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Finite-Element Model with Stress-Dependent Support

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ABSTRACT

The finite-element model presented is a modified and expanded version of the model developed in 1977 for the study of jointed, slab-on-grade pavements, ILLI-SLAB. A number of modifications to the original code are described. The most important of these is the incorporation through an iterative procedure of the deflection-dependent resilient modulus of subgrade reaction (K_R). This parameter is considered more appropriate in modeling nonlinear subgrade response to rapidly moving loads. Other changes include generation of contour plots of system response, introduction of lines of symmetry, correction of the uniform subgrade stiffness matrix, specification of loaded areas in terms of global coordinates, and free-form input capability. To illustrate the impact of these innovations, results from several demonstration runs are summarized. The major effect of the proposed model is due to the higher values of K_R compared with the commonly used static k.

The determination of stresses and deflections in slab-on-grade pavements with joints or cracks or both has been a subject of major concern for several years. For many pavement structures it has been virtually impossible to obtain analytical (closed-form) solutions because of the complexity of geometry, boundary conditions, and material properties, unless certain simplifying assumptions are made. These, however, result in a modification of the characteristics of the problem. With the advent of high-speed digital computers, solution of these complex structural problems has been greatly facilitated. One of the most powerful methods that has evolved is the finite-element method. This method of analysis is widely accepted as applicable to a broad range of complex boundary-value problems in engineering.

In the calculation of stresses in slab-on-grade pavements, it is also necessary to idealize the characteristics of the supporting medium. In one of the simplest and most popular support characterization theories, the subgrade is regarded as a flexible bed with surface pressure proportional to surface deflection at each point, whereas adjacent unloaded areas are unaffected. This idealization is commonly called a dense liquid or a Winkler subgrade. Finite-element program ILLI-SLAB (1,2) employs a Winkler-type subgrade and can be used to study two-layer, cracked pavement sections, variable load transfer across joints or cracks by aggregate interlock or dowels or both, variable slab thickness, variable subgrade support, and complex multiwheel loading at any position on the pavement. This model has been validated and used extensively in various University of Illinois studies (1,3,4).

In the original version of ILLI-SLAB (1), the modulus of subgrade reaction (k) obtained from the plate load test is used for subgrade characterization. This is in conformity with general engineering practice. Several other finite-element models also use this approach (5-7). The value of k can be varied from node to node according to a pattern specified by the user at the beginning of the analysis. Note that k is assumed to be independent of stress or deflection level, being essentially a linear, low-stress modulus. Most subbase-subgrade support systems, however, display a response dependent on stress level. Typically a softer (lower-k) response is exhibited at higher magnitudes of stress or deflection.

To account for this effect, the concept of the resilient modulus of subgrade reaction (K_R) was introduced in a U.S. Air Force Office of Scientific Research (AFOSR) study (8). This is no longer the modulus k, derived from the static plate load test, but a modulus characterizing subgrade response to a repeated (impulse-type) test. The latter loading condition is considered more appropriate for the type of moving loads applied by modern-day highway and airport traffic. In the AFOSR study, relationships are developed between K_R and deflection (w) for a broad range of fine-grained soils.

In this paper a description is given of how the deflection-dependent support relations are incorporated into ILLI-SLAB to accommodate nonlinear subgrade behavior. Several other ILLI-SLAB improvements are also presented. The impact of these innovations is assessed by using a series of demonstration runs involving typical pavement sections and loading patterns.

ILLI-SLAB

ILLI-SLAB was developed at the University of Illinois in 1977 for structural analysis of jointed one- or two-layer concrete pavements with specified levels of load transfer at the joints or cracks (1,2). The ILLI-SLAB model is based on the classical theory of a medium-thick plate on a Winkler foundation (9) and can be used to evaluate the structural response of a concrete pavement system with joints or cracks or both. It employs the plate-bending element with four nodes and 12 degrees of freedom known in the finite-element literature as ACM or RPB12 (10). The Winkler-type subgrade is modeled as a uniform distributed subgrade through an equivalent mass formulation (11). This is a much more realistic representation than the four concentrated spring elements used in other programs (5-7). A work equivalent load vector is used (10).

Various types of load transfer systems, such as dowel bars, aggregate interlock, keyways, or a combination of these, can be considered at the pavement joints with ILLI-SLAB. The model can also accommodate the effect of a stabilized base or an overlay (with either perfect bond or no bond).

