REFERENCE

1. M.C. Anday and C.S. Hughes. Compaction Control of Granular Base Course Materials by Use of Nuclear Devices and a Control Strip Technique. HRB, Highway Research Record 177, 1967, pp. 136-143.

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Software Package for Design and Analysis of Acceptance Procedures Based on Percent Defective

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The trend toward statistical end-result specifications has led to the development of construction specifications based on the concept of percent defective. To analyze the risks and determine the effectiveness of the acceptance procedures associated with these specifications, operating-characteristic curves must be constructed. However, many potential users do not have a working knowledge of the noncentral t and beta distributions necessary for this development. The underlying theory, several useful references, and a conversational computer program that greatly simplifies the design and analysis of specifications of this type are presented.

The current trend toward statistical end-result specifications has been a natural step in the evolution of the highway quality-assurance system. Whereas the earlier method-type specifications outlined in detail precisely how the work was to be accomplished, the more modern approach has been to define the characteristics and quality requirements of the finished product. Contractors are allowed considerable flexibility in meeting these requirements and the specifying agency is responsible primarily for the evaluation of the finished work.

The end-result approach offers several advantages over the earlier method-type specifications. First, by recognizing the existence of both inherent and testing variability, it deals with construction parameters in a more realistic manner. Highway engineers have begun to realize that it is not unusual, nor necessarily undesirable, for a small percentage of test values to fall outside realistic specification limits. Second, by defining the control of the construction process as the contractor's responsibility and the acceptance of the work (end result) as the agency's responsibility, the likelihood of contractual disputes can be reduced. Third, by clearly defining acceptance criteria and random-sampling procedures, the risks to both the contractor and the highway agency can be controlled and known in advance. Under the earlier method-type specifications, a contractor's bid was often influenced by the reputation of the highway inspector assigned to the project. Fourth, the development of adjusted-payment schedules provides a practical means to deal with work that is substandard but not so deficient that it warrants removal and replacement. Finally, because the random-sampling plans avoid the biases that are likely to occur when an inspector attempts to select a representative sample, reliable estimates of the as-built construction quality can be made. This information can also be used as feedback to determine whether further modifications of the specifications are desirable.

One of the most important steps in the design of an end-result specification is the development of the operating-characteristic (OC) curve describing its capabilities. Although most of the necessary theory is available in one form or another, much of it is not familiar or easily accessible to highway engineers. In this paper this theory is outlined, appropriate references are cited, and a conversational computer program that greatly simplifies the design or analysis of the type of statistical acceptance procedure normally used with end-result specifications is presented.

PERCENT DEFECTIVE AS A MEASURE OF QUALITY

Although several statistical measures of quality are available, highway engineers have exhibited a strong preference for the concept of percent defective, the estimated percentage of the work falling outside specification limits (or its complement, the percent within limits). This measure is particularly appealing, not only because the amount of material falling within limits is believed to be strongly related to actual performance, but because it can be applied to virtually any construction quality characteristic. This general philosophy is promulgated in Standard 214 (1) of the American Concrete Institute (ACI), for example, although the ACI acceptance criteria do not use a purely percent defective approach.

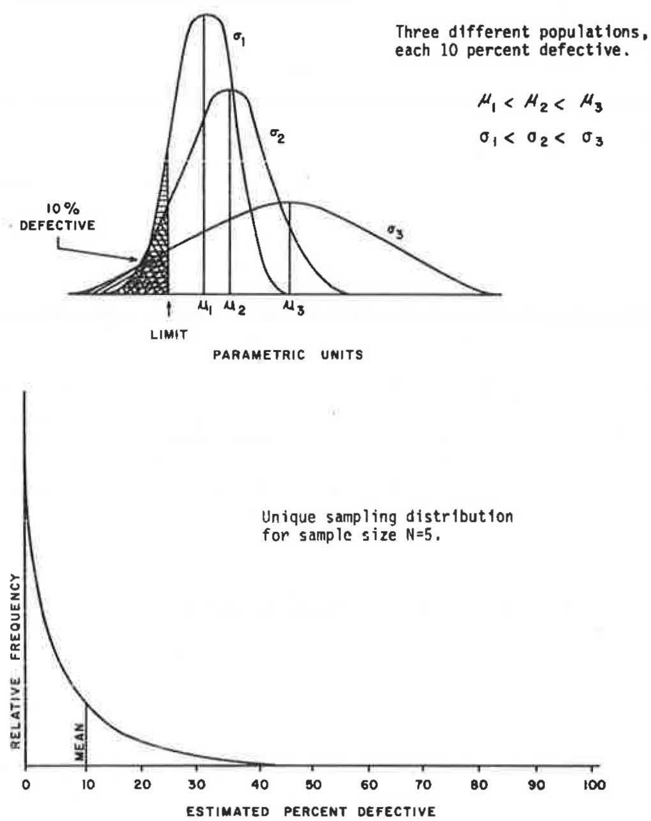
Two statistical parameters commonly used with these procedures are the process mean and standard deviation. In this paper the situation is addressed in which the values of these parameters are not known and must be estimated from sample observations. This development is appropriate for those situations in which these values may change during the course of a project.

Figure 1 illustrates three possible parent populations having identical percent defective levels and the sampling distribution associated with a sample size of 5. The sampling distribution is strongly skewed, but because the technique for estimating percent defective is unbiased, its mean is exactly at the true population percent defective. The significance of this is that although the quality of any single lot may be overestimated or underestimated, the long-term average of these estimates will be exactly equal to the true lot quality. This is of particular importance in developing fair and equitable construction specifications.

The theory associated with the development of specifications based on percent defective is somewhat involved and uses frequency distributions seldom encountered in introductory statistics courses.

Estimates of percent defective are derived from the symmetrical beta distribution (2), and the associated OC curves are computed with the aid of the noncentral t distribution (3). The literature on these two distributions is cited for the sake of completeness, but for practical purposes a conversational computer program (4) will be presented that tremendously simplifies the application of this theory. Acceptance plans based on percent defective can also be developed by use of Military Standard 414 (5), although the flexibility is quite limited.

Figure 1. Distinction between distribution of population parameter and distribution of percent defective estimates.



