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# Spatial Aggregation Effects in Equilibrium and All-or-Nothing Assignments

P.H.L. BOVY AND G.R.M. JANSEN

**The level of spatial aggregation (i.e., zone size and network detail) used in transportation analyses is commonly regarded as an important factor affecting the accuracy of the resulting estimates. However, the precise effects of the level of network detail are largely unknown. To investigate these effects empirically for the car traffic assignment module, an experiment was designed to study both the separate and combined effects of the level of detail and the type of assignment model. It involves the application of various assignment models at different levels of detail. Network models at three levels of detail—fine, medium, and coarse—were developed for the road network of Eindhoven, Netherlands (population 200,000). The results of equilibrium and all-or-nothing assignments are presented. This presentation is mainly confined to the sensitivity of link load estimates. For further clarification, effects on shortest route prediction are also discussed. The level of detail had a significant effect on load and route predictions with both assignment models. This effect proved to be consistent but diminishing: an increase in the level of detail always yields better results, but only marginal improvements can be obtained beyond a certain level. Compared with all-or-nothing assignments, equilibrium loads agree much better with traffic counts at all levels of spatial detail. Recommendations are given as to what combinations of level of detail and assignment model type should be applied in practice.**

Transportation systems are usually quite complex. The analysis of such systems generally requires various simplifications because of time and money restrictions. Spatially, a transportation system is simplified into what is called a network model.

Many different network models of a particular transportation system can be constructed. An essential characteristic of a network model is its level of detail, which is mainly characterized by the numbers of zones and links included. It may be assumed that the level of detail greatly affects the time and costs involved in the analysis. Furthermore, the level of detail probably has a considerable effect on the results of the analyses (e.g., estimates of the plan's consequences).

An increase in the level of detail will presumably yield an improvement in the results at an additional cost. Therefore, the transportation analyst has to determine the appropriate level of detail by trading off the accuracy of the analysis and the analysis effort in the light of the specific planning problem to be solved. In practice, however, the optimal level of detail is hard to determine because knowledge of its effects on accuracy and costs is lacking.

In this paper results of empirical research into the effect of the level of spatial detail on car traffic assignment results are presented. The results of equilibrium and all-or-nothing assignments performed on network models with widely different levels of detail are presented and compared. The findings are also examined in relation to the accuracy of both assignment models as a function of the level of detail. They will assist the transportation analyst in selecting the optimal combination of network model and assignment technique, given accuracy requirements for the results. More detailed information on this research work and its results can be found elsewhere (1-4).

## EXPERIMENTAL WORK

### Experimental Design

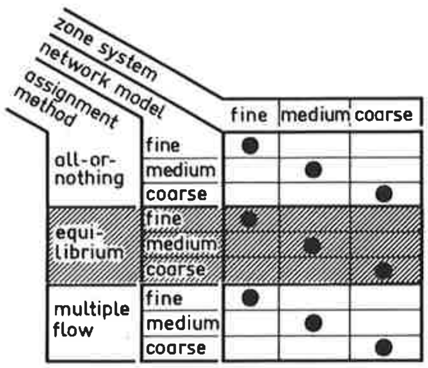
The Dutch city of Eindhoven, with its nearly 200,000 inhabitants, was chosen for the case study. Various network models of the city were constructed at different levels of detail by systematically varying zone size and network detail. At each level of detail a number of complete assignments for peak-period car traffic were performed by using different assignment methods.

Three assignment models were applied at three network levels of the Eindhoven road system. Figure 1 shows the experimental design. It can be seen that the fineness of the zone system was adapted to the degree of network detail. Only assignment models used in current practice--all-or-nothing, equilibrium, and multiple-route assignment--were applied. The results presented, however, are confined to the load and route predictions of the equilibrium and all-or-nothing models.

### Assignment Network Models

The road system was simplified by using the reduc-

Figure 1. Experimental design of spatial aggregation study.




tion method. This means that the real links of a network model were selected directly from the actual road network. The selection was based on the functional class of the links. The characteristics of the model links, such as length and capacity, are identical to those of the corresponding real-world links. The zone systems were based on the selected networks: they are the "holes" delimited by the real links selected.

The network models are strictly hierarchical, which means that a link included in a lower-level network model is also included in every higher-level network model. A real link included in more than one network model has the same characteristics at every level of detail. By analogy, a fine-level zone is always included completely in a coarse-level zone.

Models of three network levels were developed: fine, medium, and coarse. The fine-network model is nearly identical to the actual road network. It includes almost all streets and has building blocks as zones. The medium-network model was developed so that it corresponds with normal transportation planning practice. It includes all arterials and collectors. The coarse-network model only represents the arterials and may therefore be regarded as a sketch-planning network.

Table 1 gives some overall statistics of the network models. It can be seen that the respective sizes of the medium and coarse networks, measured by the number of nodes and links, are about 20 and 4 percent that of the fine network. Link length and capacity, however, appear to decrease much less--about 50 and 25 percent for the medium and coarse networks, respectively. Thus, by reducing the size to one-fifth, only half of the capacity is lost.

