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# Transportation Evaluation in Community Design: An Extension with Equilibrium Route Assignment 

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An integrated model of transportation and land use is developed for the purpose of evaluating alternative community master plans. Equilibrium route assignment is combined with the conventional four-stage transportation model to calculate the overall economic benefits of alternative urban planning decisions. Problems of measuring benefits associated with elastic trip demand and domand shifts are also examined. The model is used to evaluate planning alternatives for a 7,500 -acre suburban community. It is especially adapted to the problem of evaluating a subcommunity within the context of a larger metropolitan area. Equilibrium route assignment provides an efficient low-cost method of determining route flows and the cost implications of various road networks and land use decisions.

Land use planning and transportation planning should go hand in hand. However, with few exceptions transportation analysis is performed only after land uses and densities have been set. The analyses are often performed in order to determine which road should be improved or whether a new road should be added, but they are rarely performed before the land use decisions are made that overburden existing transportation facilities.

The purpose of this paper is to demonstrate the
application of transportation modeling to land use decision making with particular reference to master plans in communities of about 2,000 to 20,000 acres. Although other factors such as environment, soils, drainage, and public service are also an integral part of land use decision making, the current model focuses on the interrelationship between land use and transportation. An integrated model of transportation and land use is developed for the purpose of choosing among a series of land development and road network alternatives in a suburban community.

One of the major problems in transportation modeling is route assignment--the determination of which routes, among several alternatives, trip takers will choose to reach their destinations. Route assignment is particularly important in land use planning because it can be used to determine which transportation facilities will be burdened by a given land use change and when roads and other facilities will become congested.

Equilibrium route assignment as developed by

LeBlanc (1) provides an effective algorithm for route selection. The output includes traffic volumes and travel time delays on all road (or transit) links for a given transportation network. Because trip takers choose their destinations in part by their perception of how long it will take to get there (or how costly it will be), the output of the route-assignment function then becomes an input to the trip-distribution function. A general model equilibrium is reached in which trip distribution reflects the equilibrium travel times on alternative routes and route assignment reflects the equilibrium trip distribution.

The paper is divided into two parts. In the first section the general model and the related assumptions behind consumer surplus measures of transportation benefits are described. In the second section the application of the model to master plans for a 7,500-acre suburban community is described. The model proves to be an effective tool in planning land use and transportation--one that is cost-effective and easily carried out within the typical budget and data-collection constraints of submetropolitan communities.

## THE MODEL

The model uses a traditional approach that has four main steps: trip generation, trip distribution, modal split, and route assignment. The general approach is described by Wilson (2) and is found in most transportation-planning models.

When transportation modeling is used to evaluate master-plan decisions, the central problem is one of evaluation of cost-effectiveness. Certain models such as Putman's (3) provide more detail than the model described here, but the very complexity limits their usefulness in evaluating a series of alternative master-plan decisions.

Cost-effective transportation and land use evaluation depends on obtaining sufficient informáion for the required decision and foregoing detail that either requires costly data collection or does not contribute significantly to the decision. The model described here takes all of its data from the master-plan proposals for a community. Although the application is to a new community of approximately 7,500 acres ( 3 by 4 miles), the model is just as easily applied to planning decisions at a larger subregional or regional scale. It also can be used for incremental planning decisions--road improvements and zoning changes.

Unlike transportation and land use models based on a Lowry approach (4), in which residential land and other land uses are allocated alternatively, this mndel takes land use, road, and transit network design information as given. The model then solves for the aggregate transportation cost of all users within the system. Altermative land use configurations or road networks or minor changes to the original plan are evaluated in the same way. The preferred design emerges as that plan or configuration that has the lowest aggregate cost.

In the case in which implementation, such as the development of a 7,500-acre community, will take place over a number of years, the evaluation is based on the present value of projected costs to all current transportation users for each year:
$P V=\sum_{t=1}^{N}(1+r)^{-t}\left[f\left(\mathrm{TDC}_{t}\right)\right]$
where
$T D C_{t}=\sum_{i} \sum_{j} \sum_{m} \sum_{\mathrm{s}} \mathrm{T}_{\mathrm{ij} \mathrm{mst}} \mathrm{C}_{\mathrm{ijmst}}$
and
$T D C_{t}=$ total daily transportation cost in year ${ }_{t}$,
$T_{\text {ijmst }}=$ trips from origin zone $i$ to destination zone $j$ by mode $m$ along route $s$ in year $t$, and
$C_{i j m s t}=$ generalized cost of that trip.
Generalized cost is based on the mode of travel and includes all direct costs such as vehicle operating costs and fares plus all indirect costs such as travel time and waiting time. Tijmst captures all trips taken along a particular route by a particular mode. The product of $T_{i j m s t}$ and $C_{i j m s t}$ represents the total travel cost for all trips along a particular route by a particular mode. Thus, by summing these products for all routes and modes between all destinations, one obtains an estimate of total transportation cost for the system.

The total present value (PV) of all costs represents the sum of costs for all trips taken within the community over the projected life ( $t=1$ to $N$ ). If alternative master plans are to be analyzed for the same development, as in the case illustration that follows, it is sufficient to measure $\mathrm{TDC}_{t}$ under the assumption that travel demand each year is the same for different plans. However, if different growth rates for the community are likely to result from different master plans, the extension of total daily costs $\left(\mathrm{TDC}_{\mathrm{t}}\right)$ to annual costs and then to PV according to Equation 1 is necessary in order to select the best plan.