ESTIMATING PERCENT DEFECTIVE

The mechanics of estimating the percent defective of any construction parameter are exceedingly simple but require that the basic assumptions of a normal population and random sampling be satisfied. Once the sample has been taken and the test values have been obtained, the mean ($\bar{X}$) and standard deviation (S) are computed. Then, in order to estimate the percent defective below some lower limit (L), a quality index (Q) is calculated as follows:

$$Q = (\bar{X} - L)/S \quad (1)$$

All that remains is to determine the level of percent defective associated with the computed value of Q. This is accomplished by means of special tables such as that shown in Figure 2. (A different table is used for each sample size. Due to certain limitations of the underlying beta distribution, no tables exist for sample sizes smaller than 3.) For example, for a sample size of 5 and a quality index of 1.25, the estimated percent defective read from Figure 2 is 9.46.

If it were desired to estimate the percentage of material falling above an upper limit (U), the Q statistic would be computed by Equation 2 and the same procedure would be employed with the appropriate Q value table.

$$Q = (U - \bar{X})/S \quad (2)$$

For acceptance procedures with both lower and upper limits, the percent defective estimate is the sum of the results obtained by using Equations 1 and 2. The analysis is much more complicated in this case, however, and is beyond the scope of this paper.

Figure 2. Typical Q table for estimating percent defective of normal population.

SAMPLE SIZE = 5

Q	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	50.00	49.64	49.29	48.93	48.58	48.22	47.86	47.51	47.15	46.80
0.1	46.44	46.09	45.73	45.38	45.02	44.67	44.31	43.96	43.60	43.25
0.2	42.90	42.54	42.19	41.84	41.48	41.13	40.78	40.43	40.08	39.72
0.3	39.37	39.02	38.67	38.32	37.97	37.62	37.28	36.93	36.58	36.23
0.4	35.88	35.54	35.19	34.85	34.50	34.16	33.81	33.47	33.12	32.78
0.5	32.44	32.10	31.76	31.42	31.08	30.74	30.40	30.06	29.73	29.39
0.6	29.05	28.72	28.39	28.05	27.72	27.39	27.06	26.73	26.40	26.07
0.7	25.74	25.41	25.09	24.76	24.44	24.11	23.79	23.47	23.15	22.83
0.8	22.51	22.19	21.87	21.56	21.24	20.93	20.62	20.31	20.00	19.69
0.9	19.38	19.07	18.77	18.46	18.16	17.86	17.55	17.25	16.96	16.66
1.0	16.36	16.07	15.78	15.48	15.19	14.91	14.62	14.33	14.05	13.77
1.1	13.48	13.20	12.93	12.65	12.37	12.10	11.83	11.56	11.29	11.02
1.2	10.76	10.50	10.23	9.98	9.72	9.46	9.21	8.96	8.71	8.46
1.3	8.21	7.97	7.73	7.49	7.25	7.02	6.79	6.56	6.33	6.10
1.4	5.88	5.66	5.44	5.23	5.02	4.81	4.60	4.39	4.19	3.99
1.5	3.80	3.61	3.42	3.23	3.05	2.87	2.69	2.52	2.35	2.19
1.6	2.03	1.87	1.72	1.57	1.42	1.28	1.15	1.02	0.89	0.77
1.7	0.66	0.55	0.45	0.36	0.27	0.19	0.12	0.07	0.02	0.0

(Q VALUES ARE COMPUTED BY THE STANDARD DEVIATION METHOD. FOR POSITIVE Q VALUES, THE PERCENT DEFECTIVE ESTIMATE IS READ DIRECTLY FROM THE TABLE. FOR NEGATIVE Q VALUES, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.00.)

DEFINITION OF QUALITY LEVELS

In the development of statistical specifications, two quality levels are of particular significance. These are the acceptable quality level (AQL) and the rejectable quality level (RQL), defined as follows: AQL is the maximum percent defective that (for the purposes of the acceptance specification) can be considered satisfactory as a process average. RQL is the percent defective value that if equaled or exceeded represents a seriously defective or potentially dangerous level of quality.

A common setting for the AQL is 10 percent defective. The RQL is usually set at a point at which the specifying agency reserves the option to require removal and replacement of the work at the contractor's expense. Typical values might be in the range of 40 to 60 percent defective. (It should be noted that it is possible to develop an acceptance procedure without explicitly defining an RQL.)

STATISTICAL QUALITY INFERENCES

In making an inference about the quality of any particular lot, two types of error are possible. AQL lots may be rejected or RQL lots may be accepted. The risks of making these errors are known as the producer's and consumer's risks, respectively, and are defined as follows: Alpha (α) is the producer's risk that AQL material will be rejected. Beta (β) is the consumer's risk that RQL material will be accepted.

Obviously, it is desirable that both risks be as small as possible. However, the cost of sampling, the consequences of accepting defective work, and other factors tend to dictate the levels of risk that are considered acceptable.

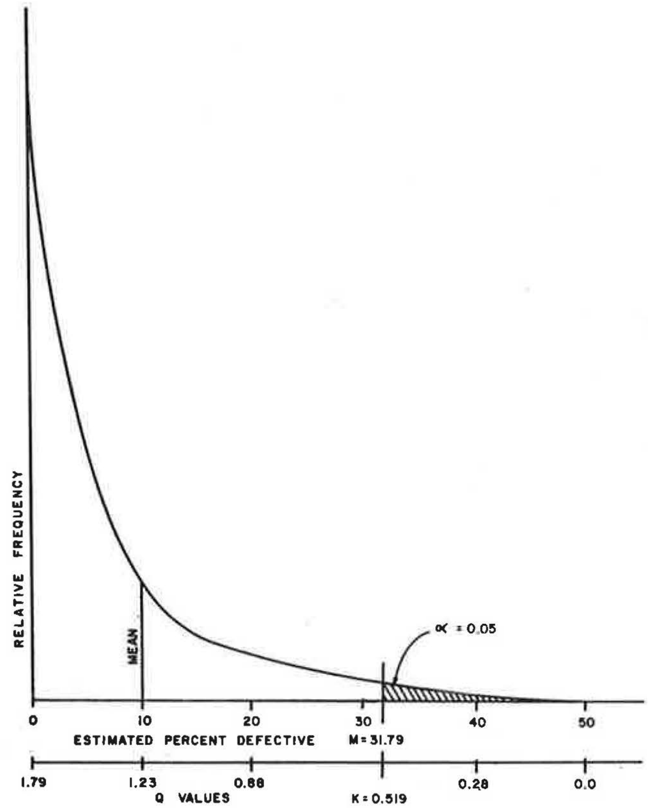
The percent defective estimate sampling distribution for a 10 percent defective quality level and a sample size of 5 is shown in Figure 3. As with any frequency distribution, there exists some limit on the estimated percent defective axis that cuts off 5 percent of the area in the upper tail. As shown in Figure 3, this limit occurs at 31.79 percent defective. If a lot is inferred to be AQL whenever a sample size of 5 estimates the quality as 31.79 percent defective or less, truly AQL material will be rejected only 5 percent of the time. In other words, if a sample size of 5 is used and a producer's risk of 0.05 is desired, the tolerable percent defective (M) estimated by a sample is 31.79 percent. For practical purposes, a value of M = 32 percent would probably be used.