MODIFICATION OF ILLI-SLAB

For the benefits of the finite-element method to be fully realized, it is highly desirable that programs using this method of analysis

1. Accept easy-to-compile, user-oriented input data restricted to the absolute minimum required and in which, where possible, potential pitfalls for the user have been eliminated;
2. Employ carefully selected default values that will reduce the amount of input data required;
3. Perform error checks, especially in the case of default values, so that errors that are concealed by the otherwise normal execution of the program will be avoided;
4. Be free of code errors;
5. Organize the output so that it is neat, meaningful, and user-oriented;
6. Incorporate skillful data-base management for the efficient utilization of available memory core;
7. Provide basic and higher-level routines; and
8. Present the results in a summary or a graphical form.

The ILLI-SLAB modifications presented in the following are aimed at providing these capabilities.

Stress-Dependent Subgrade Definition

The general expression for the relation between K_R and w as developed in this study (30-in.-diameter plate) is (8)

$$K_R = (1/w)\left[A_1\left(1 - \exp\left\{-A_2\left[(w/D_y) - A_3\right]\right\}\right) + A_4\left[(w/D_y) - A_3\right] + 2\right]$$
$$= A_5/D_y \quad \text{if } (w/D_y) < A_3 \qquad (1)$$

where A_1, A_2, A_3, A_4, A_5, and D_y are regression parameters determined from plate load tests simulated by using ILLI-PAVE (12), a stress-dependent (nonlinear) finite-element program developed at the University of Illinois. By specifying these parameters, the user can define a stress-dependent subgrade. Parameter sets for the following broad subgrade types have been developed and are now an integral part of the revised version of ILLI-SLAB:

1. Very soft (K_R = 300 psi/in.),
2. Soft (K_R = 425 psi/in.),
3. Medium (K_R = 725 psi/in.), and
4. Stiff (K_R = 1,000 psi/in.).

The figures in parentheses are recommended initial (small-deflection) values. These are significantly higher than the normally accepted static k-values, reflecting the increased subgrade stiffness that results from dynamic or rapidly moving loads.

Other options available in modified ILLI-SLAB are as follows:

1. Other: The user specifies the regression parameters individually to obtain a different relation of K_R versus w.
2. Winkler: This option is the stress-independent, uniform Winkler subgrade, available in the original version of ILLI-SLAB.
3. Springs: Support is provided by four springs at the corner of each element (stress independent). This option allows validation by direct comparison with other programs but is not recommended for general use.

An iterative procedure, which compares support values (K_R) corresponding to calculated deflections with previously assumed or determined values, is used in the modified version of ILLI-SLAB. New support values are assigned for each subsequent iteration until compatibility is achieved between support system deflections and the user-prescribed support pattern. Furthermore the new model allows the user to assign different support values to selected (or all) nodes. When one of the stress-dependent subgrade types is used, subroutine ITERATE provides a procedure for checking convergence, updating support values, and proceeding with additional iterations as necessary. Subroutine ITERATE is structured to allow easy modification of the regression equations or addition of other subgrade options. The user controls the iterative procedure by three variables:

1. ITMAX: This variable specifies the maximum number of iterations desired. Usually three iterations are sufficient; a value of ITMAX = 6 is recommended.
2. TOL1: Convergence tolerance for updated K_R compared with K_R from the previous iteration. A value of TOL1 = 0.05, i.e., 5 percent, is recommended.
3. TOL2: Convergence tolerance for percentage of nodes at which TOL1 is not satisfied. Again, a value of TOL2 = 0.05, i.e., 5 percent, is recommended. The recommended values of ITMAX, TOL1, and TOL2 are used as default values.

Contouring Capability

During a study conducted in summer 1981, the facility to generate contours of stresses and deflections was incorporated into the ILLI-SLAB package (4). This was done through auxiliary program CONT, which accepts ILLI-SLAB results as input data. Subroutine CONTOUR in this auxiliary program passes these results to a number of subroutines developed at the National Center for Atmospheric Research (NCAR). The software used in these subroutines has been developed and is made available with the restriction that NCAR be acknowledged as the source in any resulting research or publications. The most important family of NCAR subroutines used in ILLI-SLAB is CONRAN. CONRAN, the standard version, plots contour lines by using random, sparse, or irregular data sets; adds line labeling and contour dash patterns; and plots relative highs and lows. The data are triangulated and then contoured. Contouring is performed by interpolating the triangulated data. Typical contour plots are shown in Figure 1.

