The average effectiveness factors for OAF, California, and Utah are close to the expected 1.0, and the range of variation for OAF is not unreasonable. The Louisiana and Mississippi methods both result in substantially higher effectiveness factors and have more variability than the other methods. It should also be noted that the Louisiana, Mississippi, and Utah methods result in negative factors; i.e., the predicted overlay based on after-overlay measurements is greater than that based on before-overlay measurements.

Although the evidence is circumstantial, the results of the above analysis indicate that the proposed method is successful in evaluating the in situ layer properties and in determining the required overlay thicknesses.

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Publication of this paper sponsored by Committee on Pavement Rehabilitation.

Analysis of Stresses in Unbonded Concrete Overlay

BHAGIRATH LALL AND G. LEES

The results of a laboratory investigation in which static load tests were conducted on unbonded concrete overlays are presented. The investigation included study of the critical stresses and deflections that occur when an unbonded overlay of a concrete slab on subgrade is loaded at corner, edge, and interior positions and static loads are applied through varying contact areas. The distribution of stresses in the two slabs (overlay and underlying), whose ratio of thickness was varied, was investigated. The investigation was conducted on model-scale slabs. The construction material used for the model slabs was a microconcrete with high flexural strength. Concrete overlays are seen as an important means of strengthening existing concrete pavements. The use of multiple-layered systems for the new construction of thick slabs is regarded as an innovation in pavement design. The limitation of existing methods for the design of concrete overlays established the need for the investigation. Load-stress characteristics of the overlay systems are presented, and the effect of the warping of the slabs on these characteristics is analyzed to provide an understanding of the behavior of the pavement. The analysis presented is a forerunner to the establishment of a new method of design.

A laboratory investigation was conducted to study the stresses in an unbonded overlay of concrete pavement. The model slabs, whose dimensions were 3x9 ft (0.92x2.75 m), were constructed of microconcrete on sand subgrade. The stress measurements were made in both the underlying slab and the overlay in the three critical locations--corner, edge, and interior--when static loads were applied in these locations through circular contact areas. The scale of the model slabs was carefully selected not only to ensure simple geometric similarity but also to simulate as closely as possible the theoretical assumptions that can be applied to the analysis of the prototype. The assumptions and the resultant theory developed by Westergaard (1,2) were used to exercise dimensional control over the model in relation to the prototype. The horizontal dimensions of the model slabs were selected to ensure infinite behavior in both directions for the interior loading conditions.

The investigation focused mainly on the unbonded overlay providing frictionless interface to simulate the theoretical assumptions. To keep the friction at the interface of the underlying slab and the overlay as low as possible, an MGA pad was introduced between the two slabs to act as a slip layer. The MGA pad consists of a Melinex Polyester film (gage 100), Molyslip grease (containing 3 percent molybdenum disulphide), and a hardened aluminum sheet 0.003 in. (0.075 mm) thick. MGA pads were used by Hughes and Bahramian (3) to reduce end restraint in cube tests for the determination of uniaxial compressive strength of concrete and were reported to be effective.

The strain gages were fixed to the upper surface of the pavement slabs. Strain gages were so located for all three loading positions to determine maximum principal stresses. In addition, deflection measurements were made along several lines to establish the pattern of bending of the slabs. Unless otherwise specified, all references in this paper to stresses and deflections refer to maximum values for the location of loading under discussion. A hypothesis on the behavior of the overlay-pavement-earth system is presented in this paper, along with a description of the state of stress of an A/B overlay system, which consists of an overlay slab A on an existing slab B resting on subgrade, where A and B have equal or unequal thicknesses. In the following discussion, 1/1 and 1/1.33 overlay systems refer to (a) a 1-in. (25-mm) thick overlay on a 1-in.-thick slab and (b) a 1-in.-thick overlay on a 1.33-in. (34-mm) thick slab on subgrade.

CORNER LOADING OF A/B OVERLAY SYSTEM

In analyzing a single slab on subgrade, Westergaard (1) accounted for the degree of subgrade support in presenting the load-deflection and load-stress relations. That analysis provides corrections to be applied to values computed by using the equations of the original analysis (2). It is necessary to determine certain quantities experimentally in order to compute these corrections.

Teller and Sutherland (4) reported that for various reasons it was not possible to make a satisfactory determination of these quantities from their investigation. They reported stresses in the pavement under conditions of upward warping, no warping, and downward warping. Even then, they stated that it was possible that, even when the corners were warped downward, full contact with the subgrade was not established.

