- K.R. Peattie. Stress and Strain Factors for Three-Layer Elastic Systems. Bull. 342. HRB, National Research Council, Washington D.C., 1962, pp. 215-252.
- 6. W.R. Barker and W.N. Brabston. Development of a Structural Design Procedure for Flexible Airport Pavements. Report FAA-RD-74-199. U.S. Army Corps of Engineers Waterways Experiment Station, Vicksburg, Miss., Sept. 1975.
- D.N. Little. The Dual Parametric Approach to the Analysis of Surface Deflection Data. Texas Transportation Institute, Texas A&M University System, College Station, June 1980.
- Flexible Pavement Design and Management, Materials Characterization. NCHRP Report 140. TRB, National Research Council, Washington, D.C., 1973.
- H.F. Southgate and R.D. Deen. Temperature Distribution within Asphalt Pavements and Its Re-

- lationship to Pavement Deflection. Report HRP-1(3), Part II: KYHPR-64-20. Division of Research, Kentucky Department of Highways, Lexington, 1964.
- W. Barker et al. Description of BISDEF Program. U.S. Army Corps of Engineers, Aug. 1982.
- 11. N.K. Vaswani. Method for Separately Evaluating Structural Performance of Subgrades and Overlying Flexible Pavements. <u>In</u> Highway Research Record 362, TRB, National Research Council, Washington, D.C., 1971, pp. 48-62.

Publication of this paper sponsored by Committee on Strength and Deformation Characteristics of Pavement Sections.

Stresses in Full-Depth Granular Pavements

S.R. DODDIHAL and B.B. PANDEY

ABSTRACT

Highway pavements for moderate to low-volume traffic often consist of granular materials topped with thin bituminous surfacing. The particulate bases transfer the stresses to the underlying layer by grainto-grain interaction and cannot resist tensile stresses. The usual analysis of multilayer pavements requires the granular layers to possess tensile strength. A finite-element solution is presented for determination of stresses in pavements made up of full-depth granular layers. In this method the tensile stresses computed in the elements of the granular material by the elastic approach are eliminated during each iteration until the solution converges. The vertical stresses on the subgrade are found to be significantly higher than those obtained from linear-elastic analysis.

Design of highway and airfield pavements involves selection of materials and determination of the thickness of various layers that should be used in pavement construction so that the pavement layers are stable and carry the traffic during the design period without any major maintenance. Highway pavements in India and elsewhere for moderate to low-volume traffic consist essentially of granular materials in the form of water-bound macadam or crushed rock with thin bituminous surfacing. The pavements in such cases are practically full-depth granular construction if the strength of the thin

bituminous surfacing is ignored. Analysis of such pavements requires special consideration because of the particulate nature of the materials and the absence of confinement for want of a thick bituminous surfacing.

Various organizations have developed computer programs such as BISAR (1), CHEVRON (2), ELSYM 5 (3), and so on, based on multilayer elastic theory, which requires the granular materials to withstand tensile stresses. Materials like crushed rock or water-bound macadam have little or no strength in tension. The little tensile strength that they may have is because of the interlocking and the intergranular friction. Hence the linear or nonlinear elastic analyses used by different researchers require modification to account for the limited tensile strength of granular layers of flexible pavements.

Some of the design procedures $(\underline{4-6})$ assign definite values of elastic moduli to the granular bases depending on the thickness of the layers, and the pavement is analyzed as an elastic-layered system. The maximum tensile strain in asphalt concrete and the vertical stress or strain on the subgrade are evaluated in order to ensure the safety of the pavement from fatigue and rutting, respectively.

In this paper the development of an analytical method for estimating stresses in full-depth granular pavements is described that takes into account the limited tensile strength of granular materials.

