

Horizontally Curved I-Girder Bridge Analysis: V-Load Method

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ABSTRACT

The V-load method is a widely used approximate method for the analysis of horizontally curved I-girder highway bridges. Previously, this method had been proven valid only for noncomposite I-girder bridges with radial supports. Thus, a study was made to extend the method to composite I-girder bridges with any general support configuration. V-load-analysis results for noncomposite and composite I-girder bridges under dead load and live load were compared with the corresponding results from three finite-element curved bridge models with different combinations of radial and skewed supports. The dead-load V-load results were extremely accurate. The live-load V-load results were strongly influenced by the lateral distribution factors that were used; specification distribution factors gave acceptable V-load results for exterior girders and conservative results for interior girders. More accurate results were obtained with more realistic factors. It is also shown that the V-load method is not a valid approximation for closed-framed I-girder systems with horizontal lateral wind bracing. Approximate expressions are presented to compute the warping stresses in the girder flanges, an important consideration in curved I-girder bridges.

Horizontally curved composite I-girder bridges are being increasingly used for highway interchanges and river crossings. Curved bridge girders offer several inherent advantages. They are more aesthetically pleasing than a series of straight girders along the chords of a roadway curve, and curved girders allow designers to use longer spans, thus eliminating much of the substructure. Curved bridges may also result in simpler and more uniform construction details because girder spacing and concrete slab overhang are generally constant along the length of the structure. One problem with curved bridge design, however, has been the difficulty in mathematically analyzing curved girders; curvature causes torsional loadings that complicate the stress analysis. Rigorous methods have been available for the structural analysis of curved bridges, but highway engineers generally prefer simplified techniques.

In 1963 a report was published (1) that presented a simplified approximate analysis technique for open-framed curved I-girder bridges [open framing was defined originally in that report as noncomposite I-girders connected by diaphragms (floorbeams) or cross frames (K- or X-bracing) with no horizontal lateral bracing near, or in, the plane of the bottom flanges]. The method was modified and greatly simplified for multigirder systems a few years later (2). Agreement with more complicated analyses was

excellent. This approximate method eventually became known as the V-load method because a large percentage of the torsional load on the girders is approximated by sets of vertical shears known as V-loads. The V-load method has been widely used in consulting engineering offices. According to a 1969 survey the method was used for the design of approximately 75 percent of the curved steel I-girder bridges in the United States (3).

One limitation of the V-load method, however, was that it had only been proven valid for noncomposite curved open-framed systems with the bridge piers on a radial alignment. In reality, curved I-girder bridges have a reinforced concrete slab and often depend on composite action to resist the loads. Also, horizontal lateral bracing is sometimes included near, or in, the plane of the bottom flanges to help resist wind loads (a so-called closed-frame system). Furthermore, many curved bridges have foundations on skewed alignments because of geometric restrictions. Thus, for the V-load method to be general, it must be established that the V-load-analysis results are valid approximations for these cases.

Recent efforts to extend the V-load method to composite curved open-framed bridges (no horizontal lateral bracing) with any general support configuration by comparing the V-load-method results to the results from several finite-element bridge models are discussed. Noncomposite and composite bridges, with combinations of radial and skewed supports, were analyzed with both methods under the correct dead and live loadings. The effect of horizontal lateral bracing was also studied for selected cases.

Another important consideration in the design of curved I-girder bridges is the warping stresses (lateral bending stresses) that develop in the girder flanges. These stresses arise from resistance to the out-of-plane warping of an I-girder cross section that is caused by the applied torsional loads. The approximate calculation of these warping stresses is also presented.

V-LOAD THEORY

Conceptually, the V-load method can be considered as a two-step process. First, the curved structure is straightened out so that the applied vertical loads are assumed to induce only longitudinal girder stresses. Next, external forces are applied to the straight structure such that the resulting internal forces are the same as those that exist in the curved structure subjected to only vertical load. To satisfy static constraints, the applied external forces must be determined such that they result in no net vertical, longitudinal, or transverse forces on the total structure. Thus, in the V-load development, the curvature forces on the equivalent straight structure are treated as externally applied loads.

To illustrate this, the curved bridge system shown in Figure 1 consists of two prismatic girders continuous over one interior support with full-depth cross frames spaced a distance d along Girder 1. The cross frames provide the primary resistance to the

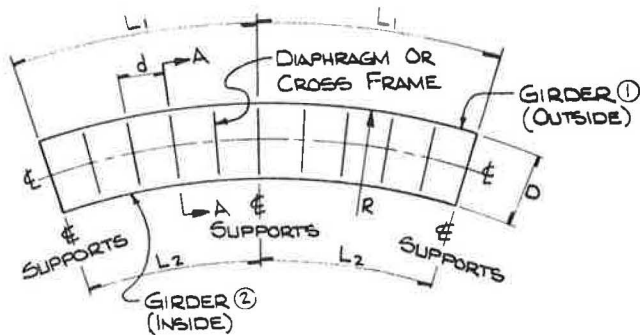


FIGURE 1 Curved bridge—plan view.

torsional loads caused by the bridge curvature. They must, therefore, be designed as primary load-carrying members. Girder 1 has a radius of R, and the distance between the girders is D. In the V-load method, the curved girders are analyzed as isolated straight girders with developed span lengths equal to their respective arc lengths, L_1 and L_2 .

Torsional Load

A plan view of the top flange of one of the girders in the example curved bridge is shown in Figure 2.