During Lall's investigation (5) of a single slab on subgrade before the overlay was constructed, similar observations were made; thus, it was established that the contact surface between the slab and the subgrade remains unpredictable due to the warping of the slab. The stresses in the pavement slab depend to a great extent on this degree of contact.

In the case of an unbonded overlay, another unpredictable factor is introduced in the contact surface between the slabs (apart from the contact surface with the subgrade). This obviously makes the analysis of an overlay system extremely complex. In this paper, an attempt is made to analyze this system by assuming that the slabs remain continuously in contact with each other and the subgrade, forming frictionless surfaces. Although these conditions are not likely to exist in practice, beginning with these assumptions helps to reduce the complexity of the problem. Modifications can then be introduced as the analysis progresses to make it realistic.

The ensuing analysis is based on the following assumptions:

1. When unloaded, the slabs are in contact with each other and the subgrade.
2. The surfaces at point of contact between the slabs and between the slab and the subgrade are frictionless.
3. Each slab behaves independently in the system when loaded and carries a stress proportional to its section modulus.
4. The middle plane of the slab forms the neutral plane so that the tensile stress at the top is equal to the compressive stress at the bottom of the slab.
5. Maximum stress occurs along a line at right angles to the bisector of the corner angle.
6. Subgrade support develops from zero to full support gradually.
7. If there were no subgrade support, the corner would act as a cantilever of uniform strength with the load concentrated at the apex.
8. If A and B were bonded, the system would act like a monolithic slab of thickness (A + B) with its middle plane as a neutral plane.

Based on these assumptions, a logical sequence of load-stress relations is shown in Figure 1 for the corner loading of single slabs and overlay systems when slabs A and B have equal thickness. In Figure 1,

1. Case (i) presents a load-stress relation for slabs A and B in which each is loaded independently and receives no subgrade support.
2. Case (ii) presents a load-stress relation for slabs A and B in which each is loaded independently and rests on a subgrade that provides gradually increasing support.
3. Case (iii) presents a load-stress relation for slabs A and B in which they are bonded so as to make the system act like a monolithic slab of thickness (A + B) that receives no subgrade support.
4. Case (iv) presents a load-stress relation for slabs A and B in which the system acts as a monolithic slab resting on a subgrade that provides gradually increasing support.
5. Case (v) presents a load-stress relation for slabs A and B in which A forms an overlay of B on a frictionless contact and the system receives no subgrade support.
6. Case (vi) presents a load-stress relation for A and B in which A forms an overlay of B on a frictionless surface and B in turn rests on a subgrade that provides gradually increasing support.

Figure 2 shows a similar sequence of load-stress relations for the corner loading of single slabs and A/B overlay systems when slabs A and B have unequal thickness.

During the investigation, unpredictable variation in the warping of the slabs was observed. To take into account the effect of the warping of the slabs on the load stresses, the following special variations of case vi in Figures 1 and 2 were considered,

Figure 1. Corner loading: expected load-stress relation of A/B overlay system where A = B.

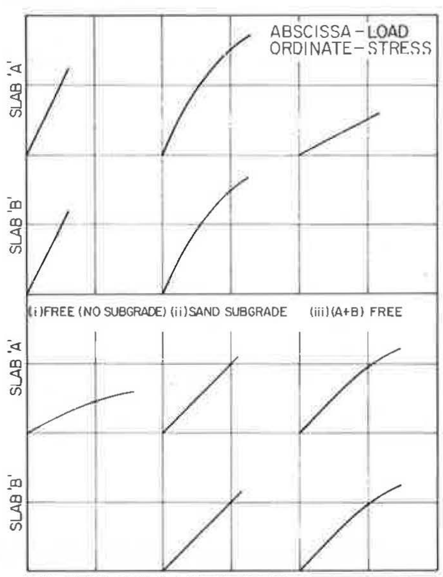


Figure 2. Corner loading: expected load-stress relation of A/B overlay system where A < B.