More Efficient Memory Core Utilization

A major problem encountered by ILLI-SLAB users is that any attempt to refine the mesh used, especially when investigation of the stability and convergence of the numerical solution is desired or when several slabs are used, faces the possibility of exceeding machine memory core capacity. This problem has been addressed by the introduction of the capability to take advantage of any symmetry lines that may exist. In the modified ILLI-SLAB version, the user has the following options:

1. No lines of symmetry exist (ISYM = 0),
2. The x-axis is a line of symmetry (ISYM = 1),
3. The y-axis is a line of symmetry (ISYM = 2), and
4. The x- and y-axes are both axes of symmetry (ISYM = 3).

Care was taken to introduce these options without imposition of a burden on the user during the preparation of the input data. Particularly undesirable are requirements to include the node numbers for the nodes along the line or lines of symmetry. In the new version of ILLI-SLAB, the various options re-

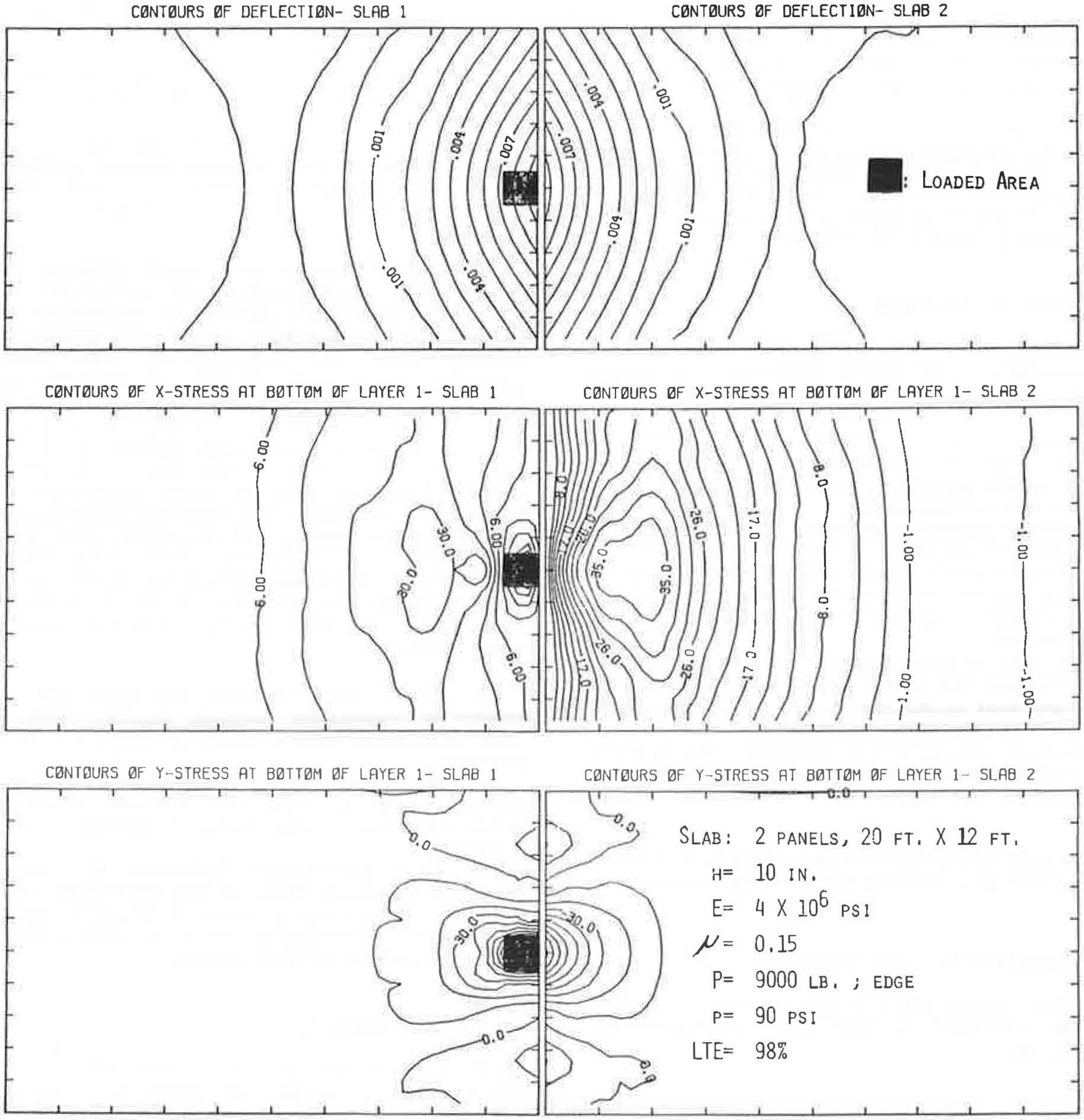

FIGURE 1 Typical contour plots.

lated with symmetry are specified by using a single input variable (ISYM), which may even be omitted if no symmetry exists.