The Q values used to estimate percent defective may also be scaled on the abscissa of the sampling distribution for percent defective as shown in Figure 3. For a given sample size, there is a unique correspondence between any Q value and a percent defective estimate. It is as meaningful to say that 5 percent of the sampling distribution lies beyond the Q value of k = 0.519 as it is to say that 5 percent of the percent defective estimates exceed M = 31.79 percent. The limit k of the Q scale, which corresponds to the limiting percent defective estimate (M), is defined as the acceptability constant.

Specification of either M or k along with a sample size and a lower or upper limit uniquely identifies an acceptance plan for a single-limit statistical specification that uses the percent defective approach. There are three ways in which the acceptance plan developed above could be stated:

1. Accept a lot as AQL (10 percent defective) if the estimated percent defective based on a sample size of 5 is less than or equal to M = 32 percent.

Figure 3. Sampling distribution for true population percent defective of 10 percent and sample size of 5.



2. Accept a lot as AQL if the Q statistic, based on a sample size of 5, is greater than or equal to k = 0.519.

3. Accept a lot if

$$\bar{X} \geq L + kS \tag{3}$$

where

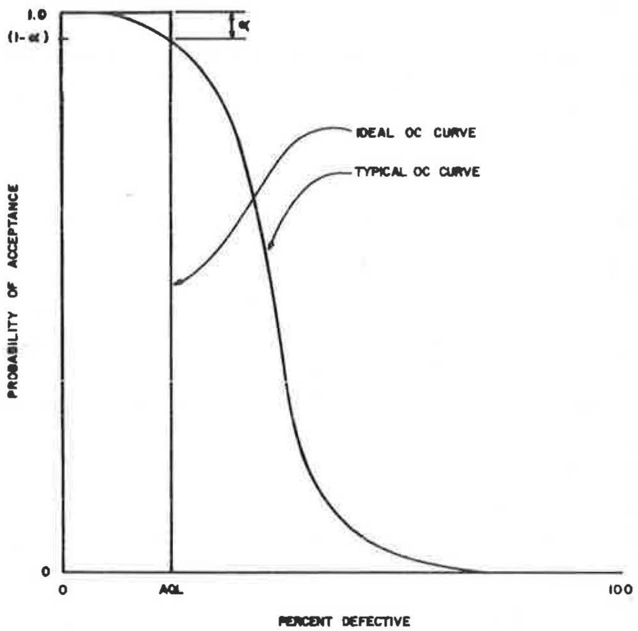
$\bar{X}$ = average value of N = 5 tests,
S = standard deviation of N = 5 tests,
L = a lower specification limit, and
k = the acceptability constant, 0.519.

If exactly AQL material were submitted under any one of these acceptance procedures, it would be accepted approximately 95 percent of the time.

The OC curve is a graphical representation of the manner in which an acceptance plan actually works and is uniquely identified by two parameters: the sample size and either k or M. It relates the probability of acceptance to the entire range of the percent defective quality measure. It will, at a glance, indicate the error risks that are incurred and it permits the meaningful comparison of alternative acceptance procedures.

An ideal OC curve is shown in Figure 4. It consists of two horizontal tails and a vertical line directly above the AQL. This curve indicates that AQL material (or better) will always be accepted. This ideal OC curve also implies that a wrong inference will never be made. With a sample size of 5, however, it is indicated in Figure 3 that the likely range of percent defective estimates extends from 0.0 to nearly 50.0 percent when the true quality is 10 percent defective. For example, if the allowable percent defective is 32 percent, it can be seen in Figure 3 that about 5 percent of the estimates will

Figure 4. Ideal and typical OC curves.



exceed M. Thus, real-world OC curves pass through the point (AQL, 1 - α) and have a more gradual slope. This more gradual slope reflects the element of risk associated with statistical acceptance procedures. It is through the construction and analysis of curves such as this that fair and effective specifications are developed.

NONCENTT PROGRAM

An interactive software package, named NONCENTT after the noncentral t distribution, has been developed to facilitate the design and evaluation of acceptance plans based on percent defective. It is written in the standard FORTRAN language and should be compatible with most computer installations. All the necessary subroutines have been incorporated into the coding so that the program is completely self-contained. Once the program has been loaded and compiled onto a computer system, it may be executed without further assistance from systems-level personnel.

NONCENTT is typically accessed by the instructions "run noncentt." As shown in Figure 5, the computer will respond with the program's title, a brief description of the program's purpose, instructions concerning the interaction procedure, and a request for information. Note the convention, which will be followed in all subsequent examples, of printing all input information in lowercase letters against the left-hand margin and all output information indented at least 10 spaces and formatted in uppercase letters. Note also the conversational nature of the expected interaction. Program requirements, as well as all diagnostic error messages, are always expressed in an easily understood conversational manner.

Detailed explanations of the input requirements and the calculations performed are available at any input stage of a NONCENTT session. In response to the instruction SELECT THE OPTION OF INTEREST, suppose the word "help" were entered. This would cause a more detailed explanation of the available options to be printed out, which is also shown in Figure 5.