DEVELOPMENT OF COMPUTER PROGRAM

A finite-element technique has been adopted to develop computer programs for calculation of stresses in granular pavements. A program designated EPAVE

was first developed for elastic analysis of a layered pavement. This formed the basis for development of computer programs that considered the no-tension nature of the granular layers. EPAVE was subsequently modified to account for the low tensile strength of granular materials and the modified program was termed NPAVE. When the results of NPAVE were compared with experimental values for full-depth granular pavements the NPAVE program was further refined, and the new program for the full-depth granular pavements was named MPAVE.

EPAVE Program

For developing this basic program, a layered system is idealized as an axisymmetric solid with finite boundaries in both radial and vertical directions. Nodal points on the vertical boundaries at the center line and at a distance of 12 radii from the center have been constrained from radial movement and those on the bottom boundary were not allowed to move in either the vertical or the horizontal direction. The bottom boundary has been fixed at a depth of 18 radii in the case of elastic half-space analysis and at a depth of 50 radii for layered pavements as suggested by Duncan et al. (7). The axisymmetry body has been divided into a set of 360 triangular ring elements. Meshes are closer near the axis of symmetry, and they have been gradually widened toward the boundaries as shown in Figure 1. The displacements within an element are represented by linear polynomials and one-point integration has been used for finding the stiffness of each element. The global stiffness of the system is banded and symmetric and the nodal displacements are obtained by a modified Gaussian elimination technique to suit the storage of the stiffness in half-band form. analysis has been carried out for a single wheel load of 40 131 N (9,000 lb) distributed over a circular area with a tire pressure of 0.55 MPa (80 psi) by using a consistent load vector. The stresses calculated by this program agree well with those obtained from elastic half-space analysis (8) as shown in Figure 2 and indicate the validity of the formulation. Poisson's ratio has been taken as 0.35 for all the layers.

NPAVE Program

The elastic analysis invariably exhibits tensile stresses in unbound granular layers, which they cannot resist. These tensile stresses are eliminated by adopting the following steps based on the principle of stress transfer developed by Zienkiewicz et al. (9):

- 1. Elastic analysis is carried out.
- 2. Principal stresses $(\sigma_1, \sigma_2, \sigma_3)$ at the centroid of each element in granular layers are calculated by using the formulas given as follows:

$$\sigma_1, \sigma_2 = [(\sigma_z + \sigma_r)/2] \pm \{ [(\sigma_z - \sigma_r)/2]^2 + \tau_{rz}^2 \}^{\frac{1}{2}}$$
 (1)

$$\sigma_3 = \sigma_\theta \tag{2}$$

where σ_1 , σ_2 , and σ_3 are the principal stresses at the centroid of each element and σ_z , σ_r , σ_θ , and τ_{rz} are, respectively, vertical, radial, tangential, and shear stresses at the centroid of each element.

- 3. The angles of inclination (θ) of principal planes are determined.
- 4. Principal tensile and compressive stresses are identified.

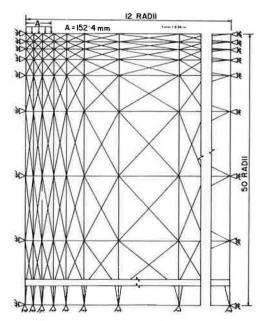


FIGURE 1 Finite-element idealization of pavement.

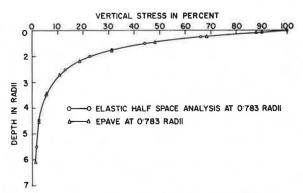


FIGURE 2 Comparison of vertical stress at 0.783 times radius of loaded region from EPAVE and elastic half-space analysis.