Unexpectedly, the number of intrazonal trips is very small at all levels of detail. This may largely be attributed to the rather widespread use of the bicycle for short trips in Holland. This means that intrazonal trips may be neglected as an important factor affecting the differences between the assignment outcomes at various levels of detail.

## PREDICTION OF LINK LOADS

### Network Load Kilometers

Some networkwide results are shown in Figures 2 and 3. The findings on total load kilometers show only small differences between the three levels of detail. They suggest that the estimation of this variable is nearly insensitive to the level of detail used.

However, the estimates by link type show significant changes. Because each network model contains a different selection of links, a comparison of results is only meaningful for identical parts of the road system. Figure 2 shows the corresponding classes for these link groups: class 1, arterials; class 2, collectors; and class 3, minor roads. These groups are also termed primary, secondary, and local roads.

It can be seen that the kilometrage on the eliminated links is partly taken over by the remaining links. The mean estimated link load, therefore, increases when the level of spatial detail is increased. This effect is quite noticeable for the primary roads in switching from the medium to the coarse level of detail.

These findings hold for both the equilibrium and all-or-nothing assignment models. The load kilometer estimates of both models differ only marginally (Figure 2).

### Network Load Hours

The estimation of load hours by the equilibrium model is shown in Figure 3. Total load hours at the fine and medium levels are nearly equal, but the coarse-level estimate is substantially higher (by nearly 50 percent).

Load hours on functional class 1 roads, which are common to all levels of detail, show significantly higher values at the coarse level. A further explanation of this doubling of load hours will provide useful insights into the way network reduction affects the assignment process (3).

In changing the coarse level the elimination of functional class 2 links directly results in a 26 percent increase in functional class 1 kilometrage. This additional amount causes a subsequent redistribution of all loads among functional class 1 links through the equilibration mechanism, which accounts for an extra increase of 4 percent. When link

Table 1. Overall statistics of actual network and network models showing level of detail of models.

| Characteristic | Actual Network | Fine | Medium No. | Medium Percent of Fine Model | Coarse No. | Coarse Percent of Fine Model |
|---|---|---|---|---|---|---|
| Network model | | | | | | |
| No. of directional links | 10,018 | 12,871 | 2,490 | 19 | 544 | 4 |
| No. of nodes | 3,570 | 4,312 | 826 | 19 | 204 | 5 |
| Directional link length (km) | 1,245 | 1,348 | 648 | 48 | 275 | 20 |
| Length · capacity (km·vehicles/hr) | | 898,651 | 525,268 | 58 | 305,388 | 34 |
| Mean free-flow speed (km/h) | | 34 | 46 | | 52 | |
| Zone system | | | | | | |
| No. of centroids (zones) | | 1,286 | 183 | 14 | 47 | 4 |
| Mean population per zone | | 150 | 1,130 | | 5,300 | |
| No. of assigned trips | | 58,575 | 57,999 | | 56,260 | |
| No. of intrazonal trips | | 143 | 719 | | 2,458 | |

**Figure 2. Equilibrium load kilometers by link type at three levels of detail with all-or-nothing results in parentheses.**

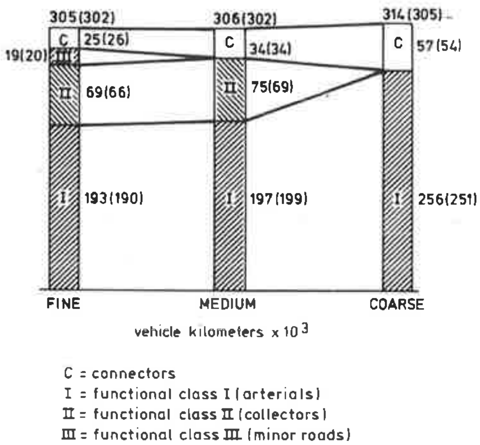
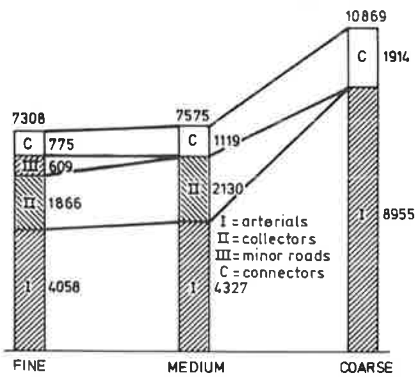
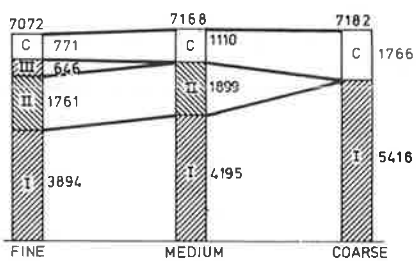



**Figure 3. Estimated load hours by link type at three levels of detail.**




A. Equilibrium




B. All-or-nothing

speeds are assumed to be unchanged, this increase in load kilometers already leads to a 30 percent increase in coarse-level load hours. The equilibration, however, also leads to a substantial drop in the mean link speed--from 46 to 29 km/hr. This fictitious congestion accounts for an additional increase of 77 percent in load hours. Figure 4 shows these effects.

