K_R: The Resilient Modulus of Subgrade Reaction

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ABSTRACT

The concept of the resilient modulus of subgrade reaction (K_R) is developed to account for the stress-dependent behavior of typical fine-grained subgrade soils. This new subgrade support parameter is defined as plate pressure or resilient deflection in an impulse plate load test simulated by using the finite-element program ILLI-PAVE. K_R is expressed in the same units as the standard static modulus of subgrade reaction (k), but the value of the former is significantly higher. This indicates increased stiffness in response to rapidly moving loads. Factors influencing K_R include plate size, deflection level, and subgrade type. A 30-in.-diameter plate was chosen in this study in conformity with general practice. Equations relating K_R and deflection level are developed for four broad cohesive subgrade soil types: very soft, soft, medium, and stiff. The effect of a granular subbase on K_R is also examined. The introduction of a granular layer increases K_R substantially. The importance of this effect, however, diminishes rapidly as subbase thickness exceeds 8 in. The beneficial effect of a granular subbase is consistent over the range of plate pressures investigated.

In the analysis of slab-on-grade pavements [portland cement concrete (PCC), high-strength cementitiously stabilized materials], it is necessary to idealize the characteristics of the supporting medium. In general, one of two fundamentally different hypotheses concerning the properties of the subgrade is used. In the first of these theories, the soil is regarded as an elastic, isotropic, and homogeneous semiinfinite body. The term "elastic solid" is often used to describe this idealization. The majority of current analyses that treat the subgrade as a semiinfinite, elastic half-space employ axisymmetric models. Thus they are only applicable to the interior condition, i.e., where the load is away from any edge or corner.

In the other support characterization theory, the subgrade is regarded as a flexible bed in which surface pressure is proportional to surface deflection at each point, whereas adjacent unloaded areas are not affected at all. This idealization is commonly called a dense liquid or a Winkler subgrade. The finite-element program ILLI-SLAB (1,2) employs a Winkler-type subgrade and can be used to study two-layer cracked pavement sections, load transfer by aggregate interlock or dowels or both, variable slab thickness, variable subgrade support, and complex multiwheel loading at any position on the pavement. This model has been validated and used extensively in various University of Illinois studies (1,3).

In the original version of ILLI-SLAB the modulus of subgrade reaction (k) based on the plate load test is used for subgrade characterization. This is in conformity with general engineering practice as well as several other finite-element models. In ILLI-SLAB the value of k can be varied from node to node according to a pattern specified by the user. Note that k is independent of stress or deflection level, being essentially a linear, low-stress modulus. Most subbase-subgrade support systems, however, display a load-deflection response dependent on stress level. Typically, a softer (lower k) response is exhibited at higher magnitudes of stress or deflection.

To develop suitable support relations to accommodate deflection-dependent subgrade behavior for ILLI-SLAB, various models proposed by others were reviewed in a study by the U.S. Air Force Office of Scientific Research (AFOSR) (4). Special attention was paid to those that could be used to simulate nonlinear subgrade response. Thompson and Robnett (5) proposed a resilient modulus characterization for the elastic-solid foundation that not only introduced soil nonlinearity but also, perhaps more importantly, accounted for the apparent increase in subgrade stiffness produced by rapidly moving, repeated loads. The aim of this study is to develop a similar resilient modulus characterization for the dense-liquid foundation.

Data for the development of the necessary algorithms were obtained by using ILLI-PAVE (6), an axisymmetric, resilient elastic-solid model, to simulate repeated plate load tests. Equations were derived relating K_R and deflection. Note that this is no longer k from the static plate load test but a modulus characterizing subgrade response to repeated (impulse-type) loading. The latter loading condition is considered more appropriate for moving wheel loads.

SOIL AND MATERIAL CHARACTERIZATION

General

The resilient behavior of a soil or material is an important property for pavement analysis and design. A commonly used measure of resilient response is the resilient modulus, defined as follows:

$$E_R = \sigma_D/\epsilon_r \quad (1)$$

where

- E_R = resilient modulus,
- σ_D = repeated deviator stress, and
- ϵ_r = recoverable axial strain.

Repeated unconfined compression or triaxial testing procedures are often used to evaluate the resilient moduli of fine-grained soils and granular materials. Resilient moduli are stress dependent: Fine-grained soils experience resilient modulus decreases with increasing stress, whereas granular materials stiffen with increasing stress level.

Fine-Grained Soils

Two models of stress-dependent behavior have been proposed for the stress-softening behavior of fine-grained soils. The arithmetic model is demonstrated in Figures 1 and 2 [AASHTO A-7-6(36)] and the semilog model is shown in Figure 3 [AASHTO A-7-6(36)]. Extensive resilient laboratory testing, nondestructive pavement testing, and pavement analysis and design studies at the University of Illinois have indicated that the arithmetic model (Figure 1) is adequate for pavement analysis activities.

In the arithmetic model the value of the resilient modulus at the breakpoint in the bilinear curve (E_{Ri}) is a good indicator of a soil's resilient behavior. The slope values (K_1 and K_2) display less variability and influence pavement structural response to a smaller degree than E_{Ri}. Thompson and Robnett (5) developed procedures for predicting the resilient behavior of fine-grained soils based on soil classification, soil properties, and moisture content. They suggested the following regression equations relating E_{Ri} with static soil modulus (E) and unconfined compressive strength (q_u):

$$E_{Ri} = 3.36 + 1.9E \quad (2)$$

$$E_{Ri} = 0.86 + 0.307q_u \quad (3)$$

where

E_{Ri} = breakpoint resilient modulus (ksi),
E = static modulus of elasticity (ksi), and
q_u = unconfined compressive strength (psi).

These correlations indicate that E_{Ri} is substantially greater than a static E.

