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# Finite-Element Program for Analysis of Folded-Plate Bridge Superstructures

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ABSTRACT

The behavior of bridge superstructures such as box girders and T-beams is similar to that of a folded-plate structure. A simplified finite-element program, FAP, specifically developed for the elastic analysis of constant depth and straight folded-plate type structures is presented. Being a specific-purpose program, it can be used by a bridge engineer without the extensive training, knowledge, and effort that may be required for finite-element programs developed for the analysis of a wide range of structural types. Most of the data for FAP analysis is generated by the program from minimal and straightforward input information. FAP has been developed with particular emphasis on practical design and construction considerations. It has already been used for the design of several bridge superstructures. FAP can facilitate the design of many folded-plate type bridge superstructures, especially in those cases in which the design may otherwise be a difficult and time-consuming effort because of the complex geometrical, loading, support, or construction conditions. The illustrative examples presented indicate that the results of analyses using FAP are in good agreement with those based on more exact theories and experimental data.


The finite-element method of structural analysis has become progressively more practical and economical as the availability and use of digital computers have increased. In the finite-element method complicated geometric forms, arbitrary loading and support conditions, and other structural parameters can be accurately and readily represented without extensive use of simplifying assumptions. This method, therefore, offers several advantages over conventional methods of structural analysis.

In recent years several computer programs based on the finite-element method have been developed for structural analysis and design. These programs have been developed for the analysis of a wide range of structural types and usually involve a large number of variables and complicated and extensive input data. This in turn requires a substantial amount of user effort, computational time, and computer capacity, which may not be necessary for the analysis of certain types of structures for which the modeling of the structural behavior can be simplified without affecting the acceptability of results for the purpose of design.

A large number of structures can be categorized as folded plates because of their behavior under loads and their cross-sectional shapes. The spatial rigidity of a folded-plate structure is provided by the out-of-plane (plate bending) and in-plane (membrane) behavior of its component plates, which join at folds (1-6). The width of these plates between folds in the transverse direction of the structure is small in comparison with their respective lengths between supports of the structure. As a result, the bending of these plates is predominantly a one-way behavior in the transverse direction of the structure.

The behavior of box girders, T-beams, and similar types of bridge superstructures is similar to that of a folded-plate structure. A finite-element computer program, FAP, for the elastic analysis of constant depth and straight folded-plate structures under static loads is presented. The program is based on a simplified plate-bending finite element specifically modeling the folded-plate behavior (6). As a result, the number of computations and variables is substantially reduced in comparison with general-purpose finite-element programs. FAP requires less computer time and memory capacity and a minimal amount of input data and user effort. It has been developed with emphasis on the practical concerns of a structural engineer and facilitates the design of folded-plate type bridge superstructures in terms of both structural and functional considerations. The program is capable of the analysis of a folded-plate structure subjected to surface, gravity, or concentrated loads; prestressing forces; temperature loading; and arbitrary boundary conditions. It has been used for the analysis and design of folded-plate type bridge superstructures (7-9) and for determination of the lateral distribution of wheel loads for multi-T-beam bridges (10).

## DESCRIPTION OF PROGRAM

The program is based on the displacement (stiffness) approach of the finite-element method of structural analysis. The theory and the development of mathematical models used in the program to represent plate-bending and membrane (plane stress) actions of component plates of the structure are described in detail elsewhere (6). The following sections provide an overview of the development of the simplified plate-bending finite-element analysis used in FAP, input data requirements, output of analytical (response) results, and other highlights of the program.

### Simplified Plate-Bending Model

The differential equations of the classical two-way plate-bending theory are expressed as follows (11,12):

$$M_x = -D[(\partial^2 w/\partial x^2) + \mu(\partial^2 w/\partial y^2)]$$
$$M_y = -D[\mu(\partial^2 w/\partial x^2) + (\partial^2 w/\partial y^2)]$$
$$M_{xy} = -D(1-\mu)(\partial^2 w/\partial x \partial y) \qquad (1)$$

where D is the plate rigidity and $\mu$ is Poisson's ratio.