Subgrade Stiffness Matrix Correction

One of the major advantages of ILLI-SLAB over other available programs is that the Winkler-type foundation is no longer modeled by four concentrated springs at the corners of each slab element. Through an equivalent mass formulation, a uniform distributed subgrade is provided under each element. The formulation for the derivation of the stiffness matrix for this subgrade (1) follows the same steps as the ones presented by Dawe (11), who first derived this matrix. In Dawe's equivalent mass formulation, the product of mass per unit area and plate thickness (ρh) replaces the subgrade modulus k (or K_R). Similar derivations using different sign conventions are also presented by Przemieniecki (13) and Zienkiewicz (10).

The subgrade stiffness matrix used in ILLI-SLAB was compared with each of the matrices presented in these publications, which were further compared with each other, with due allowance for differences in sign convention. The original formulation of the stiffness matrix in ILLI-SLAB (1) was thereby corrected. For earlier users of the program, the corrections in the stiffness matrix are most obvious in the results from solutions of symmetric problems, where identical responses are now obtained at corresponding points, as expected. Although the change in the results of a typical run may only range from 3 to 5 percent, it is important to have a balanced formulation to ensure the good behavior and convergence of the numerical solution.

Specification of Loaded Areas in Terms of Global Coordinates

In the original version of ILLI-SLAB, input data specifying loaded area or areas had to be in terms of local (element) coordinates. In this sytem the origin is set at the lower left corner of each element and the axes extend from 0 to 2a in the x-direction and 2b in the y-direction for a typical element of dimensions 2a x 2b. The result was that the user had to go through the following steps when the loaded areas for the problem were specified:

1. Determine the element numbers of the loaded elements adhering to a fixed numbering sequence, i.e., from bottom to top and from left to right. It should be noted that depending on the fineness of the mesh used, each loaded area (such as a wheel imprint) might apply a load on four or more elements. Thus, a large number of partially or fully loaded elements might be needed to define the loading pattern in all but the simplest situations.
2. For each of the loaded elements, determine the extent of the loaded subarea in terms of the local element coordinate system. These coordinates should then be specified, one subarea per card, together with the load intensity in each case.
3. Whenever there is a change in the finite-element mesh used, such as when the mesh is made finer, the process must be repeated.

As a result of the complexity of this process, especially when it was used, as it was, with the fixed-form input format, a large portion of all problems encountered by ILLI-SLAB users was related to specifying loading pattern data. To overcome these difficulties, subroutine SUBAREA was coded to allow input data specification in terms of the global coordinate system. In this system the origin is located at the lower left corner of the slab arrangement and the axes extend to the extreme corners of the arrangement of all the slabs in both the x- and y-directions. The advantages of this system are obvious:

1. Element numbering, although retained internally, does not enter into input data preparation;
2. Only as many loaded areas as actually exist need be specified; the global coordinates and the extent of each loaded area now acquire a more realistic physical meaning for the user; and
3. The global coordinate system is independent of the mesh used, being solely determined by the arrangement of slab or slabs analyzed.

Free-Form Input Capability

With the addition of several new subroutines, ILLI-SLAB can now accept free-form input data by using a problem-oriented language (POL) consisting of simple, easy-to-remember English-type statements. This has been made possible by accessing the SCAN library of routines developed at the Civil Engineering Systems Laboratory (CESL) at the University of Illinois (14-16). SCAN has been used as a teaching and research tool for a number of years at the University of Illinois. It is also the front end of the POLO system, including FINITE (7), and is used in a number of production systems at CESL.

Only those parameters that are different from the default values need be provided when the free-form input capability is used. This will save time in preparing the input data file and executing the program. The free-form subroutines are set up to issue diagnostic error messages before execution in the event of improper input data. These greatly facilitate debugging the input data file and are particularly useful to new users of ILLI-SLAB.

