Mathematical Model for Predicting Pavement Performance

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A computer program is developed for predicting the performance of a road as a function of loads, climate, and structural design. The fundamental principle is to divide the road into 0.3-meter lengthwise subsections. Parameter values are assigned to the subsections through a second order autoregressive process. For each subsection the dynamic load and the resulting fatigue and permanent deformations are calculated week by week over the desired period of time. The output as well as the input to the program is stochastic. This means that the deterioration as a function of time will be different for subsequent simulations of nominally identical pavements. The program computes the slope variance, the rut depth, and the amount of cracking, which is summarized into the Present Serviceability Index (PSI). The submodels are based on theoretical considerations and to some degree on adaptation of laboratory tests and field observations. To test the computer program, 180 sections of the AASHO Road Test were selected for simulation. The results of these simulations were encouraging. In addition to structural design of flexible pavements the program may be used to determine the most economical maintenance strategy, to study the effect of changes in legal axle loads, and to evaluate new pavement materials, such as waste materials. The program may also be used to transfer experience from the industrialized countries to the developing countries. Finally the program may, with some changes, be used for design of, and maintenance planning for, airfield pavements.

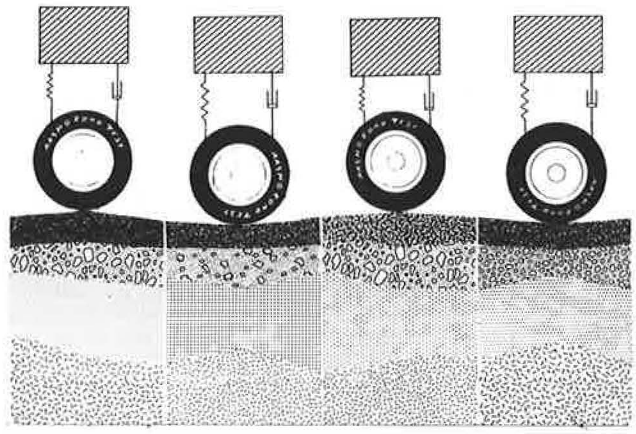


Figure 1. Illustration of the program.

In 1978 a computer program was developed for predicting flexible pavement performance (1). The program was developed on a microcomputer and is capable of predicting longitudinal roughness, rutting, and cracking of three-layer pavements, consisting of asphalt, gravel base, and subgrade. The program was presented at the annual meeting of the Association of Asphalt Paving Technologists in 1979 (2).

In 1980 a committee was formed by the National Road Directorate in Denmark to work out new design models for pavement structures. These new models were to be used to work out improved methods for pavement design. The committee therefore decided to select models that were as complete as possible. For flexible pavements the predictive design model mentioned above was selected. The computer program was rewritten for the National Road Directorate's Burroughs computer and has since undergone extensive testing and modification. The modified model is described in this paper.

OUTLINE OF THE MODEL

The structure of the model is the same as in the original version of the model and is illustrated in Figure 1. The main principle is to consider the road as consisting of a number of short sections, which again are divided into subsections 0.3 m long. Each section is considered separately, and the structural and functional condition are calculated week by week for the desired period of time. Variation of the permanent deformations of the subsections will result in longitudinal roughness, which is combined with the mean level of permanent deformation (rut depth) and structural deterioration (percentage of cracked subsections) into a Present Serviceability Index (PSI) value. Variations in permanent deformations and crack propagation for each subsection are caused by variations in materials parameters and in thicknesses of the layers as well as variations in the resulting dynamic force from subsection to subsection.

First the materials parameters are determined from the climatic conditions of the week considered (for example, temperature of asphalt and effects on unbound materials of frost melting). The loads are then applied through a system consisting of two sets of mass, spring and shock absorber (a quarter car model), simulating a heavy vehicle load. The resulting dynamic force is calculated at points spaced 0.3 m apart (one subsection apart). The force at each point will depend on the shape of the surface (the longitudinal roughness) and on the velocity of the vehicle. For each subsection the effects of the loading are calculated in terms of permanent deformation of each material and reduction of the remaining life of the asphalt layer caused by fatigue. The new PSI value is then calculated and the procedure is repeated for the following week, under a new climatic condition and with a different shape of the surface.

The program may also be used in connection with stage construction, where an overlay is applied either at a given point in time or when the pavement has reached a certain condition. In this version the program may also be used for maintenance purposes. First the previous performance of the pavement is simulated; and when this agrees with the known performance, the model may be used with some confidence for extrapolation to study the effect of applying overlays of different thicknesses. The model could also be modified for use with flexible airfield pavements, where the effects of pavement irregularities on pilot, passengers, and aircraft could be determined directly.

GENERATING PARAMETERS

The input to the program (and the output from it) is stochastic, i.e., parameters such as surface elevation, elevation of interfaces, bitumen content, and so forth, vary from point to point. To obtain a pattern of variation similar to the variations in real pavement structures, a second order autoregressive process is used to generate the parameters (3).

In a second order autoregressive process the value for X_t is determined from

$$X_t = \Phi_1 \cdot X_{t-1} + \Phi_2 \cdot X_{t-2} + a_t \quad (1)$$

where Φ_1 and Φ_2 are constants, and a_t is an independent random variable with mean value = 0 and a constant variance of ($E\,|a_t| = 0$, $\mathrm{var}\,|a_t| = \sigma_a^2$).

The constants Φ_1 and Φ_2 are determined from

$$\Phi_1 = \left\{ [\rho_1(1-\rho_2)]/(1-\rho_1^2) \right\} \text{ and } \Phi_2 = [(\rho_2-\rho_1^2)/(1-\rho_1^2)] \quad (2)$$

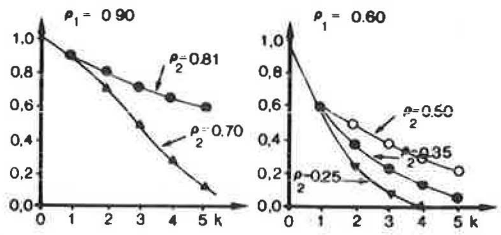


Figure 2. Autocorrelation coefficients.