The basic assumptions used in the development of a plate-bending model are as follows (Figure 1):

1. Each finite element is rectangular; of uniform thickness; and made of an elastic, isotropic, and homogenous material.
2. The force-displacement relationship is linear, so that the principle of superposition is valid.
3. The displacement field within each element can be uniquely defined in terms of its out-of-plane nodal displacements.
4. The plate-bending moments within each element (Equation 1) can be determined from its displacement field.

The following 12-term polynomial, therefore, may be used to represent the displacement field of the two-way plate-bending model shown in Figure 1:

$$w(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \qquad (2)$$

In light of the dominant one-way bending behavior of the plates of a folded-plate structure, the following additional assumptions are used in the development of the simplified plate-bending element shown in Figure 1:

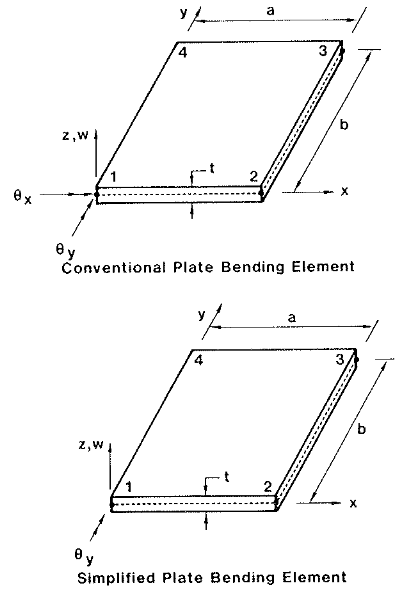


FIGURE 1 Conventional and simplified plate-bending elements.

5. The plate-bending strain ($\partial^2 w/\partial y^2$) in each plate in the longitudinal (y) direction of the structure contributes little to the total strain energy of deformation of the structure and therefore can be neglected.
6. The transverse bending moment within each finite element is constant with respect to the longitudinal direction of the structure; that is, $\partial M_x/\partial y = 0$.
7. The twisting moment ($M_{xy}$) in each element is constant.

By using these additional assumptions, the 12-term displacement field (Equation 2) is reduced to the following 6-term displacement field:

$$w(x,y) = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 x^2 + \beta_5 xy + \beta_6 x^3 \qquad (3)$$

The following conditions are then used to define the six-term displacement function (Equation 3) in terms of the following boundary displacements (Figure 1):

1. At a typical node i, $w = w_i$;
2. At x = 0 and y = b/2, $-\partial w/\partial x = \theta_0$; and
3. At x = a and y = b/2, $-\partial w/\partial x = \theta_a$.

The final displacement function for the simplified plate-bending element is

$$\begin{aligned} w(x,y) = & [1 - (x/2a) - (y/b) - (3x^2/2a^2) + (xy/ab) + (x^3/a^3)] w_1 \\ & + [(x/2a) + (3x^2/2a^2) - (xy/ab) - (x^3/a^3)] w_2 \\ & + [(-x/2a) + (3x^2/2a^2) + (xy/ab) - (x^3/a^3)] w_3 \\ & + [(x/2a) + (y/b) - (3x^2/2a^2) - (xy/ab) + (x^3/a^3)] w_4 \\ & + [-x + (2x^2/a) - (x^3/a^2)\theta_0 + (x^2/a) - (x^3/a^2)] \theta_a \end{aligned} \qquad (4)$$

Because the rate of change of $\partial w/\partial x$ with respect to $y(\partial^2 w/\partial x \partial y)$ is constant (Equation 4), $\theta_0$ and $\theta_a$ are further related to the nodal angular displacements ($\theta_{yi}$) as follows:

$$\theta_0 = (\theta_{y1} + \theta_{y4})/2$$
$$\theta_a = (\theta_{y2} + \theta_{y3})/2 \qquad (5)$$

The stiffness matrix of the total 8-df simplified plate-bending element is given elsewhere (6).