The small extra increase in load kilometers due to equilibration (4 percent) might suggest a limited spatial diversion of trips among equilibrium routes. This is not the case, however. An analysis of the equilibrium routes (5) has revealed that there is considerable route spreading. The small increase in load kilometers results from the small differences in lengths between these alternative routes.

**Figure 4. Network reduction effects on load hours for a typical functional class 1 link.**

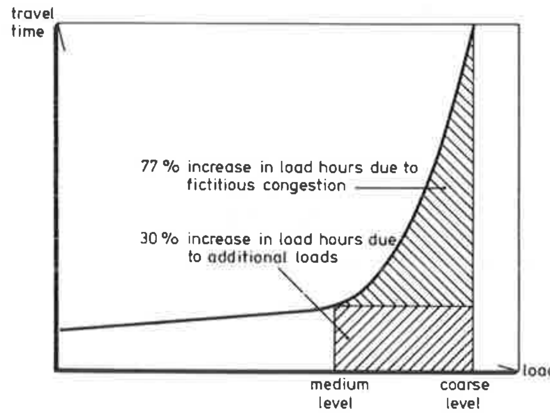
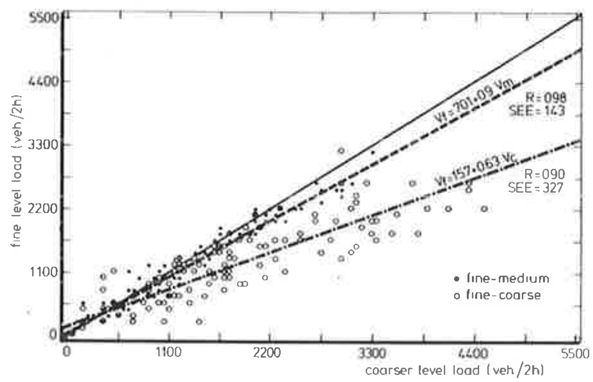



**Figure 5. Fine-level versus coarser-level link loads for primary roads (N = 144): equilibrium assignment.**




The equilibrium model gives only slightly higher load hours than the all-or-nothing model (see Figure 5) except for the coarse level, where, for obvious reasons, load hours are estimated nearly 50 percent higher with the equilibrium model. The strong similarity of the other findings stems primarily from the small differences in length between the equilibrium routes and the all-or-nothing path despite clear differences in spatial pattern.

## Individual Link Loads

The direct effect of network reduction is examined here by comparing individual link load estimates made at various levels. Differences in loads are measured by the root mean square error (RMSE):

$$\text{RMSE} = \sqrt{\Sigma_i (V_i^f - V_i^c)^2/(N-1)} \tag{1}$$

$$\text{RMSE}\,(\%) = (\text{RMSE}/\overline{V}_f) \cdot 100 \tag{2}$$

where

$V_i^f$ = finer-level volume on link i (fine or medium),
$V_i^c$ = coarser-level volume (medium or coarse),
$\overline{V}_f$ = average volume at finer level,
$\overline{V}_c$ = average volume at coarser level, and
N = sample size.

This discussion is confined to primary roads, for which each link in the sample has a fine-, a medium-, and a coarse-level estimate of the load.

**Table 2. Comparison of individual link loads predicted at three levels of spatial detail: primary roads.**

| Level-of-Detail Comparison | Link Load (vehicles/2 hr) $\bar{V}_c$ | $\bar{V}_f$ | AE | DSD | CV | RMSE | RMSE (%) | $AE^2/RMSE^2$ (%) | $DSD^2/RMSE^2$ (%) | $CV^2/RMSE^2$ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| Equilibrium Assignment | | | | | | | | | | |
| Coarse-fine | 1,668 | 1,156 | +512 | +384 | 420 | 767 | 66 | 45 | 25 | 30 |
| Coarse-medium | 1,668 | 1,206 | +463 | +326 | 390 | 688 | 57 | 46 | 22 | 32 |
| Medium-fine | 1,206 | 1,156 | +49 | +58 | 150 | 168 | 15 | 9 | 12 | 79 |
| All-or-Nothing Assignment | | | | | | | | | | |
| Coarse-fine | 1,663 | 1,194 | +469 | +306 | 638 | 845 | 71 | 31 | 13 | 56 |
| Coarse-medium | 1,663 | 1,262 | +401 | +269 | 595 | 764 | 61 | 28 | 12 | 60 |
| Medium-fine | 1,262 | 1,194 | +68 | +37 | 225 | 227 | 19 | 8 | 2 | 90 |

Note: AE = average eror, DSD = difference between standard deviations, and CV = covariance between series. N = 144.

Equilibrium Assignment

The individual link load results for the equilibrium model are given in Table 2. The RMSE (%) value for the comparison between the medium and fine levels is 15 percent, whereas coarse-level volumes show an RMSE difference of 66 percent with fine-level estimates. These figures indicate that at the level of individual link loads a much greater difference between the levels appears than was found with the aggregate load measures. Furthermore, because the fine level might be assumed to have no aggregation error, these RMSE figures also roughly indicate the level of error due to spatial aggregation that might be expected in an assignment analysis, as discussed later in this paper.