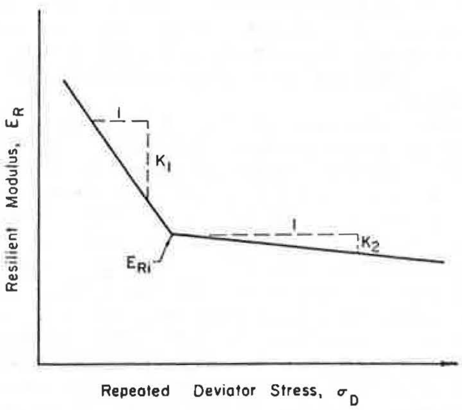

FIGURE 1 Arithmetic model for stress-dependent resilient behavior of fine-grained soils.

Granular Materials

In contrast to fine-grained soils, granular materials stiffen as the stress level increases. Repeated-load triaxial testing is used to characterize the resilient behavior of granular materials. The resilient modulus is a function of the applied stress state, defined as follows:

$$E_R = K\theta^n \quad (4)$$

where K and n are experimentally derived factors and

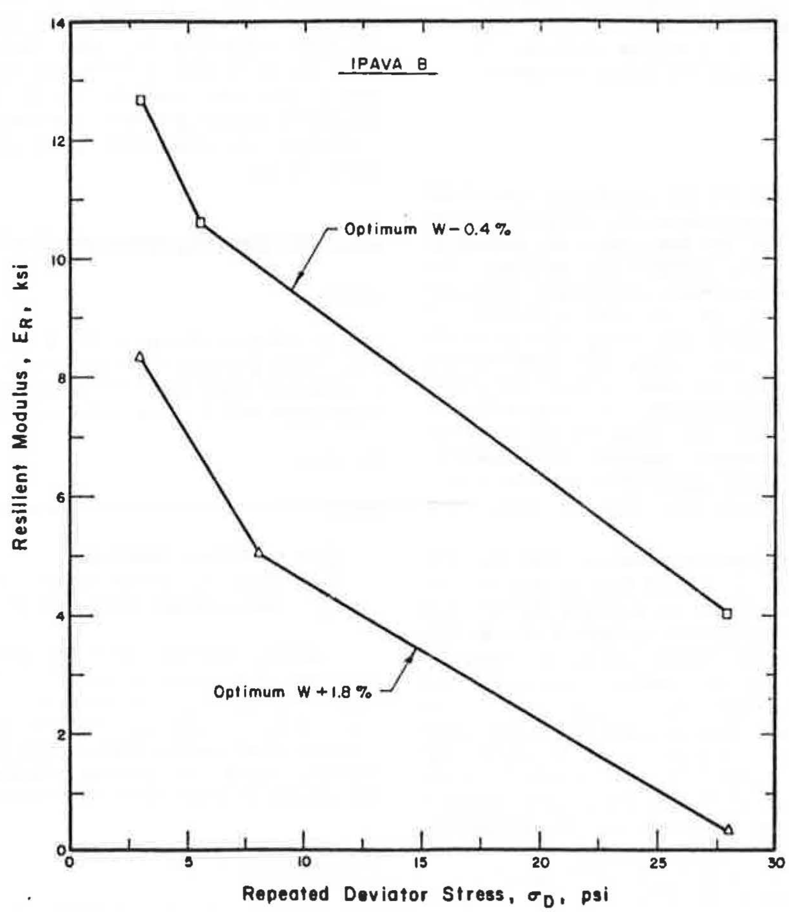

FIGURE 2 Typical stress-dependent resilient behavior of a fine-grained soil.

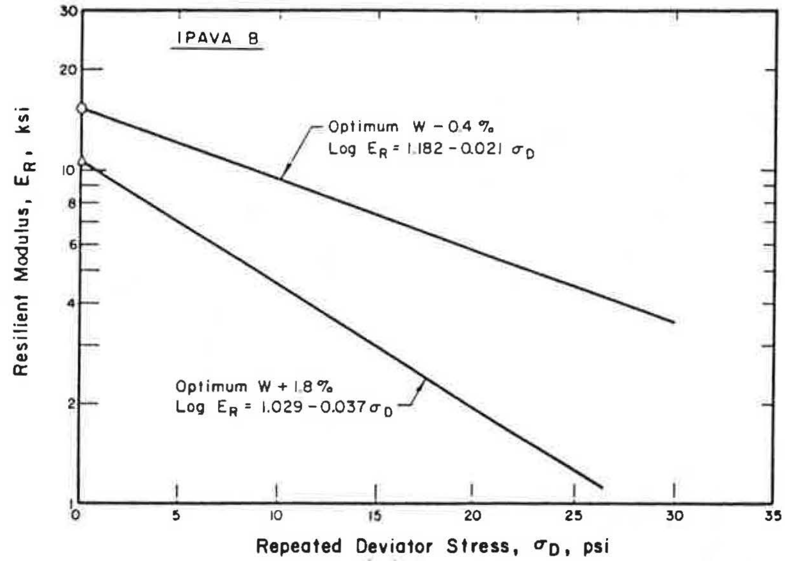

FIGURE 3 Semilog model for stress-dependent resilient behavior of a fine-grained soil.

θ is the first stress invariant ($\sigma_1 + \sigma_2 + \sigma_3$). (Note that $\theta = \sigma_1 + 2\sigma_3$ in a standard triaxial compression test.) Figure 4 is an E_R-θ relation for a sandy gravel [AASHTO A-1-b(0)].

Rada and Witczak (7) have summarized and statistically analyzed extensive published data on resilient moduli for a broad range of granular materials. The average values and ranges for K and n are given in Table 1 for several granular materials and coarse-grained soils.

Other Materials

Stabilized materials such as soil-cement, cement-aggregate mixtures, soil-lime mixtures, lime-flyash-aggregate mixtures, and similar materials that have high strength and high modulus are frequently used as base and subbase layers. These materials are normally characterized as constant-modulus materials.

