Application of a Model To Optimize Simultaneous Bus Garage Location and Vehicle Assignment

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ABSTRACT

A demonstration is presented of a new methodology that simultaneously locates and sizes bus garages and reassigns vehicles to trips with respect to the garage locations being considered. The advantage of this methodology over existing methodologies is that existing vehicle assignments are not used. The existing vehicle assignments were originally constructed with the existing garage locations in mind. Therefore, the use of existing vehicle assignments does not permit a realistic evaluation of the cost of locating at a new site. The methodology presented and demonstrated accounts for the interdependence of garage location and vehicle assignments and it gives the analyst a realistic estimate of the costs of locating at a new site. The methodology is demonstrated with a Southeastern Michigan Transportation Authority (SEMTA) case study. The north-eastern third of the the SEMTA transit network is used in the demonstration, and sites in this area are evaluated. At each iteration the methodology reassigns vehicles to trips and these reassignments are illustrated with computer plots.

In previous papers concern was expressed over the deficiencies in the existing methods to locate bus garages (1-6). Specifically, the assignment of vehicles to trips is considered fixed regardless of the garage location being evaluated. Generally, the evaluation of potential locations is conducted by estimating the deadheading cost between the site and existing pull-out and pull-in points (the points where route assignments begin and end). However, once a new bus garage is built, the scheduling department will normally redesign vehicle assignments so that deadheading between the new garage and assignments is minimized. Therefore, the existing assignment does not provide the analyst with an accurate estimate of what the deadheading costs will be if a garage were opened at a new site.

When current methodologies use existing vehicle assignments (blocks), the evaluation generally determines that the existing garage locations or nearby locations are good sites for garages. The reason for this is that when the scheduler designs vehicle assignments, he attempts to minimize deadheading from existing sites. Therefore, if the scheduler is doing a good job, existing locations will always appear to be good locations for garages. This trait has perplexed transit operators who wish to locate a new garage away from an old site but find that existing sites are well located with respect to existing vehicle assignments (7-9).

The solution to this problem that existing vehicle assignments dictate the location of new garages is to regroup trips into vehicle assignments as different combinations of new and existing garage locations are being considered. In this way the assignments are scheduled with respect to the locational characteristics of the new and existing sites considered, and the assignments are not biased by the location of an existing garage.

The purpose of this paper is to present an application of a mathematical program that simultaneously locates garages and reassigns vehicles to trips. The significance of this methodology lies in its ability to more realistically represent the transit system. In the methodology presented here it is recognized that the location of garages and the assignment of vehicles to trips are really interdependent activities. Further, failure to consider their interdependence has been shown to significantly bias the cost analysis of prospective garage sites (2,3). In this paper only a brief description of the mathematics underlying the model is presented. For the interested reader, the mathematics are thoroughly documented elsewhere (2).

METHODOLOGY

The smallest component of the methodology is the vehicle assignment (a block). A block starts with a bus pulling out from a garage and starting a trip on a route. Once the bus reaches the end of the trip, it can either pull back into the garage or hook to another trip. If the bus hooks to another trip, it has the option of pulling into the garage or hooking to another trip on the completion of the second trip. The path the bus takes from its pull-out through its hook and its pull-in is a block.

A vehicle scheduler will attempt to design blocks so that all scheduled trips are assigned a vehicle and deadheading costs between the garage and pull-out and pull-in points and those of hooks (layover and travel costs) are minimized.

In this methodology the assignment of trips to blocks and blocks to garage sites is done with a linear program the objective of which is to minimize the sum of the deadheading costs and facility costs in the combination of garages considered. The deadheading costs are the sum of pull-out travel costs, hook travel and layover costs, and pull-in travel costs. Further, a special auxiliary variable is included in the mathematical program to ensure that the bus that pulls out of one garage is returned to the same garage.

