Equilibrium Traffic Assignment on an Aggregated Highway Network for Sketch Planning

R.W. EASH, K.S. CHON, Y.J. LEE, AND D.E. BOYCE

An application of the equilibrium traffic assignment algorithm on a simplified highway network, such as might be used for sketch planning, is described. Analysis zones in the assignment are also substantially larger than in most conventional traffic assignments. The algorithm for equilibrium traffic assignment is introduced, followed by a discussion of the problems with equilibrium traffic assignment in a sketch-planning application. Next, the network coding procedures for the case study are examined. Results of the sketch-planning assignment are then evaluated against a comparable regional assignment of the same trips. Finally, there is a discussion of how this research fits into the programs of a transportation planning agency.

An application of equilibrium traffic assignment to sketch planning is presented in this paper. Trips are assigned onto an aggregated network with a limited number of links, nodes, and zone centroids. One arterial link in the sketch-planning network is equivalent to a number of links in a conventionally coded regional highway network, and one sketch-planning zone is substantially larger than a zone in the regional assignment at the same location. The traffic assignment algorithm used in the study converges to approximately equal path travel times for multiple paths between origin-destination zone pairs. The algorithm is available to most transportation planning agencies.

A major portion of the paper is spent on a comparison of this sketch-planning assignment with a regional traffic assignment of a large trip table onto a detailed coded highway network. This comparison is complicated by the different number of intrazonal trips in the two traffic assignments; therefore, a procedure was developed to determine the significance of the additional intrazonal trips in the sketch-planning assignment. Vehicle miles of capacity and travel, vehicle hours, and average speeds predicted by the two assignments are summarized at the regional and zonal levels.

In the introductory sections of the paper, the equilibrium traffic assignment algorithm and the network coding procedures for the sketch-planning network are documented. A simple method for aggregating links and summing regional link capacities into sketch-planning link capacities is then described. The question of the best network aggregation procedure is not considered. Moreover, a solution of this network aggregation problem was not an objective of the research, but rather a data requirement. The principal concern of this paper is to demonstrate a satisfactory correspondence between...
Traffic assignments on the sketch plan and regional networks. Finally, a few implications of this research for work programs of transportation planning agencies are discussed.

**Methodological Approach**

The equilibrium concept was first formulated for minimum time-path traffic assignment by Wardrop (1). Given that travel times on a network link increase with traffic, a highway network is in equilibrium if the travel times along all paths that are used between each origin-destination are equal, and no unused path has a lower time. In other words, no driver has an incentive to change paths.

Several algorithms were developed in the early 1970s to determine the equilibrium traffic flows, and one version of the algorithm is now available in the Urban Transportation Planning System (UTPS) computer programs for transportation planning supported by UMTA and FIMIN (2). The formulation of the algorithm discussed here follows the work of Nguyen (3) and LeBlanc et al. (4) and is consistent with the algorithm available in the UTPS program UROAD.

For a given trip table, the equilibrium assignment of traffic may be found by solving a nonlinear mathematical programming problem. The solution to this problem is that set of traffic flows on network links that minimizes a nonlinear convex mathematical function (called an objective function), the value of which depends on the traffic flows. These flows must also satisfy a second set of linear equations called constraints. In general terms, the constraints on the objective function ensure that all solutions are feasible trip assignments; that is, all trips in the trip table are assigned to the network, and negative link flows are prohibited.

The objective function is to minimize the sum of the areas under each link's travel-time and traffic volume congestion function from zero to the assigned flow. To understand the interest in minimizing the sum of these areas requires some mathematical analysis beyond the scope of this paper. It is only important to understand that the link flows that correspond to the minimum value of this objective function are those that satisfy the equilibrium conditions.

**Summary of Equilibrium Traffic Assignment Algorithm**

The algorithm to solve the equilibrium traffic assignment problem is based on a nonlinear optimization technique developed by Frank and Wolfe (5). Theirs is an iterative approach that starts with an initial feasible solution that satisfies the constraints, determines a feasible direction to move that improves the objective function, and then calculates how far to move in this direction. This results in a new feasible solution, and the procedure iterates until the objective function cannot be improved.