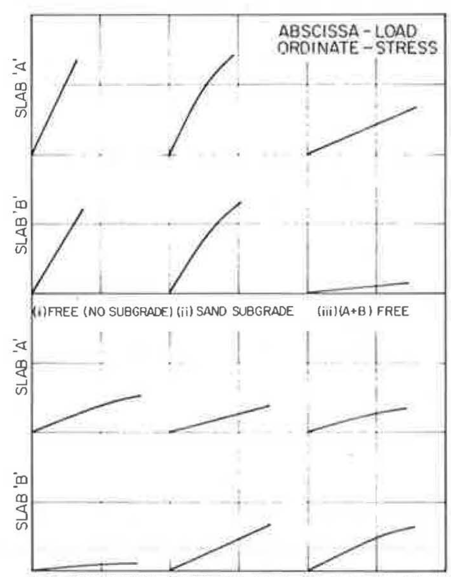


Figure 3. Corner loading: expected load-stress relation of A/B overlay system A = B and one or both slabs are warped.

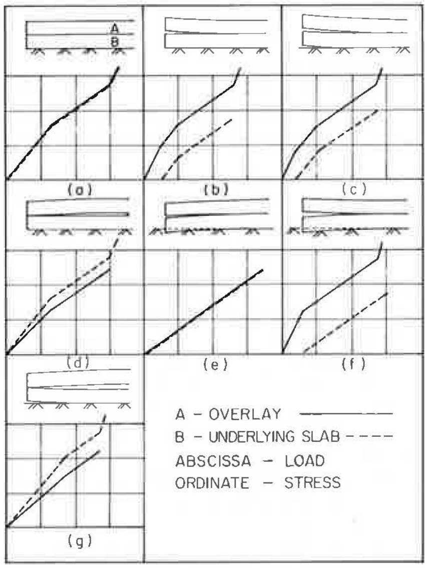
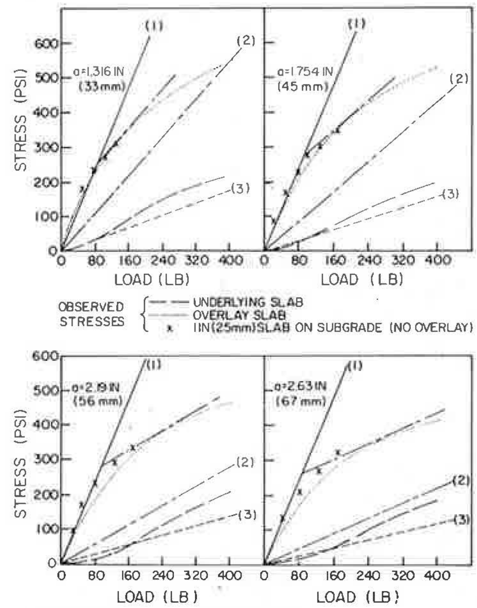


Figure 4. Corner loading: stress in 1/1 overlay system.



Note: 1 psi = 6.89 kPa; 1 lb = 0.453 kg.

in which one or both slabs may have warped up or down to a varying degree. Cases a to g in Figure 3 represent the possible variations likely to occur due to warping of the slabs. The expected load-stress relations, which are dependent on the type and extent of warping, are presented alongside. The same logic and assumptions were used in arriving at these load-stress relations as were used for the cases presented in Figures 1 and 2.

In connection with the load-stress relations shown in Figure 3, the following significance relates to the slopes of the lines. There are three states of warping in which a layer of pavement slab may be: warped up, unwarped, or warped down. Depending on the state of warping of the slab, the early slope of the load-stress relation indicates (a) free bending, (b) support from the underlying layer increasing from zero to full, or (c) full support. The successive slopes follow in the same order. The final slope indicates the plastic region, where the unrecoverable strain increase is appreciable. The load-stress relation is generally curved and not linear except when the support underneath is missing. However, the various stages in the load-stress relations have been presented as linear for the sake of simplicity.

CORNER LOADING STRESSES

Stress in 1/1 Overlay System

Load-stress relations for the 1/1 overlay are shown in Figure 4. At any value of the applied load, the overlay appears to experience a greater strain than the underlying slab. A load-stress relation for the 1-in. slab on subgrade without an overlay is also shown in Figure 4. It appears that, within the range of applied loads, the 1-in. slab experiences a similar maximum stress whether it is placed on a subgrade or whether it forms an overlay of another 1-in. slab.