- 5. A nominal stress of 0.0055 MPa (0.80 psi) equal to 1 percent of the applied surface pressure is taken as the limiting tensile strength of granular materials.
- 6. If the principal tensile stresses are greater than the limiting value, stresses (σ_z , σ_r , τ_{rz} , σ_θ) necessary on the elements to cause only principal tensile stresses are calculated from the following:

$$\sigma_{z} = [(\sigma_{1} + \sigma_{2})/2] + [(\sigma_{1} - \sigma_{2})/2] \cos 2\theta$$
 (3)

$$\sigma_{\rm r} = [(\sigma_1 + \sigma_2)/2] - [(\sigma_1 - \sigma_2)/2] \cos 2\theta$$
 (4)

$$a_{\theta} = a_{3}$$
 (5)

$$\tau_{rz} = [(\sigma_1 - \sigma_2)/2] \sin 2\theta \tag{6}$$

7. The nodal forces $\{q^{\bf e}\}$ required to develop stresses $\sigma_{\bf Z}$, $\sigma_{\bf r}$, σ_{θ} , and $\tau_{\bf rZ}$ calculated in step 6 are found by using the following equation:

$$\{q^{e}\} = \int [B]^{T} \{\sigma\} d \text{ (vol)}$$
(7)

where

- [B]^T = transpose of strain matrix of an element,
- $\{\sigma\}$ = stress vector at the centroid of the element, and
- vol = volume of the element.
- 8. The nodal forces obtained in step 7 are assembled to obtain the global load vector.
- 9. The stresses σ_Z , σ_r , τ_{rz} , and σ_θ necessary on the elements to develop only principal compressive stresses are calculated.
- 10. Once again the elastic analysis is carried out with the load vector calculated in step 8.
- 11. The stresses obtained in step 10 are added to the stress found in step 9 and strains obtained in step 10 are added to those of the previous elastic analysis.
- 12. Steps 2 to 11 are repeated until the tensile stresses in the granular layer become equal to or less than 0.0055 MPa.

It is observed that for $E_2/E_3=2$ and $H_2/A=5$, convergence occurred within four cycles, whereas for $E_2/E_3=20$, $H_2/A=2$, six cycles were required for convergence. The larger the tensile stresses computed by the elastic analysis, the greater is the number of iterations for convergence.

NPAVE RESULTS

Full-depth granular pavements have been analyzed for thicknesses ranging from two to five times the radius of the loaded area and modular ratios ranging from 2 to 20. The vertical stresses on the top of the subgrade estimated by the EPAVE and NPAVE programs are given in Table 1. It is seen that the notension analysis (NPAVE) gives higher vertical stresses than the corresponding elastic analysis (EPAVE). In order to check the validity of the analysis, the measured values of stresses by McMahon and Yoder (10), Khanna and Mathur (11), and De (12) have been compared with those predicted by the programs EPAVE and NPAVE and are shown in Tables 2, 3, and 4. To compare the results of McMahon and Yoder (10),

the modular ratio has been computed by using the equation given by Edwards and Valkering (13) because these bases were made up of crushed limestone well compacted by a gasoline-powered vibrator. The modular-ratio equation is given as follows:

$$E_2/E_3 = 0.58H_2^{0.45}$$
 (8)

where $\rm E_2$ and $\rm E_3$ are the elastic moduli of the granular base and the subgrade, respectively, and $\rm H_2$ is the thickness of the granular layer in millimeters.

The formula for computation of the modular ratio given by Dormon and Metcalf (14) has been adopted for comparing experimental results of Khanna and Mathur (11) and De (12), who used open-graded granular materials during their experiments. In the model tests reported by Khanna and Mathur, 25-mm (1-in.) aggregates were evenly spread to a loose depth of 100 mm (4 in.) in a tank 1.22 x 1.22 m (4 x 4 ft), and the layer was compacted dry. Subsequently 12.5mm (0.49-in.) aggregate and moorum with 75 percent sand were added in stages and compacted first in the dry condition and then after the surface had been wet with water. The compacted depth was 75 mm (3 in.). Various layers were constructed in multiples of 75 mm. The granular layers in De's model experiments consisted of aggregates that passed through the following sieve sizes: 19, 12.5, 9.5, 4.76, and 2.36 mm (3/4, 1/2, 1/8 in. and No. 4 and No. 8). The percent passing was 100, 93, 88, 50, and 0, respectively. The modular-ratio formula is taken as follows:

$$E_2/E_3 = 0.2H_2^{0.45}$$
 (9)

The validity of these approaches will be examined later in this paper.