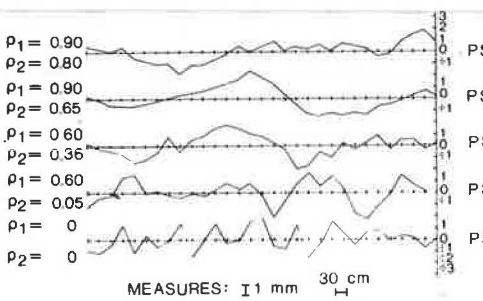


Figure 3. Road profiles as a function of ρ_1 and ρ_2.

where ρ_1 is the autoregression coefficient for a distance = Z (here Z = 0.3 m), and ρ_2 is the autoregression coefficient for a distance = 2 • Z (ρ_k is the autoregression coefficient for a distance = k • Z). The variance of a, σ_a^2, is calculated from

$$\sigma_a^2 = \sigma_x^2 \cdot (1 - \rho_1 \Phi_1 - \rho_2 \Phi_2) \quad (3)$$

where σ_x^2 is the variance of X.

By selecting different values of ρ_1 and ρ_2 it is possible to obtain different autocorrelation functions, and thus to approximate the actual autocorrelation function of the parameter considered. This is illustrated in Figure 2. The effect of selecting different values of ρ_1 and ρ_2 is also illustrated in Figure 3. In this figure the surface elevation is shown as a function of distance at points 0.3 m apart. Figure 3 shows the elevations for five different combinations of ρ_1 and ρ_2, but all with a standard deviation, σ_x, of 1 mm. The PSI values are calculated from the longitudinal profiles. Variations from 2.5 to 4.2 are possible. As the standard deviation is kept constant, the variation in PSI is solely due to the variations in ρ_1 and ρ_2.

In the program the distribution of X may be either normal or log normal. The input consists of the mean value, the standard deviation, ρ_1, ρ_2, and the type of distribution. The program then generates values at each point of the pavement, using a random number generator. Little information is available for evaluating ρ_1 and ρ_2. For the surface level the information is easily obtainable, however, and analysis of longitudinal profiles from a test pavement showed that $\rho_1 = 0.92$ and $\rho_2 = 0.75$ were reasonable values. For other parameters, values of 0.9 and 0.7 for ρ_1 and ρ_2, respectively, have been used when a large degree of autocorrelation was assumed. For low degrees of autocorrelation values of $\rho_1 = 0.6$ and $\rho_2 = 0.3$ were used.

ELASTIC PARAMETERS

The term elastic has been used to denote all recoverable (or resilient) deformations whether they are truly elastic (linear or nonlinear) or viscoelastic. Likewise E-value or E-modulus is used for the ratio between the dynamic stress and the elastic part of the strain (the secant modulus). Poisson's ratio is kept constant, and the same value (0.35) is used for all layers.

Asphalt

The E-modulus (or stiffness) of the asphalt may be calculated by two different methods; however the method to be used must be specified. One possibility is to use the method developed by Shell:

1. The volume concentration of aggregate, C_v, is calculated from

$$C_v = VA/(VA + VB) \quad (4)$$

where VA is the percentage volume of aggregate, and VB is the percentage volume of bitumen. The volume concentration is then corrected to allow for void contents larger than 3 percent (4).

$$C_v' = C_v / \left\{ (0.97 + 0.01) \cdot [100 - (VA + VB)] \right\} \quad (5)$$

If the void content is less than 3 percent this last step should be deleted.

2. Within certain limits the following relationship, which is derived from Van der Poel's nomograph (5), may be used for calculating the stiffness of the bitumen.

$$S_b = 1.157(10^{-7}) \cdot t_w^{-0.368} \cdot e^{-PI} \cdot (T_{RB} - T)^5 \quad (6)$$

where

S_b = bitumen stiffness in MPa,
t_w = loading time in seconds,
PI = penetration index (e is the base of the natural logarithm),
T_R = softening point ring and ball in degrees Celsius (or the temperature corresponding to a penetration of 800), and
T = temperature of the bitumen in degrees Celsius.

The relationship will give an approximate value of the bitumen stiffness within the following limits:

0.01 sec < t_w < 0.1 sec
-1 < PI < +1
10°C < T_{RB}-T < 70°C

Bitumen stiffnesses calculated from Equation 6 have been compared to values found from the PONOS computer program developed by Ullidtz and Peattie for Shell (6). The comparisons cover bitumens having initial penetrations ranging from 30 to 200 at temperatures from 0°C to 30°C and times of loading between 0.1 and 0.01 sec. The comparisons are shown in Figure 4.

3. The stiffness of the mix (E_1 MPa) is then calculated from (7).

$$E_1 = S_b \cdot \left\{ 1 + (2.5/n) \cdot [C_v'/(1 - C_v)] \right\}^n \quad (7)$$

where

$$n = 0.83 \cdot \log_{10}[(4 \cdot 10^4)/S_b] \quad (8)$$

Moduli determined from this method are usually in good agreement with moduli found from bending tests. At high temperatures, however, bending tests tend to give moduli that are too low. When the temperature increases the modulus of an asphalt layer in a pavement will approach the modulus of the ag-

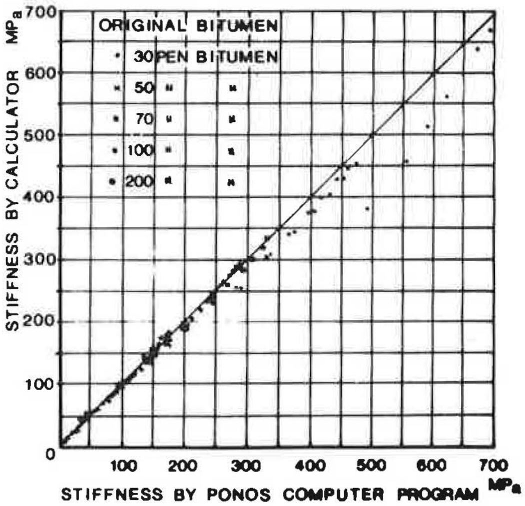


Figure 4. Comparison of bitumen stiffnesses obtained by Equation 6 and the PONOS computer program.

gregate, whereas the modulus of a beam used in a bending test will approach zero.

From the AASHO Road Test (8) a modulus-temperature relationship was found from back analysis of deflection data. Deflections corresponding to a vehicle velocity of 55 km/h at temperatures ranging from 0°C to 40°C were used. The following relationship was found.

$$E_1(t) = 15000 - 7900 \cdot \log_{10} t°C, t > 1°C \quad (9)$$

where $E_1(t)$ is the asphalt modulus in MPa at t°C.