A network composed of links with congestion functions, a trip table for assignment, and a first solution that is a feasible assignment of trips to the network are given. The equilibrium conditions are normally not met by this first trip assignment. Application of the method by Frank and Wolfe then involves the following steps.

1. Compute the travel time on each link by using volumes in the current solution.
2. Trace minimum time-path trees from each origin to all destinations by using the link times computed in step 1.
3. Assign all trips for each origin to each destination to the minimum paths computed in step 2 (this produces an all-or-nothing trip assignment).
4. Linearly combine the current link volumes \( v_a \) of the solution and the new all-or-nothing link volumes \( v_a' \) of the assignment to obtain a new current solution \( v_a'' \) that minimizes the objective function:
   \[
   \sum \int_{0}^{v_a''} s_a(x) \, dx
   \]
   where

   \[
   v_a'' = (1-\lambda)v_a + \lambda v_a' = \text{new current solution volume on link } a
   \]
   \[
   S_a(x) = \text{link congestion function for link } a, \text{ and } \lambda = \text{constant between 0 and 1.}
   \]
5. If the solution has converged sufficiently, stop; otherwise return to step 1.

The sequence of program steps is shown in the flowchart in Figure 1.

![Flowchart](chart)

**Equilibrium Traffic Assignment and Sketch Planning**

The obvious problems in applying equilibrium traffic assignment to sketch planning are how to simplify the traffic assignment network and analysis zones and the nature of the travel-time and traffic volume congestion function for such a network. Previously, researchers have constructed sketch-planning networks either by eliminating minor and lightly traveled links (6) or by aggregating links in a
detailed network into summary links (7). The sketch-planning network for this project combines these two approaches and includes all freeway and expressway links with a grid network of aggregate links for arterial streets.

A number of time and volume relationships have been developed for traffic assignment when the coded network resembles an actual highway network (8). The most widely used is the Bureau of Public Roads (BPR) formula available in the program UROAD:

\[ T = T_0 \left[ 1 + 0.15 \left( \frac{v}{c} \right)^2 \right] \]

where

- \( T_0 \) = uncongested (zero traffic flow) travel time on the link,
- \( T \) = estimated link travel time, and
- \( v/c \) = ratio of link traffic volume to link capacity.

In a convenitionally coded highway assignment network, each link is a street or highway segment, the attributes of which can be observed. To illustrate the detail coded into these networks, only local streets and rural roads used principally for local access are omitted in the regional network used by the Chicago Area Transportation Study (CATS). It is reasonable, therefore, to assert that the coded network built from all these individual links reflects the supply characteristics of the regional highway network.

If the regional network links to be combined in a sketch-planning link can be identified and acceptable regional network link congestion functions exist, then two methods for developing aggregate congestion functions appear plausible. First, the regional time and volume relationships can be mathematically combined to form an aggregate congestion function. Alternatively, a general link congestion function can be applied to a summary link, the attributes of which are aggregate quantities. Both methods were attempted in this project.

Further problems in using equilibrium assignment for sketch planning are caused by the larger analysis zones and the corresponding smaller trip table. Potential conventional regional assignment, an analyst might have a trip table with a thousand or more zones. By comparison, no more than a few hundred zones can be used in a sketch-planning application.

More trips occur within a zone when larger zones are used in a traffic assignment. Because intrazonal trips are not assigned to the highway network, this means that estimated traffic is reduced. This underassignment of trips, in turn, affects congestion in the highway network and the travel times predicted by the link congestion functions. The effect of this larger number of intrazonal trips on the sketch-planning assignment was evaluated in this project.

The smaller trip table causes cell values to increase, and more trips are loaded onto the network at zone centroids. The links immediately adjacent to centroids are then loaded with all the traffic from the larger area covered by the sketch-planning zone. These links tend to be overassigned, which also affects the travel times predicted by the link congestion functions. Fortunately, this problem is mitigated by running more iterations of the equilibrium algorithm to load more paths in the sketch-planning network.