Considerable useful information is derived from the analysis of each alternative, including trip information for each route and for each mode at peak travel times and nonpeak travel times. Weak spots in the system design emerge from the loading on individual links in the road or transit system. Because the entire planning program is evaluated for any change in the transportation or land use plan, externaiities and secondary effects such as dectinetion and route changes in distant zones are included in the evaluation for even incremental changes in land use or transit system design. The general scheme for the model is shown in Figure l. The four stages are outlined in the following.

## Trip Generation

The first stage of the model is trip generation. Trips are generated as a function of the land use and density in a given zone. A single land use may generate several trips of different purposes. For example, a residence will generate home-to-work trips, home-to-shopping trips, home-to-school trips, and home-to-recreation trips.

## Trip Distribution

The second stage of the model is trip distribution, which is calculated by using a singly constrained gravity model ( $\underline{2}, \underline{5}, \underline{6}$ ):
$\mathrm{T}_{\mathrm{ij}}=\mathrm{a}_{\mathrm{i}} \mathrm{O}_{\mathrm{i}} \mathrm{D}_{\mathrm{j}} \exp \left(-\lambda \mathrm{C}_{\mathrm{ij}}\right)$
subject to
$\sum_{j} D_{j}=\sum_{i} O_{i}$
where
$T_{i j}=$ number of trips from origin zone $i$ to destination zone $j$,
$O_{i}=$ number of trips originating in zone

$$
\begin{align*}
& D_{j}=\text { number of trips ending in zone } j, \\
& C_{i j}=\text { generalized cost of such a trip, } \\
& \exp \left(-\lambda C_{i j}\right)=\text { trip decay function, and } \\
& a_{i}=1 / \sum_{j} D_{j} \exp \left(-\lambda C_{i j}\right) \tag{4}
\end{align*}
$$

where $a_{i}$ ensures that Equation 3 is satisfied.
The land use plan to be analyzed is subdivided into geographic zones for purposes of calculating trip movements. Trip generations $O_{i}$ are a function of the residents and workers located in each zone as of year $t$. Origins and destinations are determined by trip purpose and time of day. They include home to work, home to school, home to recreation, home to shopping, work to shopping, and so forth, subdivided according to rush-hour and non-rush-hour trips. For simplification, trip takers are not differentiated by income, education, job status, and other characteristics found in the logit models of McFadden and others (7-9). Although assignment of the resident population to certain neighborhoods according to socioeconomic characteristics improves the accuracy of the trip-distribution calculations, the data requirements often ge beyond the information that is available at the time that transportation and land use decisions are being made. This is particularly true when a new development is being planned.

## Modal Split

Modal split is calculated for four modes: walking, car, bus, and taxi. Like trip distribution, modal split is a function of generalized cost, which includes operating costs, time en route, waiting time, parking, walking time to transit stops and from parking lots to buildings, modal comfort, and congestion:
$\mathrm{T}_{\mathrm{ijm}}=\exp \left[-\alpha f\left(\mathrm{C}_{\mathrm{ij} m}\right)\right] / \sum_{M} \exp \left[-\alpha \mathrm{f}\left(\mathrm{C}_{\mathrm{ijm}}\right)\right]$
where $\alpha$ is the sensitivity constant and $f\left(C_{i j m}\right)$ represents the functional relationship between generalized cost and mode $m$ for trips between zones $i$ and $j$.

## Route Assignment

Route assignment is performed by using an equilibrium route-assignment procedure developed by LeBlanc (1) and based on the Frank and Wolfe (10) algorithm for quadratic programming. The road network is catalogued as a series of road links ( $g, h$ ) between each intersection with flows ( $x_{g h}^{j}$ ), where gh represents the road links along a trial route connecting origin zone $i$ to destination zone j. Given the road network, the trip table indicating the total number

Figure 1. Flow diagram of transportation model.

of vehicles per unit time from origin zone i to destination zone $j(T i j)$, and a congestion function [ $A\left(X_{g h}\right)$ ], the equilibrium traffic flows can be found by solving the following nonlinear programming problem:

Minimize
$\sum_{\text {links gh }} \int_{0}^{X_{g h}} \mathrm{~A}(\mathrm{q}) \mathrm{dq}$
subject to
$\mathrm{I}_{\mathrm{ij}}+\sum_{\mathrm{g}} X_{\mathrm{gi}}^{j}=\sum_{\mathrm{h}}^{\sum} X_{\mathrm{jh}}^{j}$
(Flow originating at $i$ going to $j$ plus all flows passing through $i$ for $j$ on links $g i$ equals flows leaving $i$ for $j$ on links ih.)
$X_{g h}=\sum_{j} X_{g h}^{j} \quad$ (definitional)
$X_{\mathrm{gh}}^{\mathrm{j}} \geqslant 0 \quad$ (nonnegativity)
Equilibrium traffic flows among all available routes between any pair of origin (i) and destination (j) zones are calculated so that the generalized travel cost is the same for all the possible routes connecting the two zones. In equilibrium, no trip takers can save money by switching to an alternative route.

Trip distribution, modal choice, and route assignment are endogenous to the model because they are a function of generalized cost, which is of course a function of travel distance and travel time between zones according to mode and route. These are calculated through an iterative process shown in Figure 1 in which the first iteration assumes that everyone takes the shortest route and the lowestcost mode. After the first iteration, generalized costs between each pair of zones are adjusted as a function of congestion anả tiravei times calculated̉ in the first iteration. Trip destinations may be altered when distant zones become less costly to reach than nearer zones because of congestion. After several iterations, the system converges to a stable equilibrium in which no trip taker can save money by switching destinations, modes, or routes.