By using the statistical split-up of the RMSE, it is possible to gain further insight into the factors that affect the differences between the levels. The following expression holds:

$$RMSE^2 = [N/(N-1)]\ AE^2 + DSD^2 + CV^2 \tag{3}$$

where

$AE = \bar{V}_c - \bar{V}_f$,
$CV^2 = 2(1 - R) \cdot SD_f \cdot SD_c$, and
$DSD = SD_c - SD_f =$

$$\sqrt{\Sigma_i (V_i^c - \bar{V}_c)^2/(N-1)} - \sqrt{\Sigma_i (V_i^f - \bar{V}_f)^2/(N-1)}$$

This relationship indicates that the total variance between two series is a sum of three components:

1. The (squared) difference between the means of both series, which is equal to the average difference or bias;
2. The (squared) difference between the standard deviations of both series; and
3. The covariance between the series.

The first two components represent that part of the differences that is caused by a change in the general level of the outcome. In this case the change stems from forcing the same trips over a smaller network in going, for example, from a medium to a coarse network. This introduces a systematic difference (bias) between the volumes.

The third component expresses the difference that follows from interchanging and redistributing volumes between links. In this case this stems from a change in routing possibilities due to network changes and equilibration. Such differences are called dispersion around the bias. The data given in Table 2 show that in switching from the fine to the medium level differences are generally small and, for the most part (79 percent), unsystematic. However, at the coarse level the differences become very large--70 percent due to a general increase in volumes (high positive bias) and only 30 percent due to random variations.

Simple regression lines between the fine- and coarser-level loads (Figure 5) show that this is a strongly proportional increase: on the average, high volume links have a high increase and low-volume links only a small increase. At the coarse level, the standard error of estimation (SEE) after regression is relatively small: less than half the total difference between the series (SEE/RMSE = 327/767 = 0.43).

From this error analysis the following can be concluded. Assuming that the fine-level results are most accurate, an increase in network simplification leads to increasing link load errors. The greater the departure from the real network, however, the larger will be the systematic error component and, consequently, the better will be the possibility of correction for the bias resulting from network simplification. It seems possible to correct nearly 60 percent of the error in coarse-level estimates. Thus, there is evidence that it is worthwhile to develop rules of thumb for improving link load estimates from crude assignments.

All-or-Nothing Assignment

Two interesting findings emerge from the corresponding figures for all-or-nothing assignments in Table 2:

1. All-or-nothing loads show somewhat larger differences between the spatial levels.
2. The systematic component of these differences is smaller than that with the equilibrium model.

A possible explanation might be that the equilibrium model assigns more traffic to the less important functional classes. Removal of these classes then causes larger systematic differences. Moreover, the equilibrium model produces more uniform traffic patterns, which allow less covariance. This finding implies that spatial aggregation errors in the all-or-nothing link load estimates cannot be corrected to the same degree as in the case of equilibrium predictions.

## Link Loads Versus Traffic Counts

To assess the empirical validity of predictions at different levels of spatial detail, assigned volumes were compared with ground counts. The total error at each level is determined by using the same RMSE breakdown used earlier in this paper.

Equilibrium Assignment

Figure 6 shows frequency distributions of volume-count differences for the equilibrium assignment. They reveal a considerable bias of the coarse-level estimates, which sharply decreases when one switches to the medium level. The curves show also a clear reduction in the number of highly overestimated links. A further increase in the level of detail appears to yield only a marginal improvement.

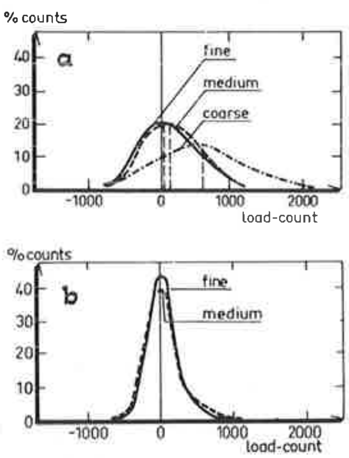
Table 3 gives detailed RMSE figures and their breakdowns for the various relevant link groups. A first important finding is the general improvement of the load estimates that results when one goes from a coarse to a more detailed network: (a) for all links as well as specific link groups, (b) for total error as well as error components, and (c) for absolute as well as relative errors. The fine level (complete network) gives the best estimates.

For primary roads (class 1) error is substantially reduced--from 87 to 45 percent--when a medium network instead of a coarse network is used. In going from the medium to the fine network level, however, only a marginal error improvement can be observed--from 45 to 41 percent--in spite of the large difference in network size. For secondary roads (class 2), which are included only in the medium and fine network levels, the relative error is improved slightly more: from 81 to 68 percent. The large reduction in error between the coarse and medium levels and the small reduction between the medium and fine levels are important findings for practice (see Figure 7).

As for the error components, there is a considerable difference between levels: at the coarse level ($AE^2/RMSE^2$ and $DSD^2/RMSE^2$ ratios in Table 3), 65 percent of the error stems from a systematic difference in the general level of the volumes, whereas the medium- and fine-level estimates show only a relatively small contribution of systematic influences--i.e., 26 and 20 percent (primary roads only). Thus, at the two finer levels the contribution of random factors is predominant.