Line 1 in the graphs shown in Figure 4 represents the load-stress relation for a 1-in. slab that is not receiving any support at the corner and thus behaves like a cantilever of uniform strength. Line 2 represents the load-stress relation for a 1-in. slab receiving full subgrade support, based on Westergaard's formula. The following equations were used to calculate the relations for lines 1 and 2 in Figure 4:

$$\sigma_c = 3P/h^2 \tag{1}$$

$$\sigma_c = (3P/h^2)[1 - (a_1/\ell)^{0.6}] \tag{2}$$

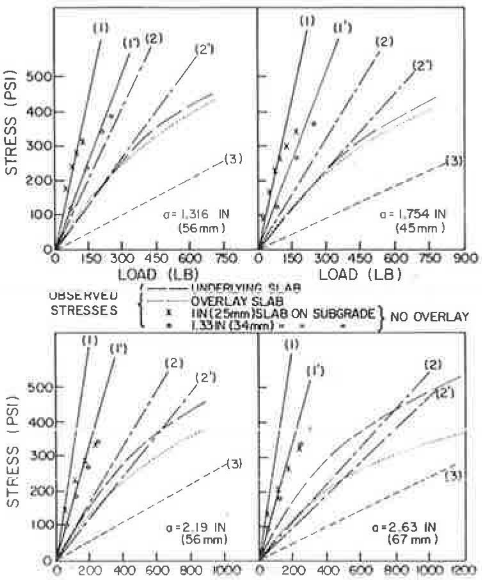
where

- σ_c = maximum stress at the corner location of the pavement slab,
- P = load at the corner location,
- h = uniform thickness of the pavement slab,
- a_1 = $a\sqrt{2}$, where a is the radius of contact area for the load, and
- ℓ = radius of relative stiffness.

Line 3 in Figure 4 represents the load-stress relation for a 2-in. (50-mm) slab on subgrade based on equation 2. The observed load-stress relation for the overlay closely follows line 1 in the early region, which indicates the missing support from the underlying layer. However, as soon as it comes in contact with the layer underneath, the overlay begins to receive gradually increasing support. In Figure 4, the first part of the load-stress relation for the overlay is shown to be made up of slopes of lines 1 and 2. It indicates a transition from the free corner to the corner receiving full subgrade support. Beyond this stage, the rate of stress increase in the overlay continues to decrease with the increase in the applied load, as would be expected.

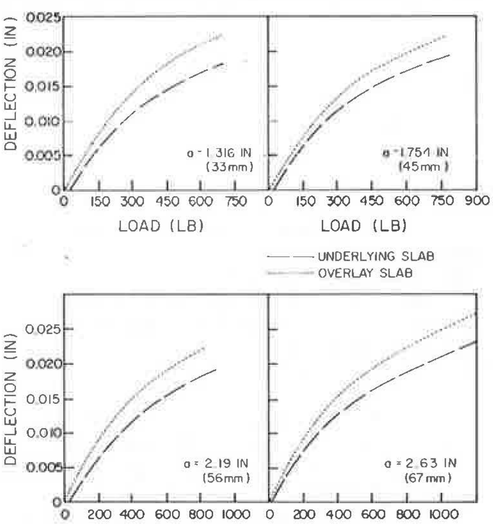
A comparison of the load-stress relation of this system with the generalized load-stress relation of an A/B overlay system shown in Figure 3 indicates that it could conform to case b, c, or f in Figure 3. In all three of these cases, the overlay slab is likely to reach the state of critical stress; therefore, the design of the system should be based on the history of stress through which the overlay slab passes. Toward the upper limit of the loads to which the system was subjected, the rate of stress increase in the overlay and the underlying slab appears to be equal.

Figure 5. Corner loading: stress in 1/1.33 overlay system.



Note: 1 psi = 6.89 kPa; 1 lb = 0.453 kg.

Figure 6. Corner loading: load deflection relations for 1/1.33 overlay system.



Note: 1 in. = 25.4 mm; 1 lb = 0.453 kg.

In the explanation of the behavior of this system, it has been assumed that the overlay was warped up and the underlying slab was either unwarped or warped only slightly. No visible gap was observed between the overlay and the underlying slab. Because the deflection measurements were made only on the upper surface of the slabs, it was not possible to define the extent of warping from the difference in maximum deflection of the two layers. However, because of the possibility of separation of the two layers, it was decided that in further work with the 1/1.33 overlay system maximum deflection measurements should be made separately for the overlay and the underlying slab.

Figure 5 shows load-stress relations for the 1/1.33 overlay system. In contrast to the 1/1 overlay system, in this system the underlying slab experiences more stress than the overlay at a specified load. In addition, whereas in the 1/1 overlay system the maximum stresses in the two layers tend to come closer as the radius of the contact plate increases, in the 1/1.33 overlay system the stresses in the two layers tend to diverge.