Facility costs of the sites consist of a fixed charge for opening a site and a construction and garage operation cost that increases linearly with the number of buses assigned to the garage. Garage sites are switched off and on to permit the evaluation of different combinations of sites. To switch a site off, its capacity is bounded to zero, and to switch a site on, its capacity is bounded by the maximum number of vehicles that can be assigned to the site. As garages are switched on, the appropriate fixed charge is added to the result of the objective function. Sites are switched off and on through a branch and bounding process until the optimal combination is found. [For a complete discussion of the model, see the report by Maze et al. (2).]
The decision rules used in the branch and bounding process start with all sites in an undecided set, \( k_0 \), where members have not been fixed open or closed [the decision rules are taken from a paper by Khumawala (10)]. Then, through the delta decision rules, sites in \( k_0 \) are tested for candidacy to a set where all members are fixed open (set \( k_1 \)). In the garage problem this entails testing existing garages to determine whether they remain in the optimal option. Next, the omega decision rule determines which of the members remaining in \( k_1 \) can be made members of a set where all members are fixed closed (set \( k_2 \)). In the garage problem, this entails testing candidate sites to determine which can be eliminated. After sites have been tested with both rules, the sites that still remain in \( k_2 \) can be evaluated by enumerating all combinations of members of \( k_2 \) plus all members of \( k_1 \). The two decision rules are as follows:

1. If the least possible increase in total variable costs (the sum of deadhead, variable garage operating, and variable garage construction costs) due to not opening a facility is greater than the fixed charge, the facility should remain open (delta decision rule). The least total variable cost decrease of opening a site can be estimated by solving a linear program including all sites \( (L[\{ k_1 \cup k_2 \}] \) \) (\( k_1 \) may be empty at this point) and then solving a linear program without the facility in question \( (L[\{ k_1 \cup k_2 \} - j]) \).

2. If the largest possible decrease in total variable cost due to opening a facility is less than its fixed charge, the site should remain closed (omega rule). The greatest total variable cost decrease can be estimated by solving a linear program with only those facilities that have been fixed open through the delta rule \( (L[\{ k_1 \}] \) and then adding one site and solving that linear program \( (L[\{ k_1 \} + j]) \).

In both of the foregoing rules, \( L[\{ \}] \) is the linear program including the set of sites inside the brackets, \( k_0 \) is the set of sites fixed closed, \( k_1 \) is the set of sites fixed open, and \( k_2 \) is the set of sites that have not yet been fixed open or closed.

**CASE STUDY**

This hypothetical case study is presented to demonstrate the application of the model. The northeast third of the transit network of the SEMTA (Southeastern Michigan Transportation Authority) [Southeastern Michigan Transportation Authority (SEMTA)] is used in the demonstration. The chosen portion of the network includes routes that are currently serviced from SEMTA’s Macomb garage. Included are nine routes with a combined total of 447 trips.

**System Characterization**

The portion of the SEMTA system considered consists of all fixed routes that fall roughly between Lake St. Clair on the east, downtown Detroit on the south, and Van Dyke Road on the west. These boundaries cover a triangular area including the east side and eastern suburbs of the Detroit metropolitan area. This region does not completely enclose the entire transit system. There are minor exceptions; for example, there is a crosstown route considered that follows 9 Mile Road. The two termini of this route are the box (Figure 1) indicating a route pull-in and pull-out point about a mile north of the 9 Mile Road and Gratiot Avenue and the box on the west side just northwest of the 8 Mile Road and US-10 (Lodge Freeway).

**Route Description**

All the routes considered except the one crosstown route mentioned earlier are radial commuter-oriented routes. Many of these have a broad variety of travel patterns. For example, the bus route that operates on Van Dyke Road has five different patterns. Although all five patterns have common portions of travel (they all travel along Van Dyke Road between 8 Mile Road and 15 Mile Road), some trips on the route have a southern terminus downtown and others have a southern terminus near Jefferson Avenue and 8 Mile Road. Hence, although only nine routes are considered, the coverage of the east side is greater than what might be imagined because of the fragmented nature of the route patterns.

Although some of the routes are located along the western boundary of the triangular region considered, the most densely covered portion is the eastern portion of the east side, in the Gratiot-Jefferson corridor. For example, the Gratiot Avenue route (Route 560) has more than 100 inbound and outbound trips daily. Other routes that lie in the same area include Kercheval-Mack (Routes 610 and 615), Jefferson (Routes 630 and 635), and Harper avenue (Route 563) which have 96, 61, and 15 trips daily, respectively.

The clustering of service in the Gratiot-Jefferson corridor is also reflected by the position of the pull-in and pull-out points shown in Figure 1. The pull-out and pull-in points are clustered along the corridor. There is a total of 25 such points in Figure 1. There is only one in the central business district (CBD) of Detroit, although there are actually several places where routes begin or end in the CBD. Most of these points are at best a few blocks away from one another, but because of the scale of the map, they are represented by one box.

**Site Identification**

Within the study area there exists one SEMTA garage. This is the Macomb garage (site C in Figure 1) and it is located on 15 Mile Road just to the east of Gratiot Avenue. The Macomb garage is considered to have a capacity of 100 buses.