Coding the Sketch-Planning Network

The first step in coding the sketch-planning network was selection of the system of analysis zones. The zone system used in the project was developed by combining the CATS regional zones into a suitable number of areal units. Each sketch-planning zone usually includes four to nine regional zones. The resulting zone system covers the eight-county northeastern Illinois (six counties) and northwestern Indiana (two counties) region; it is shown in Figure 2. There are 317 sketch-planning zones compared to the 1,797 zones used in the CATS regional traffic assignments.

The basic areal unit in the region is the survey township, a roughly 6-mile² land unit originally surveyed in the mid-1800s. All of the CATS zone systems, including the sketch-planning zones, make use of these survey townships. A majority of the sketch-planning zones are quarter-townships (approximately 9 miles²), with full townships as the next largest group of zones. At the state line between Indiana and Illinois, a few zones are slightly larger than full townships, and several smaller odd-sized zones are along the lakefront.

Zones are covered by a network of bidirectional arterial and freeway links (9). Each zone's centroid is located at the center of a zone and is connected by two to four arterial street links to produce a fairly regular grid network over the region. Freeway and expressway links are then coded on top of this regular grid of arterial street links, with interchanges placed approximately at their actual locations. A portion of the sketch-planning network is shown in Figure 3.

Links are coded as either arterials or freeways (expressways). Attributes coded for each sketch-planning link include beginning and ending node numbers, link length, type of area where link is located, link free speed, and link capacity. The type of area where the link is located is coded by using municipal boundaries and zone populations. Link free speed is then estimated for each link by using the area and facility types. All coding was done in the usual UTPS format, except that the traffic count field was used for link capacity and UROAD was altered to accept link capacities in this field.

Arterial Link Capacity and Congestion Functions

The original approach in the project to develop
sketch-planning arterial street network congestion functions was to aggregate mathematically the BPR formula congestion functions used in the regional network, as described by Morlok (10). This approach can straightforwardly be applied for two or more consecutive links, or for two parallel links between the same two nodes. The intent was to construct the sketch plan arterial network congestion functions from the regional network congestion functions by repeated aggregation using these two relationships. This approach proved far too time consuming to be completed manually, and it was believed that preparing suitable software to accomplish the work required substantial efforts beyond the scope of the project.

Given the geometry of the sketch-planning network and the arrangement of zones (two regular grid patterns offset so that each zone boundary is usually crossed by only one summary arterial street link), an obvious method for estimating sketch-planning arterial link capacities was to sum regional arterial network capacities along the edge of a sketch-planning zone. This was accomplished by first overlaying the sketch-planning zones on the regional highway network to identify the regional arterial street links crossing a zone boundary, and then summing the appropriate regional link capacities. This procedure is shown in Figure 4. Note that in this example the capacity of the first regional link is shared with the adjacent zone.

**EVALUATION OF SKETCH-PLANNING ASSIGNMENT**

Because there are separate zone systems in the regional traffic assignment and in the sketch-planning traffic assignment, intrazonal trips in the two assignments are different. This makes it difficult to compare the two assignments because fewer trips are assigned onto the sketch-planning network and fewer vehicle miles of travel are produced. To remove this bias from the comparison of the regional and sketch-planning assignments, an estimate of these added intrazonal trips and missing vehicle miles was needed.

A second assignment of trips onto the regional highway network was performed with a trip table that contained only the additional intrazonal trips in the sketch-planning assignment, i.e., the trips that became intrazonal when the regional zones were aggregated. This partial trip table was created by scanning the regional trip table and eliminating all entries that would be intrazonal in the sketch-planning zone system. The resulting intrazonal trip table was then assigned onto the same minimum time paths used in the regional traffic assignment. The proportion of the intrazonal trip table assigned to each minimum time path was the same as the proportion of the full trip table assigned to that minimum time path. Link volumes from the intrazonal trip assignment were then subtracted from the original link volumes of the regional assignment to produce a revised vehicle mile estimate.