For an explanation of the behavior of this system, refer to the cases of the generalized A/B overlay system discussed earlier in this paper. Due to the experience with the testing of the 1/1 system, it appears that the 1/1.33 overlay system is somewhat better understood. In testing the 1/1.33 system, dial gages were suitably placed at the corner to record the maximum deflection of the two layers of the system separately. An observation was also made during testing of the maximum deflection experienced by the overlay before the underlying slab started to register any downward movement.

Load-deflection relations for the 1/1.33 overlay system are shown in Figure 6. It appeared from the deflection observations that the underlying slab started to register deflection when the overlay recorded, on the average, a downward movement of 0.001 in. (0.025 mm). This gap was considered unlikely to have a significant effect on the stress. Again strain was registered in the two layers at the same time. For the smaller two of the four contact areas used, the stress in the two layers was nearly the same at small loads. From this it can be inferred that the underlying slab was equally well supported by the subgrade from the start. In light of the discussion of the special cases shown in Figure 3, the observed behavior of the 1/1.33 overlay system limits us to the consideration of four of those cases: a, d, e, and f. On further examination, the number of cases can be reduced as follows.

In addition to the load-stress relation of the 1/1.33 overlay system, Figure 5 shows observed load-stress relations of the 1-in. slab on subgrade and the 1.33-in. slab on subgrade. It also shows load-stress relations for the 1-in. slab on subgrade as lines 1 and 2 based on Equations 1 and 2, assuming free corner and a fully supported corner. Relations 1' and 2' shown in Figure 5 are the corresponding expressions for the 1.33-in. slab based on the same equations. If one compares the observed load-stress relation of the 1/1.33 overlay system with relations 1 and 2 and 1' and 2', it appears that (a) the 1-in. slab in the system receives full or more than full support and (b) the 1.33-in. slab in the system receives full or less than full subgrade support to start with.

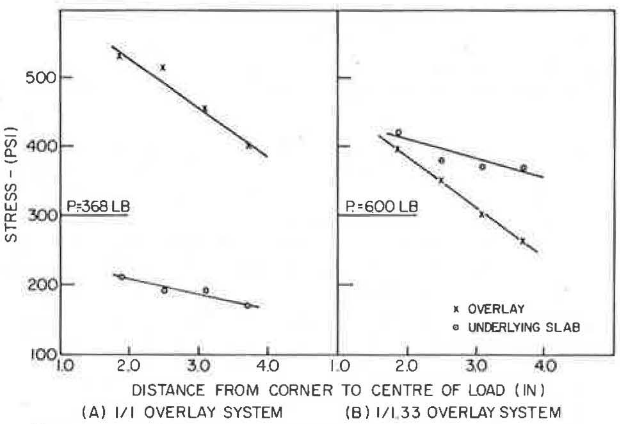
It seems fair to assume then that the 1-in. slab was warped down whereas the 1.33-in. slab was relatively unwarped. This corresponds to case d in Figure 3. When the expected load-stress relation for the system is modified so that the two layers experience stresses proportional to their section moduli, the trend remains essentially the same as shown in Figure 3. The observed load-stress relations of the 1/1.33 overlay system show a similar trend.

Effect of Bearing Area

The effect of the bearing area on the stresses in the two layers of the overlay systems is shown in Figure 7. In both the 1/1 and 1/1.33 overlay systems, for the range of contact areas used, the stress reduction appears to be greater in the overlay than in the underlying slab. This can be attributed to the load-spreading effect of the thickness of the overlay, which increases the bearing area for the underlying slab.

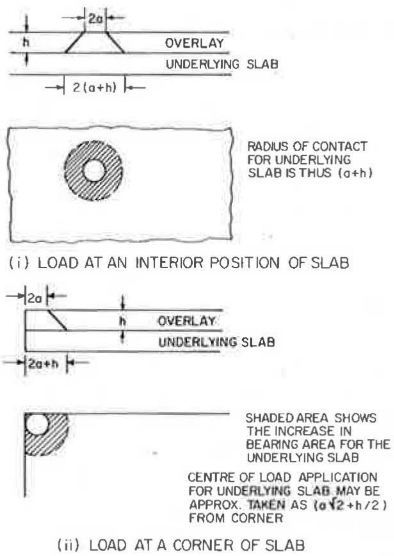
Figure 8 shows the increase in the bearing area for the underlying slab for a load position at the

Figure 7. Corner loading: effect of bearing area on maximum stress in overlay systems.