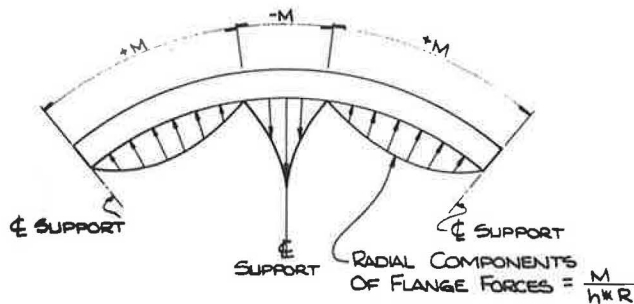


FIGURE 2 Curved top flange—plan view.

When the flanges are assumed to resist the full moment, the axial force in the flange at any point is equal to the bending moment on a transverse section, M, in the girder at that point divided by the depth, h, between the midthickness of the top and bottom flanges. Because of the bridge curvature, these axial forces are not collinear along any given segment of the flange. Thus, to maintain equilibrium, radial components of the internal flange forces are developed. These distributed radial forces have a magnitude of $M/(h \cdot R)$ for small angles. Note that the radial forces are directed outward where the flange is in compression (positive bending), and inward where the flange is in tension (negative bending). The corresponding radial forces in the bottom flange are in the opposite direction. It is these equal opposing forces times the depth, h, that cause twisting of the girders about their longitudinal axes.

Development of V-Loads

Consider a segment of the curved top flange of the outside girder, a distance $d/2$ on either side of a cross frame. To determine the torsional load re-

sisted internally by the girder at the cross frame, the assumption is made (2) that the distributed radial flange force has a constant value of $M/(h \cdot R)$ over the segment, where M is the bending moment in the curved girder at the cross frame. The force exerted on the flange by the cross frame is, therefore, equal to $(M \cdot d)/(h \cdot R)$ for small angles. Equal and opposite reaction forces are developed in each flange of each curved girder in the system (H_1 and H_2) at every cross frame (it is assumed that no internal reactions are developed in the web). H_1 and H_2 are shown in Figure 3 acting on the flanges of each girder on a free-body diagram of the cross frame at Section A-A of the example curved bridge. These forces create counterclockwise torsional couples at each end of the cross frame and corresponding clockwise resisting couples in each girder.

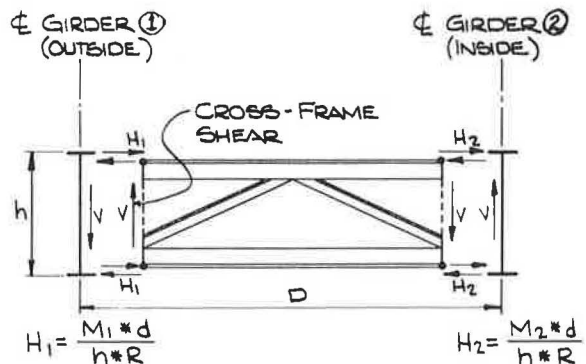


FIGURE 3 Section A-A of curved bridge.

To equilibrate the torsional couples on the cross frame, vertical shear forces, V, develop at each end of the cross frame as a result of cross-frame rigidity and end fixity. These shear forces then react on the girders resulting in a set of self-equilibrating girder shears. The net effect of the shears is to shift the total load on the curved bridge toward the outside girder. These girder shears, which are applied as the external loads to the equivalent straight structure to account for the curvature, are known as the V-loads. Application of the external V-loads ensures that the internal forces in the straight structure will be nearly the same as those that exist in the curved structure under applied vertical loads.

Thus, in a V-load analysis of a system, the bending moments at the cross frames, M_{1p} and M_{2p} , in each of the isolated developed straight girders, caused by applied vertical loads, are first determined by applying those loads to the straight girders. These vertical bending moments will hereafter be referred to as primary moments. The corresponding V-load moments caused by the V-loads, M_{1v} and M_{2v} , are then determined by applying the V-loads, in the proper directions, to the straight girders at the cross frames. The final moments in the curved girders, M_1 and M_2 , are then obtained by simply summing the respective straight-girder primary and V-load moments. The shears, reactions, and deflections in the curved girders are also determined in the same manner. First, however, a method is needed to calculate the V-loads.

Calculation of V-Loads

In the calculation of V-loads, the distribution of

cross-frame shears across the section becomes important, and the relative stiffnesses of the adjacent girders must be considered. The problem is simplified if it is assumed that (a) all the girders in the section have nearly the same vertical stiffness, (b) the girder shears across the section are self-equilibrating, and (c) the loading on girders outside the longitudinal centerline of the system is increased, and the loading on girders inside the longitudinal centerline is decreased. An important assumption can then be made about the apportionment of the cross-frame shears to the individual girders. The assumption is that the shear on a girder is proportional to the distance of that girder from the longitudinal centerline of the bridge, which, based on the preceding assumptions, implies a linear distribution of girder shears across the section. This is a reasonable approximation because the assumption that all girders have nearly the same vertical stiffness is good for curved bridges under normal highway loadings. However, for curved bridges under exceptional conditions, which result in girders with large stiffness variations, the girder-shear distribution may be nonlinear and the method would not give a valid approximation of actual behavior. For such cases, a more detailed analysis should be made.

The calculation of the V-loads, based on the previous assumptions, will be illustrated with reference to the four-girder system shown in Figure 4. All girders in the cross section are assumed to be equally spaced. It is assumed that the section is subjected to a net torque from the internal radial forces, H, in each of the girder flanges. These forces (not shown) are in the same direction as the forces in Figure 3. It is also assumed that the cross frames are rigid enough that the torques in the individual girders at the section can be summed.