Three other sites are considered potential locations for garages. The identification of the candidate sites is based on the following criteria:

1. The candidate site should be close (e.g., 1 mile or less) to a major arterial or freeway, and
2. The site should not be occupied by a structure that is currently in use and the adjacent area should be of a similar land use (e.g., industrial, warehouse, or light commercial).

Three candidate sites are selected based on these criteria and based on knowledge of the transit network. The model could be expanded to include more sites and an even larger network, but in this study the data are coded by hand for batch entry to the mathematical programming package. Because the model requires two deadheading cost estimates for each trip (one for the pull-out and one for the pull-in) and one deadhead cost estimate for each potential hookmate for each trip from each garage, the coding becomes a rather lengthy task if the number of either trips or potential garage locations is large. For example, in the case study there are 4 garage sites, 447 trips, and 3 potential hookmates for each trip. This results in about 9,000 cost parameters to be coded for entry into the mathematical program. The model can easily be expanded to include more sites and trips if future users wish to apply it to.
FIGURE 1 Detroit area base map.

larger-scale systems. On consideration of the hand coding involved in the case study and on trading off this effort with the loss in realism in proving a small-scale example, a decision was reached to examine only four sites. It is suggested that future users wishing to consider large systems create an automatic coding system for data input.

Site A in Figure 1 is in an area that was cleared for urban renewal and is still largely vacant. Besides a few warehouses, the central maintenance facility for the Detroit Department of Transportation (the transit operator for the city of Detroit), and a small public housing development, the area is largely vacant land. The site has good access to I-75 and is close to I-94, Gratiot Avenue, and downtown Detroit. Site B, located adjacent to I-696, is in an area of predominantly light industry and there exists ample vacant land. Site D is located near the intersection of 14 Mile Road and Van Dyke. There is nearly a quarter section of vacant land that abuts a factory.

Admittedly some of these sites may be unavailable for use as garage locations, and if this study were being conducted for an actual transit operator, investigations would need to be conducted to determine ownership, zoning, political feasibility, and other characteristics of the site. Because the sites are identified only for the purposes of demonstrating the model, this is unnecessary. Once the sites to be considered have been identified, the next step is to estimate the cost parameters based on these sites and based on the portion of the transit network considered.

Model Parameter Generation

The model parameters that need to be generated can be divided into three types: garage operating costs, both variable and fixed; garage construction costs, both variable and fixed; and deadhead costs, which include travel time and distance costs of pull-outs, pull-ins, and hooks and the time cost of layovers. The methodology used to estimate each of these costs is described in the following subsections.

Garage Operating and Construction Costs

Garage operating and construction cost functions were estimated as part of a previous UMTA University Research and Training Grant (2). Both costs are annualized and the functions are shown in the following. The cost functions are modeled after those derived by actual engineering cost studies of bus garages, and the functions are estimated using SEMTA.
cost information (11,12). In the derivation of the cost functions, a 10 percent spare factor is assumed. The annual construction cost function is based on a 25-year design life and a discount rate of 9 percent.

Total annual garage operating cost = $192,412 + $10,340 (number of buses assigned to a garage).

Total annual garage construction cost = $192,412 + $3,190 (number of buses assigned to a garage).

Deadhead Costs

Deadhead costs include the costs of travel from the garage to pull-out points, travel between hookmates, travel from pull-in points to the garage, and the driver's time during layovers. To estimate the deadhead costs for the Detroit case study, the Urban Transportation Planning System (UTPS) computerized highway network of the metropolitan area is used (13). The network contains average speed of travel and the length of highway links. The mileage cost (the cost to operate a bus per mile) and the time cost (the driver's wage per minute) are applied to the Detroit metropolitan area's UTPS highway network. UTPS module UROAD is used to determine the minimum-cost paths and to accumulate the costs from every centroid to every other centroid.

Centroids can be moved or created so that they are located approximately at every pull-out and pull-in point and at every garage location. In this way the costs of traveling to every pull-out point and from every pull-in point to all garage sites and between all potential hookmates can be estimated. The advantage of this methodology is that transportation costs are estimated with a simulation that closely approximates the actual costs between every pair of points of interest within the system. The distance and time costs are derived from the computerized highway network by using actual SEMTA travel costs of 30 cents per mile and average driver wages, including all benefits, of 22 cents per minute. The deadhead cost estimates are annualized for all possible pull-outs and pull-ins and for the three best hookmates for each trip.

