Abridgment

Introduction to Fuzzy Sets in Pavement Evaluation

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ABSTRACT

The theory of fuzzy sets is introduced to assist pavement evaluation. Many of the phenomena that control the performance of highways are not precisely defined, and engineering judgment and subjectiveness are inherent components of performance evaluation. This makes attractive the use of the mathematics of fuzzy sets to divide the performance evaluation into simpler questions and relate verbal statements and subjectiveness to quantitative statements. Techniques are presented to develop a "fuzzy pavement serviceability rating" that contains more information than the conventional pavement serviceability rating and incorporates each rating panel member's perceptiveness of pavements. The notion of a fuzzy binary relationship between roadmeter reading and pavement serviceability reading is introduced. Roadmeter readings can be composed with this relationship to give fuzzy pavement serviceability indices for different pavement sections.

Many of the properties that control the performance of highways are not precisely defined. This uncertainty requires that experience and engineering judgment supplement scientific knowledge in performance evaluation. The combination of objective information and subjective judgment can be performed methodically by the use of fuzzy sets mathematics (1).

Three kinds of uncertainty are encountered in engineering practice: random uncertainty, human-based uncertainty, and system uncertainty. Human and system uncertainty both derive from a lack of precision (or understanding) of an event. This results in the use of linguistic variables such as "economical" or "safe." The fact remains that these imprecisely defined or "fuzzy" classes play an important role in engineering decisions. The theory of fuzzy sets has proved to be an effective tool in handling these types of uncertainty.

A fuzzy set A in the space X is characterized by a membership function $\mu_A(x)$ that associates with each point in X a real number in the interval [0,1]. The value of $\mu_A(x)$ at x represents the "grade of membership" of x in A (i.e., the degree of support or belief that the element x belongs to the subset A). The closer the value of $\mu_A(x)$ is to unity, the higher is the grade of membership of x in A. The membership function is usually written as

$$A = \bigcup_j \mu_A(x_j)/x_j = \mu_A(x_1)/x_1 + \ldots + \mu_A(x_n)/x_n \quad (1)$$

where the plus sign is used in place of "union" as in ordinary set theory, and $\mu_A(x_j)$ is the degree of support for any value x_j.

Many complex problems can be divided into a sequence of simpler questions, which can be answered by experienced engineers using descriptive phrases. The membership functions can be used to methodically quantify such linguistic variables; they can then be manipulated using the axioms of fuzzy set theory to obtain a meaningful answer to the original problem (2). In this paper, these techniques are applied to pavement serviceability.

PAVEMENT SERVICEABILITY

Roughness has the greatest impact on the serviceability of a pavement, and in many states it is used as the initial screening criterion of road sections in determining maintenance needs. The pavement serviceability rating (3) of a particular road section (PSR) is the mean of the ratings of a panel of road users (4,5) in the interval [1,5].

Judgments of the panel members are subjective and involve human instead of random uncertainty. To use fuzzy logic to develop a PSR that reflects the human uncertainty and the relative significance or perceptiveness of the various judgments, the panel is separated into groups of individuals with similar backgrounds, to account for the differences in perceptiveness. Furthermore, to avoid any differences within a group such as experience or age, each group is subdivided into a sufficient number of subgroups. Experienced engineers have a deeper insight into the road condition, so far as maintenance requirements are concerned, and their opinions are weighted more in arriving at the combined opinion of their group.

As an example, let us assume that the panel members have been divided in two groups, A of highway engineers and B of laymen (6). Group A has two subgroups: A_1 of experienced engineers, and A_2 of engineers with little experience. Similarly, the group of laymen in subdivided into B_1 and B_2, frequent and infrequent road users, respectively. Each subgroup's opinions of pavement quality are represented by the following fuzzy sets along with the relevant weights:

$A_1 = 0.6/2.7 + 0.8/2.8 + 1.0/2.9 + 0.8/3.0 \quad w_1 = 0.6$

$A_2 = 0.6/2.8 + 0.8/2.9 + 1.0/3.0 + 0.7/3.1 \quad w_2 = 0.4$

$B_1 = 0.8/2.8 + 1.0/2.9 + 0.7/3.0 + 0.5/3.1 \quad w_1 = 0.7$

$B_2 = 0.9/2.8 + 1.0/2.9 + 0.9/3.0 \quad w_2 = 0.3$

Group opinions are found by Dubois and Prade (7) to be

$$A = \bigcup_i w_i A_i \quad (2)$$

where

$$\sum_i w_i = 1$$

with

$$\mu_A(x) = \max_i [w_i \times \mu_{A_i}(x)] \quad (3)$$

Hence,

$A = 0.36/2.7 + 0.48/2.8 + 0.6/2.9 + 0.48/3.0 + 0.28/3.1$

$B = 0.56/2.8 + 0.7/2.9 + 0.49/3.0 + 0.35/3.1$

Groups A and B can now be assembled according to each group's relative significance or perceptiveness of the influence of roughness. In the previous example, assuming that the relative significance of Groups A and B are $\alpha = 2.0$ and $\beta = 0.5$, respectively, Group A is concentrated (8):