To equilibrate this net torque, the internal cross-frame shears, V_1 , V_2 , and V_3 , are developed. These shears are shown acting at inflection points in the cross frames, which are assumed to be at distances a, b, and c from the respective girders (arbitrarily shown at the cross-frame midspace in Figure 4). Because these shears develop reactions at the adjacent girders, and the shears, V, on the outside and inside girders are assumed to be equal and opposite, $V = V_1 = V_3$. Invoking the assumption that the girder shears are proportional to the distance of the girders from the longitudinal centerline of the bridge, the shears on the two interior girders are equal to $(1/3)V$ acting in opposite directions. This also means that $V_2 = V_1 + (1/3)V = (4/3)V$. The factor 1/3 is a proportionality factor that is based on the geometry of the bridge cross section. Note that the girder shears add up to zero across the section as assumed. The V-loads,

which would be applied to the straight structure, are equivalent to the girder shears in the curved structure and are shown below the section.

Next, moment equilibrium between the inflection points is enforced at the bottom of each girder and results in the following equations:

$$V_1 \cdot a = H_1 \cdot h = (M_1 d)/R \tag{1}$$

$$V_1 \cdot [(D/3) - a] + V_2 \cdot b = (M_2 d)/R \tag{2}$$

$$V_2 \cdot [(D/3) - b] + V_3 \cdot c = (M_3 d)/R \tag{3}$$

$$V_3 \cdot [(D/3) - c] = (M_4 d)/R \tag{4}$$

where M_1 , M_2 , M_3 , and M_4 are the final moments in the respective curved girders at the cross frames. Substituting $V_1 = V_3 = V$ and $V_2 = (4/3)V$, and solving the four equations simultaneously for V gives

$$V = (M_1 + M_2 + M_3 + M_4) / [(10 \cdot R \cdot D) / (9 \cdot d)] \tag{5}$$

Because the V-load moments are generally small compared to the primary moments, the simplifying assumption is made that

$$M_1 + M_2 + M_3 + M_4 = M_{1p} + M_{2p} + M_{3p} + M_{4p} \tag{6}$$

Substituting these terms in Equation 5 gives

$$V = (M_{1p} + M_{2p} + M_{3p} + M_{4p}) / [(10 \cdot R \cdot D) / (9 \cdot d)] \tag{7}$$

or the more general relationship

$$V = \Sigma M_p / (C \cdot K) \tag{8}$$

where ΣM_p is the summation of the primary moments in each girder at a particular cross frame, C is a coefficient that depends on the number of girders in the system, and K is equal to $(R \cdot D) / d$ (R and d are for the outside girder). Coefficient C for various multigirder systems assuming equal girder spacing is

No. of Girders in System	Coefficient C
2	1
3	1
4	10/9
5	5/4
6	7/5
7	14/9
8	12/7
9	15/8
10	165/81

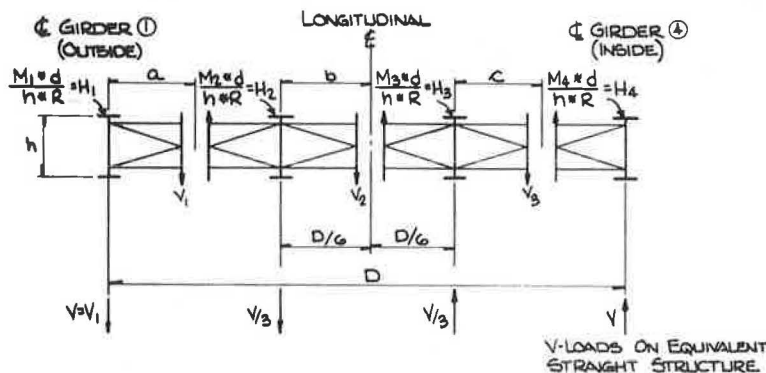


FIGURE 4 Cross section—multigirder curved system.

Equation 8 is, therefore, the general equation for the V-loads on the outside and inside girders of the equivalent straight structure at the cross frames. The proper proportionality factors are applied to these values to obtain the V-loads on the other equivalent straight girders in the section. Special attention must always be given to the direction of the loads.

FINITE-ELEMENT MODELS

Mathematical finite-element models of the preliminary designs of three curved I-girder bridges (non-composite and composite) with different combinations of radial and skewed supports were developed to check the accuracy of the V-load method. Three curved bridge schemes, representative of common highway structures, were chosen for the study. They will hereafter be referred to as

- Scheme A--radial supports,
- Scheme B--parallel skewed supports, and
- Scheme C--two parallel skewed supports and one radial support.

Plan views of the schemes (noncomposite) are shown in Figure 5.

All three bridges are two-span, continuous, four-girder structures with unequal spans and compound radii. All schemes are initially open framed, that is, with no horizontal lateral bracing. The span lengths are generally different among the three schemes and are shown in Figure 5. The radii, R_1 and R_2 , of each girder in each span are the same for all schemes and are also shown. The cross-frame spacings along the outside girder are shown on each bridge. The spacings were chosen to minimize the lateral bending stresses in the flanges caused by warping of the girder cross sections. The skew angle of the supports in Schemes B and C is approximately 41 degrees. All three bridge schemes have the same basic cross section shown at the top of Figure 6. The specified compressive strength of the concrete, f'_c , modular ratio, n , and rebar area, A_s , are shown in Figure 6. A typical cross frame is shown at the bottom of Figure 6. The cross frames are K-braces made up of structural tees (WT5x12.5).