In addition, an examination of the absolute error components shows noticeable differences: the level of the covariance (CV in Table 3) is relatively constant compared with the other components. This absolute as well as relative increase in the systematic error component when coarser networks are used can only stem from spatial aggregation.

Figure 6. Frequency distribution of volume-count differences by equilibrium assignment: (a) primary roads at three levels of detail (N = 57) and (b) secondary roads at two levels of detail (N = 83).




All-or-Nothing Assignment

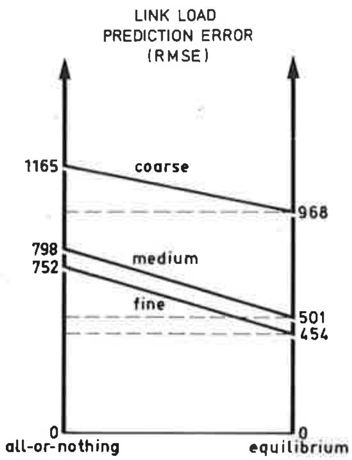
As the data given in Table 3 show, the level of detail has a similar effect on all-or-nothing load predictions. However, the results clearly show the superior predictive quality of the equilibrium model, not only for the total network but also for specific link classes. The largest differences between both models were found at the finer levels of detail. These findings are illustrated for primary roads in Figure 7 and are discussed further later in this paper.

Table 3. Load-count agreement at three levels of detail for various link groups.

| Level of Detail | Functional Class | N | Link Load (vehicles/2 hr) $\bar{V}_o$ | $\bar{V}_e$ | AE | DSD | CV | RMSE | RMSE (%) | $AE^2/RMSE^2$ (%) | $DSD^2/RMSE^2$ (%) | $CV^2/RMSE^2$ (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Equilibrium Assignment | | | | | | | | | | | | |
| Coarse | 1 | 57 | 1,107 | 1,698 | +591 | +505 | 572 | 968 | 87 | 38 | 27 | 35 |
| Medium | 1 | 57 | 1,107 | 1,265 | +158 | +197 | 432 | 501 | 45 | 10 | 16 | 74 |
| | 2 | 83 | 342 | 386 | +44 | +115 | 250 | 278 | 81 | 3 | 17 | 80 |
| | 1 + 2 | 140 | 654 | 744 | +90 | +152 | 341 | 383 | 59 | 5 | 16 | 79 |
| Fine | 1 | 57 | 1,107 | 1,228 | +121 | +167 | 405 | 454 | 41 | 7 | 13 | 79 |
| | 2 | 83 | 342 | 373 | +32 | +82 | 216 | 233 | 68 | 2 | 12 | 86 |
| | 3 | 50 | 76 | 73 | -3 | +16 | 88 | 89 | 117 | 0 | 3 | 97 |
| | 1 + 2 | 140 | 654 | 721 | +68 | +121 | 310 | 340 | 52 | 4 | 13 | 83 |
| | 1 + 2 + 3 | 190 | 502 | 551 | +49 | +107 | 270 | 295 | 59 | 3 | 13 | 84 |
| All-or-Nothing Assignment | | | | | | | | | | | | |
| Coarse | 1 | 57 | 1,107 | 1,726 | +619 | +655 | 734 | 1,165 | 105 | 28 | 31 | 41 |
| Medium | 1 | 57 | 1,107 | 1,360 | +253 | +403 | 640 | 798 | 72 | 10 | 25 | 65 |
| | 2 | 83 | 342 | 354 | +12 | +131 | 261 | 292 | 85 | 0 | 20 | 80 |
| | 1 + 2 | 140 | 654 | 764 | +110 | +286 | 465 | 557 | 85 | 4 | 26 | 70 |
| Fine | 1 | 57 | 1,107 | 1,285 | +178 | +363 | 634 | 752 | 68 | 6 | 23 | 71 |
| | 2 | 83 | 342 | 339 | -3 | +108 | 254 | 276 | 81 | 0 | 15 | 85 |
| | 3 | 50 | 76 | 61 | -15 | +14 | 109 | 111 | 146 | 2 | 2 | 97 |
| | 1 + 2 | 140 | 654 | 724 | +71 | +244 | 459 | 525 | 80 | 2 | 21 | 77 |
| | 1 + 2 + 3 | 190 | 502 | 550 | +48 | +204 | 403 | 454 | 90 | 1 | 20 | 79 |

Note: $\bar{V}_o$ = mean observed volume and $\bar{V}_e$ = mean estimated volume.

**Figure 7. Link load prediction error (RMSE) of functional class 1 links by assignment model type and level of network detail (N = 57).**




## ANALYSIS OF ROUTES

### Purpose and Method

An analysis of routes predicted at various levels of detail offers a promising possibility of explaining the network aggregation mechanism. This sensitivity was investigated along two lines (6):

1. Minimum-time routes predicted at different levels of detail were compared with each other.
2. The empirical validity of predicted routes at each level was assessed by comparing them with observed routes.

The analysis was performed only for the all-or-nothing assignment model.