Now that all cost parameters have been estimated and the system has been characterized, the model can be applied to the system, which is described next.

Model Application

The first step is to develop a list of sites to be evaluated. The algorithm enters the delta rule phase and existing sites are tested to determine whether they can either be fixed open (see (K1)) or remain undecided (set (K2)). Next the algorithm enters the omega rule phase and all sites remaining in (K2) are tested to either become fixed closed (set (K0)) or remain in set (K2). Sites remaining in (K2) enter a branch and bound phase and the result is an optimal solution.

Delta Decision Rule

The first step in the delta rule is to run a linear program with all garage capacities upper bounded at their maximums. The second step is to set the upper bound of the capacity of one garage equal to zero, rerun the linear program, and calculate the difference in the total variable cost from the first to the second run. If the variable costs increase by a sum greater than the removed site's fixed charge, the site is fixed open. The third step is to iterate back to step two until the list of sites is exhausted.

The running of the problem with all upper bounds set equal to the site's maximum capacity is the only time the problem is to be run with a raw set of data. From that point onward, the basis of the previous run is revised. The initial solution is as follows:

<table>
<thead>
<tr>
<th>Site</th>
<th>Capacity (no. of buses)</th>
<th>Active Buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C (Macomb)</td>
<td>100</td>
<td>72</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Total variable cost = $2,040,783,
Total cost = $2,048,947.

To help interpret the result, Figures 2-5 are used. Figure 2 indicates the relative locations of the four sites. The size of the box indicating each site is proportional to the number of buses assigned to each site. As can be seen, the largest box is located at site C and it is proportional to the 72 buses assigned to that site.

To interpret Figures 3, 4, and 5, it must be remembered that at each iteration the model not only assigns buses to the garages considered at that iteration but also optimally assigns buses to trips. Figure 3 shows the pull-out assignments from each of the sites considered. To understand how to interpret Figure 3, note the location of site A, the southern site, closest to downtown Detroit, and the wide isosceles triangle pointing downward from site A and terminating in downtown Detroit. The box at the tip of the triangle indicates a trip terminus point, as do all the other similar boxes. The base of the triangle is proportional in length to the number of buses that are assigned to pull out from site A for trips starting in downtown Detroit. As can be seen in Figure 3, there are a large number of pull-outs from site C (the site of the Macomb garage). The proportion of activity originating from site C corresponds to the relatively large quantity of buses assigned to this site.

Figure 4 shows the quantity of flow of buses between trip termini. More specifically, the triangles shown are proportional to the hooking activity between trip termini. Note in Figure 4 that the majority of the hooking activity is between the trip terminus along Gratiot Avenue and to the east (see Figure 1 for the location of Gratiot Avenue). The majority of the hooking activity corresponds to the location of the majority of the routes.

Figure 5 defines the pull-in activity. This figure is read in the same fashion as the other figures showing flows of buses.

In the first run of the model, only one bus is assigned to site B because this garage is relatively close, in terms of travel cost, to site C. Therefore, if a bus is to be assigned to site B, there must be a deadhead cost savings, as compared with site C, equal to or greater than the cost of constructing a new space at site B. To demonstrate the trade-offs between the facility costs and the deadhead costs, the optimization model is run with the facility costs at site C increased so that they are equal to the facility costs at a new site (variable garage operating costs plus variable garage construction costs). Site B is assigned 12 buses in stead of only one, whereas the number of buses assigned to the existing garage at site C is decreased from 72 to 42.
FIGURE 2  Relative number of buses assigned to sites.

FIGURE 3  Pull-outs with all garages included.

FIGURE 4  Hooks between routes with all garages included.
The next step in the delta decision rule would be to switch off the existing site and determine whether it should be fixed opened (made a member of \( \mathcal{K}_1 \)). To prove that the existing garage at site C should remain open, another linear program is run with all sites except site C switched on. The linear program with only sites A, B, and D results in assignments of 38, 27, and 25 buses, respectively. The sum of the facility variable costs and deadhead costs in the solution is $2,338,750. The difference between the cost of the combination with sites A, B, and D switched on and the cost with all sites switched on is $297,967. Because this is greater than the fixed charge of $192,412 (because the garage is already built there is no construction cost fixed charge), the delta value is greater than zero and site C will be the optimal solution.

The next step in the process is to eliminate candidate sites from consideration. Candidate sites are eliminated through the omega rule in the following.