The preliminary designs of the girders were prepared with the bridge-design program SIMON (4). SIMON is a straight-girder design program; therefore, some adjustments had to be made to the program

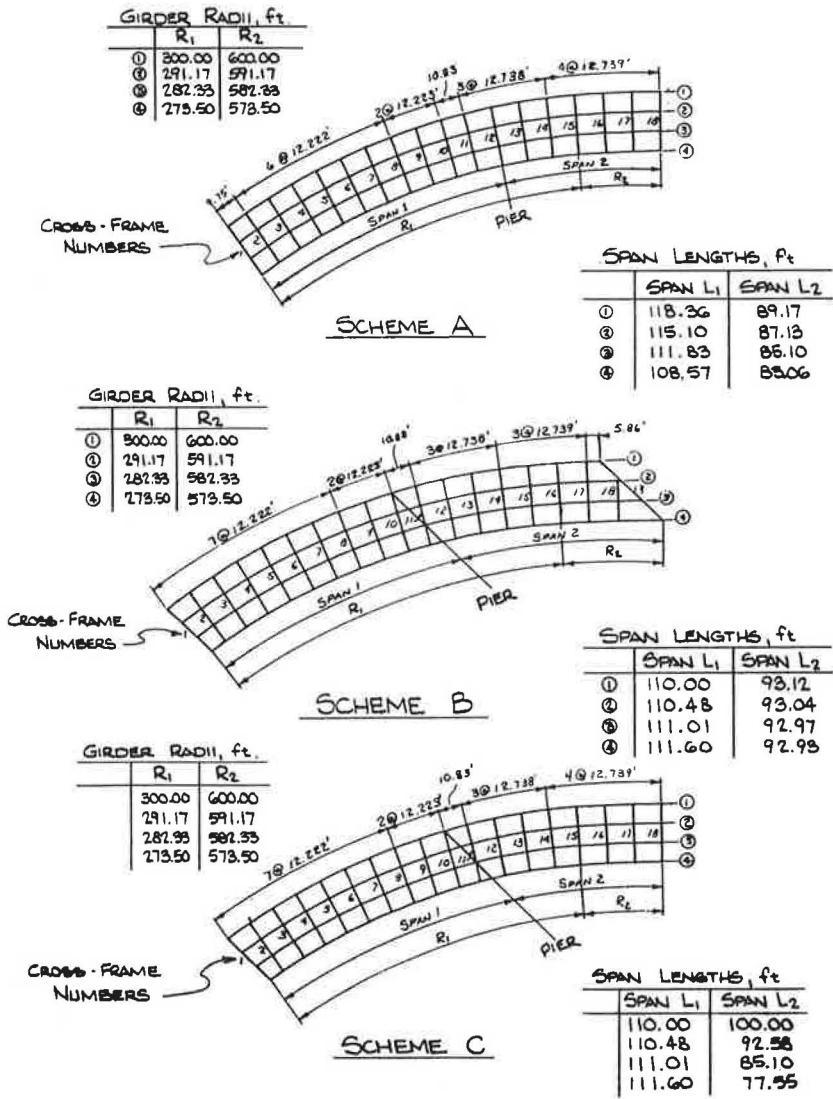


FIGURE 5 Curved-bridge schemes.

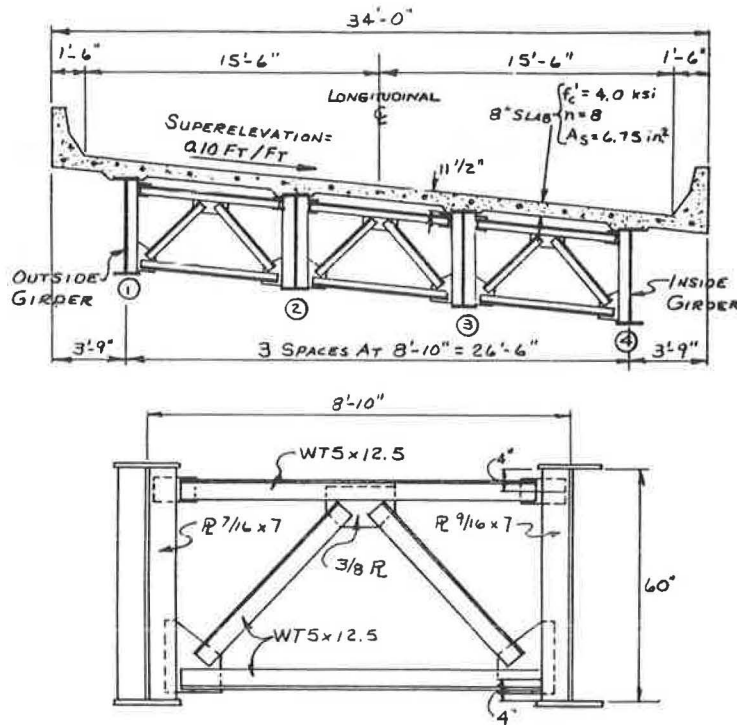


FIGURE 6 Bridge-scheme cross sections.

input to achieve an appropriate design. The girders were designed for noncomposite dead load and composite dead and live loads using 50-ksi yield-point steel and the load factor design method in the AASHTO Standard Specifications for Highway Bridges (5). All girders are nonprismatic with thicker flanges over the interior pier. All webs are 60 in. deep and transversely stiffened. The girder designs for all three schemes are similar--the only difference is minor variations in the flange thicknesses. In regions of negative bending (over the pier), the composite moment of inertia of the girder and rebar only was used in the analysis and design; the concrete was assumed to be ineffective. Computer models of each of the three curved bridge schemes were developed using the finite-element program MSC/NASTRAN (6). Noncomposite models were generated

for all three schemes. Composite models, including concrete slab elements, were generated for Schemes A and C only. The bridges were supported vertically and radially at all three supports, and longitudinally at the interior support only.