The similarity of two route sets is assessed by comparing each route of the first set with the corresponding route of the second set. One route is called the base route and the other the test route. The test route is interpreted as a prediction of the base route. The similarity is expressed by the route identity (RI) ratio, which is the ratio of the length of the common part to the base route length averaged over all zone pairs.

In the example shown in Figure 8, the length of the common part is 4, which results in a route identity ratio of 4/9. The contribution of a certain link class k to the commonality can be evaluated by the partial route identity ratio ($RIP_k$), which is similar to RI except that the common part is confined to links of class k. In the example used here the RIP value for link type 1 is 2/9. The $RIP_k$ values add up to RI.

The specific route identity ratio for class k ($RIS_k$) expresses the degree to which links of that type in the base route are part of the test route. $RIS_k$ has the same definition as the RI ratio except that both lengths refer to type k links only. In this example the specific identity ratio for type 1 links is 2/3.

### Identity of Routes Predicted at Different Levels of Detail

Table 4 summarizes the $RIP_k$ ratios for the inter-level route comparisons. The medium-fine case shows an RI ratio of 81.4 percent. This high value indicates that the fine-level route is well matched by the medium-level route, especially if the highest possible value (93.5 percent) is taken into account. About one-third of the difference between these routes is due to the elimination of locals (100 - 93.5 = 6.5 percent); the remainder is caused by rerouting (93.5 - 81.4 = 12.1 percent). The major part of the overlap appears to be on arterials because their $RIP_k$ ratio is 61.1 percent.

The data given in Table 5 show that the $RIS_k$ ratios for arterials in the medium-fine comparison are extremely high (±90 percent). This implies that nearly every arterial link in the fine-level route is also part of the medium-level route and vice versa. The values for the collectors are somewhat lower (±70 percent).

The analysis thus shows that the spatial pattern of fine- and medium-level shortest routes coincides to a large extent, especially on important roads.

**Figure 8. Comparison of a test route and a base route.**

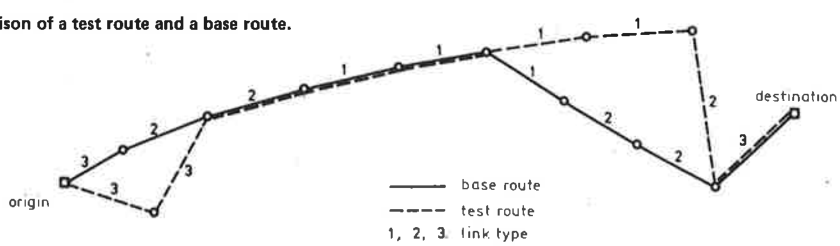



**Table 4. Average commonalities between shortest routes predicted at different levels of spatial detail.**

| Item | Medium/Fine | | Coarse/Medium | | Coarse/Fine | |
|---|---|---|---|---|---|---|
| | Common Link Length (m) | $RIP_k$ (%) | Common Link Length (m) | $RIP_k$ (%) | Common Link Length (m) | $RIP_k$ (%) |
| Type of common links | | | | | | |
| Arterials | 2,894 | 61.1 | 2,309 | 49.3 | 2,158 | 45.6 |
| Collectors | 962 | 20.3 | - | - | - | - |
| Sum | 3,856 | 81.4 | 2,309 | 49.3 | 2,158 | 45.6 |
| Max identity[a] | 4,431 | 93.5 | 3,325 | 71.0 | 3,131 | 66.1 |
| Base route level | Fine | | Medium | | Fine | |

Notes: For RIP, total length of base route minus connectors is 100 percent.
N = 232.

[a]Length of base route minus connectors and eliminated links.

**Table 5. $RIS_k$ of shortest routes predicted at different levels of spatial detail.**

| | $RIS_k$ by Base Route Level (%) | | | | | |
|---|---|---|---|---|---|---|
| Item | Fine | Medium | Medium | Coarse | Fine | Coarse |
| Type of common links | | | | | | |
| Arterials | 92.5 | 87.0 | 69.4 | 50.3 | 68.9 | 47.0 |
| Collectors | 74.1 | 70.9 | – | – | – | – |
| Sum | 87.1 | 82.4 | 69.4 | 50.3 | 68.9 | 47.0 |

Note: Total length of class k links in base route is 100 percent.
N = 232.

**Table 6. Average overlap of observed and modeled routes at three levels of detail.**

| | Fine/Observed | | Medium/Observed | | Coarse/Observed | |
|---|---|---|---|---|---|---|
| Item | Common Link Length (m) | $RIP_k$ (%) | Common Link Length (m) | $RIP_k$ (%) | Common Link Length (m) | $RIP_k$ (%) |
| Type of common links | | | | | | |
| Arterials | 2,221 | 43.8 | 2,300 | 45.3 | 2,001 | 39.4 |
| Collectors | 764 | 15.1 | 733 | 14.5 | – | – |
| Locals | 83 | 1.6 | – | – | – | – |
| Sum | 3,069 | 60.5 | 3,033 | 59.8 | 2,001 | 39.4 |
| Max identity | 5,075 | 100.0 | 4,696 | 92.5 | 3,184 | 62.7 |

Note: Observed routes are the base.
N = 232.