Although the stresses obtained by the NPAVE program are higher than those computed by the EPAVE program, they are still significantly lower than the measured values. Hence the NPAVE program requires further modification to take into account the load-spreading behavior of granular materials.

From the pressure distribution diagrams obtained

TABLE 1 Vertical Stresses on Top of Subgrade

Slab No.		Vertical Stresses (% compression)							
	Thickness of Base Course (1adii) (H ₂ /A)	$E_2/E_3 = 2$		$E_2/E_3 = 10$		$E_2/E_3 = 20$			
		EPAVE	NPAVE	EPAVE	NPAVE	EPAVE	NPAVE		
1	2,0	16.75	19.00	10.50	13.75	5.50	9.00		
2	3.0	9.75	11.00	5.50	6.25	2.50	3.25		
3	5.0	3.00	3.25	2.25	2.50	1.25	1.25		

Note: A = radius of loaded area = 152.4 mm (6 in.); H_2/A = thickness of granular layer in radii; E_2/E_3 = ratio of the moduli of the granular layer and the subgrade.

TABLE 2 Comparison of Measured and Predicted Values of Vertical Stresses on Top of Subgrade: Results of McMahon and Yoder (10)

Slab No.	Di . Di			Vertical Stresses			
	Plate Diameter and Base Thick- ness (mm)	H ₂ /A	E_2/E_3 (0.58 $H_2^{0.45}$)	McMahon and Yoder (10)	NPAVE	EPAVE	Elastic Half-Space Analysis ^a 60.00 23.74
1	182.6 and 101.6	1.113	4.64	51.0	52.03	44.57	60.00
2	182.6 and 203.2	2.226	6.34	17.0	12.97	11.63	23.74
3	182.6 and 304.8	3,339	7.61	12.5	5.76	5.32	12.60
4	304,8 and 203,2	1.330	6.34	38.0	33.66	28.61	48.80
5	304.8 and 304.8	2.000	7.61	26.0	14.86	12.87	28.45
6	457.2 and 304.8	1.330	7.61	38.0	31.59	26.01	48.80

Note: 1 mm = 0.04 in.

 $a_{E_2/E_3} = 1$.

TABLE 3 Comparison of Measured and Predicted Values of Vertical Stresses on Top of Subgrade: Results of Khanna and Mathur (11)

Slab No.	n n.		-	Vertical Stresses (% compression)				
	Plate Diameter and Base Thick- ness (mm)	H ₂ /A	E_2/E_3 (0.2 $H_2^{0.45}$)	Khanna and Mathur (11)	NPAVE	EPAVE	Elastic Half-Space Analysis ^a	
1	152.4 and 152.4	2.00	2,000	38.00	20.37	13.75	28.45	
2	152.4 and 228.6	3.00	2.305	20.00	9.22	8.32	14.62	

Note: 1 mm = 0.04 in.

 $aE_2/E_3 = 1$.

TABLE 4 Comparison of Measured and Predicted Values of Vertical Stresses on Top of Subgrade: Results of De (12)

Slab No.				Vertical Stre	Vertical Stresses (% compression)			
	Plate Diameter and Base Thick- ness (mm)	H ₂ /A	E_2/E_3 (0.2 $H_2^{0.45}$)	De (12)	NPAVE	EPAVE	Elastic Half-Space Analysis ^a 32.63 28.45 14.62	
1	330.2 and 304.8	1.846	2.623	28.80	23.24	20.28	32.63	
2	228.6 and 228.6	2.000	2,305	34.00	20.37	17.72	28.45	
3	152,4 and 228.6	3.000	2,305	20.00	9.22	8.32	14.62	

Note: 1 mm = 0.04 in.