On review of the available options, suppose it is

Figure 5. Initial portion of interactive session with NONCENTT.

```
run noncentt
EXECUTION BEGINS...

          NONCENTT:
          ---------

          APPLICATIONS OF THE NONCENTRAL T AND
          SYMMETRICAL BETA DISTRIBUTIONS TO THE
          DESIGN OF STATISTICAL SPECIFICATIONS



               AT SELECTED POINTS IN THE EXECUTION OF THIS PROGRAM YOU
          WILL BE ASKED FOR SPECIFIC INFORMATION.  YOU WILL BE EXPECTED TO
          RESPOND IN ONE OF THREE WAYS:

                    1) INPUT THE REQUESTED INFORMATION,
                    2) TYPE 'HELP' FOR A MORE DETAILED EXPLANATION, OR
                    3) TYPE 'QUIT' TO  EXIT ANY OPTION OR TERMINATE
                       NONCENTT SESSION.


          SELECT THE OPTION OF INTEREST.


help
               THE NONCENTRAL T AND SYMMETRICAL BETA DISTRIBUTIONS
          UNDERLIE ONE-SIDED VARIABLES SPECIFICATIONS WITH THE STANDARD
          DEVIATION UNKNOWN, SUCH AS THOSE FOUND IN MILITARY STANDARD
          414.  DESIGNERS OF DOUBLE LIMIT SPECIFICATIONS ARE REFERRED TO
          MILITARY STANDARD 414, AND TO 'QUALITY CONTROL AND INDUSTRIAL
          STATISTICS', BY A.J. DUNCAN.

               THE FOLLOWING OPTIONS ARE AVAILABLE IN THIS SOFTWARE
          PACKAGE.  ADDITIONAL EXPLANATORY COMMENTS WILL BE AVAILABLE
          AS EACH OPTION IS SELECTED.

                    1) ESTIMATION OF LOT PERCENT DEFECTIVE.

                    2) PASSING AN OC CURVE THROUGH ONE POINT.

                    3) PASSING AN OC CURVE THROUGH TWO POINTS.

                    4) ESTABLISHING POINTS ON AN OC CURVE.

                    5) CONFIDENCE LIMITS FOR TRUE PERCENT DEFECTIVE.

                    6) PROBABILITY OF EXCEEDING CRITICAL PERCENT DEFECTIVE
                       LIMITS.

                    7) THE EXPECTED PAYMENT CURVE FOR A STATISTICAL
                       SPECIFICATION.

                    *) TYPE 'QUIT' TO TERMINATE NONCENTT SESSION.




          SELECT THE OPTION OF INTEREST.


two

ERROR ------- IMPROPER INPUT OR INPUT FORMAT. RETYPE LAST LINE.
```

decided to run option 2 first. If the word "two" is typed instead of the numeral 2, it will not be accepted by the computer. NONCENTT will perform an error check on all data entered for compatibility with the requested information and for logical consistency. In this example, the selected option was not properly identified. The numeral 2 should have been entered to correctly access the desired option.

The NONCENTT session illustrated by the following examples has been streamlined for conciseness. No further input errors will be made nor will help be requested. The direct interaction that follows demonstrates the efficiency available when this software package is accessible to an experienced user. A summary of the NONCENTT options currently available is given in Table 1.

EVALUATION AND MODIFICATION OF EXISTING ACCEPTANCE PROCEDURE

For the purposes of this paper, assume that an agency is currently using a specification in which the AQL is 10 percent and the sample size is 5. Further assume that the RQL has been identified as 50 percent defective and that both alpha and beta are intended to be at the 0.05 level. Option 2 of the NONCENTT program can be used to determine the ac-

Table 1. Capabilities of NONCENTT program.

Option No.	Program Function: Option Title	Possible Application
1	Estimation of lot percent defective	Converting Q statistic into percent defective estimates Converting acceptability constant (k) into maximum allowable percent defective in a sample (M)
2	Passing an OC curve through single predetermined point	Identifying (N, k) combination that results in a specified producer's risk (α) that AQL material will be rejected Identifying (N, M) combination that results in a specified producer's risk (α) that AQL material will be rejected Placing a one-tailed confidence limit on a percent defective estimate provided the true population percent defective is known
3	Passing OC curve through two predetermined points	Identifying sample size and acceptance parameter required to pass an OC curve through both (AQL, 1 − α) and (RQL, β)
4	Establishing points on OC curve	Performing a detailed investigation of probability of accepting material whose true quality may vary over a range of possible values
5	Confidence limits for true percent defective	Determining two extreme percent defective estimate distributions that could have produced, with a level of risk equal to α/2, the observed sample estimate
6	Probability of exceeding critical percent defective limits	Determining probability of misinterpreting AQL quality to be RQL, or vice versa Determining likelihood of achieving a particular pay factor
7	Expected-payment curve for statistical specification	Determining expected payment associated with stepped, continuous linear, or continuous curvilinear adjusted-payment schedules

Figure 6. Option 2 of NONCENTT: passing OC curve through single predetermined point.

```
         SELECT THE OPTION OF INTEREST:

2

         PASSING AN OC CURVE THROUGH ONE POINT
         -------------------------------------

         ENTER:    1) THE SAMPLE SIZE                    (AN INTEGER ≥ 3)
                   2) THE PERCENT DEFECTIVE AT THE AQL        (A PERCENT)
                   3) THE PRODUCER'S RISK, ALPHA       (0.0< ALPHA <1.0)

5 10 0.05

            EITHER OF THE FOLLOWING CRITERIA MAY BE USED:
            ---------------------------------------------

                                      MAXIMUM ALLOWABLE
            ACCEPTABILITY             PERCENT DEFECTIVE
            CONSTANT                  IN A SAMPLE
            -------------             -----------------

              K = 0.519                  M = 31.79
```

Figure 7. Option 4 of NONCENTT: establishing points on OC curve.

```
         SELECT THE OPTION OF INTEREST.

4

         ESTABLISHING POINTS ON AN OC CURVE
         ----------------------------------

         ENTER:    1) THE SAMPLE SIZE                     (AN INTEGER ≥ 3)
                   2) EITHER 'K= *** ' OR 'M= *** '      (A LETTER, EITHER
                                                          K OR M, FOLLOWED
                                                          BY VALUE OF K OR
                                                          M.)
                   3) THE PERCENT DEFECTIVE RANGE OF         (TWO PERCENT
                      INTEREST                                VALUES)
                   4) THE OC CURVE PLOT INCREMENT             (A PERCENT)

5 K=0.519 10 90 10

                     POINTS ON OC CURVE
                     ------------------

                        N =    5
                        K =  0.519
         ----------------------------------------
         PERCENT               PROBABILITY
         DEFECTIVE             OF ACCEPTANCE
         ---------             -------------

          10.00                  0.949983
          20.00                  0.769359
          30.00                  0.531302
          40.00                  0.313540
          50.00                  0.155191
          60.00                  0.061190
          70.00                  0.017200
          80.00                  0.002668
          90.00                  0.000099
```

ceptability constant associated with N = 5, AQL = 10 percent, and $\alpha = 0.05$.