$$A^* = A^\alpha \quad (4)$$

$$= 0.13/2.7 + 0.23/2.8 + 0.36/2.9 + 0.23/3.0 + 0.08/3.1$$

and B is dilated (8):

$$B^* = B^\beta \quad (5)$$

$$= 0.75/2.8 + 0.84/2.9 + 0.70/3.0 + 0.59/3.1$$

The values of w_i, α, and β should be obtained by consulting highway experts. The authors prepared a questionnaire to obtain the factors denoting the relative significance of possible panel groups and also the relative weights assigned to the subgroups. If α and β factors are to represent experts' collective judgment, they themselves could turn out to be fuzzy sets (9).

Final aggregation of the information contained in A* and B* is possible using a number of operations (7); algebraic product is used herein to retain every independent judgment (rating) in the PSR:

$$PSR = A^*.B^* = A^\alpha.B^\beta \quad (6)$$

$$= 0.17/2.8 + 0.30/2.9 + 0.16/3.0 + 0.05/3.1$$

This is the fuzzy PSR for the pavement section under consideration. The PSR of a section originates from subjective judgments that support a region of values rather than a single value. The conventional PSR is a discrete number and thus does not clearly indicate this region of PSR, supported by the members of the panel. On the other hand fuzzy PSR shows this region of support as well as the degree of support for each value. Thus, the fuzzy PSR is an improvement over the conventional one. Further, it incorporates each individual's perceptiveness of pavements while carrying his judgment up to the final stage of the analysis.

MEASUREMENTS WITH THE ROADMETER

The roadmeter reading varies with the path traced by the vehicle, the driver characteristics, the vehicle speed, and the gas tank level. These introduce imprecision, and representing the roadmeter reading by a fuzzy set may be more appropriate (10). A typical representation of a roadmeter reading of 800 is shown in Figure 1. In this figure 800 is the reading obtained for a section and 840-760 is the range of values obtained by repeated measurements on the same pavement section. This curve may either be a straight line or a π curve (11) depending on the experts' subjective judgment.

FORMATION OF THE RELATIONSHIP

Rather than correlating by linear regression analysis, in the fuzzy sets theory the notion of a link between two elements belonging to the same universe or two different universes is expressed by a binary fuzzy relation (7). If A and B are fuzzy sets in two universes X and Y, respectively, the most common formulation of binary fuzzy relation R is done using the cartesian product:

$$R = A \times B \quad (7)$$

with

$$\mu_R(x_i,y_j) = \min[\mu_A(x_i), \mu_B(y_j)] \quad (8)$$

When a set of data is available for correlation, the global (binary) fuzzy relation is formed by the union of fuzzy relations for each pair of data. By

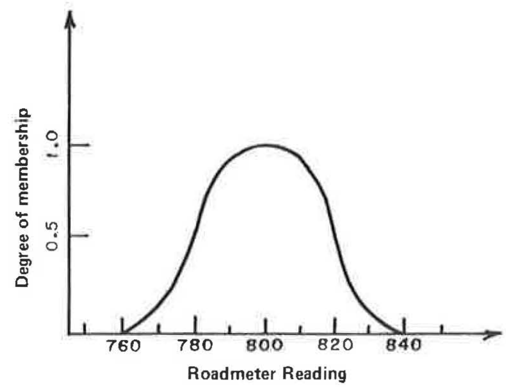

FIGURE 1 Typical fuzzified roadmeter reading.

making the highest membership 1.0 and increasing the other memberships proportionately, the fuzzy PSR in Equation 6 is normalized as

$$PSR = 0.57/2.8 + 1.0/2.9 + 0.53/3.0 + 0.17/3.1$$

and the binary fuzzy relationship between the PSR and the roadmeter reading (RR) of Figure 1 is formed according to Equation 8. As an example, μ_R (3.0, 790) is 0.53, the minimum of μ_{PSR} (3.0) = 0.53 and μ_{RR} (790) = 0.75. Repeating this operation for each set of values results in Table 1. Membership values of a fuzzy relation are analogous to the strength of the link between the corresponding RR and PSR values. For example, RR = 800 and PSR = 2.9 are strongly linked (membership values = 1.0) whereas RR = 830 and PSR = 3.1 are weakly related (membership values = 0.17). Such relationships can be formed for all the sample sections, covering wide ranges of PSR and RR. Aggregation of these data produces the global PSR-RR relationship for the highway network.