RELATIONS BETWEEN K_R AND DEFLECTION

The finite-element program ILLI-PAVE (6) was used to develop the data (resilient plate load deflections) required to establish algorithms for K_R and deflection. This program considers an axisymmetric solid of revolution. Nonlinear stress-dependent material models and failure criteria for granular materials and fine-grained soils (6,8,9) are incorporated into ILLI-PAVE. The principal stresses in

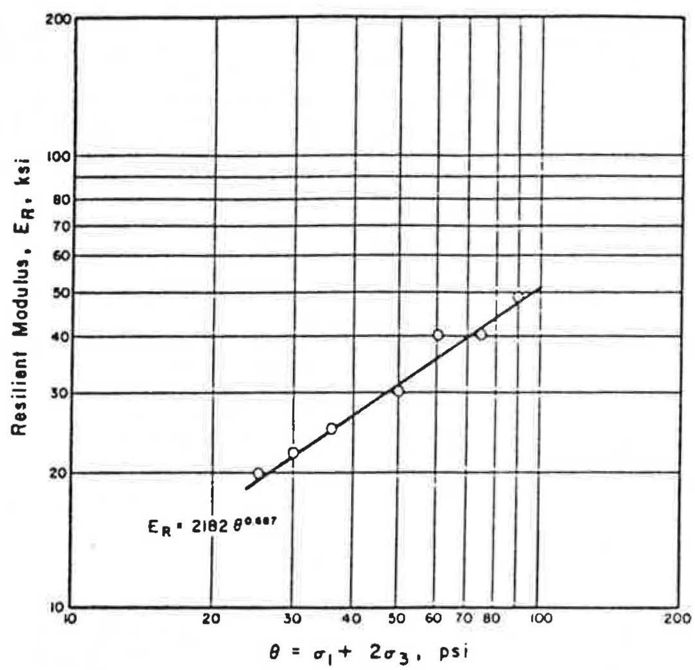

FIGURE 4 Resilient modulus versus θ for a sandy gravel.

TABLE 1 Typical Resilient Property Data (7)

Granular Material Type	No. of Data Points	K^a (psi) Mean	K^a (psi) Standard Deviation	n^a Mean	n^a Standard Deviation
Silty sand	8	1,620	780	0.62	0.13
Sand-gravel	37	4,480	4,300	0.53	0.17
Sand-aggregate blend	78	4,350	2,630	0.59	0.13
Crushed stone	115	7,210	7,490	0.45	0.23

[a] $E_R = K\theta^n$ where E_R is the resilient modulus (psi) and K,n are experimentally derived factors from repeated triaxial testing.

the granular and subgrade layers are modified at the end of each iteration so that they do not exceed the strength of the materials as defined by the Mohr-Coulomb theory of failure.

Studies comparing measured and ILLI-PAVE-predicted load-deformation responses, reported by Raad and Figueroa (6), Suddath and Thompson (10), Traylor (11), and Hoffman and Thompson (12), yielded favorable results. The ILLI-PAVE approach has been successfully used in developing a highway overlay design procedure for flexible pavement based on nondestructive-testing data analyses (13) as well as mechanistic thickness design procedures for secondary-road flexible pavements (14) and soil-lime layers (15).

ILLI-PAVE was used to simulate repeated plate load tests on various subgrades. The rigid plate condition was represented by a steel loading plate 4 in. thick ($E_s = 30 \times 10^6$ psi). Plate diameters of 30, 21, and 15 in. were considered for various plate pressures. For each loading condition, a resilient (recoverable) deflection was determined from the ILLI-PAVE analysis. K_R is analogous to k but is calculated by dividing the plate pressure by the calculated resilient plate deflection.

Applied plate pressures ranged from 2 psi to πc psi (c is the subgrade cohesion). Pressures larger than πc are not of practical interest, because at higher pressures significant permanent deformation (rutting) will occur in the subgrade.

Four fine-grained subgrade types (very soft, soft, medium, and stiff) were investigated. Pertinent subgrade properties and characteristics are summarized in Table 2. Relations of the resilient modulus versus repeated deviator stress level are shown in Figure 5.

Plate pressure versus resilient displacement data are presented in Figures 6 and 7. Relations of K_R versus plate deflection for the various subgrades are shown in Figures 8 through 11. The subgrades show a definite softening behavior (reduced K_R) with increasing pressures. The softening behavior

TABLE 2 Materials Property Summary

Property	Subgrade Very Soft	Subgrade Soft	Subgrade Medium	Subgrade Stiff	Gravel Subbase
Unit weight (pcf)	110.0	115.0	120.0	125.0	135.0
Coefficient of earth pressure at rest	0.82	0.82	0.82	0.82	0.6
Poisson's ratio	0.45	0.45	0.45	0.45	0.38
E_{Ri} (ksi)	1.00	3.02	7.68	12.34	–
E_R, model (psi)[a]	–	–	–	–	$5{,}000\theta^{0.50}$
Friction angle (degrees)	0.0	0.0	0.0	0.0	40.0
Cohesion (psi)	3.1	6.5	11.4	16.4	0.0
Estimated k^b (psi/in.)	50	100	150	200	–

[a] $E_R = K\theta^n$ (E_R, K, and θ in psi).
[b] Standard modulus of subgrade reaction.

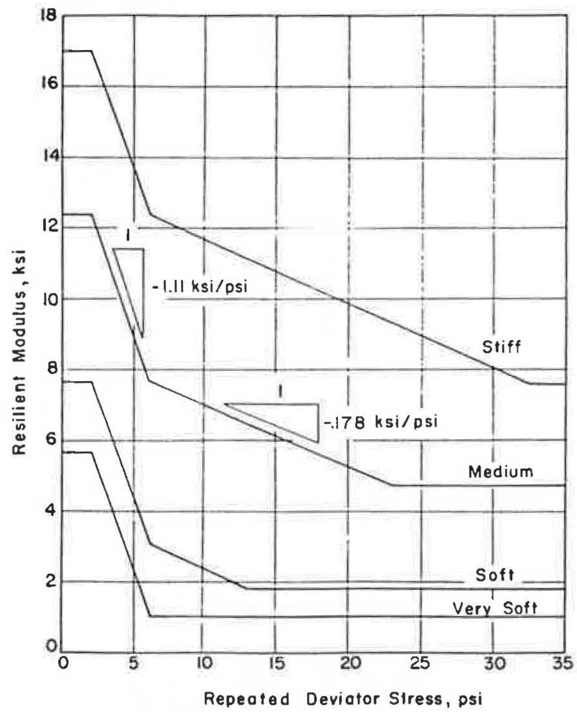

FIGURE 5 Subgrade soil material models for ILLI-PAVE analyses.

is most pronounced for the soft subgrade (Figure 9), where K_R at a pressure of πc is approximately 60 percent of K_R at 2 psi. For a given plate pressure and subgrade type, a decrease in plate size results in a stiffer plate response (K_R increases).