The printout in Figure 6 shows that for a sample size of 5, an AQL of 10 percent, and an alpha risk of 0.05, the acceptability constant (k) is 0.519. Alternatively, the tolerable percent defective as estimated by a sample of 5 tests is 31.79 percent, or approximately 32.0 percent.

The OC curve associated with the above acceptance procedure can be calculated with option 4 of NONCENTT. This option will accept either the k or the M parameter and it will compute the acceptance probabilities over a range of true percent defective values selected by the user. Assume that the OC curve is to be identified over the quality range of 10 to 90 percent in steps of 10 percent defective. The appropriate entries and the resulting output are shown in Figure 7.

The nine points on the OC curve computed by option 4 have been plotted in Figure 8. Note that at the RQL (i.e., the 50.0 percent defective quality level) the probability of acceptance is approximately 0.16, or 16 percent. Thus the consumer's risk of accepting RQL material is considerably larger than the intended value of $\beta = 0.05$. It is apparent that a sample size of 5 is simply too small to correctly recognize both the AQL and the RQL 95 percent of the time.

One may be tempted to pass the OC curve through the (RQL, β) point rather than the (AQL, $1 - \alpha$). This could be done, but it would increase the producer's risk to approximately 0.24. If the α and β risks are to be balanced near the intended level of 0.05, the sample size must be increased. To pass an OC curve through both (AQL, $1 - \alpha$) and (RQL, β), option 3 is selected as shown in Figure 9.

Properties of the OC curve for k = 0.686 indicate that this plan produces nearly the desired risks at both the AQL and the RQL and the required sample size is 9. This OC curve has also been plotted in Figure 8, and provided that the required sample size of 9 is reasonable, the acceptance procedure development process would be complete. Otherwise, if the sample size is reduced, some increase in acceptable risk levels would have to be tolerated. Further runs of option 3 could then be made to arrive at a suitable compromise.

Figure 8. Two OC curves, each passing through (AQL, 1 - α).

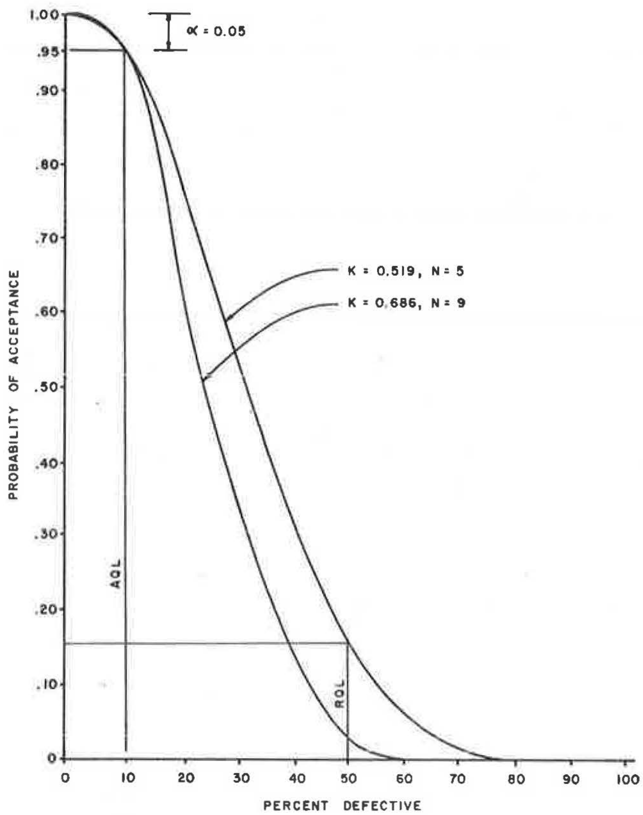


Figure 9. Option 3 of NONCENTT: passing OC curve through two predetermined points.

```
        SELECT THE OPTION OF INTEREST.

--
3

        PASSING AN OC CURVE THROUGH TWO POINTS
        --------------------------------------

        ENTER:    1) THE PERCENT DEFECTIVE AT THE AQL          (A PERCENT)
                  2) THE PRODUCER'S RISK, ALPHA          (0.0 < ALPHA < 1.0)
                  3) THE PERCENT DEFECTIVE AT THE RQL          (A PERCENT)
                  4) THE BUYER'S RISK, BETA              (0.0 < BETA < 1.0)
--
10 0.05 50 0.05

             AQL   = 10.00%   |   RQL  = 50.00%
             ALPHA = 0.0500   |   BETA = 0.0360
             -----------------+-----------------

             SAMPLE SIZE              N =    9
             ACCEPTABILITY CONSTANT   K =  0.686
             MAX. ALLOW. PCT. DEF.    M = 25.21%
```

APPLICATION TO PAY SCHEDULES

It has become common practice for highway agencies to employ adjusted-payment provisions and a recent paper succinctly states the purpose and justification for such an approach (6, p. 18):

> A construction item that falls just short of the specified quality level does not warrant rejection but neither does it deserve 100 percent payment. Accordingly, statistical specifications usually employ some form of adjusted pay schedule to award payment in proportion to the level of quality actually achieved.... Ordinarily, a pavement is designed to sustain a specified number of load applications before major repair (overlaying with bituminous concrete) is required. If, due to construction deficiencies, the pavement is not capable of withstanding the design loading, it will fail prematurely. The necessity of repairing this pavement at an earlier date results in an additional expense that, since it usually occurs long after any contractual obligations have expired, must be borne by the highway agency. It is the purpose of the adjusted pay schedule to withhold sufficient payment at the time of construction to cover the extra cost anticipated in the future as the result of deficient-quality work.