One limitation of the current model is that only single trip purposes are allowed for each trip. Thus trip takers are not allowed to combine several destinations into a single trip. Another limitation is that although trip destinations are a function of the transportation network, trip generations are not. Trip generation is fixed as a function solely of land use and density in each zone. Although these limitations do not affect one's choice amona several master-plan alternatives, they do affect the degree of preference. When transportation costs are weighed against other nontransportation factors, the degree of preference as measured by the difference in present value between plans can alter the ultimate planning decision. These limitations suggest future improvements to the model.

More detailed equations and parameters are described later in this paper.

## ISSUES IN TRANSPORTATION MODELING

Economists tend to have trouble with the traditional cost-minimization rules for transportation decision making. In one sense, the model described here uses a cost-minimization rule for evaluating alternative planning programs. However, when alternatives are being compared to a base plan, the preferred decision rule is the maximization of net benefit.

Cost-benefit analysis in transportation planning

Figure 2. Consumer surplus for inelastic demand.

is well known $(\underline{2}, \underline{5}, \underline{6}, \underline{8}, \underline{11}-\underline{13})$. When changes are made in a transportation plan, benefits to users can be divided into three parts (6): (a) benefit (costs) to those who make the same trip as before, (b) benefit (costs) to those who have changed their destination or mode of travel, and (c) benefit to those induced to travel for the first time (i.e., newly generated trips).

When demand for transportation is assumed to be perfectly inelastic in the relevant region of the demand curve, one can choose between alternative plans strictly on the basis of which plan costs less. This is so because benefit to users remains constant according to a Marshallian concept of consumer surplus.

Figure 2 shows that for inelastic demand, the consumer surplus associated with alternative plans is the area between the demand curve (D) and the price. If the complexities of transit pricing can be ignored and it can be assumed that users correctly perceive and then base their transportation choices on generalized cost, trip price can be construed to be the same as generalized cost. Consider a case in which two transportation plans, $A$ and $B$, are compared. The user population is the same for the two plans and determines the demand curve shown in Figure 2. Suppose that demand is perfectly inelastic in the region in question (O). Under these circumstances, the plan conferring the greater benefit to its users will be the one having the larger consumer surplus, namely, the one that provides equal benefit (D) at lower cost $\left(P_{B}\right)$. Thus, plan $B$ is preferred to plan A.

Major difficulties arise when demand is elastic. First mere estimation of the demand elasticity itself can pose major difficulties (14). Second measurement of the consumer surplus or benefit requires that users be divided into two groups: those who would have taken trips at the original price level under plan $A$ and those who are induced to travel by the lower prices under plan B. The benefits of lower cost to the first group of users can be measured by the approach shown in Figure 2 for inelastic demand. The problem is how to measure benefit for those induced to take new trips. If one looks only at cost, total transportation costs for all trips will likely be higher under the lower-cost plan $B$ because of the added number of new trips.

However, although total costs may increase, total benefit should at least be equal to plan $A$ or the new trips would not be taken. Presumably total benefit in plan B will in fact increase. The appropriate measure of net benefit to new trip takers is the added consumer surplus, approximated by triangle ABC in Figure 3. Various approximations have been developed for measuring net benefits from generated travel, such as the rule of half (15). The precise benefit is the integral under the demand curve ( $A B C$ ). But unless one knows precisely what the shape of the demand curve is, an accurate measurement of this area is problematical. Some investigation into the functional form for elastic demand has been performed by Lerman and Louviere (16).

A related problem in evaluating alternative transportation networks and improvements arises from changes in land use density, which may cause a shift in total travel demand. Figure 4 shows the simplified case in which the original demand (based on the original trip takers) is inelastic in the relevant region, but a land use change (say from agricultural to residential) causes a shift in demand from $D_{1}$ to $D_{2}$. If the shift in demand does not cause any change in prices (prices remain at $P_{A}$ ), any in-

Figure 3. Consumer surplus for elastic demand.


Figure 4. Consumer surplus for shift in demand.

crease in demand will confer a net increase in benefit on the community (area ACDG). However, if the shift in demand causes an increase in prices (prices increase to $P_{B}$ ), the net benefit due to the demand shift is represented by the difference between the added benefits to the new users and the added costs to the original users (area ABEG minus area $P_{A} C B P_{B}$ ). This measurement requires an explicit determination of benefit from knowledge about the actual demand curve, a problem that was avoided altogether in Figure 2 and for the most part in Figure 3.

In the model presented here, it is a simple matter to evaluate the increase in congestion costs and other costs for the original users ( $P_{A} C B P_{B}$ ), who continue to make the same trips under the new demand schedule as under the old. However, measuring the added benefit to the new users requires some assumptions concerning the value to them of making the new trips. Presumably the value of the trip is at least equal to its cost or the trip would not be taken. The measurement of net benefit associated with the trip (value minus cost) assumes a knowledge of what the trip taker would be willing to pay to take the trip rather than just what it actually costs him. Such knowledge is beyond the scope of the current model.

Another problem in applying transportation models to planning decisions concerns the fact that most small urban communities are not self-contained (17). That is, in modeling transit trips for a subarea within a larger metropolitan area, numerous trips will either originate or terminate outside the boundaries of the subject area. Once outside the area, the trip taker's benefit is independent of any planning decisions associated with the subject area. In this paper, only that portion of each trip that takes place inside the subject area is considered. There may be externalities associated with trips once they are outside, but in such cases the costs or benefits must be included as a separate part of the evaluation.