A 2-df/node rectangular plane stress element with in-plane flexural modes (13) is used in FAP to represent the in-plane behavior of the plates of the structure. The overall representation of the behavior of a folded-plate structure is therefore reduced from a conventional 6-df/node idealization to a 4-df/node idealization (Figure 2).

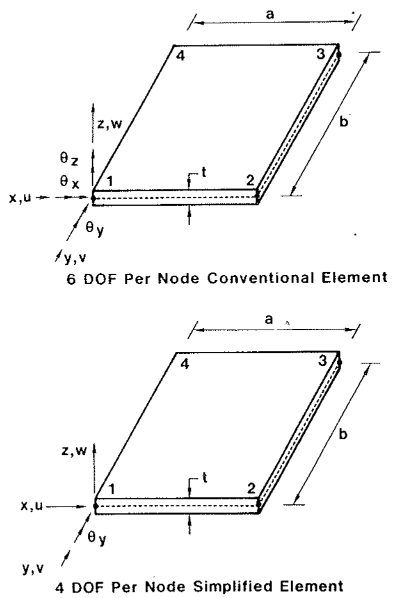


FIGURE 2 Conventional and simplified elements.

## Idealization of Prestressing Forces

Another practical feature of FAP is the representation of prestressing forces and tendon profiles in concrete structures. The exact representation of the variation of prestressing forces along the tendon length and the geometry of tendon profile for finite-element analysis is usually a complex and time-consuming effort. For the analysis by FAP, a prestressing tendon is represented by piecewise straight segments (Figure 3). The tendon is divided into as many segments as the number of elements through which it passes. The prestressing force is assumed to be constant within each segment of the tendon. This method allows the study of the structural response under a tendon of arbitrary length and facilitates the optimum use of prestressing (9). The overall effort of representing the tendon profile and the prestressing force is also reduced. The approximation converges rapidly to an almost exact condition with the refinement of the finite-element idealization of the structure. The initial and final prestressing forces along the tendon length are estimated by conventional methods (14-16); the program does not compute them.

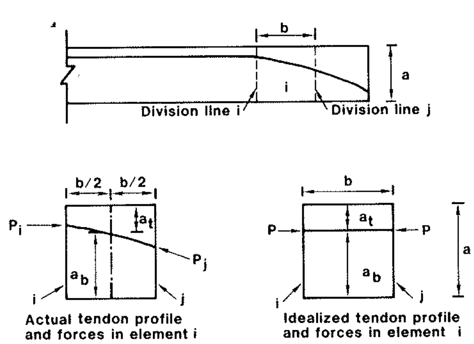


FIGURE 3 FAP idealization of prestressing forces and tendon profiles.

## Idealization of the Structure and Input Requirements

The finite-element idealization of the structure for FAP analysis is shown in Figure 4. In order to understand this idealization and the subsequent input requirements, certain terms are defined as follows:

1. Longitudinal joints are the transverse lines that divide the structure into (idealized) segments longitudinally.
2. Nodal lines are parallel to the longitudinal direction of the structure and divide the structure transversely.
3. Nodes are the location at which nodal lines intersect with longitudinal joints. Node numbering is defined once and is same for all joints.
4. A segment is a portion of the structure between two consecutive longitudinal joints.
5. An element is the rectangular plate portion of the structure bounded by two consecutive longitudinal joints and nodal lines. Element numbering is defined once and is same for all segments.
6. The structural coordinate system is shown in Figure 4. To simplify the transformations between the element and structural coordinate systems, the y-axis is always parallel to the longitudinal direction of the structure; x- and z-axes are interchangeable.
7. The element coordinate system is shown in Figures 1 and 2. Again, the element y-axis is always parallel to the longitudinal direction of the structure. In the element coordinate system, x- and z-axes are not interchangeable.
8. The degrees of freedom correspond to a nodal displacement due to the deformations of the structure. Each degree of freedom also refers to the directions of loads applied to a node. Each node can be displaced or loaded four ways: three translational displacements per load corresponding to x-, y-, and z-directions and one rotational displacement per load about the y-axis (Figure 4).
9. Constrained nodes are those at which one or more degrees of freedom are constrained because of the physical conditions existing at a support or in cases where symmetry is used in the analysis. A controlled displacement, such as that due to support settlement, is also considered a constrained degree of freedom.

