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Riprap Stability Analysis

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ABSTRACT

In the absence of wave and seepage forces, the stability of rock riprap particles on a side slope is a function of the magnitude and direction of the stream velocity next to the particles, the angle of the side slope, and the characteristics of the rock, including the geometry, angularity, and density. A method of designing riprap was developed based on a functional relation among the variables. Rock particles on side slopes tend to roll rather than to slide. Therefore, it is appropriate to consider the stability in terms of moments about a point of rotation. The functional relation has its basis in the balance of moments about a point of rotation at incipient motion between the forces trying to move the particle and the forces resisting movement. A safety factor was developed that is the ratio of the resisting moments to the moments tending to move the particle of riprap. This safety factor takes into consideration the side slope of the bank being protected; the size, density, and angle of repose of the rock; and the lift and drag forces of the following water. The method is described, examples of its use are given, and it is compared with other methods developed by the Bureau of Public Roads, U.S. Army Corps of Engineers, California Division of Highways, Bureau of Reclamation, the ASCE Task Committee on Sedimentation, and others.

Highway crossings or encroachments of rivers usually require some form of protection for the encroaching embankments, bridge abutments, and adjacent riverbanks. Long approach embankments across the floodplain may need protection also. Usually this protection is provided by rock riprap because of its low cost, flexibility, and ease of repair. The important factors in designing rock riprap are durability, size, shape, angularity, angle of repose, and density of the rock; side slope of the bank line being protected; wave action; seepage forces; and flow velocity (both magnitude and direction) close to the rock. The Bureau of Public Roads provided methods of designing riprap or bank protection in Hydraulic Engineering Circular 15. In the 1970s, the problem of riprap design for side-slope protection of banks in contact with flowing water at river crossing encroachments was investigated by Colorado State University for the Wyoming State Highway Department in cooperation with FHWA, U.S. Department of Transportation (1-6). As a result of this investigation a method of riprap design was developed based on a functional relation between the forces moving the particle and those resisting these forces. The method defines a safety factor for the rock riprap, which is defined as the ratio of the moments of the forces resisting rotation of a rock particle and of the riprap to the moments of forces tending to dislodge the particle. The critical condition is the flow for which incipient motion occurs. This critical condition has a safety factor of 1. If the moments of the forces tending to dislodge a particle are larger than the resisting moments, the safety factor is less than 1, rocks are removed from the riprap layer, and failure of the protection may occur. When the safety factor is greater than 1, the riprap is safe from failure.

The equations are developed from theoretical considerations and existing empirical information. The hydrodynamic lift (7,8) and drag forces (9) of the fluid on the rock, the submerged weight and angle of repose of the rock, and the Shields criteria as modified by Gessler (10) for incipient particle motion are considered. The magnitude of the lift force is proportional to the magnitude of the drag force but is perpendicular to the drag force. This is important in analyzing particle stability (11). The theoretical development, a design example, and calculation of the safety factor of several recommended design methods are presented.

THEORETICAL DEVELOPMENT

In the absence of waves and seepage, the stability of rock riprap particles on a side slope is a function of the magnitude and direction of the stream velocity in the vicinity of the particles, the angle of the side slope, and the characteristics of the rock, including the geometry, angularity, and density. In the following development of the safety factor several flow conditions are considered: oblique flow on a side slope, horizontal flow on a

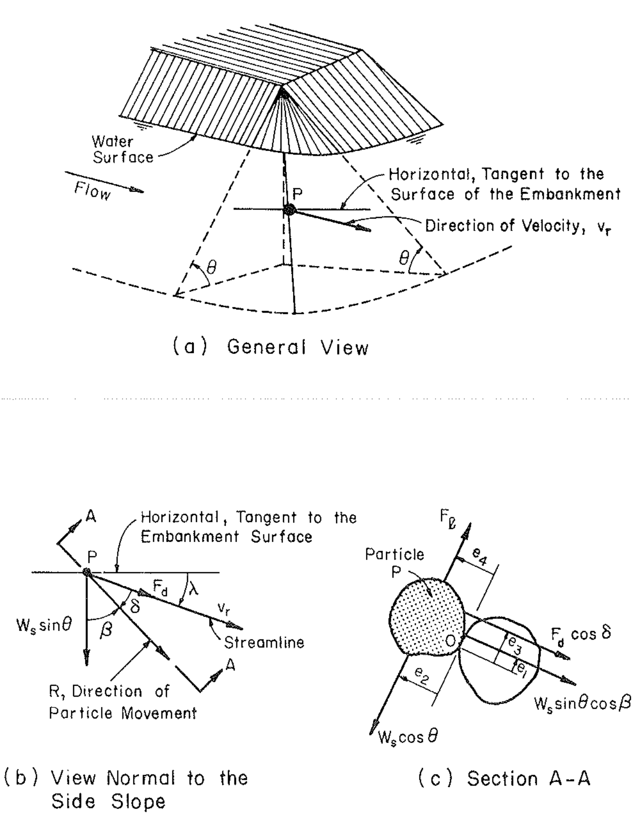

FIGURE 1 Diagrams for the riprap stability conditions.

side slope, flow on a plane sloping bed, and flow on a horizontal bed. The development was previously presented by Stevens et al. (6).