Another difference between the two assignments is the number of iterations of the equilibrium algorithm. In the regional traffic assignment, five separate all-or-nothing assignments are completed,
which correspond to four iterations of the equilibrium algorithm. For the sketch-planning assignment, 10 all-or-nothing assignments are prepared (9 iterations of the equilibrium algorithm); therefore, each interchange has the opportunity to travel 5 added paths. However, the sketch-planning assignment is still less expensive in computer costs. This points out the trade-off between detail in the assignment network and the number of paths that can be practically loaded in the equilibrium algorithm. As the network becomes more detailed, the cost of building minimum time paths increases, thereby restricting the number of iterations of the equilibrium algorithm that can be completed.

The CMTS regional and sketch-planning assignments in the project are given in Table 1. Both assignments are for a 1-hr 1975 trip table in the morning peak period. The sketch-planning network is less than one-tenth the size of the regional network, even allowing for the fact that the regional network extends slightly beyond the eight-county area covered by the sketch-planning network. The data in Table 1 indicate that the number of trips in the two assignments is slightly different because of rounding during the allocation of the regional trip table into sketch-planning zones. There are an additional 137,000 intrazonal trips in the sketch-planning assignment.

Table 1. Summary of regional and sketch-planning network assignments.

<table>
<thead>
<tr>
<th>Item</th>
<th>Sketch-Planning</th>
<th>Regional Network a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyses zones</td>
<td>317</td>
<td>1,797</td>
</tr>
<tr>
<td>Network nodes</td>
<td>820</td>
<td>12,040</td>
</tr>
<tr>
<td>Origin links</td>
<td>2,422</td>
<td>37,065</td>
</tr>
<tr>
<td>Assigned intrazonal trips</td>
<td>1,016,900</td>
<td>1,140,400</td>
</tr>
<tr>
<td>Unassigned intrazonal trips</td>
<td>192,800</td>
<td>55,800</td>
</tr>
<tr>
<td>Number of iterations (all-or-nothing assignments)</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Computing time b (CPU)</td>
<td>3 min, 45 sec</td>
<td>163 min, 7 sec</td>
</tr>
<tr>
<td>Memory required b (maximum bytes)</td>
<td>600K</td>
<td>540K</td>
</tr>
</tbody>
</table>

aThe regional network covers a slightly larger area than the eight-county Chicago region.
bIBM 3033, Operating System VS2.

The last two items in Table 1 give the relative computer costs of the two assignments. The sketch-planning assignment was accomplished with the UTPS program UROAD (slightly modified to use link capacities from the network link file and an efficient line-search procedure), whereas the regional assignment made use of the PLANPAC programs originally prepared in the mid-1960s by the FHWA (11), with a separate program for the equilibrium algorithm (12). Different programs are required because of the size of the regional network, which is too large for the version of the UROAD program used in the project. Identical functions (path building, assignment, line search between all-or-nothing assignments, and calculation of link times) are carried out in both cases.

Computer memory requirements for the two assignments are about equal because UROAD allocates memory space according to the largest node number (more than 6,000 in this case) instead of the number of nodes in the network, and also because the individual PLANPAC programs can be written more efficiently for memory use because each program performs only a single function. The computer time required to run the sketch-planning assignment is almost insignificant compared with the regional assignment, even though twice as many iterations are performed for the sketch-planning assignment.

Regional Travel Comparison

The results of the regional and sketch-planning assignments within the eight counties are given in Tables 2-5. Because the sketch-planning network does not include any ramps between freeways or between freeways and arterials, ramps are not included in the regional travel figures. Miles of capacity (Table 2). Even without ramps, slightly more capacity is available in the regional network than in the sketch-planning network. Total arterial capacity in the two networks is surprisingly close, however, considering the crude method used to estimate the capacity of the sketch-planning arterial street links.

Several reasons can be cited for the discrepancy between the freeway capacities in the two networks. A few short freeway segments, most only a mile or so in length, are omitted from the sketch-planning network. Another reason is that the sketch-planning freeway links are coded somewhat abstractly as straight links between freeway interchanges. This tends to underestimate the actual length of these links.