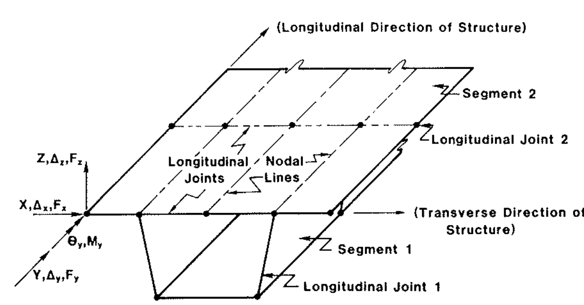


FIGURE 4 FAP idealization of folded-plate structure.

10. Nodal loads are concentrated loads directly applied to a node. These loads are indicated in the structural coordinate system.
11. Element loads are uniformly distributed surface loads on the element. These loads are indicated in the element coordinate system. Element loads also include gravity load due to the self weight of the material and forces produced by the temperature differential existing across the depth, width, or the length of the structure.
12. Stagewise analysis is automated analysis of the same structure for varying length, support, and loading conditions and changes in other structural parameters during construction of the structure or its service life (Figure 5).

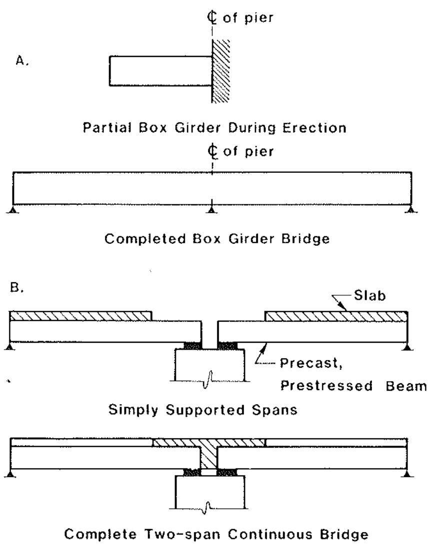


FIGURE 5 Stagewise analysis.

13. Load case is a particular combination of loads for which analysis of the structure is desired.

The flowchart in Figure 6 indicates the sequential order of input data requirements. A brief description of each requirement is as follows:

1. Structure comment cards are used to provide information about the structure, which is echo-printed and has no influence on the analysis.
2. Structure control data provide the following information: total number of segments in the structure, number of nodes per longitudinal joint, number of elements per segment, number of segment types (a segment type is defined according to segment properties, described in the following), and number of stages to be analyzed.
3. Segment type code indicates the type of each segment in the structure in sequential order according to its location in the structure.
4. Segment properties provide the following information for each type of segment: segment width between longitudinal joints and elastic modulus, thickness, Poisson's ratio, and unit weight of material for each element in the segment.
5. Element connections indicate the numbers of the two nodal lines that bound each element transversely. The first number refers to node 1 and the second number refers to node 2 of the element. These two numbers together establish the element coordinate system shown in Figures 1 and 2. This information is provided once and element locations for the whole structure are generated by the program.

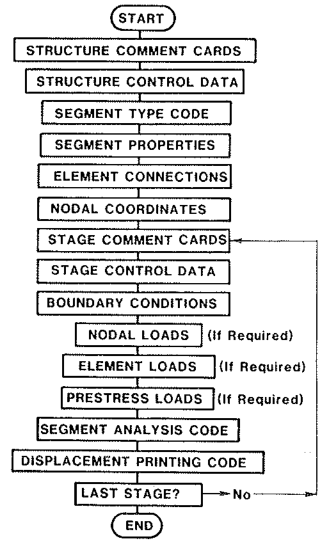


FIGURE 6 Sequence of input information for FAP.