Table 2. Sample stepped pay schedule.

Step	Range of percent defective	Pay Factor (%)
1	0.0-10.00	100
2	10.01-20.00	90
3	20.01-30.00	80
4	30.01-40.00	70
5	40.01-50.00	60
6	50.01-100.00	50[a]

[a]The agency reserves the right to require removal and replacement at the contractor's expense of any lot the percent defective of which exceeds 50.00 percent. If for practical reasons this option is not invoked, the lot receives the minimum pay factor of 50 percent.

There are two basic types of adjusted-payment schedules--stepped and continuous. Stepped pay schedules define discrete intervals of quality and award a single pay factor for each. Continuous pay schedules express the pay factor in equation form as a function of the selected quality measure. Although stepped pay schedules are more common, continuous pay schedules do offer certain advantages. Besides being more concise, they more precisely match the appropriate pay factor with the estimated quality for any given lot. This tends to minimize the harshness of having just missed the next higher pay level. Nevertheless, stepped and continuous pay schedules can be constructed that will have essentially the same long-term performance.

For demonstration purposes, suppose that a highway agency has developed the stepped pay schedule shown in Table 2 for use with a particular acceptance procedure. The first step of this pay schedule indicates that a pay factor of 100 percent will be awarded if the percent defective quality measure is less than or equal to 10 percent, the AQL. If the estimated percent defective is greater than 10 percent but less than or equal to 20 percent defective, 90 percent of the contract amount will be awarded, and so on. Note that for practical purposes this stepped pay schedule can be briefly summarized by listing only the upper limits of the quality intervals along with the associated pay factors.

It would be misleading, however, to compare alternative pay schedules purely on the basis of their indicated pay factors. That a pay factor is associated with some level of quality does not guarantee that material of that quality will, on the average, receive that pay factor. Seldom is that the case. True quality levels are estimated by the quality levels of samples, and these sample estimates are used in the pay-factor determinations. The distribution of pay factors, therefore, is influenced both by the sample-estimate distribution and by the adjusted-payment schedule. In most cases, some degree of distortion is found to occur between the respective distributions.

Expected pay factors are computed as the sum of the products of all pay factors multiplied by the probability of obtaining each pay factor (7). This computation will numerically identify the mean value of the pay-factor distribution. The expected payment (EP) curve relates probable payment to the true level of quality. This allows one to read the average pay factor directly from the Y-axis for any level of true percent defective, analogous to the OC curves already discussed.

The EP curve may be computed by option 7 of the NONCENTT program. This first example will produce the EP curve associated with the stepped pay schedule just presented. In order to perform the necessary computations throughout the entire range of percent defective, it will be assumed that all RQL lots receive the minimum pay factor of 50 percent. The input and output are shown in Figure 10.

The EP curve has been plotted in Figure 11 and provides the means to judge the probable payment from the perspectives of both the highway agency and the contractor. The underlying goals are (a) to provide sufficient incentive for the contractor to produce good-quality work and (b) to pay a fair reduced price when the work is substandard. To determine whether the first objective has been met, the highway agency must judge whether it is in the contractor's best interest to produce the desired level of quality. To judge whether the second objective has been met, various methods have been proposed (6-9). In some cases, when little information has been available relating quality measures to performance, these methods have necessarily been quite arbitrary. In other cases, for which the quality-performance relationship can be established, more logical and rational procedures can be employed (6).

Nevertheless, there is one obvious problem apparent in Figure 11. A producer who consistently supplies the AQL of 10 percent defective will not, on the average, receive 100 percent payment. Instead, the expected pay factor for AQL work is approximately 93 percent. As demonstrated in an earlier paper (10), an inequitable condition such as this imposes a severe hardship on the producer.

To correct this problem, the EP curve must be raised so that the expected pay factor is 100 percent when the quality is exactly at the AQL. To do this, it is necessary to use a pay schedule that is capable of awarding pay factors greater than 100 percent. Either a crediting provision (9), in which pay factors greater than 100 percent are used to

Figure 10. Option 7 of NONCENTT: establishing points on EP curve.

```
          SELECT THE OPTION OF INTEREST.

 7

          EXPECTED PAYMENT CURVE
          ----------------------

             THIS OPTION COMPUTES THE EXPECTED PAY FACTOR BASED ON A
          GIVEN PAY SCHEDULE AND SAMPLE SIZE.

             ENTER '1' IF A CONTINUOUS PAY SCHEDULE IS TO BE USED,
          OR ENTER THE NUMBER OF STEPS IF A STEPPED PAY SCHEDULE IS
          PREFERRED.

 6

          ENTER:      THE UPPER PERCENT DEFECTIVE        ( 6 PERCENT DEFEC-
                      LIMIT FOR EACH STEP                 TIVE VALUES)

 10 20 30 40 50 100

          ENTER:   THE  6 PAY FACTORS ASSOCIATED
                   WITH THE  6 PAY STEPS.                ( 6 PERCENT VALUES)

 100 90 80 70 60 50

          ENTER:   THE SAMPLE SIZE                       (AN INTEGER ≥ 3)

 5

          ENTER:   1) THE PERCENT DEFECTIVE RANGE OF      (TWO PERCENT
                      INTEREST                             VALUES)
                   2) THE EP CURVE PLOT INCREMENT          (A PERCENT)

 10 90 10

                      POINTS ON THE EXPECTED PAYMENT CURVE
                      ------------------------------------

                      SAMPLE SIZE          =    5

            STEP            QUALITY INTERVAL                  PAY FACTOR
            ----     ------------------------------           ----------
              1        0.0% ≤ PCT. DEF. ≤  10.0%                100.0%
              2       10.0% < PCT. DEF. ≤  20.0%                 90.0%
              3       20.0% < PCT. DEF. ≤  30.0%                 80.0%
              4       30.0% < PCT. DEF. ≤  40.0%                 70.0%
              5       40.0% < PCT. DEF. ≤  50.0%                 60.0%
              6       50.0% < PCT. DEF. ≤ 100.0%                 50.0%

                      PERCENT                  EXPECTED
                      DEFECTIVE                PAY FACTOR, %
                      ---------                -------------
                        10.00                     93.156
                        20.00                     84.453
                        30.00                     75.560
                        40.00                     67.289
                        50.00                     60.334
                        60.00                     55.169
                        70.00                     51.952
                        80.00                     50.442
                        90.00                     50.025
```

Figure 11. EP curve for six-step adjusted-pay schedule shown in Figure 10.