## CASE ILLUSTRATION

In this section a case study is presented of the application of the model to a new 7,500-acre community on the suburban fringe of Houston, Texas. At issue is the selection of the better master plan from two alternatives. The two master plans are shown in Figures 5 and 6. Plan A (Figure 5) is the final master plan that was adopted by the developer of the 7,500-acre project. Plan B (Figure 6) represents an alternative master plan showing how the community would likely have developed under typical Houston suburban sprawl (18). The road pattern in plan B tends to follow road easements in existence at the inception of development, whereas in plan $A$ the road plan was a new design around the proposed development. In terms of land use, plan A follows a design approach similar to that of Columbia, Maryland, and other new towns; intensive land uses such as shopping and industry are located at interior sites, easily accessible to residents of the community. By contrast, in plan $B$ intensive land uses are located along major access roads where they have greater visibility and are more easily reached from outside the community than under plan A. Although comparison of the two plans offers insight into the costs of urban sprawl, it is used here to illustrate the application nf the transportation model and in particular the equilibrium route-assignment pattern of the model.

The two plans are designed to serve the same population and employment. Therefore, the land use budgets are the same; that is, the acreage alotted to each major land use is the same between the two

Figure 5. Plan A.

plans. However, the road networks are different and the locations of various land uses throughout the two plans are different.

Each plan is divided into 11 zones based on the location of highways, creeks, and other major geographical features = The zones range in size from 36 to 1,424 acres and may include only one land use or several different land uses in each zone.

The master plan catalogs nine different land uses, each associated with an average density of development. The densities shown are appropriate for development in Houston. These are given in Table 1.

The distribution of land uses in each zone for plan A is shown in Table 2. A similar matrix is determined for plan $B$ based on the master plan. Greater precision can easily be obtained by dividing the subject area into more zones or more land uses. However, because the application here is to the problem of choosing between alternative master plans before any development has occurred, the choice of 9 land uses and 11 geographic zones provides sufficient detail.

Excluding open space, there are eight land uses, which are grouped into six characteristic populations based on the densities in Table 1 and the land use distribution in Table 2. Characteristic populations, which are described by such terms as number of dwelling units (DU) or square footage of industrial space, are used to determine trip generation

Figure 6. Plan B.

and trip distribution. They are a function of acres (by zone and land use) and density:
$\mathrm{C}(\mathrm{i}, \mathrm{lu})=\mathrm{ACRE}(\mathrm{i}, \mathrm{lu}) \cdot \mathrm{DEN}(\mathrm{lu})$
where $C(i, l u)$ is the number of $D U$ per acre or square footage of office or industrial space in zone $i$ of land use (lu) (one zone may have several land uses) and DEN(lu) is the density of development (e.g., 6.5 DU/acre).

Categories of similar space such as residential units are aggregated to form six characteristic populations, shown in Table 3.
$\operatorname{POP}(\mathrm{i}, \mathrm{z}=\mathrm{j})=\mathrm{C}(\mathrm{i}, \mathrm{l} \mathrm{l}=\mathrm{j} 0)+\ldots+\mathrm{C}(\mathrm{i}, \mathrm{l} \mathrm{u}=\mathrm{j} 1)$

Table 1. Population and development density by land use.

| Land Use | Density of Development |
| :--- | :--- |
| Low-density residential | $6.5 \mathrm{DU} /$ acre |
| High-density residential | $29.5 \mathrm{DU} /$ acre |
| Strip commercial | $10,000 \mathrm{ft}^{2} /$ acre |
| Town center and office | $30,000 \mathrm{ft}^{2} /$ acre |
| Research and development | $15,000 \mathrm{ft}^{2} /$ acre |
| Industrial | $10,000 \mathrm{ft}^{2} /$ acre |
| Community facilities (school) | Actual number of seats by zone |
| Neighborhood shopping | $10,000 \mathrm{ft}^{2} /$ acre |
| Open space | - |

Note: $D U=\mathrm{d}$ welling unit,

Table 2. Acreage distribution by land use and zone for plan $\mathbf{A}$.

| Zone | Acreage by Zone |  |  |  |  |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| 1 | 0 | 13 | 49 | 99 | 0 | 0 | 10 | 0 | 69 | 240 |
| 2 | 269 | 76 | 30 | 51 | 83 | 70 | 10 | 7 | 303 | 899 |
| 3 | 233 | 35 | 0 | 0 | 35 | 0 | 55 | 14 | 77 | 449 |
| 4 | 383 | 50 | 0 | 0 | 65 | 0 | 10 | 6 | 107 | 621 |
| 5 | 727 | 0 | 0 | 0 | 0 | 0 | 72 | 8 | 398 | 1,205 |
| 6 | 690 | 0 | 0 | 0 | 91 | 0 | 92 | 7 | 544 | 1,424 |
| 7 | 40 | 20 | 77 | 0 | 137 | 0 | 14 | 0 | 169 | 457 |
| 8 | 288 | 96 | 71 | 0 | 30 | 0 | 87 | 7 | 276 | 855 |
| 9 | 0 | 0 | 17 | 0 | 19 | 0 | 0 | 0 | 0 | 36 |
| 10 | 145 | 0 | 0 | 0 | 0 | 0 | 10 | 7 | 79 | 241 |
| 11 | 0 | 0 | 0 | 0 | 0 | 890 | 0 | 0 | 218 | 1,108 |
| 12 | Outside subject area | - | - | - | $-$ | 8 | - | - | - | , |
| Total | 2,775 | 290 | 244 | 150 | 460 | $\overline{960}$ | 360 | 56 | 2,240 | 7,535 |

Note: Land use categories are as given in Table 1.