6. Nodal coordinates indicate the x- and z-coordinates of each node in the structural coordinate system. This information is provided once and the nodal coordinates for the whole structure are generated by the program.

The following information is provided for each stage of analysis:

7. Stage comment cards are the same as the structural comment cards.
8. Stage control data provide the following information for a particular stage of analysis: number of segments to be included in the stage, number of constrained nodes, number of nodal load cases, number of element load cases (surface loads in any of the three directions, self weight of the material, and temperature differential between elements), and number of prestress load cases.
9. Boundary conditions indicate, at this step, each degree of freedom of the constrained nodes, as indicated in the stage control data, individually coded for its constrained or free condition.
10. Nodal loads provide the magnitude, location, and direction of each nodal load, according to the nodal load cases. The term "load" is used for either an applied force or a displacement condition corresponding to each degree of freedom. If a degree of freedom is free, the corresponding load is considered an applied force; if the degree of freedom is constrained, the corresponding load is considered a forced displacement including zero displacements.
11. Element loads provide the data for three different types of element loads, as required. It is not necessary that all the elements of a segment, or all the segments in a particular stage of analysis, be loaded for any element load case.
12. Prestress loads (tendons) can also be grouped in different load cases. For each prestress load case, the following information is provided: number of tendons in the load case, number of segments through which each tendon passes, and location of tendon and average prestressing force within each element (Figure 3).

## Output Information

The output consists of an echo print of all input data and the following response quantities for each stage of analysis. All information refers to the load case number, longitudinal-joint number, segment number, element number, and nodal-line number, as required. The response quantities are units consistent with the units of the input information:

1. Longitudinal in-plane (membrane) stresses at midpoints of the edges of each element,
2. Transverse in-plane stresses at the center point of each element,
3. Transverse bending moments at midpoints of the longitudinal edges of each element, and
4. Displacements and reactions corresponding to each degree of freedom of a node.

The preceding information is printed only for segments and longitudinal joints desired by the user. At the end of the analysis for each load case, the external equilibrium conditions in the x-, y-, and z-directions are checked and printed.

## Error Messages

Any error in the input data for which the analysis cannot be executed stops the program from further execution. In this case an error message or messages are printed indicating the possible source of error to assist the user in locating it in the input information.

## ILLUSTRATIVE EXAMPLES

The complete analysis of various types of folded-plate structures, including stagewise analysis, loading, and boundary conditions, is beyond the scope of this paper. The intent of the following examples is to provide representative illustrations of the use and the accuracy of FAP.

### Example 1

The structure for example 1 is taken from a study by Goldberg et al. (17). The dimensions and loading of the single-span, simply supported structure are shown in Figure 7. The structure has diaphragms at both ends that are assumed to be infinitely rigid in their planes but completely flexible for out-of-plane deformations. The elastic modulus and Poisson's ratio are 10,600 ksi and 0.33, respectively. Table 1 represents the vertical and horizontal displacements of point A, as shown in Figure 7. These include the elasticity and experimental results (17), results based on the simplified 4-df/node plate element, and results using the conventional 6-df/node plate element (18).

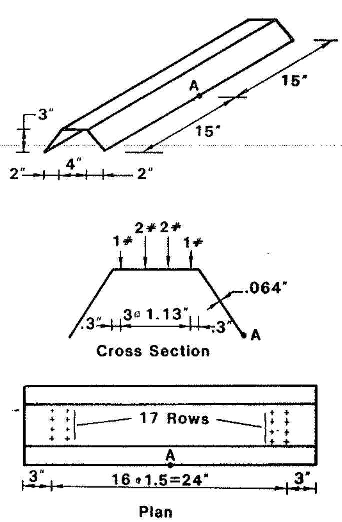


FIGURE 7 Example 1: single-span folded-plate structure with loading.