 $a_{\rm E_2/E_3} = 1$.

by Herner (15), Khanna and Mathur (11), and De (12), it is observed that the stress distribution in granular layers is confined to certain zones only. In the usual finite-element idealization all the elements up to 12 radii or more in the radial direction are considered for load transfer to the lower layers. It is seen from Figure 3 that the vertical pressure distribution on the subgrade due to the applied loads on the surface of the granular layers is confined within a zone formed by 45-degree lines, and hence the elements outside the zone do not participate in the stress distribution. They have been assigned a modular ratio one-tenth of their values in the NPAVE program. This makes the boundary of the zone nearly free and it simulates the stress distribution condition illustrated in Figure 3. This method of stiffness reduction is adopted so that the

same NPAVE program can be used with a little modification in this case. The modified NPAVE program is termed MPAVE.

MPAVE RESULTS

Figures 4, 5, and 6 give the results of some full-depth granular pavements for both the EPAVE and the MPAVE programs. Because the vertical stress on the subgrade is a major factor to be considered in the design of such pavements, the values of the vertical stresses on the top of the subgrade are shown in Figure 7 for various modular ratios and pavement thicknesses for the EPAVE, NPAVE, and MPAVE programs. It may be seen that when $\rm H_2/A$ is equal to or greater than 5, all the analyses give the same results.

COMPARISON OF MPAVE STRESSES WITH MEASURED VALUES

The stresses measured by McMahon and Yoder $(\underline{10})$ were compared with those estimated by the MPAVE program and are given in Table 5. For the comparison, the modular ratio given by Equation 8 has been used.

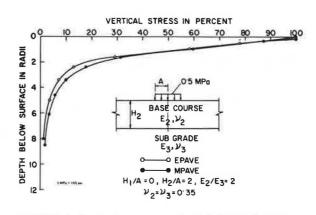


FIGURE 4 Vertical stress versus depth below surface for $H_2/A=2,\,E_2/E_3=2.$

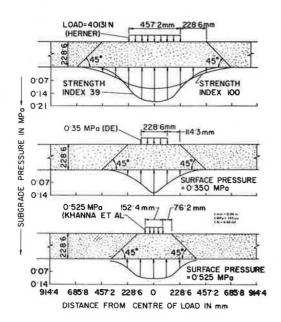


FIGURE 3 Stress distribution in granular bases.

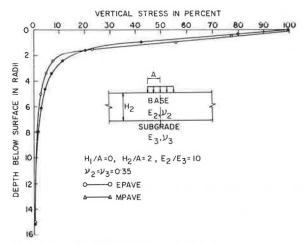


FIGURE 5 Vertical stress versus depth below surface for $H_2/A = 2$, $E_2/E_3 = 10$.

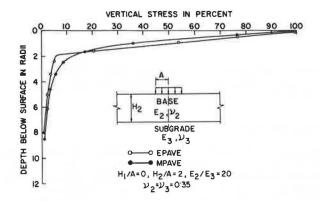


FIGURE 6 Vertical stress versus depth below surface for $\rm H_2/A=2,\,E_2/E_3=20.$

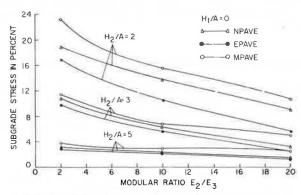


FIGURE 7 Subgrade stresses for granular pavements.

Table 5 shows clearly how the stresses on the subgrade increase as the program is improved by the consideration of the particulate nature of the base courses. The stresses predicted by MPAVE agree reasonably well with those measured by McMahon and Yoder (10) for rows 1, 2, 4, and 6, but the results in rows 3 and 5 deviate considerably from the measured values. However, it may be seen that McMahon and Yoder's results in rows 3 and 5 are close to the elastic half-space solution ($E_2/E_3 = 1$) given for those rows in the last column of Table 5. This indicates that the granular material was not effective in reducing vertical stress on the subgrade, probably because of lack of proper grading or poor compaction or both.