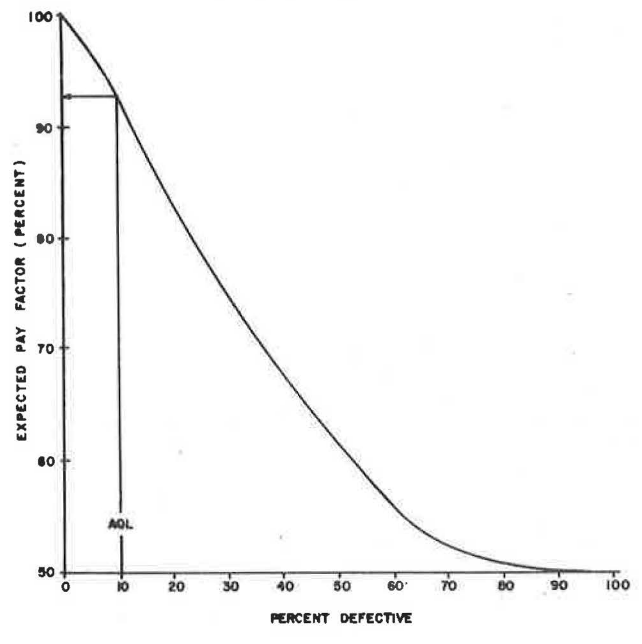


Figure 12. Points on EP curve for four-step adjusted-pay schedule.

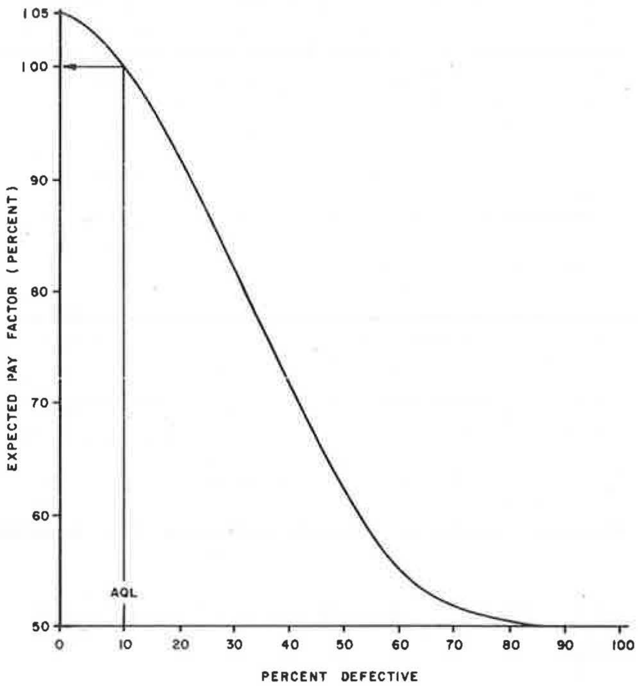
```
              POINTS ON THE EXPECTED PAYMENT CURVE
              ------------------------------------

              SAMPLE SIZE          =    5

    STEP            QUALITY INTERVAL                  PAY FACTOR
    ----     ------------------------------           ----------
      1        0.0% ≤ PCT. DEF. ≤  10.0%                105.0%
      2       10.0% < PCT. DEF. ≤  20.0%                 97.0%
      3       20.0% < PCT. DEF. ≤  40.0%                 90.0%
      4       40.0% < PCT. DEF. ≤ 100.0%                 50.0%

              PERCENT                  EXPECTED
              DEFECTIVE                PAY FACTOR, %
              ---------                -------------
                10.00                     97.826
                20.00                     92.143
                30.00                     82.207
                40.00                     71.571
                50.00                     62.253
                60.00                     55.584
                70.00                     51.031
                80.00                     50.337
                90.00                     50.013
```

Figure 13. EP curve for four-step pay schedule shown in Figure 12.



offset pay factors below 100 percent, or a true bonus provision (11) may be used. It will be assumed in the next example that a bonus provision is in effect.

Option 7 of the NONCENTT program was run once again to produce the output shown in Figure 12. As before, it is assumed that the minimum pay factor of 50 percent is assigned when the lot is estimated to be at or below the RQL. The associated EP curve has been plotted in Figure 13.

Three points are worthy of note concerning the EP curve in Figure 13. First, the AQL (10 percent defective) now receives an expected payment of virtually 100 percent, whereas large percent defective values retain their previous expected payment. Second, the quality intervals identified in the pay schedule in Figure 12 need not be directly associated with the AQL and RQL definitions. This emphasizes that the adjusted-payment schedule addresses the level-of-quality estimates, whereas the AQL and RQL definitions pertain to true levels of quality. Consequently, it is the EP curve that should be analyzed, not the pay schedule itself. Finally, the pay schedule has been simplified. Four pay levels are now specified rather than six.

At this point, the highway agency must judge whether the acceptance procedure is suitable. Recent publications (6-8,11) provide guidance in the development of equitable and effective specifications, but the ultimate decision must rest with the agency itself. The NONCENTT program has served its purpose by providing the information on which this decision can be based.

A continuous (equation-form) pay schedule could also be used. Again by using option 7, it was found by trial and error that Equation 4 is essentially equivalent to the stepped pay schedule shown in Figure 12. Here again, it is assumed that the minimum pay factor of 50 percent is assigned whenever the percent defective estimate is greater than or equal to 40 percent.

$$PF = 105.0 - 0.5PD \quad (4)$$

where PF is the pay factor in percent and PD is the estimated percent defective.

For this particular example, the linear function used in Equation 4 produces essentially the same EP curve as that shown in Figure 13. In other cases, it may be necessary to include a quadratic term in the pay equation. Accordingly, option 7 of NONCENTT provides this capability.