ALGORITHM DEVELOPMENT

A recent study by AFOSR at the University of Illinois (4) indicated that an empirical relation of plate pressure versus deflection proposed by Butterfield and Georgiadis (16) best represented the ILLI-PAVE-generated data on K_R and deflection. The basis of the equation is an idealization proposed by Burland and Lord (17). The equation is characterized by three parameters: an initial stiffness (k_0), a final stiffness (k_f), and a pressure-axis intercept (q_u) (Figure 12). The form of the equation is as follows:

$$q = q_u \left\{1 - \exp\left[-(k_0 - k_f)(w/q_u)\right]\right\} + k_f w \quad (5)$$

where w is plate deflection. The equation was modified for presenting the ILLI-PAVE data for plate pressure versus resilient displacement. A normalized deflection parameter (w/D_y) was substituted into the equation for w, where D_y represents the deflection at a plate pressure of πc psi. The resulting equation is as follows:

$$p = A_1 \left(1 - \exp\left\{-A_2\left[(w/D_y) - A_3\right]\right\}\right) + A_4\left[(w/D_y) - A_3\right] + 2 \quad (6)$$

where

p = plate pressure,
D_y = deflection factor for a given subgrade type (very soft, soft, medium, stiff), and
$A_1, A_2, \ldots$ = subgrade constants.

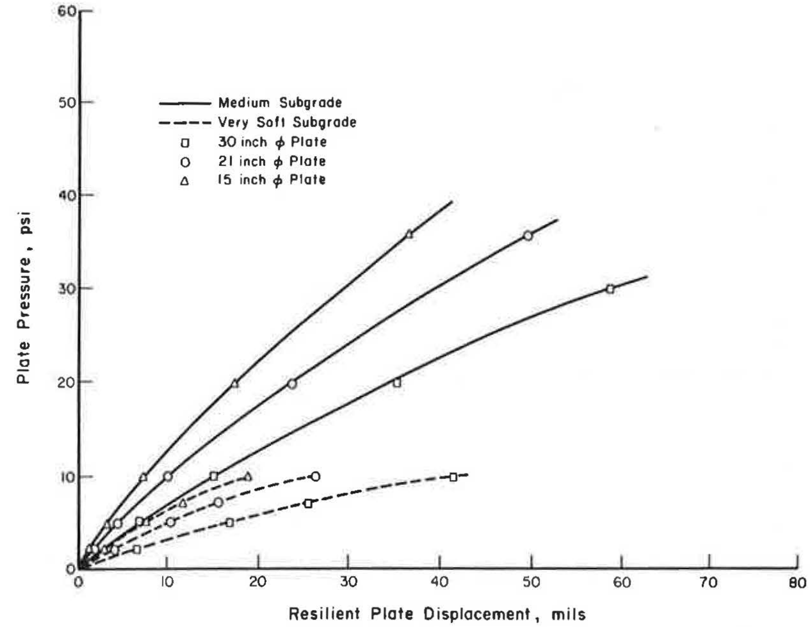

FIGURE 6 Plate pressure versus resilient displacement for medium and very soft subgrades.

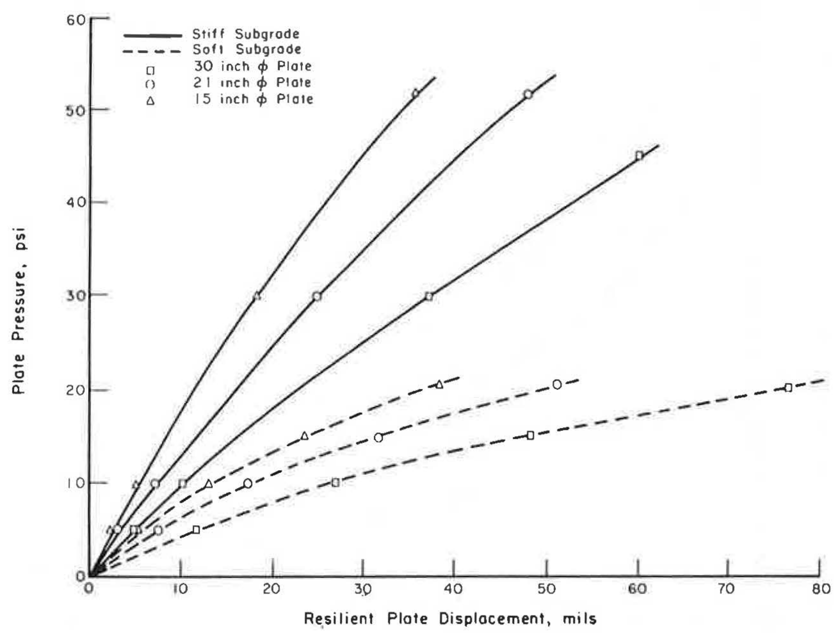

FIGURE 7 Plate pressure versus resilient displacement for stiff and soft subgrades.

If this equation is divided through by the plate deflection (w), the p/w term is K_R, the resilient modulus of subgrade reaction. The final K_R algorithm is as follows:

$$K_R = (1/w)\Big[A_1\Big(1 - \exp\{-A_2\,[(w/D_y) - A_3]\}\Big) + A_4\,[(w/D_y) - A_3] + 2\Big]$$
$$= A_5/D_y \quad \text{if } (w/D_y) < A_3 \qquad (7)$$

Regression analyses were used to develop four equations, one for each subgrade type studied. The resulting equation parameters for the 30-in.-diameter plate are summarized in Table 3. Values of the correlation coefficient (R), standard error of estimate, and coefficient of variation for the equations are also presented in Table 3. To be consistent with the relations of subgrade resilient modulus versus stress level (see Figure 5), K_R is assumed to be a constant for pressures less than 2 psi.