Oblique Flow on a Side Slope

The fluid forces on a rock particle identified as P in Figure 1 result primarily from fluid pressures around the surface of the particles. The lift force (F_ℓ) is defined here as the fluid force normal to the plane of the embankment. The drag force (F_d) is defined as the fluid force acting on the particle in the direction of the velocity field in the vicinity of the particle. Both forces are zero when the fluid velocity is zero.

Gravity acts on the particle and on the fluid surrounding it. The effect of gravity on the particle and fluid is a force equal to the submerged weight of the rock particle (W_s). Forces act at the contact points between particle P and its immediate neighbors. Rotation occurs at one of these points of contact. The forces at the other points of contact are neglected here. The assumption is warranted for mild side slopes, say, 2.5H to IV. At steeper slopes, many particles are subjected to a significant force from particles upslope. This force produces a stabilizing moment on the particle. It is the most prominent force when rocks are hand placed to form a vertical wall.

Rock particles in contact with water tend to roll rather than slide, so it is appropriate to consider the stability of rock particles in terms of moments about a point of rotation. In Figure 1b, the direction of movement is defined by the vector R. The point of contact about which rotation in the R direction occurs is identified as point O in Figure 1c.

The forces F_d and $W_s \sin\theta$ act in the plane of the side slope as shown in Figure 1b. The angle θ is the side-slope angle. The lift force and the component of submerged weight ($W_s \cos\theta$) act normal to the side slope as shown in Figure 1c. There is a balance of moments of these forces about the point of rotation at incipient motion such that

$$e_2 W_s \cos\theta = e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell \quad (1)$$

The moment arms (e_1, e_2, e_3, and e_4) are defined in Figure 1c; angles and forces are defined in Figure 1b.

The factor of safety (SF) of the particle P against rotation is defined as the ratio of the moments resisting particle rotation to the moments tending to rotate the particle out of the bank, or

$$SF = e_2 W_s \cos\theta / (e_1 W_s \sin\theta \cos\beta + e_3 F_d \cos\delta + e_4 F_\ell) \quad (2)$$

With no flow and a side slope equal to the angle of repose φ for the rock particles, SF = 1, θ = φ, β and λ = 0 degrees, δ = 90 − λ − β = 90 degrees (see Figure 1b) and Equation 2 reduces to

$$(e_2 W_s / e_1 W_s)/(\cos\theta/\sin\theta) = 1 \quad (3)$$

or

$$\tan\phi = e_2/e_1 \quad (4)$$

That is, the ratio of the moment arms (e_2/e_1) is characterized by the natural angle of repose (φ). Further, it is assumed that the ratio e_2/e_1 is invariant to the direction of particle motion indicated by the angle β.

If both numerator and denominator are divided by $e_2 W_s$, Equation 2 is transformed into

$$SF = \cos\theta \tan\phi / (\eta' \tan\phi + \sin\theta \cos\beta) \quad (5)$$

in which

$$\eta' = (e_3 F_d / e_2 W_s) \cos\delta + (e_4 F_\ell / e_2 W_s) \quad (6)$$

The variable η' is called the stability number for the particles on the embankment side slope and, as will be shown, is related to the Shields parameter,

$$T = \tau_0 / (S_s - 1)\gamma D \quad (7)$$

where

τ_0 = average tractive force on the side slope in the vicinity of particle P,
S_s = specific weight of the particle,
γ = unit weight of water, and
D = diameter of the rock particles.