SUMMARY AND CONCLUSIONS

Statistical acceptance procedures based on percent defective are now in common use by many highway agencies. In order to develop effective specifications in an expeditious manner and minimize costly and time-consuming field trials, it is necessary to develop and compare the OC curves for the various plans under consideration. This requires the use of statistical theory and special frequency distributions unfamiliar to many potential users. With the aid of the conversational computer program presented in this paper, however, these steps can easily be performed by individuals who have only a basic theoretical background.

This new capability should have several effects. First, it will greatly simplify the work of agencies planning to develop additional statistical specifications. Second, it will make it possible to more formally check existing specifications the risk levels of which may be far from optimal. Finally, this added convenience may serve to overcome the reluctance of the relatively few agencies who have yet to realize the advantages of statistical quality assurance.

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REFERENCES

1. Recommended Practice for Evaluation of Strength Test Results of Concrete (ACI 214-77). American Concrete Institute, Detroit, Mich., 1977.
2. J.H. Willenbrock and P.A. Kopac. The Development of Tables for Estimating Percentage of Materials within Specification Limits. Pennsylvania Transportation Institute, Pennsylvania State Univ., University Park, Pa., 1976.
3. G.J. Resnikoff and G.J. Lieberman. Tables of the Noncentral t Distribution. Stanford Univ. Press, Stanford, Calif., 1978.
4. R.T. Barros. The Theory and Computerized Design of Statistical Construction Specifications. Pennsylvania State Univ., University Park, Pa., M.S.C.E. thesis, 1982.
5. Sampling Procedures and Tables for Inspection by Variables for Percent Defective (Military Standard 414). Office of the Assistant Secretary, U.S. Department of Defense, 1957.
6. R.M. Weed. Method to Establish Pay Schedules for Rigid Pavement. TRB, Transportation Research Record 885, 1982, pp. 18-24.
7. J.H. Willenbrock and P.A. Kopac. Development of Price-Adjustment Systems for Statistically Based Highway Construction Specifications. TRB, Transportation Research Record 652, 1977, pp. 52-58.
8. Statistically Oriented End-Result Specifications. NCHRP, Synthesis of Highway Practice 38, 1976, pp. 8-9.

9. R.M. Weed. Unbiased Graduated Pay Schedules. TRB, Transportation Research Record 745, 1980, pp. 23-28.

10. R.M. Weed. Optimum Performance Under a Statistical Specification. TRB, Transportation Research Record 697, 1979, pp. 1-6.

11. R.M. Weed. Development of Multicharacteristic Acceptance Procedures for Rigid Pavement. TRB, Transportation Research Record 885, 1982, pp. 25-36.

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Correlation of Quality-Control Data and Performance of PCC Pavements

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The interrelationship between concrete pavement quality indicators and pavement performance is presented. In the study reported here, a literature review was conducted to help identify pavement quality indicators, such as water/cement ratio, strength, slump, air content, and so forth. A detailed field investigation was carried out in five states to collect quality-indicator data. A pavement-condition-rating (PCR) procedure was developed to collect PCR data for various pavement sections. Linear and nonlinear statistical analyses were conducted to develop models interrelating quality-control data with PCR data. The results of the statistical analyses and the nature of the models developed are discussed in detail.

The development of statistically based performance specifications as part of quality assurance programs in highway construction and maintenance is geared toward establishing construction and material quality levels based on expected performance. Payment adjustment schedules can then be adopted by which contractors are paid according to the performance of the final product. Payment penalties are based on failure to meet performance specifications rather than on material specifications. Such programs reduce the need for materials testing as well as the necessity for revising or creating materials-based specifications, and contractors have more latitude in their choice of materials and construction methods as long as the final product performs as expected. Nevertheless, the development and implementation of such specifications for pavement quality-control variables have raised two questions: How do material variables relate to pavement performance, and are these variables adequate indicators of pavement performance and quality?

Establishing interrelationships between pavement performance and quality-control criteria requires a basic understanding of the parameters affecting performance, an identification of those parameters indicative of quality, and a knowledge of their statistical variations. These parameters are usually classified into several categories--environmental, geometrical, boundary, material, construction, traffic loading, and design variables. The degree to which each variable influences performance is often affected by the interaction of numerous parameters, which requires sophisticated statistical analyses of the data in order to establish the relative significance of each variable.

The reliability of such interrelationships is highly dependent on the nature of the data collected, the statistical significance, and the validity. Many sources of material and construction quality data can prove to be biased or inaccurate. This is particularly true when subjective judgments are used to reject on site some materials suspected of not meeting specifications whereas other materials deemed to be in compliance are accepted and used without actual testing to verify whether they meet specifications. To establish accurate relationships between material quality indicators and performance, truly unbiased estimates of those parameters that affect pavement quality must be obtained.

The validity of these relationships also depends on having a reliable method for estimating pavement performance. Ideally, performance should be evaluated through detailed measurements, both destructive and nondestructive, to determine remaining life. Because this is a time-consuming and expensive process, a rapid, cost-effective, reliable pavement condition evaluation system that reflects actual conditions is needed.

In this paper the results of a recent study (1) of the interrelationships between quality indicators and performance of concrete pavements are reported. In that study, historical and construction data on selected quality variables were collected for 104 concrete pavement projects in five states. In addition to these data, the 104 projects were subjected to pavement condition evaluations to establish current performance levels. Statistical analyses were performed to establish relationships between performance rating and quality-indicator data, and 30 models were developed and tested.

A general model and representative data from the Ohio projects in that study are presented here to illustrate the types of performance and quality data required to develop statistically reliable relationships, the types of results that can be obtained from such analyses, and the impact of missing data on model development and reliability. A brief description of quality indicators known to affect concrete pavement quality and performance is presented in the next section. In the third section the pavement condition evaluation system used to rate performance of the pavement projects is discussed. Data collection is outlined in the fourth section, and in the last section the statistical analyses performed and results obtained are summarized.

QUALITY INDICATORS IN PCC PAVEMENTS

When quality-assurance programs are carried out that use statistically based quality-indicator specifications to meet performance requirements, it is neces-