Note that K_R obtained from these algorithms has values much greater than the corresponding static subgrade modulus (k) for any given soil. This is consistent with the observation that soils exhibit a much stiffer response when loaded by rapidly moving loads rather than by static loads. A similar observation was made regarding the relation between the values of E_{Ri} and static E.

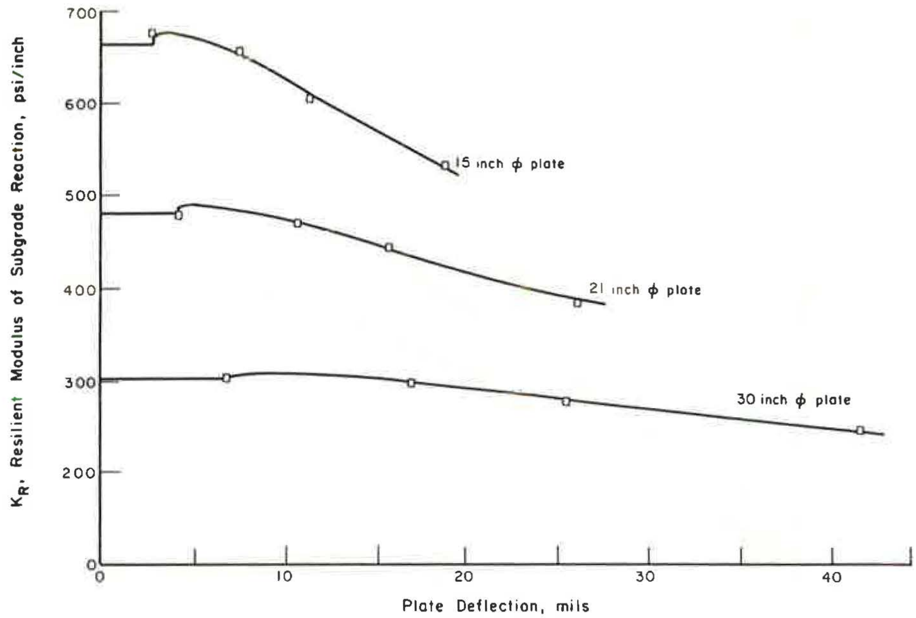

FIGURE 8 K_R versus deflection for various plate sizes (very soft subgrade).

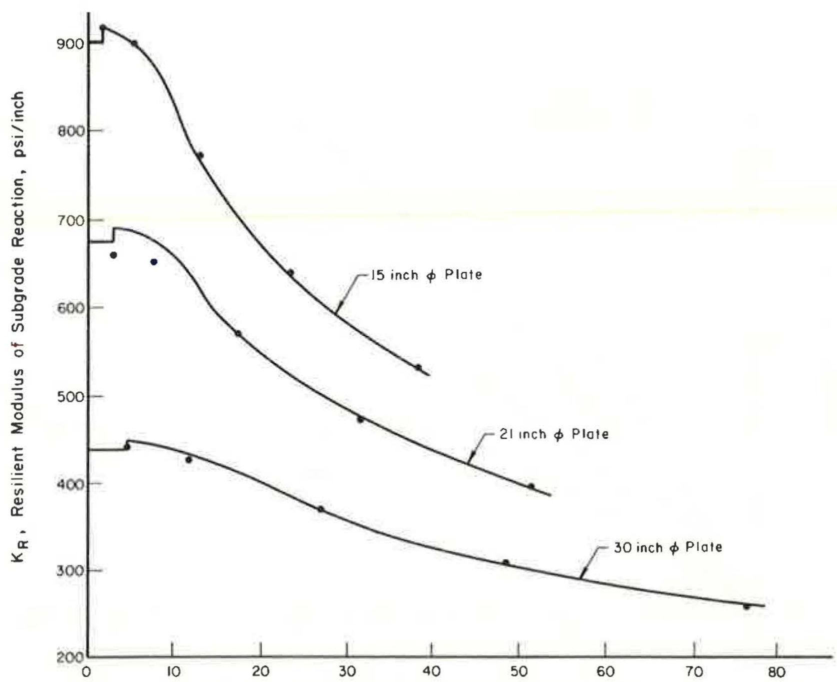

FIGURE 9 K_R versus deflection for various plate sizes (soft subgrade).

SUBBASE EFFECTS

Effect of Granular Subbase

A layer of granular material is frequently used as a subbase in PCC pavement construction. The structural contribution of the granular layer is generally acknowledged by assigning an increased design k to the combination of granular layer and subgrade (18-20).

Additional plate load tests employing a 30-in.-diameter plate on a granular subbase and subgrade soil support system were also simulated by using ILLI-PAVE. The properties of the granular subbase (gravel) are listed in Table 2. Three different granular subbase thicknesses (8, 16, and 24 in.) were considered. The applied plate pressure was 2c. Figure 13 shows the effect of granular layer thickness on K_R for each of the four subgrades. It may be concluded that the introduction of a granular