Miscellaneous Changes

In addition to providing a user-oriented input data capability, it is desirable to have a user-oriented presentation of the results from a given run. Early in this study particular attention was directed toward improving the output format by the introduction of appropriate carriage control characters, elimination of unnecessary lines of output, and replacement of these by other meaningful output information. The changes incorporated in the revised version of ILLI-SLAB are aimed at providing a well-organized, clear echo of the input file so that the parameters and loading conditions used can be easily checked as well as at giving the user a neat, usable output. Of great interest to the user is the summary of maximum values of deflection and stresses and the nodal numbers at which these occur, which is given at the end of the output.

A second group of changes involved the elimination of several code errors ("bugs") that were revealed by numerous error checks. Comparisons with FINITE show that at least the major routines of ILLI-SLAB, such as stiffness matrix assembly, inversion, and solution and determination of stresses and deflections, are free of any code errors.

TYPICAL EFFECTS WITH MODIFIED ILLI-SLAB

To illustrate the capabilities of ILLI-SLAB and the impact of the modifications described earlier, a number of demonstration runs are presented. Two typical pavement cross sections are considered in this investigation. The first is a 10-in. portland cement concrete (PCC) pavement consisting of panels that are 20 ft square and 20 ft by 12 ft, with or without load transfer between adjacent slabs. These dimensions are typical of airport and highway pavements, respectively. The contour plots shown in Figure 1 were obtained from one of the demonstration runs with a highway pavement. For the cases involving load transfer, a second, mirror-image panel was added to the right of the panels shown in Figure 1. The second pavement section considered is a 12-in. pavement incorporating a stabilized base layer with a modulus (E) of 1.5×10^6 psi. This pavement consists of panels 15 ft by 12 ft and is typical of pavements proposed for the U.S. Air Force (USAF) alternate launch and recovery surface (ALRS) program.

The two pavement sections are loaded by a 9-kip highway load or typical USAF aircraft loading patterns, namely, the F-4, the C-130, and the F-111. A typical soft subgrade is assumed with two alternative characterizations. The first is the conventional static subgrade modulus (k), which was assigned a value of 120 to 150 psi/in. by using the WINKLER option in ILLI-SLAB. The second is the proposed resilient modulus (K_R), which was set at 425 psi/in. by using the SOFT option. Associated with the latter opinion is stress dependence, provided by the iterative scheme in ILLI-SLAB.

In an effort to clarify the picture presented by the results of these demonstration runs, three distinct effects are identified and discussed separately in the following.

Effect of Load Transfer

To investigate the impact of load transfer systems, load transfer by aggregate interlock was provided in some runs and these are compared with those in which only one panel was used. It was intended to investigate the two extreme cases, that of no load transfer efficiency (LTE = 0 percent) and that of full load transfer efficiency (LTE = 100 percent). For the latter an aggregate interlock factor (AIF) of 1×10^6 was specified, producing LTEs between 97 and 99 percent.

Under conditions of full load transfer, maximum deflection is reduced to half its value for the condition of no load transfer. The effect of load transfer on maximum bending stress is shown as a stress ratio in Table 1, which indicates that full-load-transfer stress is about 0.6 times the no-load-transfer stress. It is also observed that the proposed change to a resilient modulus subgrade characterization has only a minor effect in this respect. As expected, LTE is more pronounced (albeit only slightly) in the case of the less stiff ALRS pavement.

TABLE 1 Effect of Load Transfer

Pavement	Loading	Subgrade	Stress Ratio[a]
PCC	9-kip	k = 150 psi/in.	0.61
		SOFT	0.63
	F-4	k = 150 psi/in.	0.61
		SOFT	0.62
	C-130	SOFT	0.62
ALRS	F-4	k = 120 psi/in.	0.59
		SOFT	0.61
	C-130	SOFT	0.62

Note: All runs are for edge-loading condition. PCC = portland cement concrete; ALRS = alternate launch and recovery surface.

[a] Stress ratio = σ_{max} for LTE = 100 percent divided by σ_{max} for LTE = 0, where LTE = load transfer efficiency (deflection across joint on unloaded side divided by maximum deflection along joint on loaded side). The corresponding deflection ratio is 0.50 for LTE = 100 percent. All comparisons are for the first iteration.