To account for cross-sectional warping, a modeling technique was used whereby the two flanges and web are modeled as separate BEAM elements (7). Two groups of three straight flange and web BEAM elements along the arc length of the curve were used for each girder between each cross frame to approximate the curved girders. Because the model is three dimensional, the full-depth cross frames can also be modeled using BEAM elements. This is shown for the noncomposite Scheme A model in Figure 7.

In the composite models, QUAD4 plate elements with bending stiffness through the thickness were

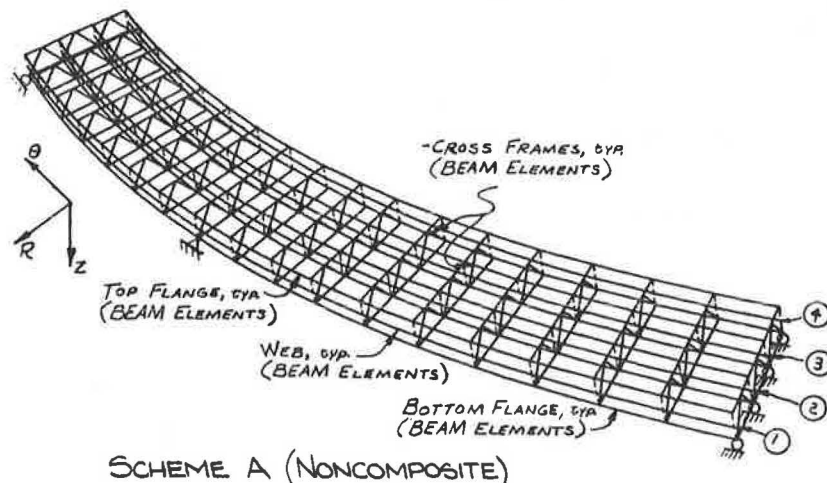


FIGURE 7 MSC/NASTRAN finite-element model.

used to model the concrete slab. The flange and web BEAM elements were offset the proper distances from the slab-element grid points, so each girder actually behaved compositely. For composite dead load, the elastic modulus of the concrete elements was reduced by two-thirds (equivalent to 3n concrete) to account for creep. A Poisson's ratio of 0.15 was assumed for the concrete.

DEAD-LOAD RESULTS

Noncomposite

The noncomposite uniform dead load for all the schemes is 1.210 k-ft on the outside and inside girders, and 1.227 k-ft on the two interior girders. These loads include an assumed weight for the girders and cross frames. The maximum and minimum noncomposite MSC/NASTRAN bending moments, for the four girders in all three noncomposite models, were compared to the corresponding V-load-analysis moments. The results for the girders of Scheme C (the most complicated scheme) are summarized in Table 1. The corresponding primary moments are also presented. The difference between the primary and V-load moments represents the effect of curvature. Note that the curvature generally increases the moments in Girders 1 and 2 and decreases the moments in Girders 3 and 4. All of the V-load noncomposite results for all three schemes were within 7.5 percent of the

MSC/NASTRAN results. The skewed supports had little effect on the accuracy of the V-load results.

Composite

The uniform superimposed dead load on all the composite girders for all the schemes is 0.411 k-ft. The maximum and minimum composite MSC/NASTRAN bending moments, for the four girders in the composite Scheme A and C models, were compared to the corresponding V-load-analysis moments. The results for the girders of Scheme C are summarized in Table 2. The corresponding primary moments are also presented. All of the V-load composite dead-load results for both schemes were within 8 percent of the MSC/NASTRAN results. The composite slab had a negligible effect on the accuracy of the V-load results.

LIVE-LOAD RESULTS

For this particular study, it was not practical to develop and compare the full live-load moment envelopes in the girders calculated from influence surfaces in the curved bridge (using MSC/NASTRAN) and influence lines in the developed straight girders (using the V-load method). Therefore, the live-load bending moments at only three locations were investigated: the negative moments at the pier, and at points near the maximum moment in each span. Influ-

TABLE 1 Noncomposite Dead-Load Bending Moments (Scheme C)

<u>Location</u>	<u>Primary Moment, k-ft</u>	<u>MSC/NASTRAN Moment, k-ft</u>	<u>V-Load-Analysis Moment, k-ft</u>
<u>Girder 1</u>			
Maximum Moment in Span 1	+975.1	+1393.1	+1392.6
Minimum Moment - Pier	-1947.7	-2132.9	-2144.7
Maximum Moment in Span 2	+672.6	+607.7	+617.8
<u>Girder 2</u>			
Maximum Moment in Span 1	+1060.8	+1205.6	+1196.9
Minimum Moment - Pier	-1819.8	-1945.5	-1894.2
Maximum Moment in Span 2	+552.9	+533.6	+536.8
<u>Girder 3</u>			
Maximum Moment in Span 1	+1120.0	+1007.2	+990.1
Minimum Moment - Pier	-1706.6	-1687.0	-1633.4
Maximum Moment in Span 2	+412.9	+415.9	+425.6
<u>Girder 4</u>			
Maximum Moment in Span 1	+1159.3	+770.5	+808.5
Minimum Moment - Pier	-1571.3	-1287.9	-1378.6
Maximum Moment in Span 2	+288.2	+319.6	+302.8