The angle λ shown in Figure 1b is the angle between the horizontal and the velocity vector (and drag force) measured in the plane of the side slope. Then

$$\delta = 90 - \lambda - \beta \quad (8)$$

so

$$\cos\delta = \cos(90 - \lambda - \beta) = \sin(\lambda + \beta) \quad (9)$$

Also,

$$\sin\delta = \sin(90 - \lambda - \beta) = \cos\lambda \cos\beta - \sin\lambda \sin\beta \quad (10)$$

It is assumed that the direction of particle motion is along R. This assumption means that the moments of the drag force F_d and the component of submerged weight $W_s \sin\theta$ normal to the path R are balanced. Thus,

$$e_3 F_d \sin\delta = e_1 W_s \sin\theta \sin\beta \quad (11)$$

It follows then from Equations 10 and 11 that

$$\sin\beta = e_3 F_d \sin\delta / e_1 W_s \sin\theta = e_3 F_d (\cos\lambda \cos\beta - \sin\lambda \sin\beta)/e_1 W_s \sin\theta \quad (12)$$

or

$$\tan\beta = \cos\lambda / [(e_1 W_s / e_3 F_d) \sin\theta + \sin\lambda] \quad (13)$$

From Equation 6, the stability number η for particles on a plane bed ($\theta = 0$) with $\delta = 0$ would be

$$\eta = (e_3 F_d / e_2 W_s) + (e_4 F_\ell / e_2 W_s) \quad (14)$$

Equation 5 becomes

$$SF = 1/\eta \quad (15)$$

For incipient-motion conditions for flow over a plane bed, SF = 1.0 by definition, so from Equation 15, $\eta = 1.0$. When the flow along the bed is fully turbulent, the Shields parameter for incipient motion has the value of 0.047 according to Gessler (10). That is, with $\eta = 1.0$,

$$\tau_0 / (S_s - 1)\gamma D = 0.047 \quad (16)$$

For flow conditions other than incipient the stability number is

$$\eta = 21\tau_0 (S_s - 1)\gamma D \quad (17)$$

For convenience, let

$$M = e_4 F_\ell / e_2 W_s \quad (18)$$

and

$$N = e_3 F_\ell / e_2 W_s \quad (19)$$

In terms of these new variables, Equation 6 becomes

$$\eta' = M + N \cos\delta \quad (20)$$

and Equation 14 becomes

$$\eta = M + N \quad (21)$$

Thus, η' and η are related by the following expression:

$$\eta'/\eta = [(M/N) + \cos\delta]/[(M/N) + 1] \quad (22)$$

Equation 22 is represented graphically in Figure 2.

The problem is to select the proper value of the ratio M/N so that the stability factor on a side slope (η') can be related to the stability factor on a plane horizontal bed (η), which in turn is related to the Shields parameter. The assumption that the drag force F_d is zero means that M/N is infinite, β is zero, and $\eta' = \eta$. The assumption of zero lift force F_ℓ means that M/N is zero and $\eta'/\eta = \cos\delta$.

In considering incipient motion of riprap particles, the ratios F_ℓ/F_d and e_4/e_3 vary depending on the turbulent conditions of the flow and the interlocking arrangement of the rock particles. In referring to Figure 1c, assume that

$$e_4/e_3 \simeq 2 \quad (23)$$

Then a choice for F_ℓ/F_d is

$$F_\ell/F_d \simeq 1/2 \quad (24)$$

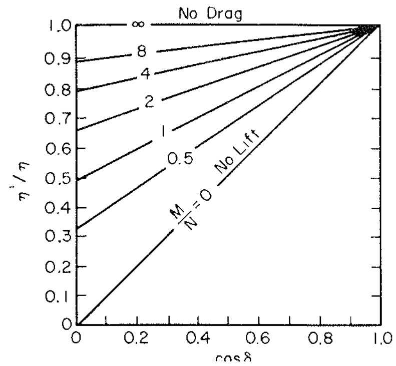

FIGURE 2 Ratio of stability factors.

so that

$$M/N = (e_4/e_3)/(F_\ell/F_d) \simeq 1 \quad (25)$$

With M/N = 1, Equation 22 becomes

$$\eta'/\eta = (1 + \cos\delta)/2 \quad (26)$$

or by using Equation 9,

$$\eta'/\eta = [1 + \sin(\lambda + \beta)]/2 \quad (27)$$

In Equation 13 the term e_1W_s/e_3F_d can be written according to Equations 4 and 19 as follows:

$$e_1 W_s / e_3 F_d = (e_2 W_s / e_3 F_d)/(e_1/e_2) = (1/N)(1/\tan\phi) \quad (28)$$

For M/N = 1, Equation 21 becomes

$$N = \eta/2 \quad (29)$$

If we substitute Equations 28 and 29 into Equation 13, the expression for β becomes

$$\tan\beta = \cos\lambda / [(2 \sin\theta / \eta \tan\phi) + \sin\lambda] \quad (30)$$

In summary, SF for rock riprap on side slopes where the flow has a nonhorizontal velocity vector is related to properties of the rock, side slope, and flow by Equation 5, in which

$$\beta = \tan^{-1}\left\{\cos\lambda / [(2 \sin\theta / \eta \tan\phi) + \sin\lambda]\right\} \quad (31)$$