In Figure 5 this relationship is compared to moduli calculated from data on the asphalt mix using Shell's method. The difference at low temperatures could result from the strains in bending tests being lower than the in situ strains. The difference at low temperatures is not very important for the simulation, whereas the difference at high temperatures

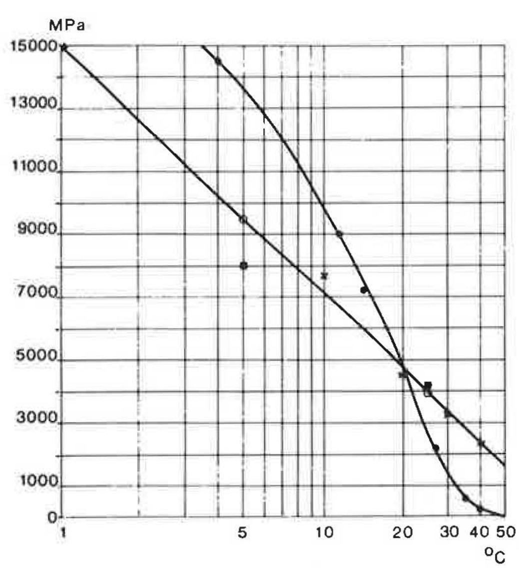


Figure 5. Moduli of asphalt mix as a function of T.

is of vital importance. Using the values determined by Shell from sections of the AASHO Road Test results in severe deterioration during the summer period; such a deterioration was not observed during the tests.

For the input one may therefore choose between Shell's method and a relationship similar to that given by Equation 9. In the latter the two constants in the relationships should be input. No matter which method is selected for calculating the asphalt modulus, a lower limiting value that corresponds to the modulus of the aggregate should be used.

Unbound Materials

For the unbound materials it is also possible to use two different inputs for the elastic parameters. One method is to input the E-value for layer No. 2 (base course) and No. 4 (subgrade) and the ratio between the moduli of layers No. 3 and No. 4 (E_3/E_4). In this method the plastic parameters discussed in the following section will have to be input for each material.

The other method is a standard input developed in connection with the simulation of the AASHO Road Test. In this method the input consists of the CBR value of each of the unbound materials. The modulus of the subgrade E_4 is then calculated from

$$E_4 = 10 \cdot \text{CBR MPa} \quad (10)$$

The accuracy of this equation, related to deformation and strength parameters, is disputed. Experiments carried out by the National Danish Road Laboratory (9) indicate that a better agreement is obtained with the relationship

$$E_4 = 10 \cdot \text{CBR}^{0.73} \text{ MPa} \quad (11)$$

Because Equation 10 is the most widely accepted relationship it is retained here.

The modulus of the subbase, E_3, is found from a relationship developed by Dorman and Metcalf (10)

$$E_3 = 0.2 \cdot h_3^{0.45} \cdot E_4 \quad (12)$$

where h_3 is the thickness of layer No. 3 in mm. This relationship was developed from deflection measurements and, therefore, corresponds to the horizontal direction. A factor 3 has been introduced to account for the anisotropy often found in granular materials. From analysis of stresses measured by Veverka (11), van Cauwelaert (12) found that the ratio between vertical and horizontal modulus, n, was 3 for a gravel and 5 for a sand. Theoretical considerations on compaction of granular materials leads to a minimum value of n = 2.25 for a material with an angle of internal friction θ = 30 degrees. Misra (13) suggests a value for shear sensitivity of 6, which leads to n = 3.8 (14). Gerard and Wardle (15) use n = 3. For stresses, strains, and deflections measured in the Danish Road Testing Machine (16), Ullidtz (14) has shown that an excellent agree- ment with calculated values is obtained when n = 3 is used.

Ideally the pavement response should be calculated using a theory incorporating anisotropy of the granular materials. In this instance, however, a modified version of the method of equivalent thicknesses is used. This method has been found to agree well with anisotropic theory as represented by the computer program CIRCLY (17,18). Finally the modulus of the base course, E_2, is calculated using the equation

$$E_2 = 0.2 \cdot h_2^{0.45} \cdot E_3 \quad (13)$$

where h_2 is the thickness of layer No. 2 (base course) in mm.

PLASTIC PARAMETERS

Plastic deformations include all nonrecoverable (irreversible or permanent) deformations and viscous deformation. An index p, is used to distinguish plastic from elastic parameters, e.g., E_p denotes the ratio of the dynamic stress to the plastic strain and is called the plastic modulus.

Asphalt

A simple method of evaluating the permanent deformations in the asphalt layer has been developed by Hill, Brian, and van de Loo for Shell (19), and a slightly modified version of this method has been used in the simulation program. In this approach the plastic strain in the asphalt is assumed to be purely viscous; i.e., the strain depends on the accumulated loading time only and is independent of the number of load applications.

To use this method a correlation between the plastic modulus of the mixture, $E_{1,p}$, and the plastic or viscous part of the bitumen stiffness, $S_{bit,p}$, must be established. This is most easily done by creep tests. In the vicinity of the total accumulated loading time the relationship may be expressed as

$$E_{1,p} = A_1 \cdot S_{bit,p}^{B1} \tag{14}$$

where A_1 and B_1 are constants. For most of the tests reported by Shell, A_1 was in the region of 70 to 90 and B_1 from 0.3 to 0.5 with moduli in MPa. Because of a certain nonlinearity of some bituminous mixtures (20), the relationship should preferably be determined at a realistic stress level. Tests carried out by Celard (21) and Francken (22) show that the strain is not purely viscous but depends on the number of load applications in addition to the accumulated loading time. It might, therefore, be more correct to use the same kind of relationship given in the section on unbound materials.

The viscous part of the bitumen stiffness may be calculated from

$$S_{bit,p} = 3\eta/t_a \tag{15}$$

where η is the viscosity of the bitumen, and t_a is the accumulated loading time, equal to the number of load repetitions, N, multiplied by the loading time, t_w, of each wheel passage. In the program an approximate value of η is found from

$$\eta = 1.3 \cdot 10^{[3+(T_{RB}-T/10)]} \mathrm{Nsec/m^2} \tag{16}$$

Unbound Materials

For unbound materials the stress level and the time of loading (or number of load applications) seem to be the most important factors. For constant volume unconfined compression tests Goldstein (23) found that the strain-time relationship could be described by the following general equation:

$$\dot{e} = K \cdot \sigma^n \cdot t^{-\alpha} \tag{17}$$

$\dot{e}$ being the axial strain velocity, σ the axial stress, t the time of loading and K, n, and α soil parameters (24). Assuming that the loading time of each wheel passage is constant and $\alpha = 1$ this relationship may be written as

$$\epsilon_p = A \cdot N^B \cdot (\sigma_1/\sigma')^C \tag{18}$$

where

N = number of load applications,
σ' = reference stress equal to 0.1 MPa, and
A,B,C = constants;

or as

$$E_p = 1/A \cdot N^{-B} \cdot (\sigma_1/\sigma')^{1-C} \cdot \sigma' \tag{19}$$

for combinations of N and σ_1 that do not approach failure.