TABLE 1 Fuzzy PSR-Roadmeter Reading Relationship

PSR \ RR	770	780	790	800	810	820	830
2.8	0.125	0.50	0.57	0.57	0.57	0.50	0.125
2.9	0.125	0.50	0.875	1.0	0.875	0.50	0.125
3.0	0.125	0.50	0.53	0.53	0.53	0.50	0.125
3.1	0.17	0.17	0.17	0.17	0.17	0.17	0.17

COMPOSITION OF THE FUZZY RELATIONSHIP

If A is a fuzzy set in the universe X and R is a fuzzy relation in the universe X x Y, the fuzzy set B (in the universe Y) induced from A through R is defined as

$$B = A.R \quad (9)$$

with

$$\mu_B(y) = \sup_x \min [\mu_A(x), \mu_R(x,y)] \quad (10)$$

Roadmeter-pavement serviceability rating data can be used to evaluate the pavement serviceability index (PSI) for a particular pavement section, if the roadmeter reading is known. As an example, suppose that the fuzzified form of the roadmeter reading for a different pavement section is given by

$$RR' = 0.6/810 + 1.0/820 + 0.6/830$$

The corresponding fuzzy PSI for this section is obtained according to Equation 10 by composing RR' with the fuzzy relation in Table 1, which is assumed to be the global relationship for simplicity:

$$PSI = 0.57/2.8 + 0.6/2.9 + 0.53/3.0 + 0.17/3.1$$

Decision theory techniques using fuzzy sets will be developed to compare the fuzzy pavement serviceability indices of pavement sections in the state and determine their maintenance priorities.

CONCLUSION

The initial stages of a procedure that enables the methodical manipulation of human-based and system uncertainties inherent in a pavement management system have been outlined. The concept of fuzzy PSR is described in detail with reference to the formation of membership functions and the incorporation of the perceptiveness of every member in the rating panel. This is accomplished by obtaining experts' opinions at different stages, in the form of relative significance factors and relative weights to be attached to the panel members' judgments.

Further, the need to fuzzify the roadmeter reading is emphasized. This facilitates gathering of pavement serviceability rating-roadmeter reading data for sample pavement sections, by means of a fuzzy binary relationship. Pavement serviceability indices for pavement sections can be extracted from this data base if the roadmeter readings are known.

The work discussed here is the preliminary stage of the development of fuzzy sets techniques to help pavement management.

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REFERENCES

1. J.L.A. Chameau, A.G. Altschaeffl, H.L. Michael, and J.T.P. Yao. Potential Applications of Fuzzy Sets in Civil Engineering. Presented at Workshop of North American Fuzzy Information Processing Group, Logan, Utah, May 19-21, 1982.
2. C.B. Brown and J.T.P. Yao. Fuzzy Sets and Structural Engineering. Journal of Structural Engineering Division of ASCE, Vol. 109, No. 5, May 1983.
3. W.N. Carey and P.E. Irick. Pavement Serviceability Performance Concept. Bull. 250. HRB, National Research Council, Washington, D.C., 1960, pp. 40-58.
4. Pavement Management. Transportation Research Record 846. TRB, National Research Council, Washington, D.C., 1982.
5. E.J. Yoder. Development of a System for the Evaluation of Pavements in Indiana. Joint Highway Research Project JHRP-81-18 Final Report. Purdue University, West Lafayette, Ind., 1981.
6. J.L. Chameau and M. Gunaratne. Performance Evaluation in Geotechnical Engineering Using Fuzzy Sets. Proc., ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Berkeley, Calif., Jan. 11-13, 1984, pp. 264- 267.
7. D. Dubois and H. Prade. Fuzzy Sets and Systems: Theory and Applications. Academic Press, New York, 1980.
8. D.G. Elms. Use of Fuzzy Sets in Developing Code Risk Factors. Presented at ASCE Convention, St. Louis, 1981.
9. J.L. Chameau, M. Gunaratne, and A.G. Altschaeffl. Type-2 Fuzzy Sets in Engineering. Presented at First International Conference of the Fuzzy Information Processing Group, Kauai, Hawaii, July 1984.
10. E. Prugovecki. Fuzzy Sets in the Theory of Measurements of Incompatible Observables. Foundations of Physics, Vol. 4, No. 1, 1974.
11. L.A. Zadeh. Calculus of Fuzzy Restrictions. In Fuzzy Sets and Their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975.

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