η is given by Equation 17, and

$$\eta' = \eta \left\{[1 + \sin(\lambda + \beta)]/2\right\} \quad (32)$$

Given a representative rock size D of specific weight S_s and angle of repose ϕ and given a velocity field at an angle λ to the horizontal producing a tractive force τ_0 on the side slope of angle θ, the set of four equations (Equations 5, 17, 31, and 32) can be solved to obtain SF. If SF is greater than unity, the riprap is safe from failure; if SF is unity, the rock is at the condition of incipient motion; if SF is less than unity, the riprap will fail.

Horizontal Flow on a Side Slope

In many circumstances, the flow on the side slope is nearly horizontal; i.e., $\lambda \simeq 0$. Then Equations 17 and 31 reduce to

$$\beta = \tan^{-1}(\eta \tan\phi / 2 \sin\theta) \quad (33)$$

and

$$\eta' = \eta\ [(1 + \sin\beta)/2] \tag{34}$$

When Equations 33 and 34 are substituted into Equation 5, the expression for SF for horizontal flow on a side slope is

$$SF = (S_m/2)\ [(\xi^2 + 4)^{1/2} - \xi] \tag{35}$$

in which

$$\xi = S_m\ \eta \sec\theta \tag{36}$$

and

$$S_m = \tan\phi/\tan\theta \tag{37}$$

The term S_m is SF for riprap on a side slope with no flow. Unless the flow is up the slope, SF for the riprap cannot be greater than S_m.

If Equations 35 and 36 are solved for η,

$$\eta = [(S_m^2 - SF^2)/(SF)\,S_m^2]\ \cos\theta \tag{38}$$

Flow on a Plane Sloping Bed

Flow over a plane bed at a slope of α degrees in the downstream direction is equivalent to oblique flow on a side slope with $\theta = \alpha$ and $\lambda = 90$ degrees. Then, according to Equation 31, $\beta = 0$ and from Equation 32

$$\eta' = \eta\ \{[1 + \sin(90^\circ + 0^\circ)]/2\} = \eta \tag{39}$$

It follows from Equation 5 that

$$SF = \cos\alpha \tan\phi/(\eta \tan\phi + \sin\alpha) \tag{40}$$

for flow on a plane bed sloping α degrees to the horizontal. Alternatively solving for η in Equation 40,

$$\eta = \cos\alpha\ [(1/SF) - (\tan\alpha/\tan\phi)] \tag{41}$$

Flow on a Horizontal Bed

For fully developed rough turbulent flow over a plane horizontal bed ($\alpha = 0$) of rock riprap, Equation 40 reduces to

$$SF = 1/\eta \tag{42}$$

If the riprap particles are at the condition of incipient motion, SF = 1, so $\eta = 1$ and the Shields criterion for the initiation of motion is obtained from Equation 16.

Representative Grain Size for Riprap

In using Equations 5, 17, 31, and 32 to determine SF for a given riprap or to design a riprap with a preselected SF, the magnitude of a representative grain size (D) is needed.

In full-scale experiments with rock riprap below culvert outlets, Stevens (5) developed the following expression for a representative grain size for well-graded materials:

$$D = \left(\sum_{i=1}^{10} D_i^3/10\right)^{1/3} \tag{43}$$

in which

$D_{(1)} = (D_0 + D_{10})/2$,

$D_{(2)} = (D_{10} + D_{20})/2 \ldots D_{(10)} = (D_{10} + D_{100})/2$

The terms D_0, D_{10}, ..., D_{100} are the sieve diameters of the riprap for which 0 percent, 10 percent, ..., 100 percent of the material is finer by weight.

The concept of a representative grain size for riprap is simple. A uniformly graded riprap with a median size (D_{50}) (riprap with a narrow range of sizes) scours to a greater depth than a well-graded mixture with the same median size (Figure 3). Riprap of uniform size scours to a depth at which the velocity is slightly less than that required for the transportation of D_{50} rock. The well-graded riprap, on the other hand, develops an armor plate. That is, some of the finer materials, including sizes up to D_{50} and larger, are transported by the high velocities, leaving a layer of large rock sizes that cannot be transported under the given flow conditions. Thus, the size of rock representative of the stability of the riprap is determined by the larger sizes of rock. The representative grain size (D) given in Equation 43 for riprap is larger than the median rock size (D_{50}). Stevens (5) compared the scour produced in two widely different gradations of riprap (shown in Figure 3) having the same median diameter, D_{50} = 1.2 in.