Triaxial tests on granular materials reported by Barksdale (25) indicate that the constant C ranges from 1 to 2 (average 1.67), i.e., from a plastic modulus which is independent of the stress level to a modulus inversely proportional to the major principal stress. One of the tests shows a value of 0.13 for the constant B. Similar tests on cohesive soils (including the AASHO Road Test subgrade) carried out by Poulsen (26) give the same range for C (average 1.66) and a range of 0.07 to 0.15 for B (average 0.11). When the constants B and C are known, they may be input to the program; if they are not known, a standard input of B = 0.1 and C = 1.6 is recommended.

For stress levels approaching the short term strength or for large numbers of stress repetitions Equation 18 is no longer satisfactory. If a road structure is designed such that the limit of Equation 18 is never reached, then the rate of permanent deformations in the structure will be decreasing with the number of loadings as it did in the VESYS II M program (27). The permanent deformations developing shortly after opening of the road may, of course, be large, depending on the stress-plastic strain relationship; but after the first resurfacing or leveling of the roughness or rutting, only small plastic deformations will develop. To predict road failures due to progressive plastic deformations, a more adequate model capable of describing the material behavior at higher stress levels is needed.

According to Vyalow and Maksimyak (28) the deformation of a clay material as a function of time may be divided into three phases: phase 1, decreasing strain rate (corresponding to the above relationship); phase 2, constant strain rate; and phase 3, accelerated strain rate eventually leading to failure, see Figure 6. In the simulation program passing from phase 1 to phase 2 has been assumed to take place when the plastic strain has reached a critical level. The equations for the plastic strain thus become

$$\epsilon_p = A \cdot N^B \cdot (\sigma_1/\sigma')^C \quad \text{for } \epsilon_p < \epsilon_o \tag{20}$$

continuing along the tangent to this relationship:

$$\epsilon_p = \epsilon_o + (N-N_o) \cdot A^{1/B} \cdot B \cdot \epsilon^{1-1/B} \cdot (\sigma_1/\sigma')^{C/B} \tag{21}$$

for $\epsilon_p > \epsilon_o$

$$\text{where } N_o = \epsilon_o^{1/B} \cdot A^{-1/B} \cdot (\sigma_1/\sigma')^{-C/B} \tag{22}$$

A relationship for the third phase has not been included in the program. Using this model on tests carried out by Lashine and reported by Pell and Brown (29), the points marked o or x in Figure 7 are obtained for a limiting strain of 5 percent and 4.5 percent, respectively. Although the agreement with experimental results is quite satisfactory, limited evidence exists to support this model. One reason for using the model is that it is simple enough to be used in a simulation program where a very large number of calculations have to be carried out. For cohesive soils Poulsen (26) found that the stress level at which 10^5 load repetitions would result

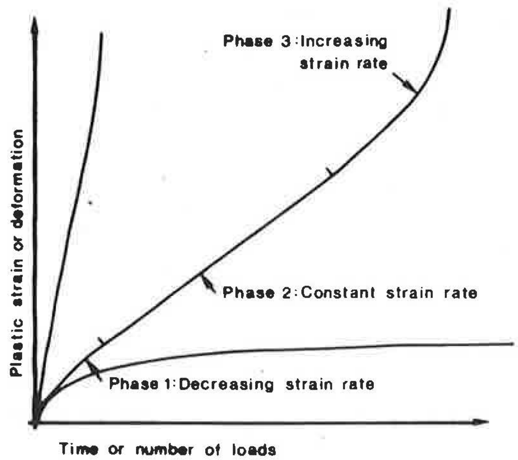
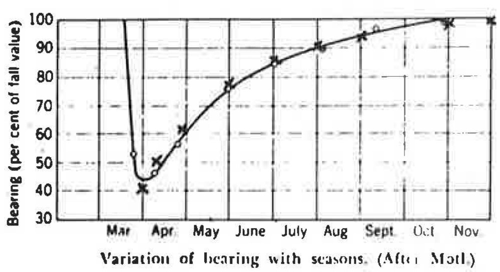
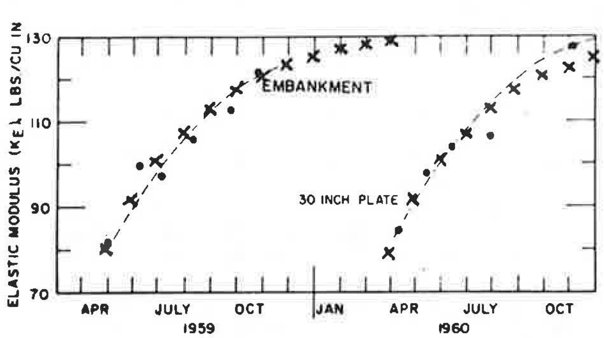


Figure 6. Three phases of deformation.



(Yoder & Witczak, 1975)



(The AASHO Road Test, 1962)

Figure 7. Measured strains compared to calculated strains.

in failure or a plastic strain of 2 percent was proportional to $(M_o/1\ \mathrm{MPa})^{1.16}$, where M_o is the elastic modulus at a stress level of 0.01 MPa. This relationship with the modulus agrees well with Kirk's relationship between permissible stress and modulus (30). Assuming this relationship to the elastic modulus to be correct, one obtains

$$A = \left\{ 1/[(E/E')^{1.16} \cdot C] \right\} \cdot A' \tag{23}$$

where

A,C = constants corresponding to Equation 20,
E = elastic modulus of the material, and
E' = reference modulus.

As described in the section on unbound materials two different types of input may be used for the elastic parameters. When using the first alternative, A in Equation 20 should be input for each unbound layer corresponding to the respective reference modulus (E'). With the second alternative A' is calculated from the CBR value

$$A' = 5/\mathrm{CBR} \cdot 0.017 \tag{24}$$

The constant 0.017 was the mean A-value determined for the AASHO subgrade soil by Poulsen (26) and corresponds to a mean subgrade CBR of 5 percent.

The critical strain, ε_o, may be found from experiments such as those shown in Figure 7; or it may be estimated from a known relationship between number of loads and permissible stress or strain based on permanent deformations (7,26,31). The additional plastic strain caused by the loads from N_i to N_j at a constant stress level is calculated with the strain hardening procedure, see Figure 8 (32). To use the time hardening procedure would not be practical because of the excessive amount of computer storage required.