Effect of Resilient Modulus Characterization

As explained earlier, it is considered that a resilient modulus subgrade characterization would be more appropriate for airfield pavement systems under transient loads than the conventional static plate load test subgrade modulus. The k-value used to characterize the subgrade in finite-element programs like ILLI-SLAB should be replaced by a stress-dependent K_R-value, which at low stress levels is substantially grater than that of the static k. In the cases analyzed in this paper, the WINKLER subgrade was assumed to have a static k-value of 120 to 150 psi/in. This is considered equivalent to the stress-dependent SOFT subgrade option in ILLI-SLAB. The low-stress-level K_R-value for this subgrade is 425 psi/in., according to the algorithms described previously.

The effect of this change is shown in Table 2 by comparing the responses of the SOFT and WINKLER subgrades in the form of deflection and stress ratios. Deflection ratios are seen to vary between 0.40 and 0.56, whereas stress ratios have values between 0.80 and 0.93. Thus, the proposed resilient modulus subgrade characterization leads to lower calculated deflections and stresses; stresses are affected to a smaller extent than deflections. Table 2 also shows that the impact of the proposed change is more significant as the load becomes more severe (C-130 instead of F-4) or if the pavement system is less stiff (ALRS rather than PCC pavement; no load transfer).

TABLE 2 Effect of Resilient Modulus

Pavement	Loading	Specified LTE (%)	Deflection[a,b] Ratio	Stress[b,c] Ratio
PCC	9-kip	0	0.53	0.91
		100	0.54	0.93
	F-4	0	0.55	0.88
		100	0.56	0.90
	C-130	100	0.49	0.81
ALRS	F-4	0	0.46	0.87
		100	0.47	0.89
	C-130	100	0.40	0.80

Note: All runs are for edge-loading condition. PCC = portland cement concrete; ALRS = alternate launch and recovery surface.

[a] Deflection ratio = maximum deflection for SOFT divided by maximum deflection for WINKLER.

[b] All comparisons are for the first iteration.

[c] Stress ratio = maximum stress for SOFT divided by maximum stress for WINKLER.

Effect of Stress Dependence: Iterative Scheme

Associated with the stress-dependent options in ILLI-SLAB, including the SOFT option employed in these demonstration runs, is an iterative scheme. In this scheme at the end of each iteration a check is performed for the compatibility of calculated deflections and assumed support pattern (i.e., K_R-values). If this compatibility is poor, a new iteration is performed after the support pattern has been updated until specified convergence tolerances are achieved. Usually no more than three iterations were required to achieve convergence within a tolerance of 5 percent.

Table 3 is an attempt to filter out the effect of the iterative scheme by presenting in terms of de-

TABLE 3 Effect of Stress Dependence

Pavement	Loading[a]	Specified LTE (%)	Deflection[b,c] Ratio	Stress[c,d] Ratio
PCC	F-4	0	1.03	1.01
		100	1.00	1.00
	C-130	0	1.10	1.05
		100	1.03	1.02
	F-111	0	1.04	1.07
ALRS	F-4	0	1.06	1.02
		100	1.01	1.00
	C-130	0	1.14	1.07
		100	1.05	1.03
	F-111	0	1.12	1.05

Note: All runs are for the SOFT subgrade and edge-loading condition. PCC = portland cement concrete; ALRS = alternate launch and recovery surface.

[a] Iterative scheme has no effect for 9-kip highway loading.

[b] Deflection ratio = maximum deflection for last iteration divided by maximum deflection for first iteration.

[c] Convergence tolerances: TOL1 = 5 percent; TOL2 = 5 percent.

[d] Stress ratio = maximum stress for last iteration divided by maximum stress for first iteration.

flection and stress ratios the responses after the first and after the last iteration. Deflection ratios range between 1.00 and 1.14, whereas stress ratios fall between 1.00 and 1.07. Thus the effect of the iterative scheme is to increase the maximum deflections and stresses obtained after the first iteration, thereby counterbalancing some of the change produced by the resilient modulus described previously.

Because the application of the iterative scheme increases execution time, it is important to draw some conclusions as to when such an increased expense is justified by the changes in response produced. Table 3 shows that the iterative scheme effect becomes substantial (i.e., 10 percent or more) for the more severe loading patterns (edge rather than interior; F-111 or C-130 rather than F-4) on the less competent pavement systems (ALRS rather than PCC section; LTE = 0 percent rather than LTE = 100 percent). The iterative scheme has no effect in the case of the 9-kip highway load, and only one iteration is required to achieve the 5 percent specified tolerance.