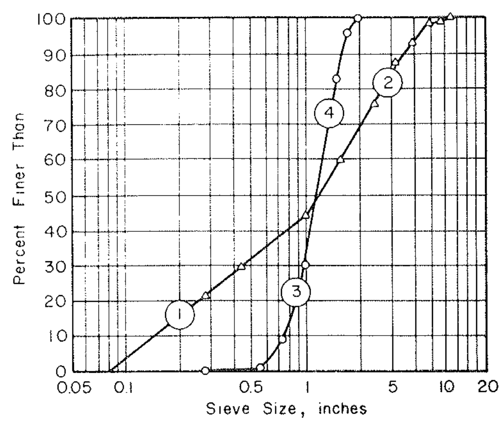

FIGURE 3 Gradation curves.

To illustrate the effect of the larger sizes on the representative grain size, D was computed for four gradations by using Equation 43. The representative grain size (D) and the ratio of representative diameter to median diameter (D/D_{50}) are given as follows for the four gradations (refer to Figure 3):

Curve	D (in.)	D/D_{50}
1-2	2.70	2.25
3-2	2.71	2.26
1-4	1.21	1.01
3-4	1.27	1.06

Inspection of the foregoing values shows that the larger sizes in the gradation have a dominant effect in the determination of the representative grain size. The large sizes of a gradation are the important sizes for stability.

Riprap Gradation and Placement

Riprap gradation should follow a smooth size-distribution curve such as that shown in Figure 4. The ratio of maximum size to median size (D_{50}) should be about 2.0 and the ratio between median size and the 20 percent size should also be about 2.0. This means that the largest stones would be about 6.5

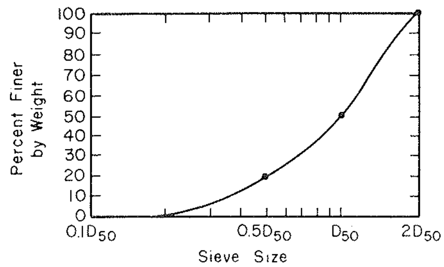

FIGURE 4 Suggested gradation for riprap.

times the weight of the median size and small sizes would range down to gravels. The representative rock size (D) for the gradation shown in Figure 4 is $1.25D_{50}$ (calculated by using Equation 43), which is approximately equal to the D_{67}.

With a distributed size range, the interstices formed by the larger stones are filled with the smaller sizes in an interlocking fashion, preventing formation of open pockets. Riprap consisting of angular stones is more suitable than that consisting of rounded stones. Gradation of the riprap is almost always controlled by visual inspection.

If it is necessary, rock with poor gradation can be employed as riprap provided the proper filter is placed between the riprap and the bank of bed material. Two types of filters are commonly used: gravel filters and plastic filter cloths.

Gravel Filters

A layer or blanket of well-graded gravel should be placed over the embankment or riverbank before riprap placement. Sizes of gravel in the filter blanket should be from 0.187 in. to an upper limit depending on the gradation of the riprap with a maximum size of about 3 to 3.5 in. Thickness of the filter may vary depending on the riprap thickness but should not be less than 6 to 9 in. Filters that are one-half the thickness of the riprap are quite satisfactory. Suggested specifications for gradation are as follows:

1. $D_{50}(\text{filter})/D_{50}(\text{base}) < 40$,
2. $5 < D_{15}(\text{filter})/D_{15}(\text{base}) < 40$, and
3. $D_{15}(\text{filter})/D_{85}(\text{base}) < 5$.

Plastic Filter Cloths

Plastic filter cloths are being used beneath riprap and other revetment materials such as articulated concrete blocks with considerable success. The cloths are generally in rolls 100 ft long and 12 to 18 ft wide. Overlap of 8 to 12 in. is provided with pins at 2- to 3-ft intervals along the seam to prevent separation in the case of settlement of the base material. Some degree of care must be exercised in placing riprap over the plastic cloth filters to prevent damage. Experiments and results with various cloth filters were reported by Calhoun et al. (12), who listed specific manufacturers and brand names. Stones weighing as much as 3,000 lb have been placed on plastic filter cloths with no apparent damage.

Filters can be placed subaqueously by using steel rods as weights fastened along the edges. Additional intermediate weights would assist in sinking the cloth into place. Durability of filter cloths has not yet been established because they have only been in use since around 1967. However, inspections at various installations indicate that little or no deterioration has occurred in the few years that have elapsed for test installations.