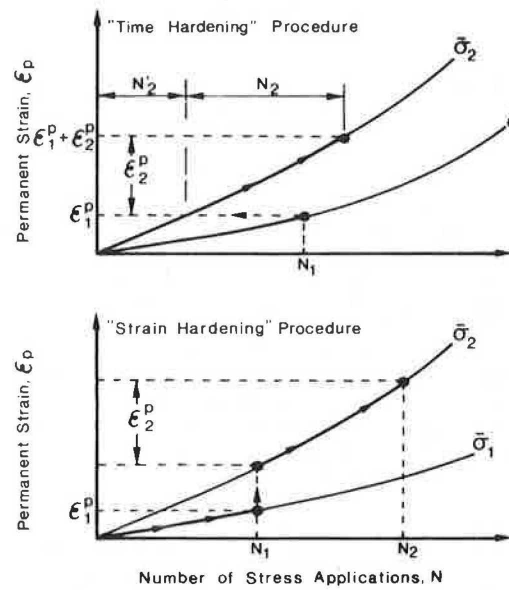


Figure 8. Procedures to predict cumulative loading from simple tests.

FATIGUE PARAMETERS

The general relationship for determining fatigue characteristics developed by Cooper and Pell (33) was used to predict the fatigue life of the asphalt layer:

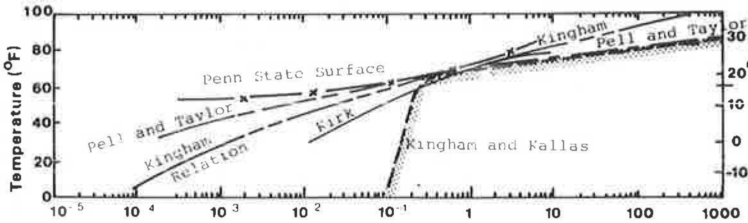
$$\log \varepsilon_r = [(14.39) \cdot (\log V_B + 24.2) \cdot (\log SP_i - 42.7 - \log N)] \div [(5.3) \cdot (\log V_B + 8.63) \cdot (\log SP_i - 15.8)] \tag{25}$$

where

ε_r = maximum allowable tensile strain, in microstrain, for N load applications,
V_B = percentage volume binder in the mix, and
SP_i = initial Ring and Ball softening point of the bitumen.

The logarithms are to base 10.

In Equation 25, N corresponds to the number of loads determined in laboratory tests. In the original equation given by Cooper and Pell the laboratory-determined fatigue life had been multiplied by a factor of 100 to allow for the influence of rest periods (factor of 5) and crack propagation (factor



a. Plots of normalized values $K_1(T)$ versus temperature $K_1(T)/K_1(21°C)$

Figure 9. Temperature correction factor for asphalt fatigue life.

of 20). In the present version of the simulation program these two factors are input values.

In connection with the work on the simulation program there was an attempt to make a theoretical estimate of the crack propagation time in the relationship

$$CP = 10^{(N/N_o - f)} \cdot 100 \quad (26)$$

where CP is the crack propagation in percent of total thickness, and f is the factor for crack propagation.

To follow the crack propagation in the asphalt and thus make possible a gradual decrease of the modulus, the above relationship has been used in the simulation program. The crack propagation factor was found to increase with decreasing asphalt thickness. For the AASHO Road Test simulation, however, a constant factor of 2.7 has been used. A factor of 3 has been used for rest periods, giving a combined factor of only 8.1 between in situ and laboratory determined fatigue life. No factor has been included for the lateral distribution of the traffic loads. According to Verstraeten et al. (34) the fatigue life should be multiplied by a factor of 5 when the traffic is not canalized.

To allow for the influence of temperature on fatigue life, the revision suggested by Rauhut et al. (35) has been used. Of the temperature-fatigue life relationships shown in Figure 9 those resulting in the longest fatigue life have been selected and have been approximated by three straight lines, shown dotted in the figure. The correction factors are given below:

$$0.1 \cdot 10 \exp [(T+17.8)/98.7] \quad \text{for } T < 16°C \quad (27)$$

$$0.22 \cdot 10 \exp [(T-16)/7.6] \quad \text{for } 16°C<T<21°C \quad (28)$$

$$1.0 \cdot 10 \exp [(T-21)/3] \quad \text{for } T>21°C \quad (29)$$

CLIMATIC FACTORS

Climatic factors affect the performance of the pavement in two different ways. The frost and thaw cycles change the bearing capacity of the unbound layers and changes in temperature alter the elastic and plastic parameters of the asphalt layer.

Temperature of Asphalt Layer

As discussed in the previous chapter the temperature of the asphalt layer will influence the elastic modulus of the material, the plastic strains, and the fatigue life and is therefore important for the performance of the pavement. For known climatic conditions the temperature input could be the mean temperature of each week of the year, possibly supplemented by the standard deviations. To reduce the amount of input, however, an approximate relationship in the form of a cosine function has been chosen:

$$T = [(T_1 + T_2)/2] + [(T_1 - T_2)/2] \cdot \cos\{[(U - U_0)/26] \cdot \pi\} \quad (30)$$

where

T_1 = maximum temperature during the year, in degrees Celsius,
T_2 = minimum temperature in degrees Celsius,
U = week number (counted from New Year), and
U_0 = number of weeks from the beginning of the year to the week of maximum temperature.

The temperature found from the above relationship is the mean weekly air temperature; whereas the asphalt temperature during the daytime hours, when most of the loads are applied, is somewhat higher. According to Barker et al. (36), the asphalt temperature, T_{asp}, may be estimated from the air temperature, T_{air}, as

$$T_{asp} = 1.2 \cdot T_{air} + 3.2 \quad (31)$$

where temperatures are in degrees Celsius.

Effect of Frost Melting on Unbound Materials

If unbound materials are allowed to freeze, the pavement may be damaged by frost heave and/or by loss of bearing capacity during the frost melting period in the springtime. Of these two types of distress, the latter is usually by far the most important and is the only one accounted for in the program. This is done by multiplying the modulus of each of the unbound materials by a factor R, corresponding to the frost sensitivity of that material:

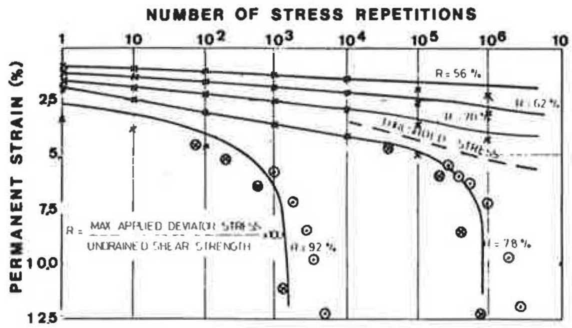
$$R = [1-(1-R_o)] \cdot [e^{A \cdot U_T}] \quad (32)$$

where

R_o = minimum factor corresponding to the week of thaw, the factor is a function of the soil type and the freezing index value;
U_T = number of weeks after the week of thaw; and
A = (negative) constant.