Note: 1 psi = 6.89 kPa; 1 in. = 25.4 mm; 1 lb = 0.453 kg.

Figure 8. Effective bearing area for underlying slab of an overlay system.



interior and a corner of the slab, assuming that the load spread occurs at an angle of 45°. Apparently, a large change in the bearing area for the overlay slab results in a small change in the bearing area for the underlying slab, which may be directly reflected in the stresses.

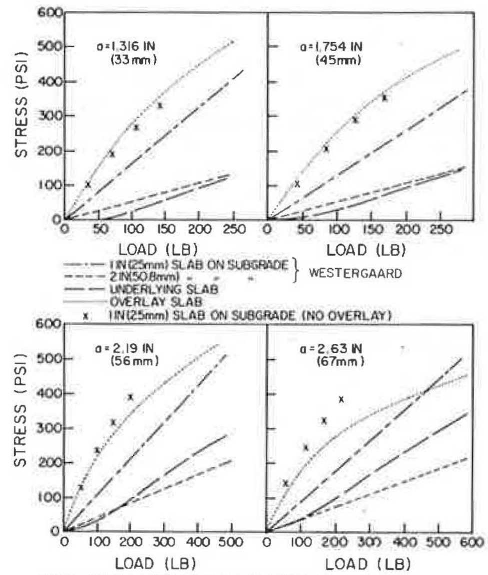
EDGE LOADING

Stress in 1/1 Overlay System

Figure 9 shows load-stress relations for the 1/1 overlay system under edge loading. In light of the discussion of corner loading for this system, the following observations can be made about its behavior under edge loading:

1. In view of the upward warping noticed and accounted for at the corner, it seems reasonable to assume that the edges of the system were also warped up.
2. Maximum stress under a specified load at the edge of the 1-in. slab is nearly the same whether it rests on the subgrade or forms an overlay of another 1-in. slab.

Figure 9. Edge loading: stress in 1/1.33 overlay system.



Note: 1 psi = 6.89 kPa; 1 lb = 0.453 kg.

3. Maximum stress in the 1-in. overlay slab appears to be more than the stress given by the Westergaard formula. This may be due to upward warping at the edge.
4. Maximum stress in the overlay slab decreases with the increase in the radius of the bearing area. In comparison, the stress reduction in the underlying slab with the increase in bearing area is quite small at any specific load. This may be attributed to the load-spreading effect of the thickness of the overlay.

Stress in 1/1.33 Overlay System

Figure 10 shows load-stress relations for the two layers of the 1/1.33 overlay system. The following observations can be made about this relation:

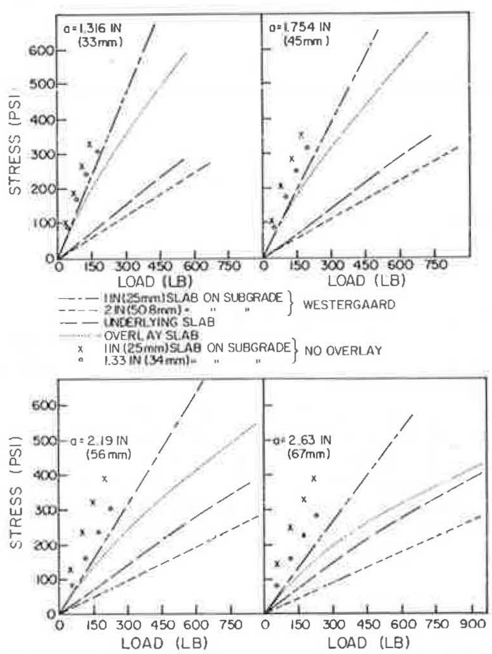
1. In the discussion of the corner loading of this system, it was hypothesized that both slabs were warped down. It is reasonable to assume, therefore, that the edges were also warped down.
2. Maximum stress in the 1-in. overlay slab was observed to be less than the maximum stress observed in the 1-in. slab on subgrade. The stress given by the Westergaard formula was also greater than the stress experienced by the overlay slab. This may be due to an increase in support resulting from the downward warping of the slab.
3. Stresses in both layers of the system decrease with the increase in the radius of the contact plate. However, over the range of contact areas used, the stress reduction in the underlying slab is smaller than that in the overlay. The answer may again lie in the load-spreading effect of the thickness of the overlay; the change in the effective bearing area for the underlying slab is thus not as great as that for the overlay slab.