Riprap Thickness

If a riprap gradation has a wide range of sizes, the riprap layer must be thick enough to permit the loss of some fines (armorplating) without allowing the protected materials (filter or bank material) to become uncovered. The recommended thickness for the recommended gradation (Figure 4) is D_{100}. For gradations with large gradation coefficients, the thickness must be at least D_{100}. For very large gradation coefficients ($G > 3.0$), the thickness should be increased to $1.5D_{100}$ to provide enough material for armorplating.

Determination of Tractive Force on Riprap

In order to design riprap, it is necessary to be able to determine the tractive force (τ_0) acting on the riprapped bed or bank. This can be done by the relation between the fluid velocity (v) in the vicinity of the riprap and τ_0. For fully turbulent flow, the Prantle-Von Karman relation for the local velocity (v) at a distance (y) above the bed for the hydraulically rough boundary is given by

$$v = 2.5V_* \ln [30.2 (y/D)] \quad (44)$$

in which V_* by definition is the shear velocity:

$$V_* = (\tau_0/\rho)^{1/2} \quad (45)$$

and D is the representative grain size.

If the velocity at a distance y = D above the bed is selected as the reference velocity v_r, then

$$v_r = 2.5V_* \ln 30.2 \qquad v_r = 8.5 V_* \quad (46)$$

and from Equation 45,

$$\rho v_r^2 = 72\tau_0 \quad (47)$$

This relation is strictly valid only for uniform flow in wide prismatic channels in which the flow is fully turbulent. For the purposes of riprap design, Equation 47 can be employed when the flow is accelerating, for example, on the nose of a spur dike. The equation should not be used where the flow is decelerating or below energy-dissipating structures. In such regions, the shear stress is larger. Also, the equation is not valid for flows with a small y/D ratio.

By substituting Equation 47 into Equation 17, the expression for the stability factor η becomes

$$\eta = 0.30\, v_r^2/(S_s - 1)gD \quad (48)$$

For riprap-size material the depth-averaged velocity in the vertical (V) can be written as follows:

$$V = 2.5V_* \ln [12.3(y_0/D)] \quad (49)$$

in which y_0 is the depth of flow adjacent to the riprap. The ratio of the reference velocity v_r to the depth-averaged velocity is

$$v_r/V = 2.5V_* \ln(30.2)/2.5V_* \ln [12.3\,(y_0/D)] \quad (50)$$

or

$$v_r/V = 3.4/\ln [12.3\,(y_0/D)] \quad (51)$$

Then the expression for the stability factor η can be written in terms of the depth-averaged velocity:

$$\eta = \epsilon V^2/(S_s - 1)gD \quad (52)$$

in which

$$\epsilon = 0.30 \left\{ 3.4/\ln [12.3 (y_0/D)] \right\} \quad (53)$$

The value of ϵ varies from 0.30 for relatively shallow flows to 0.04 for y_0/D = 1,000 (Figure 5). In Hydraulic Engineering Circular 11 (13) the following expression is used to determine the velocity against the stone:

$$v_s/\bar{V} = 1/[0.958 \log (y_0/D) + 1] \quad (54)$$

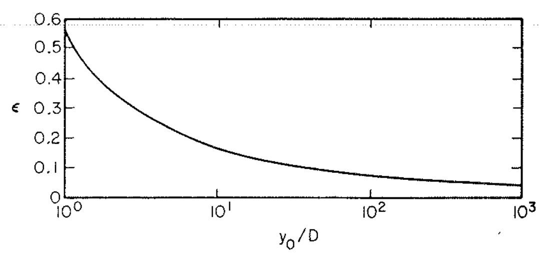

FIGURE 5 Relation between ϵ and y_0/D.

where

v_s = velocity against the stone,
$\bar{V}$ = mean velocity in the channel, and
log = logarithm to the base 10.

In wide channels, the depth-averaged velocity and the mean velocity in the channel are nearly equal; i.e., $\bar{V} \simeq V$. Then the velocity against the stone is related to the reference velocity by the following expression according to the Equations 51 and 54:

$$v_r/v_s = 3.4 [0.958 \log (y_0/D) + 1]/\ln [12.3 (y_0/D)] \quad (55)$$

For values of y_0/D between 1 x 10^0 and 1 x 10^6, the value of the v_r/v_s is nearly 1.4. By letting v_r/v_s = 1.4, the expression for the stability factor η becomes

$$\eta = 0.60\, v_s^2/(S_s - 1)gD \quad (56)$$

In summary, the following expressions for η are employed by definition--Equations 17, 48, 56, and 52--and ϵ is defined in Equation 53.