The data in Table 3 give vehicle miles of travel for the two networks. Vehicle miles on ramps are included in the regional assignments, even though their capacity was not coded in the sketch-planning network, the vehicle miles of travel that would occur on ramps are approximated by additional travel to reach the single interchange node. Vehicle miles on ramps that connect freeways are included in the freeway category, and vehicle miles on ramps between freeways and arterials are split evenly between both route types in the regional assignment figures. Only vehicle miles within the eight counties are tabulated.

The data in Table 3 also describe the impact of the intrazonal trips in a comparison of the two assignments. When the sketch-planning and regional assignments are first compared, there is a difference of 4 percent in the division of vehicle miles between freeways and arterials. Twenty-nine percent of the unadjusted regional vehicle miles are assigned to freeways, whereas 31 percent of the sketch-planning vehicle miles occur on freeways. When the regional assignment is adjusted for the different number of intrazonal trips, part of this difference is explained. The great majority of trips in the intrazonal trip table is assigned onto arterials because these trips are short and are not likely to use a freeway.

The difference between the total vehicle miles in the sketch-planning assignment and the adjusted regional assignment is about 5 percent, and the extra vehicle miles on sketch-planning network freeways account for nearly all of the difference. After reviewing the coding of the two networks, it is clear that freeways in the sketch-planning network have some advantages that freeways in the regional network do not have. In addition to the slight undercoding of distance along sketch-planning freeway links noted earlier, the only radial links included in the sketch-planning network are freeway links, so paths made up only of arterial links must be longer than comparable paths in the regional assignment network.

Vehicle hours of travel for the assignments are given in Table 4. These estimates follow the pattern established in Table 3. Although total vehicle hours in the sketch-planning assignment and in the adjusted regional assignment are nearly equal, the distribution of the vehicle hours between freeways and arterials is somewhat different. In the sketch-
Table 2. Vehicle miles of capacity for eight-county region.

<table>
<thead>
<tr>
<th>Highway</th>
<th>Sketch Plan</th>
<th>Regional</th>
<th>Intrazonal</th>
<th>Regional Less Intrazonal</th>
<th>Sketch Plan/Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway</td>
<td>3,870</td>
<td>4,241</td>
<td>NA</td>
<td>NA</td>
<td>0.91</td>
</tr>
<tr>
<td>Arterial</td>
<td>14,286</td>
<td>14,584</td>
<td>NA</td>
<td>NA</td>
<td>0.98</td>
</tr>
<tr>
<td>Total</td>
<td>18,156</td>
<td>18,825</td>
<td>NA</td>
<td>NA</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Note: NA = not applicable.

Table 3. Vehicle miles of travel for eight-county region.

<table>
<thead>
<tr>
<th>Highway</th>
<th>Sketch Plan</th>
<th>Regional</th>
<th>Intrazonal</th>
<th>Regional Less Intrazonal</th>
<th>Sketch Plan/Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway</td>
<td>3,476</td>
<td>2,985</td>
<td>3</td>
<td>2,982</td>
<td>1.17</td>
</tr>
<tr>
<td>Arterial</td>
<td>7,001</td>
<td>7,315</td>
<td>289</td>
<td>7,026</td>
<td>1.00</td>
</tr>
<tr>
<td>Total</td>
<td>10,477</td>
<td>10,300</td>
<td>292</td>
<td>10,008</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 4. Vehicle hours of travel for eight-county region.

<table>
<thead>
<tr>
<th>Highway</th>
<th>Sketch Plan</th>
<th>Regional</th>
<th>Intrazonal</th>
<th>Regional Less Intrazonal</th>
<th>Sketch Plan/Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway</td>
<td>104,962</td>
<td>81,549</td>
<td>103</td>
<td>81,446</td>
<td>1.29</td>
</tr>
<tr>
<td>Arterial</td>
<td>261,048</td>
<td>298,704</td>
<td>12,040</td>
<td>286,664</td>
<td>0.91</td>
</tr>
<tr>
<td>Total</td>
<td>366,010</td>
<td>380,253</td>
<td>12,143</td>
<td>368,110</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 5. Average travel speed for eight-county region.