**Table 7. $RIS_k$ of observed routes and shortest routes at three levels of detail.**

| | $RIS_k$ by Base Route Level (%) | | | | | |
|---|---|---|---|---|---|---|
| Item | Observed | Fine | Observed | Medium | Observed | Coarse |
| Type of common links | | | | | | |
| Arterials | 69.8 | 70.9 | 72.2 | 69.2 | 62.8 | 43.6 |
| Collectors | 50.5 | 58.8 | 48.5 | 54.1 | – | – |
| Locals | 21.9 | 27.0 | – | – | – | – |
| Sum | 60.5 | 64.8 | 64.6 | 64.8 | 62.8 | 43.6 |

Note: N = 232.

As expected, coarse-level shortest routes are much different: in comparison with fine-level routes, the RI ratio for coarse-level routes now is only 45.6 percent (Table 4). This reduction is mainly caused by the large share of collectors and locals in the fine-level routes (34 percent), which cannot be matched by the coarse-level routes. Rerouting leads to additional path differences on the remaining arterials.

The $RIS_k$ ratios are lower, too (Table 5). However, an arterial link of the fine-level route still has a rather high probability (nearly 70 percent) of being part of the coarse-level route. It is interesting to note, however, that the probability that an arterial link of the coarse-level route is also on the fine-level route is considerably lower (47.0 percent). This is because the average coarse-level route length on arterials is substantially greater.

## Identity of Modeled and Observed Routes at Different Levels of Detail

To assess the empirical validity of the minimum-time paths as a function of level of detail, the predicted paths were compared directly with the observed routes. Let us first examine the fine-level routes, which are assumed to be unaffected by spatial aggregation. The RI ratio for fine-level routes (see Table 6) is about 60 percent, which is a rather high value compared with that obtained by using other transportation models. By far the largest contribution to the overlap is attributable to the arterials. This link category also shows high $RIS_k$ ratios (see Table 7), which means that most of the arterials in the observed route are part of the predicted fine route. Together with the large share of arterials in the routes this yields the large contribution to the overlap mentioned previously. The local streets exhibit a marked contrast: a poor $RIS_k$ ratio (±22 percent), together with a small share in the routes, gives rise to a small contribution to the overlap (±2 percent).

It appears that the predictive quality of minimum-time routes increases sharply with the importance of the roads (Table 7). This important finding implies that the smaller error in load estimates on major roads is not only due to a compensation of errors but is also a result of more accurate route predictions.

How does network reduction affect this predictive performance? Medium-level routes show the same overall RI ratio despite a marked reduction in network model size. Further simplification down to the coarse level causes, however, a substantial decrease in this performance index, from 60 to 40 percent. This can be explained as follows.

At the medium level only local streets were eliminated. They are many but their share in the observed and fine-level routes is small and, moreover, route predictions on these streets are bad. In addition, as the discussion in the previous section of this paper showed, routes do not change much on the remaining roads. Elimination of the collectors, however, has far more serious consequences because both their share in the routes and their $RIS_k$ ra-

Table 8. Breakdown of total link load prediction error (error sum of squares) by error type and source: primary roads.

| Error Component | Fine Level | | | | Medium Level | | | | Coarse Level | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | All-or-Nothing | | Equilibrium | | All-or-Nothing | | Equilibrium | | All-or-Nothing | | Equilibrium | |
| | Amount | Percent | Amount | Percent | Amount | Percent | Amount | Percent | Amount | Percent | Amount | Percent |
| Spatial aggregation bias[a] | - | - | - | - | 228 | 6 | 73 | 5 | 2,459 | 32 | 2,350 | 45 |
| Assignment model bias[a] | 221 | 7 | 102 | 9 | 221 | 6 | 102 | 7 | 221 | 3 | 102 | 2 |
| Total bias | 221 | 7 | 102 | 9 | 450 | 12 | 175 | 12 | 2,681 | 26 | 2,453 | 47 |
| Spatial aggregation variance[a] | - | - | - | - | 181 | 5 | 180 | 13 | 2,053 | 26 | 1,741 | 33 |
| Assignment model variance[a] | 2,308 | 72 | 358 | 31 | 2,308 | 69 | 358 | 26 | 2,308 | 30 | 358 | 7 |
| Trip input variance[b] | 63 | 2 | 63 | 1 | 63 | 2 | 63 | 4 | 63 | 1 | 63 | 1 |
| Count data variance[b] | 631 | 19 | 631 | 8 | 631 | 17 | 631 | 45 | 631 | 8 | 631 | 12 |
| Total random variance | 3,002 | 93 | 1,053 | 91 | 3,183 | 88 | 1,232 | 88 | 5,055 | 65 | 2,794 | 53 |
| Total sum of errors squared | 3,223 | 100 | 1,155 | 100 | 3,633 | 100 | 1,407 | 100 | 7,736 | 100 | 5,247 | 100 |

Notes: Squared error expressed in (vehicles/2 hr)$^2$ x $10^2$.
N = 57.

[a]Derived from assignments.
[b]Determined exogenously.

tios are significant. For these reasons the $RIS_k$ ratios of arterials and collectors are only marginally affected by the level of detail.