Variations in moduli found from Equation 32 have been compared to values reported by Yoder and Witczak (37) and to values from the AASHO Road Test (8) in Figure 10. For the subgrade the factor is applied only to the upper part of the subgrade that has been exposed to frost. The frost penetration is calculated from an approximate relationship developed from Schweizerische Normenvereinigung (38). Expressed in centimeters the frost penetration, D_f, is

$$D_f = 4.5 \cdot \sqrt{Ig} + 0.5 \cdot D_0 \quad (33)$$



$+\ \epsilon_p = 0.04125 \times N^{.0757} \times (\sigma_1/\sigma_B)^{2.8}, \epsilon_p < \epsilon_o$

⊗ Constant strain rate, $\epsilon_p > \epsilon_o = 4.5\%$

⊙ Constant strain rate, $\epsilon_p > \epsilon_o = 5.0\%$

Figure 10. Seasonal variation of subgrade modulus.

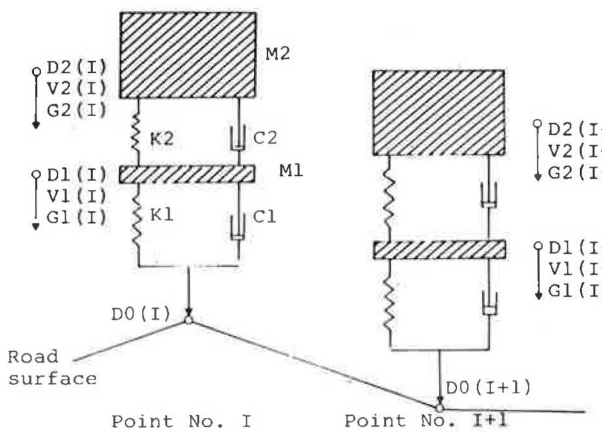


Figure 11. Mechanical analogue.

where Ig is the freezing index value (°C • days), and D_0 is the pavement thickness in centimeters. Equation 33 corresponds to silty or clay subgrades with a moisture content of 18 to 25 percent and a dry density of 1.6 t/m³. For materials that are less frost susceptible, correct determination of the frost penetration is less important.

LOADING

The performance of the pavement is affected by the number of vehicles, the size of the loads, and the speed of the vehicles. In addition to the static load, the vibrations of the vehicle will cause a dynamic load. The dynamic load will depend on the spring and shock absorber system of the vehicles as well as on the longitudinal profile of the pavement surface.

Mechanical Analogue

For loading of the structure a simple mechanical analogue consisting of two systems of mass, spring, and shock absorber as shown in Figure 11 has been chosen. The lower system (M1, K1, and C1) represents the mass of axle and wheel, the spring constant of the tire, and the tire damping; and the upper (M2, K2, and C2) the mass of vehicle body, and the load transferred to one wheel, and the spring constant of the suspension. All relationships are assumed to be linear, but it would not be complicated to introduce nonlinear relationships, e.g., as suggested by Osman (39) and used by Ullidtz (40).

To calculate the force exerted on the surface of the road, the wavelength of the vibration of the mechanical analogue at the reasonant frequencies must be long compared to the distance between two consecutive points in which the force is calculated. When this is the case the acceleration of the masses may be assumed to remain constant between two points. In the program a distance of 50 mm is used between the points. If the vehicle velocity is low, the masses small, and/or the spring constants large, the mechanical analogue may only be used if the distance between the points is decreased.

Loading Time

The influence of loading time on the performance of the asphalt layer is similar to the influence of temperature; i.e., elastic, plastic, and fatigue behavior are affected. For the unbound layers, on the other hand, the loading time has no influence according to the material models used in this program. The loading time corresponding to the middle of the asphalt layer is therefore used and is calculated on the assumption that the load at that depth ($h_1/2$, where h_1 is the thickness of the asphalt layer) is uniformly distributed over a circular area with radius a + $h_1/2$, where a is the radius of the contact area between the tire and the road surface:

$$t_w = (2a + h_1)/V \tag{34}$$

where t_w is the loading time, and V is the velocity of the vehicle. In calculating the loading time no reduction has been made for the influence of dual tires or for lateral distribution of the loads. The results, therefore, should be on the conservative side.

Summation of Loads

For the unbound materials the compaction during construction of the pavement is assumed to correspond to a certain number of wheel passes. This is an input parameter; but a value of 1,000, as used in the simulation of the AASHO Road Test, appears to give reasonable results.

For the permanent deformation of unbound materials, the summation of loads is restarted at the compaction number each time the material has been frozen, i.e., each spring. The large suction values during freezing followed by a redistribution of the moisture during melting causes a certain remolding of the unbound materials. To account for this effect it is necessary to restart the summation after each frost period. All loads are summed for the asphalt layer except loads occurring during periods when the unbound materials are frozen because the load-associated damage to frozen asphalt is negligible.

PAVEMENT PERFORMANCE

Calculating Stresses and Strains

Several programs have been developed during the last decade for calculating stresses in layered structures. Most programs are based on a generalization of Burmister's equations for a two-layered structure (41). The programs developed by Chevron and Shell are widely used and are capable of calculating the stresses, strains, and vertical deflections at an arbitrary point of an n-layer, linear-elastic system loaded at the surface. Few road building materials, however, are linear elastic; practically all materials have nonlinear stress-strain relationships.

Finite-element methods (FE methods) can be used to handle this nonlinearity (42-45). Although the finite-element method is not exact, results close to the correct values may be obtained provided the number of elements is adequate. In addition to calculating stress and strain in nonlinear elastic structures, the finite-element methods may also be used to calculate plastic (or permanent) deformation (46).

Both exact elastic theory and finite-element methods require large digital computers. In deterministic design of road structures this is usually not a problem, because only a few structures will have to be evaluated; but a stochastic design based on a simulation with thousands of calculations would be prohibitively expensive. For the purposes considered in this study calculations must be based on simple equations, capable of providing approximately correct answers.

The elastic stresses and strains could be estimated using the methods developed by Westergaard (47) for calculating stresses in concrete pavements. This has two drawbacks: (a) the subgrade condition is unrealistic (a Winkler foundation), which is of more importance to a flexible structure than to a concrete pavement, and (b) the plastic strains cannot be computed.