Stresses measured by Khanna and Mathur (11) and by De (12) are compared with those predicted by MPAVE and elastic half-space analysis. The comparison is shown in Tables 6 and 7. For these cases Equation 9 is used for modular-ratio calculation. Measured values by Khanna and Mathur (11) are considerably higher than those predicted by the elastic half-space solution (E_2/E_3 = 1), which indicates that the base course was of poor quality. The stress predicted by MPAVE nearly agrees with that measured by De in row 1, but other results deviate considerably from the measured values. However, it may be seen that the measured values in rows 2 and 3 are much higher than those predicted by the elastic half-space solution. This indicates that the granular layers had no strengthening effect because of lack of grading or improper compaction or both.

It is thus generally found that the vertical stresses on the top of the subgrade predicted by the MPAVE program are reasonably close to the experimental values for the base courses that are well graded and properly compacted.

It may be noted that the computation has been done with the assumption that the load is uniformly distributed over a circle, whereas experimental results were obtained from rigid-plate tests. But the numerous tests by Herner (15) on granular bases of various thicknesses indicated that the vertical pressure transmitted to the subgrades from the loads applied on the bases by rigid plates or by pneumatic tires are nearly the same, except for the tests on thin bases. In such cases the stress distribution curves from rigid-plate loading are a little flatter than those obtained from pneumatic-tire loading.

The measured values of vertical subgrade stress for the thinnest base layer in slab 1 of Table 5 are lower than the MPAVE or NPAVE values. On the basis of Herner's test results (15) discussed earlier, the experimental results of McMahon and Yoder (10) for the thin layer would have been still closer to those predicted by the MPAVE program if the load had been applied by pneumatic tires. The comparison of the theoretical values with experimental results substantiates the validity of the theory within reasonable engineering accuracy in spite of unavoidable experimental errors in pressure measurement in the

TABLE 5 Comparison of Experimental Results of McMahon and Yoder (10) with Computed Results

Slab No,	m / p:			Vertical Stresses	Vertical Stresses (% compression)			
	Plate Diameter and Base Thick- ness (mm)	H_2/A	E_2/E_3 (0.58 $H_2^{0.45}$)	McMahon and Yoder (10)	EPAVE	NPAVE	MPAVE	Elastic Half-Space Analysis
1	182.6 and 101.6	1.113	4.64	51.00	44.57	52.03	55.42	60.00
2	182,6 and 203,2	2.226	6.34	17.00	11.63	12.97	15.22	23.74
3	182.6 and 304.8	3.339	7.61	12.50	5.32	5.76	7.04	12.60
4	304.8 and 203.2	1.330	6.34	38.00	28.61	33.66	37.28	48.80
5	304.8 and 304.8	2,000	7.61	26.00	12.87	14.86	17.18	28.45
6	457,2 and 304,8	1.330	7.61	38.00	26.01	31.59	34.94	48.80

Note: 1 mm = 0.04 in.

subgrade. McMahon and Yoder $(\underline{10})$ have found the experimental errors to be within 5 percent of the actual values in clay soil.

VALIDITY OF DIFFERENT PROCEDURES

The measured values of stresses obtained by McMahon and Yoder (10) for well-compacted granular bases have been compared with those calculated by MPAVE by using modular ratios given by Equations 8 and 9; they are given in Table 8. It may be seen from columns 4, 5, and 6 of Table 8 that the vertical stresses on the subgrade by using the modular-ratio formula of Edwards and Valkering (13) are generally closer to the measured values than those computed by using Equation 9. Modular-ratio values given by others (5,16) generally give vertical stress values intermediate between those in columns 5 and 6. As mentioned earlier, the experimental values of Mc-Mahon and Yoder (10) in rows 3 and 5 are close to elastic half-space analysis, and this indicates that the bases were poorly constructed. The stresses computed with the modular ratio given by Dormon and Metcalf (14) for bases in rows 3 and 5 are closer to the measured values, whereas those computed by using the modular ratios given by Edwards and Valkering (13) are much lower. Hence the modular ratios given by Dormon and Metcalf (14) are suitable for poorly graded base courses like the water-bound macadam so commonly used in India. If a good computing facility is not available, elastic half-space analysis for full-depth granular pavements similar to water-bound macadam may be carried out to obtain a reasonable estimate of the vertical stress on the top of the subgrade.