TABLE 2 Composite Dead-Load Bending Moments (Scheme C)

Location	Primary Moment, k-ft	MSC/NASTRAN Moment, k-ft	V-Load-Analysis Moment, k-ft
<u>Girder 1</u>			
Maximum Moment in Span 1	+369.6	+520.5	+536.8
Minimum Moment - Pier	-571.1	-641.0	-655.9
Maximum Moment in Span 2	+271.2	+247.7	+261.4
<u>Girder 2</u>			
Maximum Moment in Span 1	+397.0	+459.3	+451.9
Minimum Moment - Pier	-520.1	-565.1	-549.7
Maximum Moment in Span 2	+222.5	+220.9	+219.7
<u>Girder 3</u>			
Maximum Moment in Span 1	+416.0	+362.8	+362.1
Minimum Moment - Pier	-490.3	-478.7	-460.9
Maximum Moment in Span 2	+172.1	+172.6	+173.1
<u>Girder 4</u>			
Maximum Moment in Span 1	+436.9	+267.0	+282.6
Minimum Moment - Pier	-459.5	-351.0	-380.0
Maximum Moment in Span 2	+127.9	+129.8	+129.6

ence lines, generated by program SIMON for each girder, were used to determine the approximate position and direction of the loads for the minimum (negative) moment at the pier and the maximum (positive) moment in each span. Because the influence surfaces for the curved bridges were not readily available, it was initially assumed that the three loading positions are the same for the curved-bridge and straight-girder models.

The composite MSC/NASTRAN models were loaded with two AASHTO HS 20 trucks at each of the three loading positions determined from SIMON. The two trucks were placed side by side within their design lanes according to the AASHTO rules. For the V-load analysis, AASHTO straight-girder live-load lateral distribution factors were used to distribute the wheel loads to the individual girders. Applying the AASHTO rules for the calculation of these distribution factors, the values for the girders in Schemes A and C are

- Exterior Girders 1 and 4: 1.423 wheels and
- Interior Girders 2 and 3: 1.606 wheels.

These factors are used to determine the original primary live-load moments in the developed straight girders.

A separate lateral distribution factor is needed to compute the V-load live-load moments. In using the AASHTO distribution factors, it is assumed that a truck (or trucks) will eventually be positioned

over each individual girder to cause the maximum possible moment to occur in that girder. However, the V-loads on each girder--for the truck (or trucks) in that loading position--act concurrently; therefore, using the AASHTO distribution factors to compute the V-loads would overestimate the V-loads. Because the V-loads act concurrently, the summation of V-load distribution factors across the section should equal the number of wheels on the structure. Because the lateral placement of the wheels has little effect on the summation of primary moments across a section, a wheel-load lateral distribution factor for V-loads can be simply computed as

$$\text{Distribution Factor}_{V\text{-load}} = (2 \cdot N_L) / N_G \quad (9)$$

where N_L is the number of lanes loaded, and N_G is the number of girders in the section. There is only one V-load distribution factor for all the girders in a particular bridge.

A comparison of the approximate MSC/NASTRAN and V-load maximum and minimum live-load bending moments, for the girders of Scheme A, is given in Table 3. All moments are factored and include impact. These results are typical because in all cases the V-load-analysis live-load moments in the interior girders were anywhere from 20 to 55 percent conservative compared to the MSC/NASTRAN moments. The V-load-analysis moments in the exterior girders were, in most cases, within 15 percent of the MSC/NASTRAN values.

TABLE 3 Live-Load Bending Moments (Scheme A)

Location	Primary Moment, k-ft	MSC/NASTRAN Moment, k-ft	V-Load-Analysis Moment, k-ft
<u>Girder 1</u>			
Maximum Moment in Span 1	+2681.1	+3648.0	+3562.6
Minimum Moment - Pier	-1032.9	-1525.4	-1335.3
Maximum Moment in Span 2	+2035.5	+2541.6	+2400.0
<u>Girder 2</u>			
Maximum Moment in Span 1	+2973.7	+2342.2	+3267.8
Minimum Moment - Pier	-1330.6	-1155.6	-1438.3
Maximum Moment in Span 2	+2222.0	+1787.3	+2367.5
<u>Girder 3</u>			
Maximum Moment in Span 1	+2906.9	+1895.2	+2627.4
Minimum Moment - Pier	-1358.6	-875.3	-1246.3
Maximum Moment in Span 2	+2137.3	+1528.2	+2076.0
<u>Girder 4</u>			
Maximum Moment in Span 1	+2525.4	+1914.5	+1711.2
Minimum Moment - Pier	-1213.8	-944.9	-866.6
Maximum Moment in Span 2	+1833.0	+1680.7	+1572.6

Therefore, in a further study, the V-load live-load analyses for Schemes A and C were redone using different wheel-load lateral distribution factors to compute the primary moments in the exterior and interior girders. These revised factors were simply calculated from the equilibrium of a straight bridge cross section undergoing a rigid-body rotation. This rotation is due to the torsional moment on the bridge section caused by the eccentricity of the wheel loads (with respect to the longitudinal centerline of the bridge) as the trucks are shifted toward the outside girder. These recalculated distribution factors were

- Exterior Girders 1 and 4: 1.611 wheels and
- Interior Girders 2 and 3: 1.204 wheels.

The percentage errors between the MSC/NASTRAN- and V-load-analysis moments for the interior girders, using these factors, were all reduced from a range of 20 to 55 percent to well within 10 percent. The percentage errors for the majority of the moments in the exterior girders were slightly increased, but almost all errors were on the conservative side.