Table 3. Characteristic populations by zone for plan A.

| Zone | Population by Zone |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 1 \\ & \text { Office and Industry } \\ & \left(\mathrm{ft}^{2} 000 \mathrm{~s}\right) \end{aligned}$ | $\begin{aligned} & 2 \\ & \text { Shopping } \\ & \left(\mathrm{ft}^{2} 000 \mathrm{~s}\right) \end{aligned}$ | 3 <br> No. of Dwelling Units | 4 Recreation (no. of people) | 5 <br> No. of School Seats | 6 <br> No. of Hotel <br> Rooms |
| 1 | 2,970. | 490. | 377. | 8,000. | 1,000. | 1,000. |
| 2 | 3,475. | 370. | 3,952. | 4,000. | 1,000. | 0. |
| 3 | 525. | 140. | 2,529. | 3,000. | 1,000. | 0. |
| 4 | 975. | 60. | 3,939. | 4,000. | 1,000. | 0. |
| 5 | 0. | 80. | 4,725. | 4,000. | 4,000. | 0. |
| 6 | 1,365. | 70. | 4,485. | 1,000. | 5,000. | 0. |
| 7 | 2,055. | 770. | 840. | 2,000. | 1,000. | 0. |
| 8 | 450. | 780. | 4,656. | 1,000. | 4,000. | 0. |
| 9 | 285. | 170. | 0. | 0 . | 0. | 400. |
| 10 | 0. | 70. | 942. | - 1,000. | 1,000. | 0. |
| 11 | 8,900. | 0. | 0 | 0. | 0. | 0. |
| $12-15$ <br> (outside subject area) | 0. | 0 . | 46,777. ${ }^{\text {a }}$ | 0. | 0. | 0. |
| Total | 21,000. | . 3,000. | 73,222. | 28,000. | 19,000. | 1,400. |

${ }^{a^{a}} 46,777$ DU are assumed to exist outside the community in order to supply the 56,000 jobs not filled internally ( 1.2 workers/DU).
where $j 0$ and $j 1$ denote land uses that are aggregated to form characteristic population $j$.

Trip generations are then calculated from Equation 11 according to nine trip purposes ( $k$ ):
$\mathrm{P}(\mathrm{i}, \mathrm{k})=\mathrm{POP}(\mathrm{i}, \mathrm{z}) \cdot \mathrm{RP}(\mathrm{k})$
where

$$
\begin{aligned}
P(i, k)= & \text { trips generated from zone } i \text { by trip } \\
& \text { purpose } k, \\
\operatorname{POP}(i, z)= & \text { characteristic population } z \text { in zone } i, \\
& \text { and } \\
R P(k)= & \text { rate of trip production by trip purpose } \\
& k .
\end{aligned}
$$

There are nine trip purposes, each associated with a rate of trip production based on one of the characteristic populations. For example, a DU will generate three types of trips: home to work, home to recreation, and home to school. Trip production rates are shown in Table 4. Outside trip generations are actually assigned to four exterior zones (12, 13, 14, 15), one for each main access point to the community from each direction.

Finally, the origin-destination ( $O-D$ ) trip matrix is derived from Equation 2:
$\mathrm{T}_{\mathrm{ijk}}=\mathrm{A}_{\mathrm{ik}} \mathrm{O}_{\mathrm{ik}} \mathrm{D}_{\mathrm{jk}} \exp \left(-\lambda \mathrm{C}_{\mathrm{ij}}\right)$
where
$T_{i j k}=$ trips from zone $i$ to zone $j$ by trip purpose k,
$O_{i k}=$ trips originating from zone $i$ by trip purpose $k$,
$D_{j k}=$ trips attracted to zone $j$ by trip purpose
$A_{i k}=1 / \sum_{\mathrm{j}} \mathrm{D}_{\mathrm{jk}} \exp \left(-\lambda_{2} \mathrm{C}_{\mathrm{ij}}\right)$
$\lambda=$ parameter for decay rate of attraction between zones, and
$C_{i j}=$ function of generalized travel cost between zones $i$ and $j$.

Table 4. Parameters for trip production by characteristic population.

| Trip Purpose |  |  |
| :--- | :--- | :--- |
| From | To |  |
| Work | Work |  |
| Work | Shopping | 0.25 trip/ $/ 100 \mathrm{ft}^{2}$ of office and industrial space |
| Home | Work | 1.2 trips/DU |
| Home | Shopping |  |
| Home office and industrial space | Recreation | 1.0 trip/DU |
| Home | School | 0.75 trip/DU |
| Hotel | Work | 0.5 trip/hotel room |
| Hotel | Shopping | 0.25 trip/hotel room |
| Hotel | Recreation | 0.25 trip/hotel room |

## Traffic Flows

Trips by trip purpose are aggregated into three main traffic flows (Q) representing rush-hour travel, non-rush-hour travel, and recreation or non-workhour travel.
$Q(i, j, v)=\sum_{k} T_{i j k}$
where $k$ is trip purpose (see Table 4). For rushhour travel $(v=1), k=1,3,6,7$; for shopping travel ( $v=2$ ), $k=2,4,8$; and for recreation travel ( $v=3$ ), $k=5,9$. For purposes of evaluating sytem capacity and congestion, rush-hour trips (v = l) are of primary concern.

## Modal Split

Modal split is determined from the trip matrix that results from Equation 14. Although the availability of particular modes may conceivably affect the trip distribution (for example, transit availability may contribute to destination choice), the current model is simplified in this regard. All modes are considered to be available for all trips. Clearly a route-specific public transit system would enhance this part of the model.

Modal split is determined by multiplying the trips from zone $i$ to zone $j$ by an admittance factor for mode $m$ :
$T(i, j, v, m)=Q(i, j, v) \cdot A F(m)$
where $T(i, j, v, m)$ is trips from zone $i$ to zone $j$ by traffic flow $v$ and mode $m$ and $A F(m)$ is the admittance factor from mode $m$. The admittance factor is the ratio of the relative admittance factor for mode m to the sum of relative admittance factors for all modes:

$$
\begin{equation*}
A F(m)=R A F(i, j, m) / \underset{m}{\operatorname{R}} \mathbf{R A F}(\mathrm{i}, \mathrm{j}, m) \tag{16}
\end{equation*}
$$

where RAF(i,j,m) is the relative admittance factor for mode m ( $0 \leq R A F \leq 1$ ).