INTERIOR LOADING

Stress in 1/1 Overlay System

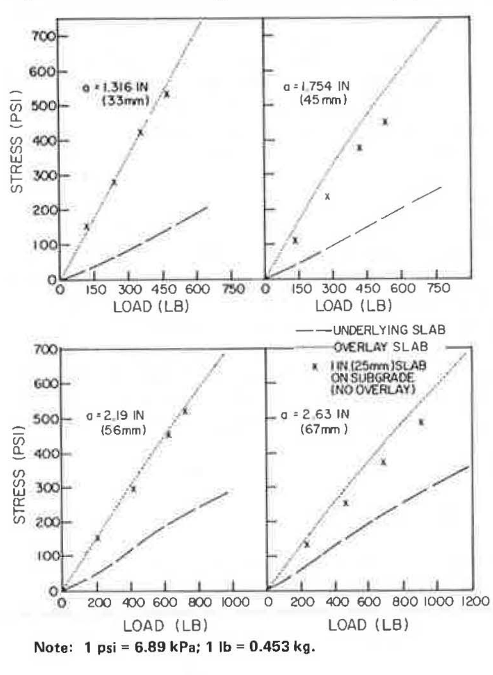
Figure 11 shows load-stress relations for a 1/1 overlay system with interior loading. The load-stress relations for the interior loading of the 1-in. slab when it rested on subgrade are also introduced in the figure. It appears that stresses in the 1-in. slab are the same whether it is placed directly on the subgrade or whether it forms an overlay of a 1-in. slab. It also appears from Figure 11 that, when the 1-in. slab forms an overlay of another 1-in. slab, the maximum stress in the overlay decreases as the area of contact increases but the maximum stress in the underlying slab remains practically constant with the increase in the bearing area. This is more clearly illustrated in section A of Figure 12. It could be argued that the load-spreading characteristics of the overlay make the effect of the contact area on the underlying slab negligible.

Figure 10. Edge loading: stress in 1/1.33 overlay system.



Note: 1 psi = 6.89 kPa; 1 lb = 0.453 kg.

Figure 11. Interior loading: stress in 1/1 overlay system.

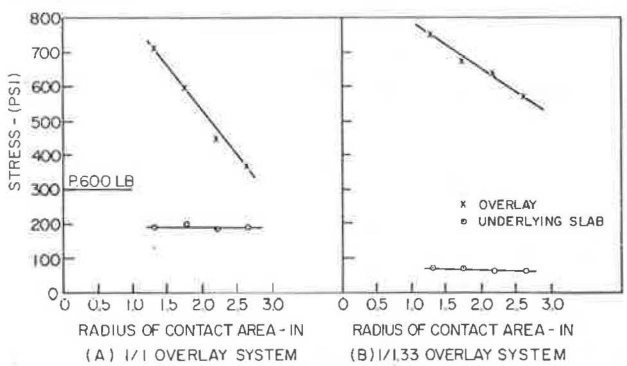


Note: 1 psi = 6.89 kPa; 1 lb = 0.453 kg.

Stress in 1/1.33 Overlay System

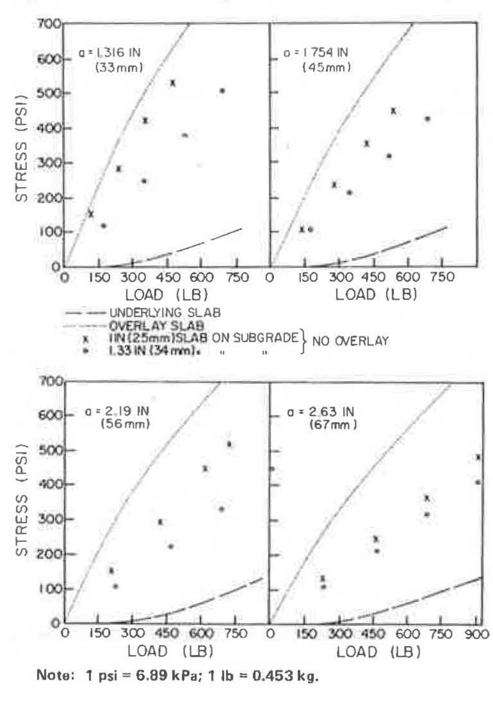
Load-stress relations for the interior loading of a 1/1.33 overlay system are shown in Figure 13. The maximum stress in the overlay again appears to decrease with the increase in the contact area whereas the stress in the underlying slab remains practically constant. The effect of the size of the bearing area on the maximum stress in the system is shown in section B of Figure 12. From a comparison of maximum stress in the 1-in. slab when it rests on subgrade and when it forms an overlay of the 1.33-in. slab, as shown in Figure 13, it appears that stress in the overlay system is greater. This is coupled with the observation that the underlying slab does not experience any stress until the load on the system is approximately 200 lb (91 kg). This leads to consideration of the possibility of the existence of a gap between the two slabs, which may have been the result of an upward warping of the overlay slab at