CONCLUSION

The stresses and strains predicted by the EPAVE program agree with those obtained by elastic half-space analysis. The values of the vertical stresses on the top of the subgrade predicted by the NPAVE program are higher than those predicted by the EPAVE program. The maximum increase in vertical stress on the top of the subgrade is 63.6 percent for a pavement with $\rm H_2/A = 2.0$ and $\rm E_2/E_3 = 20$. The difference in the vertical stress estimated by the programs decreases with greater base-course thickness, and little difference is observed if the thickness of the granular layer is equal to or greater than five times the radius of the loaded area.

Measured vertical stresses on the top of the subgrade agree well with those predicted by MPAVE except for the cases in which the measured values are nearer or higher than those predicted by elastic half-space analysis. The measured values closer to or higher than the Boussinesq solution indicate that the grading or compaction or both of base courses are poor and there is no significant strengthening effect of the granular layer.

The modular-ratio equation given by Edwards and Valkering $(\underline{13})$ is suitable for well-graded and properly compacted base courses. The values given by Dormon and Metcalf $(\underline{14})$ appear to be more appropriate for poorly graded granular layers.

In the absence of a good computing facility elastic half-space analysis may be used for the computation of vertical stresses on the top of the subgrade for full-depth granular pavements in which granular materials are open graded.

TABLE 6 Comparison of Experimental Results by Khanna and Mathur (11) with Predicted Values

Slab No.	DI - DI	H_2/A		Vertical Stres	ses (% com	pression)
	Plate Diameter and Base Thick- ness (mm)		E_2/E_3 (0.2 $H_2^{0.45}$)	Khanna and Mathur (11)	MPAVE	Elastic Half- Space Analysis
1	152.4 and 152.4	2.00	2,000	38.00	22.87	28.45
2	152.4 and 228.6	3.00	2.305	20.00	11.12	14.62

Note: 1 mm = 0.04 in

TABLE 7 Comparison of Experimental Results by De (12) with Predicted Values

Slab No.				Vertical Stresses (% compression)			
	Plate Diameter and Base Thick- ness (mm)	H_2/A	E_2/E_3 (0.2 $H_2^{0.45}$)	De (12)		Elastic Half- Space Analysis	
1	330,2 and 304,8	1.846	2.623	28.80	25.47	32.63	
2	228.6 and 228.6	2,000	2.305	34.00	22.52	28.45	
3	152.4 and 228.6	3.000	2,305	20.00	11.12	14.62	

Note: 1 mm = 0.04 in.

TABLE 8 Measured and Computed Vertical Stresses by Using Modular Ratios Calculated by Different Equations

Slab No.	Di de Di		Vertical Stresses	s (% compression))	
	Plate Diameter and Base Thick- ness (mm)	H ₂ /A	McMahon and Yoder (10)	Dorman and Metcalf (14)	Edwards and Valkering (13)	Elastic Half- Space Analysis
1	182,6 and 101,6	1.113	51.00	63,41	55.42	60.00
2	182,6 and 203,2	2.226	17.00	18.80	15.12	23.74
3	182.6 and 304.8	3.339	12.50	9.08	7.04	12.60
4	304.8 and 203.8	1,330	38.00	46.06	37.28	48.80
5	304.8 and 304.8	2.000	26.00	22,16	17.18	28.45
6	457.2 and 304.8	1.330	38.00	45.05	34.94	48.80

Note: 1 mm = 0.04 in.