The relative admittance factor is in turn calculated from cost data for mode $m$ and the distance from zone i to zone $j$ :
$\operatorname{RAF}(\mathrm{i}, \mathrm{j}, \mathrm{m})=\exp \left(-\mathrm{ak}_{4}\right)$
where a is the sensitivity constant (0.003 assumed) and

$$
\begin{equation*}
\mathrm{k}_{4}=\operatorname{CosT}(\mathrm{i}, \mathrm{j}, \mathrm{~m}) / \mathrm{SAF}(\mathrm{~m}) \tag{18}
\end{equation*}
$$

$\operatorname{CoST}_{\mathrm{m}}$ is defined for each mode as follows (parameter values are shown in Table 5):

Walking:
$\operatorname{Cost}(\mathrm{i}, \mathrm{j}, 1)=\operatorname{TIMECOST} \cdot \operatorname{CT}(1) \cdot \mathrm{D}(\mathrm{i}, \mathrm{j})_{\mathrm{s}}$
Car:
$\operatorname{COST}(\mathrm{i}, \mathrm{j}, 2)=\mathrm{PK}+\operatorname{TIMECOST} \cdot \mathrm{CT}(1) \cdot \mathrm{WD}(2)$
$+\mathrm{D}(\mathrm{i}, \mathrm{j})_{\mathrm{s}}[$ TIMECOST $+\operatorname{OPCOST}(\mathrm{m})] \mathrm{CT}(2)$
Bus:

$$
\begin{align*}
\operatorname{COST}(\mathrm{i}, \mathrm{j}, 3)= & \mathrm{F}(3)+\operatorname{TIMECOST}[\mathrm{WT}(3)+\operatorname{CT}(\mathrm{I}) \cdot \mathrm{WD}(3)] \\
& +\mathrm{D}(\mathrm{i}, \mathrm{j})_{s}[\operatorname{TIMECOST}+\operatorname{OPCOST}(\mathrm{m})] \mathrm{CT}(3) \tag{21}
\end{align*}
$$

Taxi:

$$
\begin{align*}
\operatorname{COST}(\mathrm{i}, \mathrm{j}, 4)= & \mathrm{F}(4)+\operatorname{TIMECOST} \cdot \mathrm{WT}(4) \\
& +\mathrm{D}(\mathrm{i}, \mathrm{j})_{\mathrm{s}}[\operatorname{TIMECOST}+\operatorname{OPCOST}(\mathrm{m})] \operatorname{CT}(4) \tag{22}
\end{align*}
$$

Table 5. Main parameters for modal-split equations.

|  | Mode |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
|  | Walking | Car | Bus | Taxi |  |  |  |  |  |
| Parameter | 3 | 30 | 15 | 30 |  |  |  |  |  |
| Average speed (miles $/ \mathrm{hr})[\mathrm{R}(\mathrm{m})]$ | - | - | 0.25 | 0.80 |  |  |  |  |  |
| Fare $(\$)[\mathrm{F}(\mathrm{m})]$ | - | 0.15 | - | 1.25 |  |  |  |  |  |
| Operating cost $(\$ /$ mile $)[\mathrm{OP}(\mathrm{m})]$ | - | - | - | - |  |  |  |  |  |
| Parking cost $(\mathrm{PK})$ | - | 50 | 1,000 | 0 |  |  |  |  |  |
| Average walking distance $(\mathrm{ft})[\mathrm{WD}(\mathrm{m})]$ | - | 10 | 15 |  |  |  |  |  |  |
| Average waiting time $(\mathrm{min})[\mathrm{WT}(\mathrm{m})]$ | - | $-\overline{0}$ |  |  |  |  |  |  |  |
| Social acceptability factor $[\mathrm{SAF}(\mathrm{m})]$ | 0.25 | 2.0 | 1.0 | 0.5 |  |  |  |  |  |

where

$$
\begin{aligned}
\mathrm{D}(\mathrm{i}, \mathrm{j})_{\mathrm{S}}= & \text { distance between zone } i \text { and zone } j \\
& \text { along route s measured in seconds at } \\
& 30 \text { mph, } \\
\text { TIMECOST }= & \text { value of time in cents per second } \\
& \text { (\$3.60/hr assumed), } \\
\text { OPCOST }(m)= & \text { operating cost of mode } m, \\
\mathrm{CT}(\mathrm{~m})= & \text { speed conversion factor for mode } \mathrm{m} \\
& \text { relative to automobile, } \\
\mathrm{F}(\mathrm{~m})= & \text { fare of mode } \mathrm{m}, \\
\mathrm{R}(\mathrm{~m})= & \text { speed of mode } \mathrm{m}(\mathrm{mph}), \\
\mathrm{WD}(\mathrm{~m})= & \text { walking distance to mode } \mathrm{m}, \\
\mathrm{WT}(\mathrm{~m})= & \text { waiting time for mode } \mathrm{m}, \\
\mathrm{PK}= & \text { parking cost, } \\
\mathrm{OP}(\mathrm{~m})= & \text { operating cost or per-mile fare of } \\
& \text { mode } m, \text { and } \\
\mathrm{SAF}(\mathrm{~m})= & \text { social acceptability factor for mode } m \\
& \text { (a measure of the relative convenience } \\
& \text { and availability of mode } m \text { ). }
\end{aligned}
$$