Figure 12. Interior loading: effect of bearing area on maximum stress in overlay systems.



Note: 1 psi = 6.89 kPa; 1 in. = 25.4 mm; 1 lb = 0.453 kg.

Figure 13. Interior loading: stress in 1/1.33 overlay system.



Note: 1 psi = 6.89 kPa; 1 lb = 0.453 kg.

the interior. This would agree with the observation made in the discussion of corner loading for the system--i.e., that there was reason to believe that the corners of the overlay slab were warped down.

It has been emphasized in this paper that not enough is yet known about the relation between the initial state of the pavement system (condition of warping) and the stress or deflection due to loads to be able to predict accurately the magnitudes of either. It is hoped that subsequent work will show that the pavement earth system behaves linearly at a certain stage defined arbitrarily as offering a degree of consistent support. The stress in the system when this stage is reached is important in predicting system performance.

SUMMARY AND CONCLUSIONS

The load-stress relation for a pavement slab is not a straight line. In order to understand the behavior of the slab, it is necessary to consider the history of warping through which it passes. A slab may be warped up, unwarped, or warped down. Depending on the state of warping of the slab, the early slope of the load-stress relation indicates (a) free bending, (b) support from the underlying layer increasing from zero to full, or (c) full support. The successive slopes follow in the same order.

In the context of the overlay systems, the upper layer is affected by warping whereas the underlying slab remains relatively unaffected. The stress in the system is dependent on the type and extent of warping of the two layers in the system.

Stresses in both layers of the system decrease with the increase in the radius of the contact plate. However, over the range of contact areas used, the stress reduction in the underlying slab is smaller than the stress reduction in the overlay. This is attributed to the load-spreading effect of the thickness of the overlay.

ACKNOWLEDGMENT

Presented in this paper are the results of a laboratory investigation originally conducted under the sponsorship of the Science Research Council of Great Britain.

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Publication of this paper sponsored by Committee on Pavement Rehabilitation.

Management System for Pavement Maintenance and Rehabilitation Based on Analytic Methods of Pavement Evaluation

PER ULLIDTZ

A management system for pavement maintenance and rehabilitation developed in Denmark is described. The system uses nondestructive testing with a falling weight deflectometer and elastic layer theory to evaluate the structural condition of existing pavements. The functional condition is evaluated in terms of root mean square values of vertical acceleration, which can be directly related to road user costs. For a given budget, the optimum maintenance strategy is determined by using integer programming. The outputs from the system are (a) an inventory of the structural and functional condition of the existing road network, (b) lists of maintenance and rehabilitation measures to be carried out to ensure the maximum reduction in user costs in view of annual budget constraints over a 3- to 5-year period, (c) prediction of the changes to the future functional and structural condition of the road network caused by alterations in the available yearly budget, and (d) the future budget needed to minimize the combined costs to the highway agency and road users.

The short-term goal of a pavement maintenance and rehabilitation management system is to assist the highway agency in answering the question, Which maintenance and rehabilitation (M&R) measures should be carried out, given a certain budget? The more long-term goal is to aid in answering the question, What will be the future standard of the road network, depending on the available budget? For society as a whole, the most important question is, Which maintenance strategy will minimize the combined costs to the highway agency and the road users?

This paper describes a pavement maintenance and rehabilitation management system developed in Denmark, the DMS, which is capable of providing answers to the above questions on both project and network levels. The system is based on analytic methods of pavement evaluation and design as stipulated in Sections 7.40.01 and 7.30.01 of the Danish Standards for Pavement Construction and Maintenance by the Danish Ministry of Transport (1).

The procedure consists of four steps:

1. Inventory--The functional and structural condition of the existing pavements is evaluated by using objective methods wherever possible;
2. Benefit/cost analysis--The functional and structural effects of each possible maintenance or rehabilitation alternative are determined and the