In general the effect of the iterative scheme is not dramatic. This may be attributed partly to the development of the algorithms used in the current version of ILLI-SLAB by simulating rigid plate load tests with ILLI-PAVE. The plates used in these tests are much stiffer than any ordinary pavement slabs, and their radius of relative stiffness (l) is much higher than the values encountered in pavement slabs. Westergaard (17) and other investigators have pointed out the effect of the radius of relative stiffness on the response of the subgrade-pavement system.

Finally Table 4 presents the combined effects of the resilient modulus and of the iterative scheme. The deflection ratios range between 0.42 and 0.57 and are in general substantially lower than the corresponding stress ratios, which lie between 0.82 and 0.93. This indicates that the impact of the proposed changes is much more significant with respect to deflections than stresses. Furthermore, the effects are more pronounced in the case of the more severe load patterns or the less competent pavement systems.

TABLE 4 Combined Effect of Proposed Changes

Pavement	Loading	Specified LTE (%)	Deflection[a] Ratio	Stress[b] Ratio
PCC	9-kip	0	0.53	0.91
		100	0.54	0.93
	F-4	0	0.57	0.89
		100	0.56	0.90
	C-130	100	0.50	0.83
ALRS	F-4	0	0.49	0.89
		100	0.47	0.90
	C-130	100	0.42	0.82

Note: All runs are for edge-loading condition. Changes consist of subgrade characterization by resilient modulus K_R (= 425 psi/in.; SOFT) instead of static subgrade modulus k (= 120 or 150 psi/in.) and stress dependence-iterative scheme. PCC = portland cement concrete; ALRS = alternate launch and recovery surface.

[a] Deflection ratio = maximum deflection after changes divided by maximum deflection before changes.

[b] Stress ratio = maximum stress after changes divided by maximum stress before changes.

SUMMARY

Classical slab-on-grade pavement analysis procedures (such as those proposed by Westergaard) cannot accommodate nonlinear subgrade support conditions, complex loading patterns, cracked sections with varying LTEs, and subbase effects. The modified ILLI-SLAB model developed in this study alleviates many of these inadequacies.

Computer program ILLI-SLAB (1), developed at the University of Illinois, offers great flexibility in modeling loading conditions (i.e., position, size, and intensity of loaded area or areas) and load transfer systems. The objective of this study was to modify the subgrade model in ILLI-SLAB from a simple, linear spring (Winkler) type to a stress-dependent (more accurately, a deflection-dependent) model, in which the resilient subgrade modulus (K_R) decreases with increasing deflection (w). Such a model was developed and incorporated into ILLI-SLAB by using an iterative scheme. According to this scheme, a selected initial value of K_R (dependent on subgrade type) is corrected after each iteration. After a number of iterations, the values of K_R before and after the last iteration differ by only a specified small percentage.

The impact of the iterative K_R model was investigated for several typical pavement systems subjected to edge loading. Some of the sections included load transfer systems. The major effect of the proposed K_R model stems from the difference between the value of the resilient subgrade modulus (initial K_R-value assigned in the iterative procedure) and the static k typically used. The effect of iterative analysis is limited and becomes more pronounced for conditions producing more severe pavement responses (thin structural sections, traffic overloads, edge loading, no load transfer).

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Structural Model for Concrete Block Pavement

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ABSTRACT

A structural model for the calculation of stress, strain, and deflection in a concrete block pavement is described. This model is based on the ICES STRUDL computer program that was recently extended by the introduction of a so-called RIGID BODY element. It is shown that by means of this model excellent agreement between measured and calculated deflection profiles is obtained. Furthermore, time functions were derived from the observation of 20 concrete block pavements in service. These functions show the increase of the subgrade modulus, the stiffness of the joints and the bedding layer, and the decrease in the deflection with respect to the number of equivalent 100-kN single axles. By means of ICES STRUDL and the developed time functions tentative design charts are developed in which the number of years until a given rut depth is reached is related to the initial subgrade modulus and the average daily equivalent 100-kN single axle loads. Although concrete block pavements are common in western Europe (about one-third of the paved area in the Netherlands consists of such pavements), this pavement type is not well known in the United States. Therefore a general description of the most characteristic features of concrete block pavements is given.

Concrete block paving is the most recent development in element (segmental) paving, which is built by laying down small elements on the (improved) subgrade. Element paving has been used since the Middle Ages. Wooden setts were sometimes used as elements but, especially at first, nontooled natural stone was the only pavement material that had sufficient resistance to the traffic loading by steel wheels.