The V-load live-load moments were approximately the same in all the girders using both sets of factors because the distribution factors for the V-loads in either case must always add up to the number of wheels on the structure. The major difference was in the primary moments. Therefore, the V-load-method results for live load are only as good

as the distribution factors that are used to calculate the primary moments. Using the present AASHTO distribution factors, in most cases, gave satisfactory results for the exterior girders and conservative results for the interior girders. More realistic lateral distribution factors have been derived for fatigue design (8), but more research is needed on distribution factors for strength design--particularly for bridges with full-depth cross frames.

CLOSED-FRAMED SYSTEMS

All of the analyses thus far were for open-framed systems, with no horizontal lateral bracing near, or in, the plane of the bottom flanges. Though not presently required (9), bottom lateral bracing (structural tees or angles) is sometimes included in some or all of the bays of curved I-girder bridges to share in the wind-load resistance with the slab. It has also been shown that the addition of this bracing may improve the load distribution to the individual girders in the bridge (10).

To confirm this, lateral-bracing elements were added to the MSC/NASTRAN noncomposite and composite Scheme A models in the plane of the bottom flange elements. Structural tees, WT6x32.5 with a cross-sectional area of 9.54 in.², were selected. In Table 4, the approximate maximum and minimum factored live-load moments for the Scheme A bridge, with lateral bracing in every other bay and in all

TABLE 4 Closed-Framed System Live-Load Bending Moments (Scheme A)

Location	MSC/NASTRAN Moment, No Bracing, k-ft	MSC/NASTRAN Moment, Bracing in Every Other Bay (Outside Bays), k-ft	MSC/NASTRAN Moment, Bracing in All Bays, k-ft
<u>Girder 1</u>			
Maximum Moment in Span 1	+3648.0	+2504.7	+2320.6
Minimum Moment - Pier	-1525.4	-1106.3	-913.2
Maximum Moment in Span 2	+2541.6	+1768.2	+1615.6
<u>Girder 2</u>			
Maximum Moment in Span 1	+2342.2	+2175.7	+1977.4
Minimum Moment - Pier	-1155.6	-1128.0	-959.3
Maximum Moment in Span 2	+1787.3	+1591.5	+1403.4
<u>Girder 3</u>			
Maximum Moment in Span 1	+1895.2	+1834.1	+1622.0
Minimum Moment - Pier	-875.3	-817.2	-818.3
Maximum Moment in Span 2	+1528.2	+1401.5	+1262.4
<u>Girder 4</u>			
Maximum Moment in Span 1	+1914.5	+1553.1	+1579.5
Minimum Moment - Pier	-944.9	-809.0	-746.7
Maximum Moment in Span 2	+1680.7	+1331.2	+1321.7

bays, are compared to the original moments for the bridge with no lateral bracing. In almost all cases, there was a significant decrease of the bending moments.

It is apparent from these results that the bridge behaves as a different structure when lateral bracing is added; the V-load-method assumptions are no longer valid. The composite bridge section, in particular, probably behaves more like a multicellular box-girder. Equations are available to calculate an equivalent plate thickness for common configurations of truss-type lateral bracing (11). The section could then be analyzed as a multicellular box-girder under combined bending and torsion. However, this analysis may be more complicated than a V-load analysis. Also, because the lateral bracing members are now primary load-carrying members, it is important that they be carefully designed to safely carry the loads. The many additional connection details for the bracing must also be carefully designed and fabricated. All of these factors must be weighed against the advantages that might be gained from the improved load distribution (e.g., smaller girders).

TORSIONAL STRESSES

The V-load method assumes that the internal torsional load on the bridge is resisted primarily by the shears that develop in the diaphragms or cross frames. Any remaining torque, however, must be re-

sisted internally by the girders. An open section, such as an I-shaped girder, has two basic kinds of torsional stiffness--St. Venant and warping. Together, St. Venant and warping torsion cause additional bending and shear stresses on the girder sections that must be accounted for. The theory behind the development of these torsional stresses in open sections is well documented (12,13). For I-shaped girders, however, the warping stiffness is usually much greater than the St. Venant stiffness. Thus, in the V-load method, the St. Venant torque and corresponding shear stresses are neglected. It is assumed that all of the applied torque, which is resisted internally by the girders, is resisted in warping torsion only. This was also justified in an earlier development of the V-load method (1).

In a curved bridge, the cross frames reduce the lateral bending, and, consequently, the warping stresses in the girder flanges. Thus, the cross frames act as lateral supports for the flanges. The lateral bending is caused by the radial flange forces due to curvature. As before, a conservative assumption is that the lateral distributed force on each flange has a constant value of $M/(h \cdot R)$, where M is the total vertical bending moment in the girder at each cross frame. Because the cross frames are assumed to act as rigid supports for the flanges, the approximate lateral flange warping moment at each cross frame can be calculated from the expression for the fixed-end moment in a straight beam under a uniformly distributed load:

$$M_{f_w} = (M \cdot d^2) / (12 \cdot h \cdot R) \quad (10)$$

Again, M is the total vertical bending moment (primary plus V -load) in the girder at the cross frame. The flange warping moments are equal and opposite in the top and bottom flanges.

Flange warping moments computed from Equation 10 were compared to the flange warping moments at each cross frame computed from MSC/NASTRAN flange models. Each flange of Girder 1 in Schemes A and C was isolated and analyzed as a straight flange rigidly supported at every cross frame. The flange models were laterally loaded with the appropriate distributed radial flange force for that girder, $M/(h \cdot R)$. The results were generally well within 10 percent of each other.