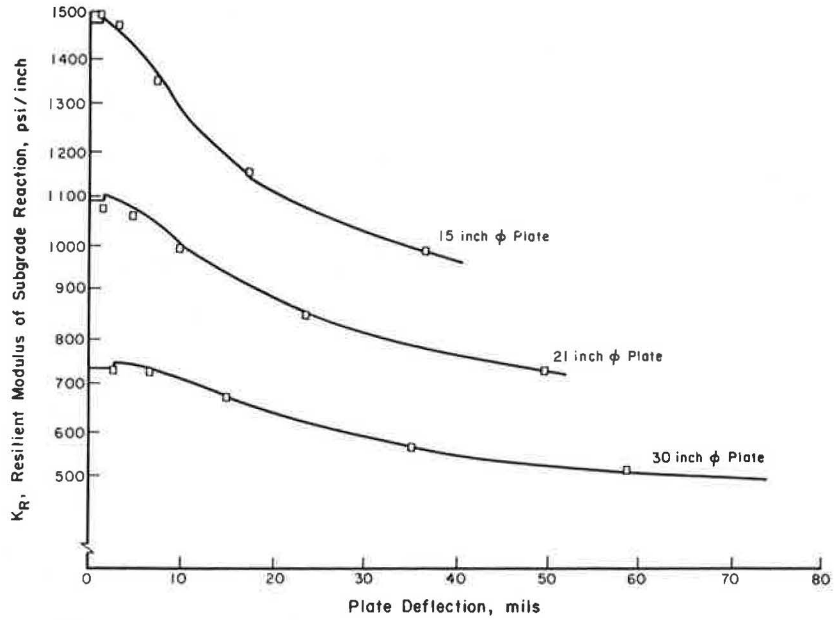

FIGURE 10 K_R **versus deflection for various plate sizes (medium subgrade).**

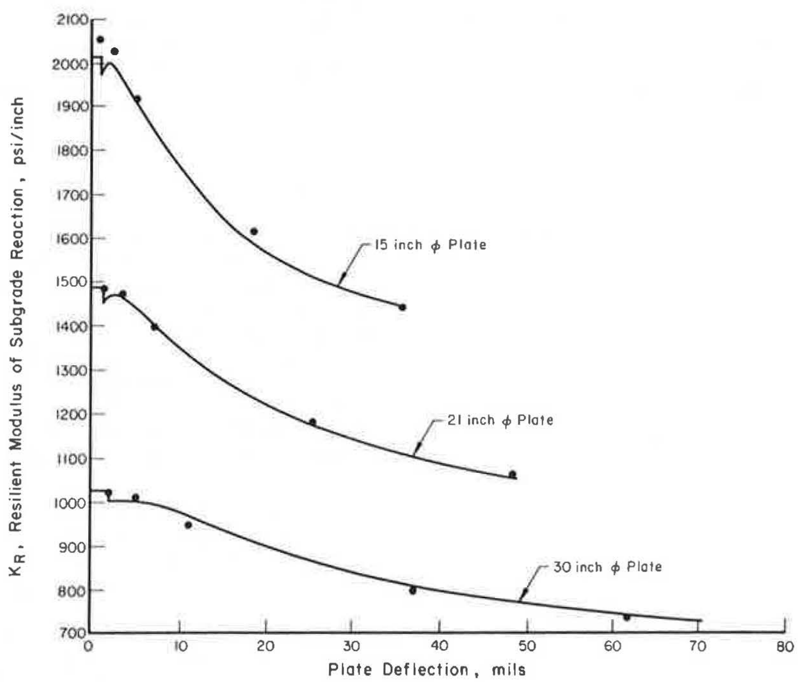

FIGURE 11 K_R **versus deflection for various plate sizes (stiff subgrade).**

TABLE 3 Regression Equation Parameters and Statistics

Subgrade	A_1	A_2	A_3	A_4	A_5	D_Y	R^a	SEE^b (psi/in.)	Coefficient of Variation (%)
Very soft	15.0	0.80	0.1680	0.62	11.9	0.0400	0.993	3.9	1.4
Soft	9.5	2.60	0.0594	10.25	33.7	0.0782	0.995	9.1	2.5
Medium	7.0	3.74	0.0377	28.10	53.1	0.0734	0.997	9.1	1.4
Stiff	5.0	5.30	0.0282	45.90	71.0	0.0707	0.995	13.9	1.5

[a]Correlation coefficient.
[b]Standard error of estimate.

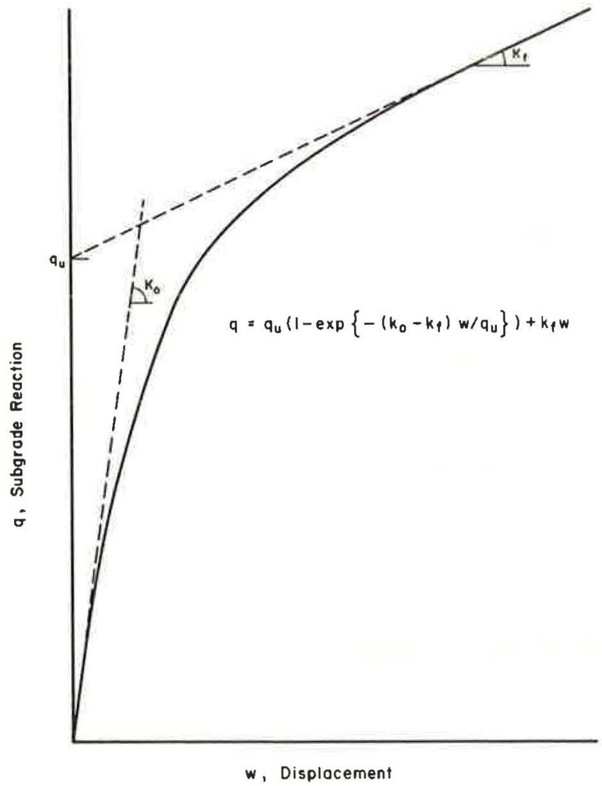

FIGURE 12 Parameters for Butterfield and Georgiadis empirical equation.

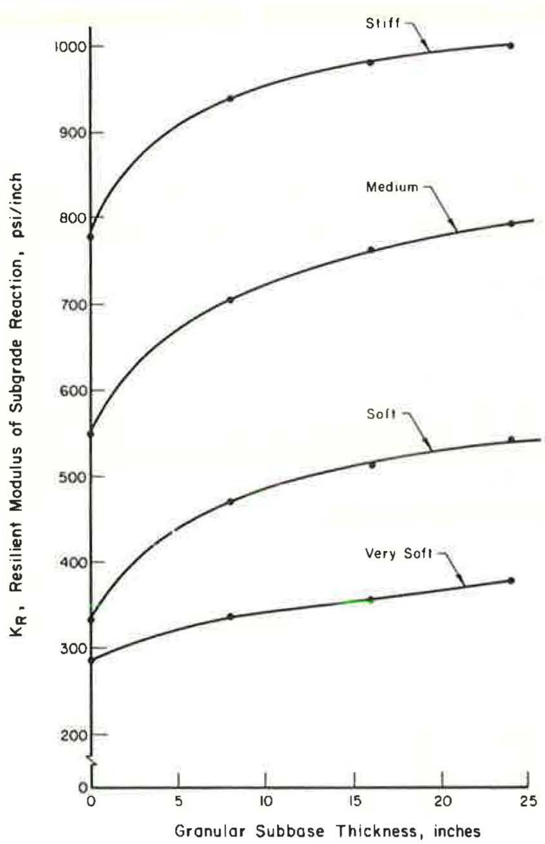

FIGURE 13 K_R versus granular subbase thickness.

subbase up to 8 in. thick has a pronounced beneficial effect on K_R. For greater thicknesses of subbase, the effect on K_R progressively decreases so that subbase thickness has only a slight effect within the 8- to 24-in. thickness range considered. The comments made in the previous section with regard to the apparently high K_R values apply here as well.