<table>
<thead>
<tr>
<th>Highway</th>
<th>Sketch Plan</th>
<th>Regional</th>
<th>Intrazonal</th>
<th>Regional Less Intrazonal</th>
<th>Sketch Plan/Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway</td>
<td>33.1</td>
<td>36.6</td>
<td>33.3</td>
<td>36.6</td>
<td>0.90</td>
</tr>
<tr>
<td>Arterial</td>
<td>26.8</td>
<td>24.5</td>
<td>24.0</td>
<td>24.5</td>
<td>1.09</td>
</tr>
<tr>
<td>Overall</td>
<td>28.6</td>
<td>27.1</td>
<td>24.1</td>
<td>27.2</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Planning assignment 29 percent of the vehicle hours are on freeways, whereas in the adjusted regional assignment only 22 percent of the vehicle hours are on freeways.

The data in Table 5 give the average network speeds computed as the ratio of vehicle miles to vehicle hours. Arterial links have higher average speeds in the sketch-planning assignment than in the regional assignment. Freeway average speeds in the sketch-planning assignment are slower than freeway speeds in the regional assignment because of the added freeway travel.

Travel at the Zone Level

Vehicle miles and average speeds from the two assignments were summarized and compared at the level of sketch-planning zones. Standard statistics were calculated for the distribution of these quantities among zones as well as the correlation between regional and sketch-planning values. All regional quantities used in this phase of the evaluation are actually adjusted quantities without the intrazonal trips added in the sketch-planning assignment.

Figures 5 and 6 are scattergram plots of the vehicle miles per sketch-planning zone and average sketch-planning zone speeds produced by the two assignments. Means and standard deviations for the vehicle mile and speed variables, and the square of the correlation coefficient between sketch-planning and regional variables, are also shown in each figure.

Implications for Planning Agencies

The question arises whether the work described in this paper is relevant for other transportation planning agencies. To a large extent, the sketch-planning zones and the geometry of the sketch-planning network used for this project are the result of the geography and township survey of the northeastern Illinois region. Because other metropolitan areas are spatially organized quite differently, it would be inappropriate to use the gridlike pattern of zones and arterial street links described here.
Sketch-Planning Capabilities

In spite of the parochial nature of the zone pattern and network geometry of the example, some general conclusions can be drawn concerning the characteristics of equilibrium traffic assignment by using larger zones and simpler network coding. The most surprising result was that the different intrazonal trip rates in the two assignments did not significantly affect assignment results. For example, even though zones were 4 to 9 times larger in the sketch-planning assignment, the error introduced in the regional vehicle miles was less than 3 percent. It is apparent that even larger zones could be used without seriously blunting the traffic estimates.

The assignment of traffic is more seriously affected by the coding of the underlying arterial street network. In this case study the grid network of arterial arterials incremented arterial travel distances; as a result, the loadings on sketch-plan freeway links exceed the regional assignment values. The method used to estimate capacity in the sketch-plan arterial street network appears adequate, given the vehicle miles and average speeds that resulted. Overall results from the sketch-planning assignment compared reasonably well with the regional assignment, and zone level assignment quantities were well correlated with regional assignment counterparts. Results from the sketch-planning assignment are, therefore, probably adequate for estimating most highway travel characteristics, including operating costs, emissions, and gasoline consumption, at regional and subregional levels.

Sketch Planning in Work Programs of Planning Agencies

Given these sketch-planning attributes relative to those of a conventional regional assignment, the sketch-planning methodology appears most applicable to long-range systems planning and strategic planning that deals with dramatic changes in transportation supply or demand characteristics. Project-level and corridor planning will almost always require more detailed network coding and smaller analysis zones. Nevertheless, the zone system and network in this sketch-planning example may be used to represent the balance of a region outside the corridor of interest.

Long-range systems planning concentrates on projected traffic or patronage for evaluation of alternative regional networks with different combinations of new major highway and transit investments. Unfortunately, the number of alternatives investigated is often limited because of the resources needed to support the conventional forecasting procedures. Less expensive approaches, such as the sketch-planning methods discussed here, will allow more alternatives to be tested and still provide reasonable estimates of traffic on major highway facilities.