TABLE 1 Example 1: Horizontal and Vertical Displacements of Point A

| Type of Displacement | Displacement (in.) Elasticity | Experimental | 4 df[a] | 6 df[b] |
|---|---|---|---|---|
| Horizontal | 0.032 | 0.030 | 0.0306 | 0.0306 |
| Vertical | 0.016 | 0.012 | 0.0146 | 0.0150 |

[a]Total df = 840.
[b]Total df = 1,260.

### Example 2

The idealization of a prestressing tendon with a varying force for a two-equal-span box beam with symmetrical prestressing is described in example 2. The structure, its FAP idealization, and the idealization of the tendon profile and the prestressing force are shown in Figure 8. The representative results of analyses by FAP are presented in Table 2. These results are compared with the results of accurate elastic analysis considering the exact tendon profile and prestressing force variation along the tendon length.

### Example 3

This example is of a three-cell, two-equal-span, symmetrically loaded box girder (Figure 9); the example is taken from a study by Scordelis (19). The finite-element idealization for FAP is also shown in Figure 9. The representative results are presented in Figures 10, 11, and 12 (in pounds-force per square foot, foot-pounds per foot, and feet x $10^5$, respectively) along with the elasticity solutions by Scordelis (19) and the results obtained by using a computer program based on a 6-df/node element (6).

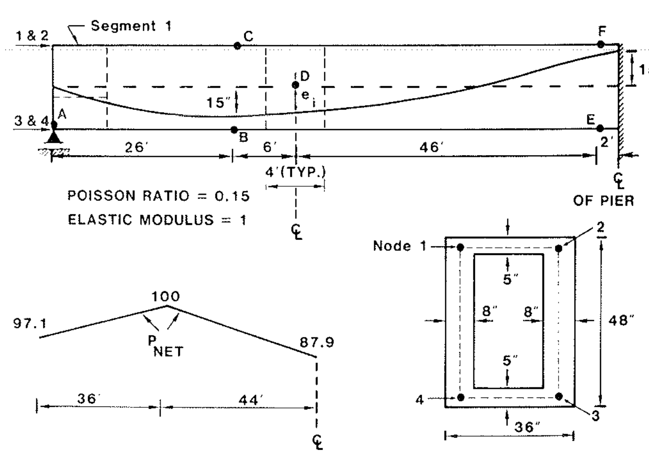


FIGURE 8 Example 2: finite-element idealization of structure and prestressing force.

TABLE 2 Example 2: Representative Results of Prestress Analysis

| Analysis | Reaction at A | Stress at B (psf) | Stress at C (psf) | Vertical Deformation at D (ftx100$^5$) | Stress at E (psf) | Stress at F (psf) |
|---|---|---|---|---|---|---|
| FAP | 0.316 | −224.7 | 23.7 | 369.83 | 45.8 | −228.5 |
| Exact | 0.316 | −224.3 | 23.4 | 371.11 | 44.8 | −227.6 |
| Difference (%) | 0 | 0.18 | 1.28 | 0.35 | 2.23 | 0.40 |

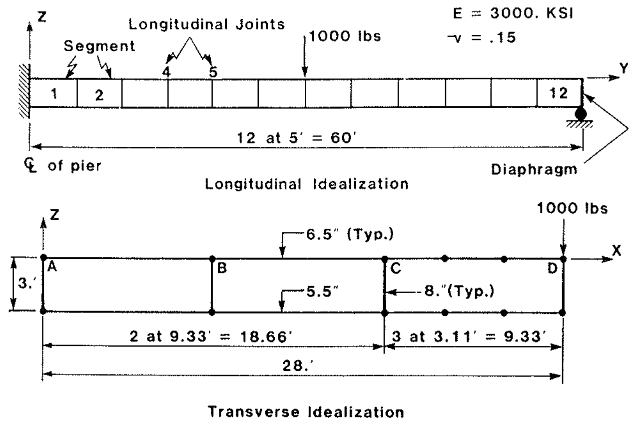


FIGURE 9 Example 3: finite-element idealization of three-cell box girder.