Plate pressure effects were also evaluated for an 8-in. granular subbase layer and the soft and very soft subgrade types for plate pressures of c, 2c, and πc psi. Comparative data for the no-subbase and subbase conditions shown in Figure 14 indicate that plate pressure has only a nominal effect. The stress-stiffening behavior of the granular material counteracts to some extent the stress-softening behavior of the fine-grained subgrade.

Effect of Stabilized Subbase

The effect of a subbase with high strength and modulus on a PCC pavement can be considered by increasing k for the stabilized subbase-subgrade system. This procedure is recommended by the Portland Cement Association (18,19) and the Federal Aviation Administration (20).

The ILLI-SLAB program, on the other hand, considers the stabilized subbase as a flexural subbase beneath the PCC layer. This is a more desirable procedure than using an increased k, because the elastic properties of the subbase and its degree of bonding with the PCC slab can be considered.

SUMMARY

The concept of the resilient modulus of subgrade reaction (K_R) is introduced to account for the stress-dependent behavior of typical subgrade soils. This new subgrade support parameter is defined as the pressure producing unit deflection in an impulse plate load test simulated by using the finite-element program ILLI-PAVE. K_R is expressed in the same units as the standard static modulus of subgrade reaction (k), but the value of the former is significantly higher. This indicates increased stiffness in response to rapidly moving loads.

Factors influencing K_R include plate size, deflection level, and subgrade type. A 30-in.-diameter plate was chosen in this study in conformity with general practice. Equations relating K_R and deflection level were developed for four broad cohesive subgrade soil types: very soft, soft, medium, and stiff. The effect of a granular subbase on K_R was also examined. The introduction of a granular layer increases K_R substantially. The importance of this effect, however, diminishes rapidly as subbase thickness exceeds 8 in. The beneficial effect of a granular subbase is consistent over the range of plate pressures investigated (c, 2c, and πc psi).

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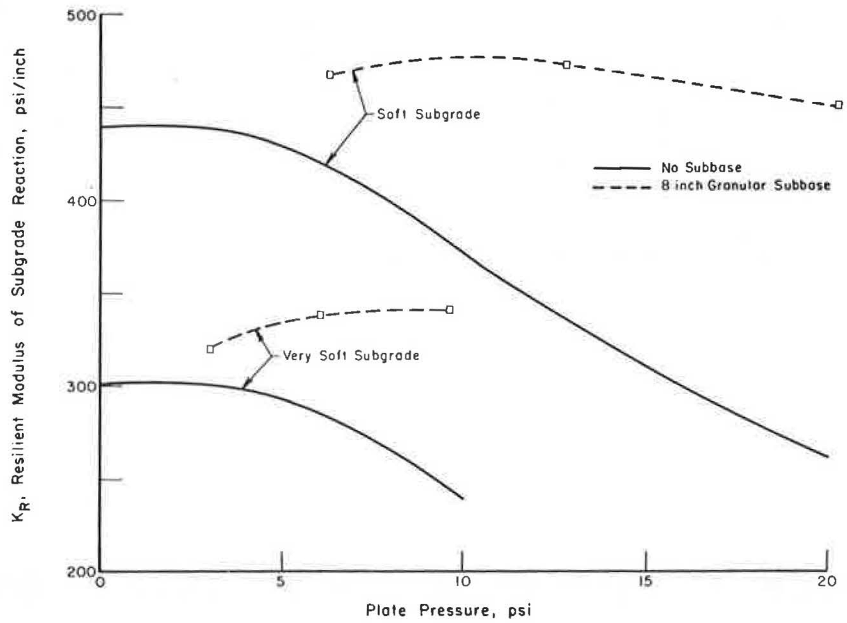

FIGURE 14 K_R versus granular subbase effects for variable plate pressures.