The maximum warping normal stress at the flange tip can then be computed as

$$\sigma_w = (M_{f_w}) / (S_f) \quad (11)$$

where S_f is the section modulus of the flange about its strong axis. This stress may be significant and should not be ignored; it is usually on the order of 5 to 10 ksi. The maximum warping normal stress is combined with the maximum longitudinal bending stress (in the flange) to determine the maximum total normal flange stress. Approximate warping shear stresses could also be similarly determined but these stresses are generally small and may be neglected.

In composite girders, the top flange and concrete slab act together to resist the top lateral warping moment. This warping moment is approximately the same magnitude as the corresponding moment in the bottom flange; however, the lateral-bending section modulus of the top flange and slab together is so large that the top warping stresses are negligible. Detailed equations are available to calculate the warping stresses in composite sections, if desired (14).

CONCLUSIONS

The V -load method is a widely used approximate method for simply analyzing horizontally curved open-framed highway bridges. The V -load method assumes that most of the internal torsional load on the bridge, due to the curvature, is resisted by self-equilibrating sets of diaphragm or cross-frame shears. The results presented in this paper have shown that the method can be extended to composite open-framed bridges with any general support configuration.

The agreement between the V -load and MSC/NASTRAN results (maximum and minimum girder bending moments) was excellent for dead load on three noncomposite and composite bridge schemes with varying combinations of radial and skewed supports. The accuracy of the V -load live-load results, however, was strongly influenced by the lateral distribution factors that were used to determine the primary moments in each of the developed straight girders due to the design truck wheel loads. The results were generally quite conservative; however, research on lateral distribution factors in the near future should greatly improve their accuracy.

Two limitations on the validity of the V -load method are noted. First, the method is only valid for loads such as normal highway loadings. For exceptional loadings, a more detailed analysis is required. Second, the present V -load method is not applicable to a closed-framed system with horizontal lateral bracing near, or in, the plane of the bottom flanges. Closed-framed systems probably should be analyzed as equivalent boxes.

The warping stresses and flange warping moments, due to lateral bending of the girder flanges, can be calculated with approximate formulas in the V -load method. For composite sections, the warping stresses in the top flanges and concrete slab are comparatively small and may be ignored. St. Venant torsional shear stresses in the girders also may be neglected.

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Prestressed Steel Beam-Concrete Slab Composite Bridge Units for County Road Use

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ABSTRACT

The results of the first three phases of a research project to study the behavior of a prestressed steel beam-concrete slab bridge unit specifically designed for county road use are presented. The units are constructed by steel fabricators in an upside-down position, trucked to the site, and turned over and set in one operation. During fabrication, heavy steel forms are hung from the beams and their weight plus the weight of the concrete provide the prestressing. The advantages of this type of construction are that it requires less structural steel, less reinforcing steel, and less concrete for the deck. In addition, because the units are prefabricated, the labor force and the construction time at the job site are reduced. Data are presented on concrete, reinforcing bar and steel beam strains, and vertical displacements for a 1-year period of observation with the unit under sustained loading. Behavior of the unit when subjected to 500,000 cycles of repeated loading is described. Data from a static 50 percent overload test are also presented. The test unit performed satisfactorily, and county road bridges are currently being constructed in Oklahoma using three or four of the units. An additional 2-year period of sustained load observation followed by static tests to failure is planned.

A research program is being conducted at the Fears Structural Engineering Laboratory, University of Oklahoma, under the sponsorship of the Oklahoma Department of Transportation, to study the strength and stiffness characteristics of a full-scale precast, prestressed steel beam composite bridge unit. Two, three, or four of the units can be used to construct county road bridges for small stream crossings. The test unit is 55 ft long and 6 ft 9.5 in. wide and consists of two W21x50 steel beams and a

7.5-in.-thick reinforced concrete slab and weighs 39,000 lb. The test unit was designed for AASHTO HS 20 loading with a 100,000 cycle fatigue rating.

The objective of the study is to experimentally investigate the behavior of the unit under various types of loading: sustained load, repeated load, and static loading to failure. The research project will take 3.5 years; it began in April 1982 with completion scheduled for December 1985. The project consists of six phases: 1 year of observation under sustained load; 3 months of repeated loading (500,000 cycles); operating rating loading; 2.5 years of observation under sustained dead load; static flexural test to failure; and transverse slab strength tests. As of this date (December 1983), the first three phases of the research project have been completed. Results are summarized herein.

BRIDGE UNIT SPECIMEN

The unit was fabricated by Robberson Steel Company, Oklahoma City, Oklahoma, using design drawings provided by Grossman and Keith Engineering Company, Norman, Oklahoma. The concrete was cast with the beams in an upside-down position using a heavy steel form hung from the beams as shown in Figure 1. When the concrete had been poured and finished, an additional steel weight was placed on the beams. After the concrete had cured, the weight and the forms were removed. The concrete was poured on April 1, 1982, and the forms were removed on April 8, 1982. The unit was moved to the Fears Structural Engineering Laboratory on April 8, 1982, in an upside-down position using a "pole" truck. The unit was turned over on delivery and was placed on small support beams on an existing concrete slab outside the laboratory. Elastomeric bearing pads were placed between the support beams and the unit beams. The bridge unit and the control cylinders were subjected to prevailing weather conditions. Three weeks after the unit was poured, 4 x 8 x 16-in. concrete blocks weighing 33 ± 0.1 lb each were placed on the unit to simulate dead load of 36 psf from an asphalt overlay.

Concrete

An air-entrained concrete with a design strength of