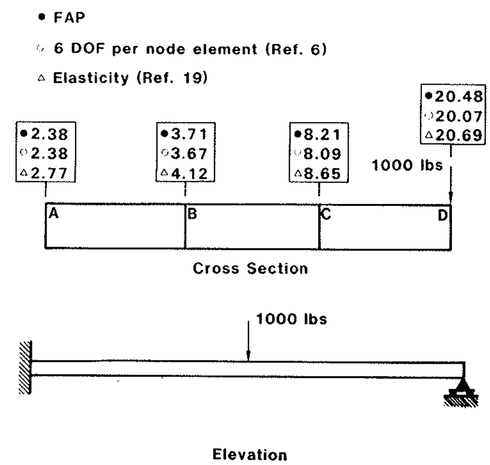


FIGURE 12 Example 3: vertical deflection at center line of span.

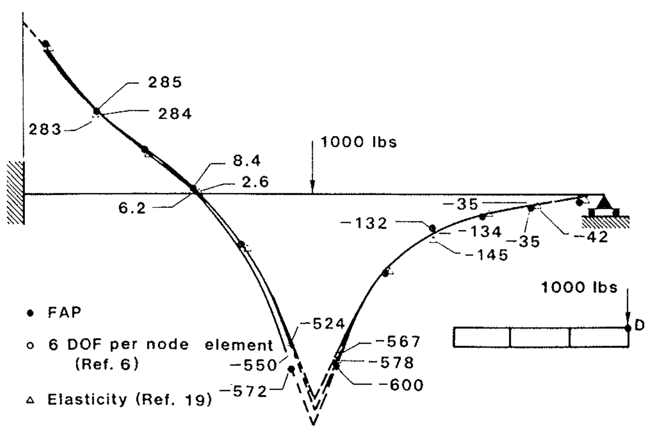


FIGURE 10 Example 3: longitudinal stresses at point D.

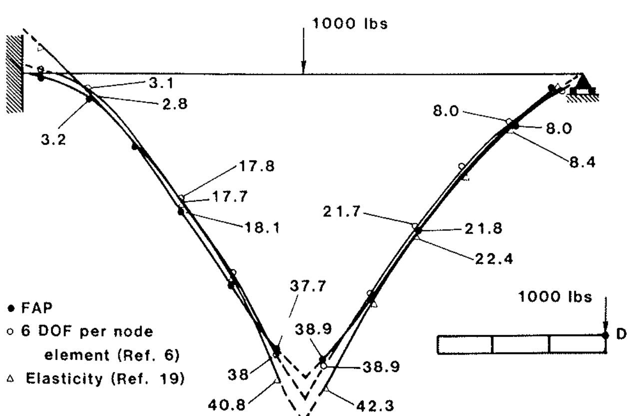


FIGURE 11 Example 3: transverse moments at point D.

## SUMMARY AND CONCLUSION

The FAP finite-element program was specifically developed for the elastic analysis of constant depth and straight folded-plate type structures. The program can facilitate the design of folded-plate type bridge superstructures such as T-beams and box girders. Most of the data for the finite-element analysis is generated by the program from minimal and straightforward input information. The bridge superstructure can be analyzed under all the loading conditions, which are common in a bridge design and for arbitrary boundary and stagewise erection conditions. The analytical results consist of deflections of the structure, transverse bending moments, and in-plane stresses in the component member plates, which are the practical strength and serviceability concerns in a design. Being a specific-purpose program, FAP may be used by a bridge engineer for the purpose of design of several types of structures without the extensive training, knowledge, and effort that may be required for the use of those finite-element programs that have been developed for the analysis of a wide range of structural types. FAP has been tested, as much as is practically possible, for accuracy of its analysis for the purpose of design of bridge superstructures.

The FAP element uses only 4 df/node compared to a conventional 6-df/node plate model. The results from FAP also converge more rapidly to acceptable results, thus requiring fewer nodes for the overall analysis. These two factors are significantly advantageous if the saving of user effort, computer time, and capacity requirements is to be considered.

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