SAMPLE RIPRAP DESIGN CALCULATIONS

With flow on the nose of an embankment, spur dike, or bridge abutment there can be an appreciable downslope component to the velocity vector. Such a flow is illustrated in Figure 6. If it is assumed that the angle between the horizontal and the velocity vector λ at the point P is 20 degrees, the reference velocity v_r is 6 ft/sec, the embankment side-slope angle θ is 18.4 degrees (3:1 side slope), the specific weight (S_s) is 2.65, the effective rock size (D) is 1.0 ft, and the angle of repose ϕ is approximately 35 degrees, the stability factor is then, from Equation 48, calculated as follows:

$$\eta = 0.30 v_r^2/(S_s - 1)gD = (0.30)(6)^2/[(2.65 - 1)(32.2) \times (1.0)] = 0.203.$$

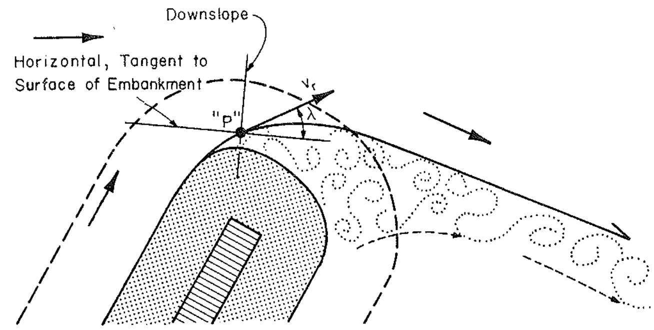

FIGURE 6 Flow around an embankment end.

From Equation 31,

$$\beta = \tan^{-1} \{\cos\lambda/[(2 \sin\theta/\eta \tan\phi) + \sin\lambda]\} = \tan^{-1} \times \{\cos 20°/[(2 \sin 18.4°/0.203 \tan 35°) + \sin 20°]\} = 11°.$$

From Equation 32,

$$\eta' = \eta \{[1 + \sin (\lambda + \beta)]/2\} = 0.203 \{[1 + \sin (20° + 11°)]/2\} = 0.154.$$

From Equation 5 SF for the rock is calculated as follows:

$$SF = \cos\theta \tan\phi/(\eta' \tan\phi + \sin\theta \cos\beta) = \cos 18.4° \tan \times 35°/(0.154 \tan 35° + \sin 18.4° \cos 11°) = 1.59.$$

Thus, this rock is more than adequate to withstand the flow velocity.

By repeating the previous calculations for a range of effective rock size D (with ϕ = 35 degrees), the curve given in Figure 7 is obtained.

Figure 7 shows that the incipient-motion rock size (SF = 1.0) is approximately 0.35 ft and that the maximum SF is less than 2.0 on the 3:1 side slope. The recommended SF for design is 1.0. Thus riprap with a representative size of 0.9 ft is required.

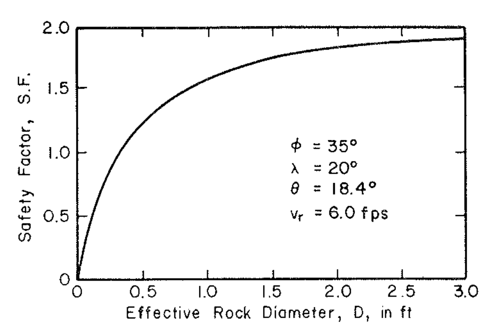

FIGURE 7 SFs for various rock sizes on a side slope.

SF of a particular side-slope riprap design can be increased by decreasing the side-slope angle θ. If the side-slope angle is decreased to zero degrees, Equation 42 is applicable and

$$SF = 1/\eta = 1/0.203 = 4.93.$$

The curve in Figure 8 relates SF and the side-slope angle of the embankment (for λ = 20°, D = 2.0 ft, and v_r = 6.0 ft/sec). The curve is obtained by employing Equations 31, 32, and 5 for various values of θ.

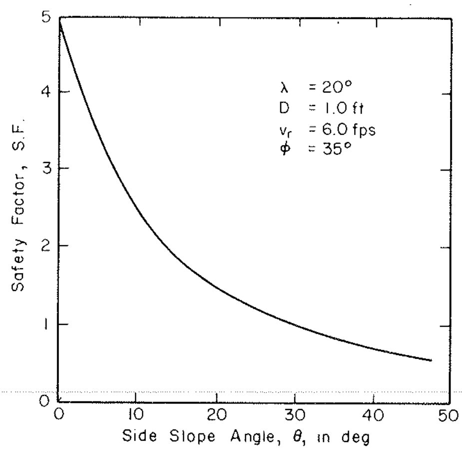

FIGURE 8 SFs for various side slopes.