**Omega Rule**

The first step in the omega decision rule is to run a linear program with only members of \( \mathcal{K}_1 \) switched. Then one member of the undecided set \( \mathcal{K}_2 \) is switched on at a time. The results of the omega rule are shown in Table 1. Because all candidate sites have omega values less than zero, all are placed in \( \mathcal{K}_0 \). The optimal solution is to keep the garage at site C and eliminate the other sites from further consideration. The flow of pull-outs, hooks, and pull-ins is shown in Figures 6, 7, and 8, respectively.

**TABLE 1 Omega Rule Results**

<table>
<thead>
<tr>
<th>Omega Value</th>
<th>Active Bus Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_A = -176,641 )</td>
<td>( N_A = 16 ) ( N_B = 0 ) ( N_C = 74 ) ( N_D = 0 )</td>
</tr>
<tr>
<td>( \Omega_B = -257,058 )</td>
<td>( N_A = 0 ) ( N_B = 4 ) ( N_C = 86 ) ( N_D = 0 )</td>
</tr>
<tr>
<td>( \Omega_D = -247,191 )</td>
<td>( N_A = 0 ) ( N_B = 0 ) ( N_C = 84 ) ( N_D = 6 )</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

In this paper an optimization model is demonstrated that simultaneously considers garage location and the assignment of vehicles to trips. The methodology was demonstrated by applying it to the transit network of the Detroit area's suburban operator. Approximately a third of the operator's network is used in the example. However, there is no reason that the model could not be expanded to consider the entire network or even a larger network. Expanding the model is simply a matter of coding more data.

The model works by opening and closing garages at prospective locations, evaluating the cost of transit operation with each combination, and iterating until a minimum-cost combination of garages is found. At each iteration plots are produced showing the flow of vehicles assigned to trips, between trips, and back to the garages. These plots provide an interesting interpretation of the model's assignments.
The importance of the model's development and application lies in its ability to realistically represent the interrelation between vehicle assignments and garage location. Other available methodologies assume that the vehicle assignments are fixed regardless of the location being evaluated. This assumption is unrealistic. In actual situations, when new garages are located, vehicles are reassigned with respect to the locational properties of the new garage. Hence, evaluating a new garage with existing vehicle assignments does not result in an accurate estimate of what it would cost to locate a garage at a candidate site. Further, existing vehicle assignments are biased toward existing garage sites. By recognizing the interrelationship between vehicle assignments and garage locations, the methodology results in realistic cost estimates of locating at a particular site and it does not suffer from the bias built into existing vehicle assignments.

The model provides a more realistic system to analyze the bus garage problem, but there are still many problems that remain to be solved. Two of the prominent problems are as follows:

1. Although vehicle assignments have been disaggregated down to the trip level, little has been done to ensure desirable driver assignments. There are reasons to believe that during run cutting good vehicle assignment will result in better driver assignments. When dealing with the bus garage problem, others have considered driver assignments by first packaging groups of blocks on the same route or route segment into desirable driver assignments (14). Then the route or route segments are allocated to garages using a mathematical program. Because this second approach does not consider the vehicle and driver assignment simultaneously, it has some of the same flaws as the method in this paper. To the best of the authors' knowledge, no feasible solution exists to the problem of simultaneous assignment of buses and drivers. Therefore, probably the best that can be done at this time is to iterate between a multifacility vehicle assignment model and a driver assignment model when new garage locations are investigated.

2. Currently, all mathematical programming techniques used to locate and size bus garages require technical knowledge that is not commonly available at transit agencies or through consulting firms. To date, the authors know of only one example where a transit agency commissioned a garage location study that included the use of mathematical programming (15). Therefore, it is unlikely that any mathematical programming techniques, in their current forms, will ever be commonly used. On the other hand, many transit agencies use complicated mathematical programs on a daily basis. ROCUS and other scheduling and run-cutting packages are commonly used and include complicated mathematical programs. What makes these packages operable by most transit agencies is that they have been automated to the point where the user does not need to understand mathematical programming. Until garage location and sizing methods become more automatic, it is unlikely that analytical methods will receive widespread use.

In conclusion, there is still much work that needs to be done in the development of bus garage planning methods. Besides resolving the two problems mentioned previously, these methods should probably
become part of the long-range transportation planning process. Typically, transportation plans are only concerned with provision of service with little regard for operational problems. On the other hand, operational planning for garages tends to occur only when there is pressure to build one new facility with little regard for long-range changes in service. These two activities need to be drawn together.

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