There is also a trend in long-range transportation planning away from the evaluation of alternative networks of major facilities. In many metropolitan areas prospects for new major investments are limited, and future planning will emphasize more general transportation investment strategies for different energy, demographic, social, and economic resource scenarios. Sketch-planning approaches appear more suited for strategic planning than conventional techniques because more scenarios can be investigated and enough detail remains to accurately predict regional and subarea transportation impacts.

ACKNOWLEDGMENT

The research described in this paper is part of a larger project to develop a family of sketch-planning models that analyze general urban transportation and location issues. This project is a cooperative effort between the University of Illinois at Urbana-Champaign and CATS, with university staff primarily responsible for theoretical and software development. The resulting prototype versions of the sketch-planning models are being used in several projects in the CATS work program. The work at the university is supported by grants from the National Science Foundation and UMTA.

REFERENCES

Network Design Application of an Extraction Algorithm for Network Aggregation

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The performance of a network extraction algorithm is described, and the algorithm is tested by using the network design problem. A network is chosen as the original network and is aggregated at different levels. The results of the optimal decision making under a common set of alternative actions are then compared against the original and the aggregated networks. The results suggest that the network aggregation algorithm is a useful tool in simplifying networks to reduce the computational burden associated with the network design problem, and to allow a broader range of policy options to be tested in a fixed amount of computer time than would be allowed by using the original disaggregated network.

Network aggregation is the art and science of condensing a given network into another one that (a) is small enough to be managed efficiently and effectively, and (b) preserves some desired characteristics or satisfies certain objectives or both (1). The usefulness of network aggregation schemes is particularly evident in instances when similar problems are to be solved on a network, or sensitivity analyses of various types are to be performed. Dealing with the detailed network in solving such problems entails high costs in terms of computer storage and time.

There are two main approaches to the network aggregation problem: network element (link or node) extraction and network element abstraction. Extraction of network elements means deletion of the elements of the network that are identified as being insignificant based on a prespecified criterion. Abstraction of the elements collapses the insignificant ones into pseudo or dummy elements. Network element extraction has the disadvantage of causing network disconnection (because of the removal of links). As a result, the remaining links of the network will be overloaded if the origin-destination (O-D) trip matrix is not adjusted appropriately. Network element abstraction is more difficult to perform. It is hard to transform the original network into an aggregate one, and moreover, it is even harder to translate the actions taken on the aggregate network into actions on the detailed network because of drastic changes in the topology of the network that occur during the aggregation process.

The primary objective in developing a network aggregation scheme should be to find an aggregation process that, when applied to a detailed network, results in an aggregate network that retains the physical appearance of the original one as much as possible. Thus when solving a decision-making problem, such as the network design problem on the aggregate network, the results should be easily transferable to the original one. With this in mind, and because the abstraction process changes the topology of the network and cannot effectively serve the process, it is proposed that an aggregation algorithm, which focuses primarily on link extraction, be used. Node extraction is a process that follows link extraction; when all links incident to a node are extracted, the node will be extracted. The algorithm is presented in the next section.

NETWORK EXTRATION ALGORITHM

Let \( N(V,A) \) be a network, where \( V \) is the set of vertices or nodes and \( A \) is the set of arcs or links. Let \( T \) be the set of destinations and \( S \) be the set of origins, \( S \subset T \). Let \( x_i^f \) be the flow over link \( i \) destined to \( t \), \( x_i^s \) be the flow over link \( i \) originating from \( s \), and \( x_i^f \) be the flow over link \( i \) that originates from \( s \) and is destined to \( t \), \( s \in S \), \( s \in S \), and \( s \in S \).

Moreover, let \( D = (a_{ij}) \) be the O-D trip matrix. Finally, let \( C_i(x_i) \) represent the average cost of travel on link \( i \) at flow \( x_i \) that is continuous, differentiable, Riemann integrable, convex, and strictly increasing.

It is assumed that the distribution of flow over a transportation network is based on Wardrop's first principle (2)—user equilibrium (3). There are some links in the network that, after the distribution of the flow has taken place, will not carry a significant amount of traffic. These links are the ones that will be focused on in the aggregation process.