Another approach--the one selected for this work--is to use a combination of Boussinesq's equations (48) and the method of equivalent thicknesses (49,50). This method may be modified to incorporate a nonlinear elastic subgrade (50) and may also be used to evaluate the plastic deformations. In the present version of the program the method of equivalent thicknesses is used to calculate the horizontal strain at the bottom of the asphalt layer and to determine the stress field in the pavement. That is, when the equivalent thicknesses are known, the stresses may be calculated at any point of the pavement. The stresses and strains are calculated as components of the loading of both wheels in a dual wheel assembly.

Calculating Permanent Deformation

To calculate the permanent deformations the separative method (51) is used. The stress field is determined by the equivalent thicknesses calculated from the elastic parameters. (The increase in permanent deformation, Δd, between the equivalent depths z_1 and z_2 and caused by the loads from N_1 to N_2 is then found by using the plastic strain-stress relationships given in Equations 20 and 21):

$$\Delta d = A \cdot (3P/2\pi\sigma')^C \cdot \{ [1/z_1{}^{(2C-1)}] - [1/z_2{}^{(2C-1)}] \} \times [(N_2^B - N_1^B)/(2C-1)] \qquad (35)$$

when z_1 is larger than that depth where the combination of stress level and number of repetitions (assuming the same stress level for all loads) would cause a permanent strain larger than the critical strain, ε_o. If the critical depth is larger than z_2, the increase in permanent deformation is found from:

$$\Delta d = A^{1/B} \cdot B \cdot \varepsilon_o{}^{(1-1/B)} \cdot (3P/2\pi\sigma')^{C/B} \times \{ [1/z_1{}^{(2C/B-1)}] - [1/z_2{}^{(2C/B-1)}] \} \cdot [(N_2 - N_1)/(2C/B-1)] \qquad (36)$$

VERIFICATION

The model has been verified by simulating the performance of the 180 four-layer flexible pavement sections in the AASHO Road Test. Details of these simulations are given by Ullidtz (52). A typical result of the simulation of one of the duplicate test sections is shown in Figure 12. The number of simulated and measured axle loads required to reduce PSI to 2.5 are compared in Figure 13. The same excellent agreement was found for all other loops and axle loads. Comparisons of predicted and actual number of loads to cause class 2 cracking are shown in Figure 14, and predicted and measured rutting are compared in Figure 15.

The model was used to extend the traffic loading over a 20-year period, rather than the 2 years of the AASHO Road Test. The results indicate that by concentrating the traffic loading into 2 years, the AASHO Road Test overestimated the permissible number of loads by a factor of about two. (Illinois uses a regional factor of about 1.8.)

Finally the program was used to transfer the AASHO Road Test to Danish climatic conditions, where winters are somewhat milder and summers are cooler than in Illinois. This resulted in an increase in the permissible number of loads by a factor of about two.

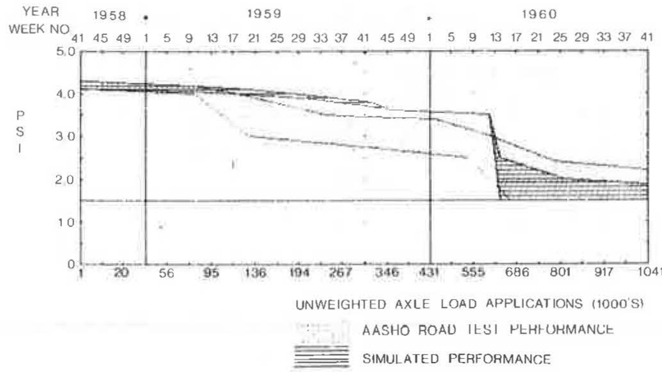


Figure 12. Loop 6, design 6-9-8, duplicated test section performance.

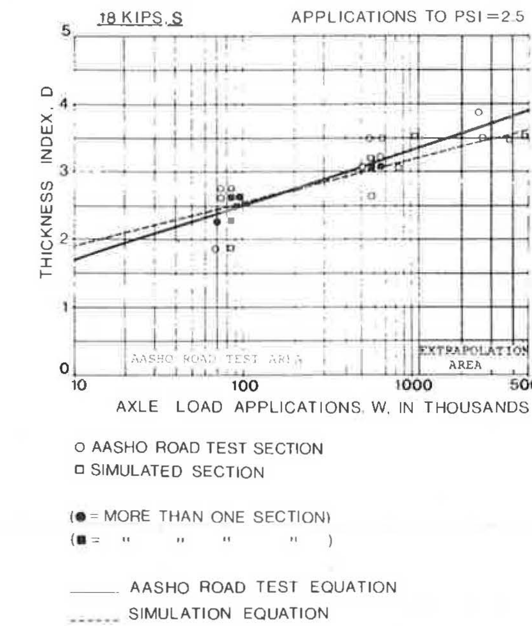


Figure 13. Relationship between design and axle application, loop 4, 18 kips, S.

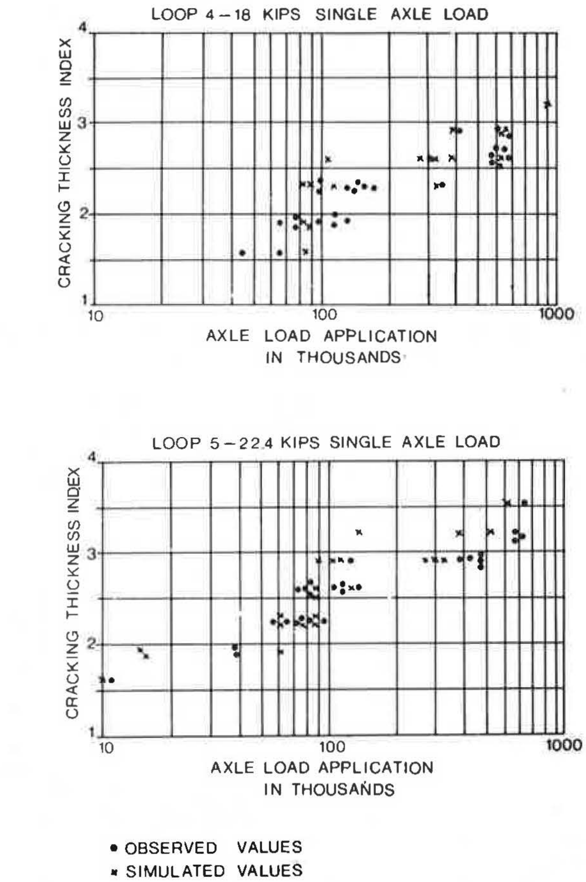


Figure 14. Axle applications to initiation of cracking.