REFERENCES

1. A.M. Tabatabaie, E.J. Barenberg, and R.E. Smith. Longitudinal Joint Systems in Slip-Formed Rigid Pavements, Vol. 2: Analysis of Load Transfer Systems for Concrete Pavements. Report FAA-RD-79-4, II. U.S. Department of Transportation, Nov. 1979.
2. A.M. Tabatabaie and E.J. Barenberg. Longitudinal Joint Systems in Slip-Formed Rigid Pavements, Vol. 3: User's Manual. Report FAA-RD-79-4, III. U.S. Department of Transportation, Nov. 1979.
3. A.M. Tabatabaie and E.J. Barenberg. Finite-Element Analysis of Jointed or Cracked Concrete Pavements. In Transportation Research Record 671, TRB, National Research Council, Washington, D.C., 1978, pp. 11-19.
4. A.M. Ioannides, M.R. Thompson, E.J. Barenberg, and J.A. Fischer. Development of a Stress-Dependent Finite Element Slab Model. Report AFOSR-TR-83-1061. Air Force Office of Scientific Research, Air Force Systems Command, Washington, D.C., May 1983.
5. M.R. Thompson and Q.L. Robnett. Resilient Properties of Subgrade Soils. Transportation Engineering Journal of ASCE, Vol. 105, No. TE1, Jan. 1979.
6. L. Raad and J.L. Figueroa. Load Response of Transportation Support Systems. Transportation Engineering Journal of ASCE, Vol. 106, No. TE1, Jan. 1980.
7. G. Rada and M.W. Witczak. Comprehensive Evaluation of Laboratory Resilient Moduli Results for Granular Material. In Transportation Research Record 810, TRB, National Research Council, Washington, D.C., 1981, pp. 23-33.
8. J.L. Figueroa. Resilient Based Flexible Pavement Design Procedure for Secondary Roads. Ph.D. thesis. University of Illinois, Urbana, 1979.
9. M.S. Hoffman and M.R. Thompson. Mechanistic Interpretation of Nondestructive Pavement Testing Deflections. Civil Engineering Studies, Transportation Engineering Series 32, Illinois Cooperative Highway and Transportation Research Program Series 190. University of Illinois at Urbana-Champaign, June 1981.
10. L.P. Suddath and M.R. Thompson. Load-Deflection Behavior of Lime-Stabilized Layers. Technical Report M-118. Construction Engineering Research Laboratory, Champaign, Ill., 1975.
11. M.R. Traylor. Nondestructive Testing of Flexible Pavements. Ph.D. thesis. Department of Civil Engineering, University of Illinois, Urbana, 1979.
12. M.S. Hoffman and M.R. Thompson. Nondestructive Testing of Flexible Pavements--Field Testing Program Summary. Civil Engineering Studies, Transportation Engineering Series 31, Illinois Cooperative Highway and Transportation Research Program Series 188. University of Illinois at Urbana-Champaign, June 1981.
13. M.R. Thompson. Concepts for Developing a Nondestructive Based Asphalt Concrete Overlay Thickness Design Procedure. Civil Engineering Studies, Transportation Engineering Series 34, Illinois Cooperative Highway and Transportation Series 194. University of Illinois at Urbana-Champaign, June 1982.
14. J.L. Figueroa and M.R. Thompson. Simplified Structural Analyses of Flexible Pavements for Secondary Roads Based on ILLI-PAVE. In Transportation Research Record 766, TRB, National Research Council, Washington, D.C., 1980, pp. 5-10.
15. M.R. Thompson and J.L. Figueroa. Mechanistic Thickness-Design Procedure for Soil-Lime Layers. In Transportation Research Record 754, TRB, National Research Council, Washington, D.C., 1980, pp. 32-36.
16. R. Butterfield and M. Georgiadis. New Interpretation of Plate-Bearing Tests. In Transportation Research Record 810, TRB, National Research Council, Washington, D.C., 1981, pp. 60-67.
17. J.B. Burland and J.A. Lord. The Load-Deformation Behavior of Middle Chalk at Mundford, Norfolk: A Comparison Between Full Scale Perfor-

mance and In-situ and Laboratory Measurements. In In-situ Investigation of Soils and Rocks, British Geological Society, 1970.
18. R.G. Packard. Design of Concrete Airport Pavement. Engineering Bull. EB050.03P. Portland Cement Association, Skokie, Ill., 1973.
19. Thickness Design for Concrete Pavements. IS010.03P. Portland Cement Association, Skokie, Ill., 1966.
20. Airport Pavement Design and Evaluation. Advisory Circular 150/5320-60. FAA, Dec. 7, 1978.

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Finite-Element Model with Stress-Dependent Support

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ABSTRACT

The finite-element model presented is a modified and expanded version of the model developed in 1977 for the study of jointed, slab-on-grade pavements, ILLI-SLAB. A number of modifications to the original code are described. The most important of these is the incorporation through an iterative procedure of the deflection-dependent resilient modulus of subgrade reaction (K_R). This parameter is considered more appropriate in modeling nonlinear subgrade response to rapidly moving loads. Other changes include generation of contour plots of system response, introduction of lines of symmetry, correction of the uniform subgrade stiffness matrix, specification of loaded areas in terms of global coordinates, and free-form input capability. To illustrate the impact of these innovations, results from several demonstration runs are summarized. The major effect of the proposed model is due to the higher values of K_R compared with the commonly used static k.

The determination of stresses and deflections in slab-on-grade pavements with joints or cracks or both has been a subject of major concern for several years. For many pavement structures it has been virtually impossible to obtain analytical (closed-form) solutions because of the complexity of geometry, boundary conditions, and material properties, unless certain simplifying assumptions are made. These, however, result in a modification of the characteristics of the problem. With the advent of high-speed digital computers, solution of these complex structural problems has been greatly facilitated. One of the most powerful methods that has evolved is the finite-element method. This method of analysis is widely accepted as applicable to a broad range of complex boundary-value problems in engineering.

In the calculation of stresses in slab-on-grade pavements, it is also necessary to idealize the characteristics of the supporting medium. In one of the simplest and most popular support characterization theories, the subgrade is regarded as a flexible bed with surface pressure proportional to surface deflection at each point, whereas adjacent unloaded areas are unaffected. This idealization is commonly called a dense liquid or a Winkler subgrade. Finite-element program ILLI-SLAB (1,2) employs a Winkler-type subgrade and can be used to study two-layer, cracked pavement sections, variable load transfer across joints or cracks by aggregate interlock or dowels or both, variable slab thickness, variable subgrade support, and complex multiwheel loading at any position on the pavement. This model has been validated and used extensively in various University of Illinois studies (1,3,4).

In the original version of ILLI-SLAB (1), the modulus of subgrade reaction (k) obtained from the plate load test is used for subgrade characterization. This is in conformity with general engineering practice. Several other finite-element models also use this approach (5-7). The value of k can be varied from node to node according to a pattern specified by the user at the beginning of the analysis. Note that k is assumed to be independent of stress or deflection level, being essentially a linear, low-stress modulus. Most subbase-subgrade support systems, however, display a response dependent on stress level. Typically a softer (lower-k) response is exhibited at higher magnitudes of stress or deflection.

To account for this effect, the concept of the resilient modulus of subgrade reaction (K_R) was introduced in a U.S. Air Force Office of Scientific Research (AFOSR) study (8). This is no longer the modulus k, derived from the static plate load test, but a modulus characterizing subgrade response to a repeated (impulse-type) test. The latter loading condition is considered more appropriate for the type of moving loads applied by modern-day highway and airport traffic. In the AFOSR study, relationships are developed between K_R and deflection (w) for a broad range of fine-grained soils.

In this paper a description is given of how the