## Passenger-Car Units

For purposes of determining congestion on various road links, the trips by mode calculated in Equation 21 àre tiransiaċed incio passenger-car unitis (FCüs) according to the following formula:
$\operatorname{PCU}(\mathrm{i}, \mathrm{j}, \mathrm{v})=\underset{\mathrm{m}}{\mathrm{z}} \mathrm{T}(\mathrm{i}, \mathrm{j}, \mathrm{v}, \mathrm{m}) / \operatorname{LOAD}(\mathrm{m})$
where PCU(i,j,v) is PCUs from zone i to zone j during traffic flow $v$ and LOAD ( $m$ ) is the load factor for mode $m$ in terms of trips taken. Load factors are as follows:

Walking: $\operatorname{LOAD}(1)=10^{10}$ (walking trips do not affect congestion),

Car: $\operatorname{LOAD}(2)=1.5$,
Bus: LOAD(3) $=3$ car lengths/25 passengers $=$ 0.12 , and

Taxi; $\operatorname{LOAD(4)}=1$.
PCUs provide a method for translating passenger trips into vehicular units. For example, a bus uses the road space of three cars but has a load factor of 25 passengers. Thus, the road space per passenger is only 0.12 car length. The PCUs become inputs to the route-assignment procedure.

## Route Assignment

Travel time [D(i,j)] between zones appears throughout the preceding equations. In the initial iteration, travel time is based on the mean free travel time between zones, assuming no congestion. Later iterations take into account congestion as a function of the PCU volume on each road link connecting each pair of zones.

Travel time per driver on each road link (gh) is a function of the mean free travel time plus congestion as shown in Figure 7:
$W_{g h}=a_{g h}+b_{g h}\left(X_{g h}\right)^{4}$
where

```
\(W_{g h}=\) average travel time per vehicle on link gh
        where \(g\) is the originating node and \(h\) is the
        terminating node for each link in the net-
        work,
\(a_{g h}=\) mean free travel time between nodes \(g\) and \(h\),
\(b_{g h}=\) congestion parameter (the parameter is a
    function of the road capacity), and
\(X_{g h}=\) traffic flow per unit time.
```

Travel time between zones is found by summing the travel times of each link along the shortest path connecting each pair of zones:
$D(i, j)_{s}=\sum_{g h=1}^{t} W_{g h}$
where $D(i, j)_{S}$ is travel time between zones $i$ and $j$ along route $s$ composed of links $g h=1$ to $t$.

At each iteration of the route-assignment procedure, a new shortest path between zones $i$ and $j$ is determined based on the sum of travel times on individual road links connecting the two zones. The procedure converges to a stable equilibrium in which the shortest path between zones cannot be improved by switching routes. The route-assignment routines allocate trips to each link so as to minimize the sum of the areas under the volume delay function (Wgh). At its minimum value, the objective function (Equation 6) is the sum of the areas under the average travel time functions shown in Figure 7 ( $W_{g h}$ ) up to the equilibrium flows $x_{g h}$. These areas have no (known) economic interpretation. Integrating Equation 24 a results in the following:
$F\left(x_{g h}\right)=\int_{0}^{x}\left(a_{g h}+b_{g h} X^{4}\right) d X=a X+(b / 5) X^{5}{\underset{0}{\mathrm{I}}}_{\stackrel{x}{l}=a x+(b / 5) x^{5} .}$
where $F\left(x_{g h}\right)$ is the area under volume delay function $W_{g h}$ at volume $x_{g h}$ and $x_{g h}$ is the number of trips on link gh.

The sum of these areas for all links is
$F(X)=\sum_{g h=1}^{t} a_{g h} x_{g h}+(b / 5) x_{g h}^{s}$
The LeBlanc route-assignment procedure determines

Figure 7. Average travel time per vehicle along link gh.

$F(X)$ for an initial trip allocation. The procedure is one of sequential solution of linearized approximations followed by a line search. This is accomplished by evaluating the gradient of $F$ for the current set $(K)$ of trips $X^{K}$ where $x^{K}=\left(x_{11}\right.$, . . , $x_{l n}$, . . , $x_{m l}$, . . . $x_{m n}$ ) for all links $g h$ in order to obtain the optimal direction of change:
$\nabla F\left(X_{g h}\right)=a_{g h 1}+b X_{g h}^{4}$
If $Y^{K}$ represents the new set of values for $X^{K}$ satisfying the optimal direction of change and conservation of flow constraints, a one-dimensional search is performed for $\alpha$ to minimize $G(\alpha)$ :
$\operatorname{Min} \mathrm{G}(\alpha)=\operatorname{Min} \mathrm{F}\left[\alpha \mathrm{Y}^{\mathrm{K}}+(1-\alpha) \mathrm{X}^{\mathrm{K}}\right]$ for $\alpha \epsilon(0,1)$
Finally a new set of route-assignment values for the next iteration of $\mathrm{X}^{\mathrm{K}}$ is found:
$\mathrm{X}^{\mathrm{K}+1}=\alpha \mathrm{Y}^{\mathrm{K}}+(1-\alpha) \mathrm{X}^{\mathrm{K}}$
The process is repeated beginning with Equation 27 until $F(X)$ in Equation 26 is minimized within a stipulated margin of error.

Although primary output of the route-assignment routine is the number of trips ( $x_{g h}$ ) on each link, the journey time between zones can easily be found from Equation 24b.

The travel times between zones are for a car traveling 30 mph . Cars, buses, and taxis are all assumed to be subject to the same congestion so that the travel time for other modes traveling at different speeds can be obtained through a simple transformation.