When the velocity along a side slope has no downslope component (i.e., the velocity vector is along the horizontal), some simple design aids can be developed. Equations 37 and 38 relate SF, stability number, side-slope angle, and angle of repose for this case. The interrelation of the variables in these two equations for the condition indicated is given in Figure 9.

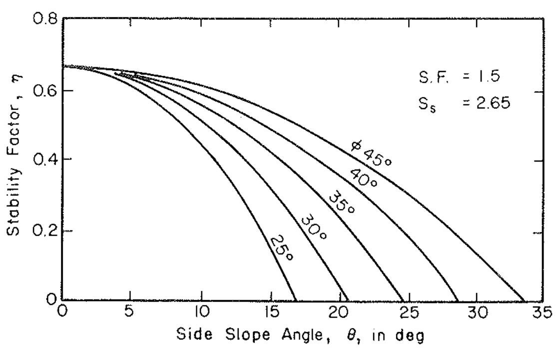

FIGURE 9 Stability factors for an SF of 1.5 for horizontal flow along a side slope θ and angle of repose φ.

SAFETY FACTORS FOR EXISTING DESIGN METHODS

In 1976 Stevens et al. (6) compared their riprap stability equations with methods for stability analysis developed by the Bureau of Public Roads (13), Corps of Engineers (14), California Division of Highways (15), ASCE Task Committee on Sedimentation (16), Bureau of Reclamation (17), Lane (9), and Campbell (18). The adequacy of the designs was judged on the basis of the computed values of SF of the riprap size given by the different methods.

In many instances the methods could not be directly compared for all ranges of flow on side slopes. Consequently, the comparisons were made for flow on a plane flat bed. The comparison indicated that in many cases SF for the riprap design was greater than 1. However, in some cases, SFs were less than 1, indicating some loss of material when the riprap was subjected to design flows. The details of the analysis were given by Stevens et al. (6) and are briefly summarized here.

Bureau of Public Roads

Searcy (13) adapted the 1948 ASCE summary on slope production (16). He further recommended riprap gradations patterned after those given by Murphy and Grace (19). These gradations called for an effective diameter of $1.08D_{50}$. The computed SFs for the design curves are less than unity.

U.S. Army Corps of Engineers

It was determined for flow on plane flat beds that the design criteria adopted by the Corps of Engineers (14) have SFs less than unity for relative depths $y_0/D \leq 1.92$ and greater than unity for larger relative depths. SF for flow on side slopes was not analyzed. The Corps no longer uses sheet 712-1 (14) for design. Most highway agencies are now using design procedures based on the laboratory tests of Anderson et al. (20).

California Division of Highways

The California design method (15) gave the same values for SF as those for the Corps of Engineers method for horizontal flow on a plane bed in a wide channel with effective diameter D of $1.2D_{50}$. SF for flow on side slopes could not be directly determined without making many simplifying assumptions.

ASCE Task Committee on Sedimentation

Analysis of this committee's recommendation (16), the design procedure for flow over a horizontal bed in a wide channel, gave an SF 20 percent greater than that for the California Division of Highways' method when the effective diameter of the riprap was $1.2D_{50}$.

Lane's Method

The analysis presented by Stevens et al. (6) determined that Lane's (21,9) design criteria for flow on a plane bed and for horizontal flow along a side slope yield SF less than 1 in most cases.

Campbell's Method

Analysis of Campbell's method (18) indicated that it always gave SFs greater than unity for flow on a plane flat bed. For flow on a side slope no direct comparison was made because his relations are so complex. However, SF for Campbell's sample computation was only 1.015, indicating that his riprap on a 6:1 side slope was at incipient motion.

SUMMARY AND CONCLUSIONS

A method of determining the size of riprap to protect highway embankments along or across rivers is developed. The method takes into consideration the forces tending to move the particle (lift and drag of the fluid and the component of submerged particle weight in the direction of movement) and the resisting force (submerged particle weight in a direction opposing motion). The method involves calculating SF of the riprap, which is defined as the ratio of the moment of the forces resisting motion to the moment of the forces producing movement. It is shown that the maximum safety factor for dumped riprap is $\tan\phi/\tan\theta$, where ϕ is the angle of repose of the riprap and θ is the side slope and this maximum occurs when there is no flow.

Several examples of the design method are given and the method is used to calculate SF of riprap designs given by the Bureau of Public Roads, U.S. Corps of Engineers, California Highway Department, Lane, and others. Because of the differences in the proposed method and these other methods, a direct calculation of SF often could not be made for all embankment side slopes or flow conditions. Where direct comparisons could be made, SFs in some cases were less than 1, indicating that there could be a loss of riprap under design flow conditions.

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