## ERROR CONTRIBUTION OF SPATIAL AGGREGATION

### Introduction

To estimate the contribution of spatial aggregation to the total link load prediction error, this error will be broken down by error source. In this case the following factors contribute to error: (a) level of spatial detail, (b) assignment model plus travel time input data, (c) trip input data, and (d) traffic counts.

The data for the error analysis resulted from all-or-nothing and equilibrium link load estimates at three levels of detail. Travel time and trip input data as well as traffic counts remained the same for all these predictions. Because of the results presented earlier in this paper, special attention will be devoted to systematic errors. Bovy (7) outlines the statistical procedure adopted for deriving the separate error contributions. This procedure performs an additive split-up of the total error sum of squares according to error type and source.

### Results

Table 8 presents the resulting error components for a sample of 57 primary roads. There are clear differences between the spatial levels and between the two models. The finer the level of detail, the smaller is the error; the equilibrium model performs much better despite the relatively low level of congestion in Eindhoven.

The first three line entries in Table 8 describe the portion of the total squared error that is due to the proportional bias in the load outcomes, and the rest of the table gives the remaining error variances.

The figures indicate that, for the primary roads and with the best model available (equilibrium), spatial aggregation accounts for 18 percent (5 percent bias + 13 percent variance) and 78 percent (45 percent bias + 33 percent variance) of the total error at the medium and coarse levels. Thus, at the coarse level spatial aggregation is by far the major source of error and more than half the aggregation error at that level stems from the proportional bias (45 percent).

The absolute effects of spatial aggregation appear to be quite similar when different assignment models are used. On the other hand, the relative contribution of spatial aggregation is much smaller when all-or-nothing assignments are performed (11 and 58 percent at the medium and coarse levels). Clearly, this stems from the much larger total error in these cases, which results from a much larger model error associated with the all-or-nothing model. Indeed, the all-or-nothing model results in a bias component two times larger and a remaining model error variance six times larger than the equilibrium model.

### Discussion of Results

The data given in Table 8 lead to the following recommendation concerning the best strategy for setting up an assignment analysis (Figure 7). The cheapest setup (all-or-nothing at coarse level) gives the worst results. The largest error contributor is the level of spatial detail (58 percent). An improvement of the setup should start with this factor (presuming that this is also less costly). At a medium level the all-or-nothing model contributes the most error (75 percent); thus, at this level a better model (i.e., equilibrium) should be chosen to improve the results most economically. Only as a last resort is it worthwhile considering an improvement of the trips input. Assignments at a fine level of detail make no sense: costs grow enormously whereas the output quality is only marginally improved.

## CONCLUSIONS AND RECOMMENDATIONS

Traffic assignment experiments in Eindhoven, the Netherlands (population 200,000), show that the level of spatial detail has a significant effect on the quality of assignment output. In the case of Eindhoven, by far the greatest contribution to spatial detail effects comes from eliminating links rather than from zone aggregation because of the negligible amount of intrazonal trips. When networks are coarsened the elimination of links implies forcing the came number of trips through a smaller network. Most of the kilometrage on the eliminated links is taken over by the connectors, but a considerable amount is added to the traffic volumes already present on the remaining links. In equilibrium assignment this increase in volume leads to an increase in travel time and a subsequent redistribution of volume by the equilibration process. In the end this results in a general proportional increase of link volumes.

The effect of spatial detail is consistent and similar for both assignment models: refining the network and zone system always improves load predictions. However, improvements diminish with increasing level of detail. Whereas the medium level represents only one-fifth of the links of the real network, the assignment predictions are only slightly worse than those at the fine level. At the coarse level, however, the extreme reduction in network size leads to significant errors in the estimations: about two-thirds of the total error in link volumes stems from spatial aggregation, although this error is mainly a systematic proportional overestimation of the volumes. Networkwide load figures are generally insensitive except for total load hours, which are highly overestimated by the equilibrium model at the coarse level.

The findings with regard to the individual link load predictions are fully consistent with the findings from a corresponding route sensitivity study. The same differences between the levels appear to exist to the same degree when individual routes are compared. The route analysis clearly shows that the overestimation of link loads with coarser networks is the result of a systematic increase of route length on primary roads whereas the major part of the modeled route on primary roads does not change.

As to the level of detail to be used in an assignment analysis, the findings indicate that it does not make sense to use very fine networks. A medium-level network, consisting of all arterials and collectors, appears to give results that can hardly be improved.

A comparison of equilibrium and all-or-nothing assignment results at three levels shows that the former model performs much better in every respect (except for coarse-level load hours). It should be remembered, however, that the costs of an equilibrium analysis are much higher; in particular, the specification of time-volume relationships for all links is difficult and laborious.

From these investigations the following recommendations can be made concerning the degree of sophistication of an assignment analysis:

1. Even in an only slightly congested network, as in this case, an equilibrium analysis will give significantly better results than an all-or-nothing analysis.
2. If one takes the simplest analysis setup as a starting point--i.e., all-or-nothing assignment at a coarse network level (only arterials)--the best way to improve output quality is first to refine the network. Once a medium assignment network that is four or five times as dense has been arrived at, further improvement can be made most economically when a better assignment model (i.e., an equilibrium procedure) is applied.

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