## Travel Cost

At each iteration of the model, total daily transportation cost (TDC) is calculated from the trip matrix $[T(i, j, v, m)]$ and cost per trip by mode $[\operatorname{CosT}(i, j, m)]:$
$\mathrm{TDC}=\sum_{\mathrm{i}} \sum_{\mathrm{j}} \sum_{\mathrm{m}} \sum_{\mathrm{v}} \operatorname{CosT}(\mathrm{i}, \mathrm{j}, \mathrm{m}) \cdot \mathrm{T}(\mathrm{i}, \mathrm{j}, \mathrm{v}, \mathrm{m})$
For simplification, trip cost is not differentiated between traffic flows (v). Travel times are based on congestion from peak-hour traffic and thus tend to bias total transportation costs toward the high side.

At successive iterations, new values for travel time $D(i, j)$ are calculated from the route-assignment routines. These values become inputs for the next iteration of the main model beginning with Equation 12, as shown by the feedback loop in Figure 1.

## RESULTS

The road networks for the two plans are shown in Figures 8 and 9. Plan A has 39 nodes with 104 links; each pair of nodes is connected by two links, one for each direction. Plan $B$ has 41 nodes and 132 links.

Both plans are connected to the region outside the subject area by the two main highways that pass through the community and that are, of course, common to both plans--US-59 and TX-6. The point at which each highway enters the community is treated as a separate zone for purposes of allocating in-ward- and outward-bound external trips. Because the major point of entry to Houston is from the northeastern end of US-59, approximately 75 percent of the external trips pass through zone 12.

TDC in the two plans is shown as follows. After
four iterations of the model, total costs converge toward a stable figure:

| Iteration | Plan A (\$) | Plan B (\$) |
| :---: | :---: | :---: |
| 1 | 497,200 | 498,600 |
| 2 | 609.400 | 692,100 |
| 3 | 608,734 | 630,080 |
| 4 | 608,750 | 630,126 |

It is immediately apparent from the preceding figures that plan A has lower transportation costs than plan B. Because the two plans serve identical user populations by assumption, they confer equivalent benefits to each user group. Therefore, plan A, which provides equal benefit at lower cost than plan $B$, is the preferred plan.

In the preceding case study, trip generation was assumed to be perfectly inelastic, that is, lower trip costs in plan A were not assumed to induce any additional travelers. However, following the argument presented earlier, the benefits of elastic demand would accrue more to plan $A$ than to plan $B$ because the lower trip costs in plan A would generate more new trips. Unless the added trips are so numerous that there is a major impact on congestion, the inclusion of elastic demand serves only to enhance the current preference for plan $A$.

Several useful observations can be made from the TDC values given previously. Because the first iteration of the model is based on travel time with zero congestion in which everyone takes the shortest path between origin and destination, iteration 1 indicates that TDC differs only marginally between the two plans. However, congestion cost can be deter-

Figure 8. Plan A road network.


Figure 9. Plan B road network.


Table 6. Comparison of traffic volumes on individual road links.

| Plan A |  | Plan B |  |
| :--- | :--- | :--- | :--- |
| Link $^{\text {a }}$ | Volume $^{\text {b }}$ |  | Link |
| $27-28$ | 7,277 | $29-33$ | Volume |
| $24-27$ | 519 | $26-29$ | 1,646 |
| $23-25$ | 4,044 | $30-34$ | 3,586 |

${ }^{a}$ Numbers refer to intersection nodes in Figures 8 and 9.
${ }^{6}$ PCUS per rush-hour period ( 3 hr in the morning and 3 hr in the afternoon).
mined from the TDC values by comparing the conges-tion-free cost of iteration 1 with that of iteration 4. It is apparent that congestion is worse in plan B, adding approximately 26.5 percent to TDC as compared with 22.4 percent in plan $A$.

At this point fine tuning the selected master plan can easily be done. Points of congestion are immediately apparent from a comparison of mean free travel time on each link with actual travel time. Land uses can be shifted, and roads can be added to the plan or widened at major bottlenecks. Table 6 shows a comparison of traffic values on three road links that are common to both plans. Because of the greater dispersal of land uses in plan $B$, traffic is more evenly apportioned over different roads. However, the absence of several connecting roads causes total transportation costs in plan $B$ to be higher than those in plan A. If one could look at the $O D$ trip matrix and the travel time on individual links, one could see how the location of land uses and the
road network each contributes to traffic flows and trip costs within the community.

## CONCLUSIONS

Transportation and land use planning is replete with complicated computer models for evaluating trip movements. Typical models have two major shortcomings when applied to land use planning: They do not provide any single summary measure of total cost for use in selecting the best plan from several alternatives and it is expensive and time-consuming to run them.

The model presented here is especially well suited for planning at the community or suburban level within a larger metropolitan area. The impact of individual land use changes or real improvements on traffic flows and total transportation costs can be determined within a context of total information. In other words the effects on all trip movements within the system can be observed for even minor changes in the development plan or transportation network. Congestion points can be seen immediately and suggested changes in the master plan can be tested easily.

The model provides key information for making planning decisions in a cost-effective manner. Input data are assembled strictly from land use and transportation network information that can be taken directly from the master plan of a community. On a CDC 6600 computer, this model executes in less than 10 sec for a plan of 15 zones, 41 nodes, and 132 links. The equilibrium route-assignment routines converge to a stable equilibrium in seconds and provide useful information concerning traffic volumes and congestion on individual links.

A major problem in land use planning today is that despite the availability of sophisticated analytical tools, few planning departments take advantage of them to test the impact of various land use and transportation decisions. Although such tools are used more often for major capital investments such as mass transit systems, they are needed most for land use and transportation decisions that cities and counties make every day. The model presented here is intended to address that need by providing key measures of the economic impact of zoning, land use, road improvement, and transportation network decisions that determine community design.

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