily sample means. Similarly, a value of 0.5 for this ratio favored the traditional method for estimating PWL because it meant that the preceding distribution was more variable than were the daily sample means.

In each simulation conducted, three approaches, designated approaches A, B, and C, were used to establish the initial estimate for the preceding distribution to be used. The approaches were

1. The use of 20 earlier production days to establish a preceding production history for the contractor;

2. The use of the first five project production days to establish the precedent for the project in question; in this approach, \bar{X}_i and s_i were used to estimate PWL in the traditional manner for the first five project days, and then the EB PWL estimators were used for days 6 through 30; and

3. No knowledge concerning the contractor's preceding production was assumed; \bar{X}_i and s_i were used on project days 1 and 2, and then EB PWL estimators were used.

Evaluation Procedure

A number of different measures for evaluating the performance of the EB PWL estimators (methods 2-5)

Table 7. Summary of results from typical computer simulation for density.

$(\sigma^2/n)/\sigma_p^2$	Approach for Establishing Preceding Distribution	Method of Estimating PWL	Avg Error for Each Project	
			Avg Daily PWL Error (PWL)	Avg Daily Squared PWL Error (PWL ²)
0.5	A	1	0.020	88.19
		2	1.814	76.45
		3	1.645	52.30
		4	1.094	66.51
		5	0.792	44.20
	B	1	0.020	88.19
		2	1.550	82.21
		3	1.396	62.30
		4	0.997	74.86
		5	0.737	55.68
	C	1	0.020	88.19
		2	1.787	82.69
		3	1.633	61.60
		4	1.169	74.56
		5	0.900	54.12
1.0	A	1	-0.011	85.50
		2	1.688	64.20
		3	1.374	40.88
		4	0.944	54.42
		5	0.581	33.22
	B	1	-0.011	85.50
		2	1.357	73.92
		3	1.106	53.04
		4	0.770	65.16
		5	0.454	45.67
	C	1	-0.011	85.50
		2	1.585	72.79
		3	1.323	51.02
		4	0.928	63.23
		5	0.596	42.86
2.0	A	1	-0.007	84.40
		2	1.511	54.93
		3	1.086	30.59
		4	0.789	44.88
		5	0.380	23.54
	B	1	-0.007	84.40
		2	1.135	61.16
		3	0.827	42.06
		4	0.548	52.92
		5	0.214	34.59
	C	1	-0.007	84.40
		2	1.321	58.72
		3	0.998	39.06
		4	0.670	49.70
		5	0.313	30.73

with respect to the traditional approach (method 1) were employed in the analysis. Two will be considered here.

For each project day, the correct PWL value for the daily population--call this p_i --was calculated from the known μ_i and σ^2 -values. By using computer simulation, it was possible to obtain the correct value of p_i , a luxury that is not possible in an actual construction situation. Also, based on the daily sample, the estimated PWL--call this \hat{p}_i --was determined for each approach (A, B, C) and each method (1-5). The performance of each PWL estimator was then evaluated by determining the difference ($\delta_i = \hat{p}_i - p_i$) between the estimated and correct PWL values. Average values of δ_i were determined for each project and pooled for the 100 projects to determine a mean PWL error for each approach and method used in each simulation. These daily PWL errors should be distributed about zero, i.e., have a mean error equal to zero. A positive or negative mean error is indicative of some bias in the method of estimating p_i . To evaluate the variability associated with each method for estimating p_i , the squared PWL error [$\delta_i^2 = (\hat{p}_i - p_i)^2$] was also determined for each project day, and then an average squared PWL error was determined for each project

and for the entire simulation for each method of estimating p_i .

Analysis of Results

The results of a typical computer simulation for density are given in Table 7 ($n = 4$). In this table, method 1 is the traditional quality-index approach (based on \bar{X}_i and s_i) and can be used as the control against which to measure the performance of the EB FWL estimators.

A review of Table 7 indicates that the traditional estimator produces a better estimate in terms of the average FWL error (δ) because the average error is nearly zero; i.e., it is an unbiased estimator. The EB estimators, on the other hand, produce average errors that are slightly biased toward the high side. Nevertheless, the EB estimators always produce a lower average squared error than the traditional method. This means that the EB estimate for FWL has a higher likelihood of being close to the true FWL value because it has less variability associated with it. Space does not allow a detailed analysis of the results in Table 7 here. A thorough discussion of the results of the computer simulation analyses may be found elsewhere (13).

CONCLUSIONS

In this paper the steps involved in the development of an acceptance approach for bituminous pavements based on EB techniques are presented. The approach developed uses a Bayesian estimator for the percentage of the lot of material within specification limits (FWL) for determining the level of quality for the pavement. The estimator was developed on the basis of three assumptions:

1. The production process follows a normal distribution,
2. The daily population means follow a normal distribution, and
3. There is a constant process variance.

GOF tests conducted on data collected from 13 bituminous runway-paving projects verified the reasonableness of the three assumptions. Computer simulation indicated that the Bayesian estimators for FWL were slightly biased toward higher estimated FWL values but that they had a lower variance for the estimated FWL value than that obtained with the traditional quality-index method.

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