REFERENCES

- D.L. DeJong, M.G.F. Poutz, and A.R. Korswagon. Computer Program BISAR: Layered Systems under Normal and Tangential Surface Loads. External Report AMSR, 006.73. Koniklijke/Shell Laboratorium, Amsterdam, Netherlands, 1973.
- J. Michelow. Analysis of Stresses and Displacements in an n-layered Elastic System under a Load Uniformly Distributed on a Circular Area. California Research Corp., Richmond, Calif., Sept. 1963.
- G. Ahlborn. ELSYM5 Computer Program for Determining Stresses and Deformation in a Five Layer Elastic System. University of California, Berkley, 1972.
- A.I.M. Claessen, J.M. Edwards, P. Sommer, and P. Uge. Asphalt Pavement Design: The Shell Method. Proc., 4th International Conference on the Structural Design of Asphalt Pavements, Vol. 1, University of Michigan, Ann Arbor, 1977, pp. 39-74.
- 5. W.R. Barker, W.N. Brabston, and Y.T. Chou. A General System for the Structural Design of Flexible Pavements. Proc., 4th International Conference on the Structural Design of Asphalt Pavements, Vol. 1, University of Michigan, Ann Arbor, 1977, pp. 209-248.
- R.C. Deen, H.G. Southgate, and J.H. Havens. Structural Analysis of Bituminous Concrete Pavements. Research Report 305. Division of Research, Kentucky Department of Highways, Frankfort, May 1971.
- J.M. Duncan, C.L. Monismith, and E.L. Wilson. Finite Element Analysis of Pavements. <u>In</u> Highway Research Record 228, HRB, National Research Council, Washington, D.C., 1968, pp. 18-33.
- R.G. Ahlvin and H.H. Ulery. Tabulated Values for Determining the Complete Pattern of Stresses, Strains and Deflections Beneath

- Uniform Circular Load on a Homogeneous Half Space. Bull. 342. HRB, National Research Council, Washington, D.C., 1962, pp. 1-13.
- O.C. Zienkiewicz, S. Valliappan, and I.P. King. Stress Analysis of Rock as a "No-Tension" Material. Geotechnique, Vol. 18, 1968, pp. 56-66.
- 10. T.F. McMahon and E.J. Yoder. Design of a Pressure Sensitive Cell and Model Studies of Pressures in a Flexible Pavement Subgrade. HRB Proc., Vol. 39, 1960, pp. 650-682.
- 11. S.K. Khanna and O.P. Mathur. Load Transmission through Base Courses of Different Types and Thicknesses. Road Research Bull. 11. Indian Roads Congress, New Delhi, 1967, pp. 1-30.
- 12. D.C. De. Development of Earth Pressure Cell and Study of Stress Distribution Characteristics of Granular Base Course Materials. Master's thesis. Indian Institute of Technology, Kharagpur, India, 1969.
- J.M. Edwards and C.P. Valkering. Structural Design of Asphalt Pavements. Shell International Petroleum Co., Ltd., London, 1970.
- al Petroleum Co., Ltd., London, 1970.

 14. G.M. Dormon and C.T. Metcalf. Design Curves for Flexible Pavements Based on Layered System Theory. In Highway Research Record 71, HRB, National Research Council, Washington, D.C., 1965, pp. 69-84.
- 15. R.C. Herner. Progress Report on Load Transmission Characteristics of Flexible Paving and Base Courses. HRB Proc., Vol. 31, 1952, pp. 101-120.
- 16. B.E. Smith and M.W. Witczak. Equivalent Granular Base Moduli: Prediction. Transportation Engineering Journal of ASCE, Vol. 107, No. TE6, 1981, pp. 635-652.

Publication of this paper sponsored by Committee on Strength and Deformation Characteristics of Pavement Sections.