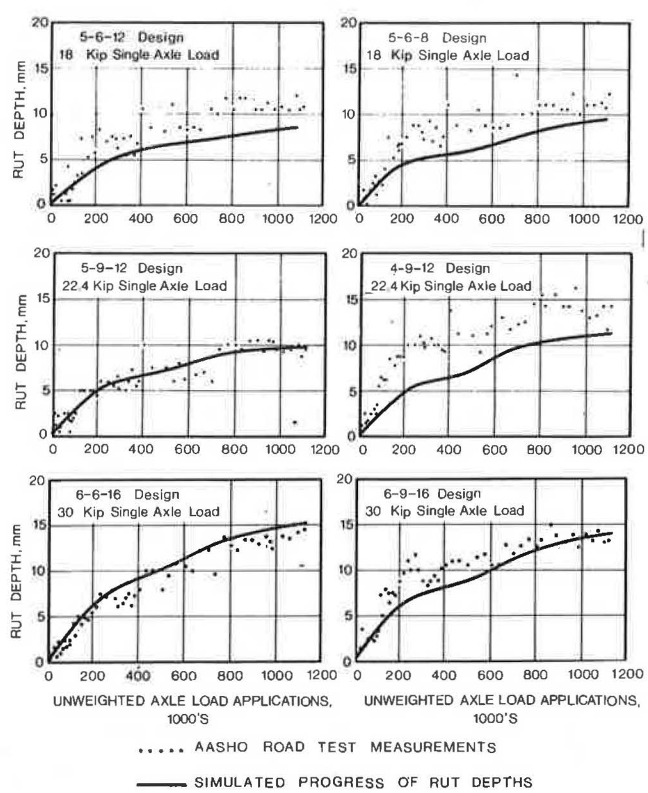


Figure 15. Measured and simulated rut depths.

CONCLUSIONS

The computer program described in this paper is capable of predicting the performance of flexible pavements, in terms of roughness, rutting, and cracking, as a function of climatic conditions and traffic loading. The input to the program consists of fundamental material parameters, which may be determined in the laboratory or through in situ testing, and the pattern of variation (autocorrelation coefficients) of the parameters along the length of the pavement. For each week of the design life of the pavement the dynamic traffic loads at points 0.3 m apart are calculated. Also the reduction of remaining life of asphalt and the permanent deformations of each layer are determined as a function of loading and of the seasonal conditions of the pavement materials.

The model has been verified by simulating the performance of the 180 four-layer flexible pavement sections in the AASHO Road Test. Situations where the model, with some modifications, may prove to be useful are

- Determining maintenance or rehabilitation requirements. Past performance of an existing pavement is simulated and the future performance, after different kinds of maintenance, is predicted. In the present program, overlays may be applied either as a function of time or as a function of pavement condition.
- Evaluating importance of supervision. The effects of variations in material parameters or layer thicknesses could be studied.
- Studying new materials. If the fundamental parameters of new materials, such as waste products, can be determined in the laboratory from dynamic triaxial testing, the model may be used to study the usefulness of these materials for pavement structures.
- Evaluating effects of different truck-wheel arrangements. Size of load, tire pressure, and suspension system may be evaluated for different pavement structures. With some modifications the effects of multiple interconnected axles may also be evaluated.
- Transferring experience. The model was used to transfer the AASHO Road Test to Danish conditions. Similarly the model may be used to transfer experience gained in the Northern hemisphere to the varied conditions of developing countries.

Finally it should be mentioned that the program could be modified to predict airfield performance and evaluate the effects of runway conditions on aircraft and passengers.

ACKNOWLEDGMENT

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Geogrid Reinforcement of Asphalt Pavements and Verification of Elastic Theory

A.O. ABDEL HALIM, RALPH HAAS, AND WILLIAM A. PHANG

The idea of reinforcing flexible pavements has existed for some years, and a few attempts have been made to use metallic and other materials. These have not been effective. Recently, however, a new, high-strength plastic geogrid material known as Tensar has become available; and pavement reinforcement has been suggested as one of its possible civil engineering applications. Consequently, the first phase of a research program initiated in early 1981 examined the potential of a variety of materials for pavement reinforcement, including geogrids. The conclusion was that these materials did indeed offer potential and should be further evaluated. A comprehensive experimental program of tests of reinforced and unreinforced pavements was carried out in the latter half of 1981 and early 1982. Descriptions are presented of the experimental and analytical program and the comparative results. The results of the unreinforced test sections were used to verify the basic elastic layer theory. The analysis shows that the theory provides a reliable tool to predict flexible pavement responses under the design load. A calibration factor that includes the effect of the dependence of elastic moduli on stress level was suggested; the result is a better agreement between predicted and measured values. The results show that the plastic geogrid used was effective as a reinforcement, in terms of carrying double the number of load repetitions or implying a substantial saving in asphalt thickness and minimizing fatigue cracking.

Many existing roads are becoming inadequate because of rapid growth in traffic volume and axle loading. The escalating cost of materials and energy and a lack of resources provide an incentive for exploring alternatives in building new roads and rehabilitating existing ones. Flexible pavement reinforcement is one such alternative. If it could reduce the thickness of paving materials or extend pavement life and be both cost and performance effective, it would be a viable alternative.

Nonmetallic materials, such as fabrics, have been used to a significant degree in some areas of North America and claims have been made that these materials have reinforcement properties. The low strength, high extensibility, and low modulus of these materials make this doubtful. Moreover, there is little if any documented evidence to demonstrate that any fabric reinforces a pavement or extends its life except in warmer climates, where some fabrics may be effective in crack reflection and waterproofing. In view of new technological developments in nonmetallic reinforcing materials, however, reinforcing flexible asphalt pavements may now be worthwhile. The reinforced flexible pavement concept described in this paper includes the initial design analysis, the experimental program carried out to verify the concept, and the analysis and verification of the elastic layer theory.

REINFORCED PAVEMENT CONCEPT

Feasibility Study

In late 1980 a comprehensive research program was initiated to evaluate existing metallic and nonmetallic reinforcing materials, including geotextiles (1). It included the design and implementation of an experimental program as a cooperative effort between Royal Military College (RMC) at Kingston, the Ontario Ministry of Transportation and Communications, Gulf Canada Ltd., and the University of Waterloo.

The primary candidate arising from the evaluation was a new high-strength, plastic mesh or geogrid material known as Tensar. This material is made from polypropylene and is biaxially oriented to give strengths on the order of mild steel in both directions.

Program Objectives

The experimental program was carried out at RMC in Kingston. The design involved varying thicknesses of reinforced and unreinforced full-depth asphalt