Geotechnical Centrifuge Modeling of Soil Erosion

D. J. GOODINGS

ABSTRACT

Geotechnical centrifuge modeling has been used to special advantage in instances where the nature of a geotechnical phenomenon has not been entirely understood and where interaction of several effects makes standard analyses difficult to apply. The influence of surface erosion on slope instability is one such condition, but it is important to determine if these events are modeled in similarity. This question is addressed in this paper with an examination of the scaling laws for laminar seepage, turbulent surface flow, initiation of erosion and rate of sediment transport in centrifuge models built of either cohesionless or cohesive soils to scale l/N. These scaling laws are then compared to the laws for conventional l g models. The conclusions are: (a) turbulent surface flow and laminar seepage can be modeled correctly in the centrifuge model if permeability is reduced by a factor of N; this similarity cannot be achieved in a l g model; (b) erosion of cohesionless soil that results from overflow can be modeled correctly in the centrifuge and in the l g model provided all particles are reduced in size by a factor of N (this cannot be achieved, however, when emerging seepage is the dominant source of flow in either case); (c) in cohesive soils, initiation of erosion may be correct in the centrifuge model, but is not expected to be so in the l g model (it is unlikely that sediment transport is correctly modeled in either case); and (d) one erosion feature peculiar to centrifuge models is that erosion may be achieved either by increasing flow at constant acceleration, or by decreasing acceleration at constant flow. 

LAMINAR FLOW

1 g Models

In a conventional l g model of prototype soil, the laminar flownet that will develop as a result of a difference in boundary water conditions will be identical to that of a geometrically similar prototype that is N times larger in all dimensions and boundary conditions. The rate of head loss per unit length will be similar, and if the prototype soil is used in the model, then the seepage velocities, \( v \), will be related as:

\[
\frac{v_m}{v_p} = N
\]

where the subscripts m and p refer to model and prototype, respectively. Because rates of seepage discharge per same unit width in model, \( q_m \), and prototype, \( q_p \), are calculated as \( q = v d \), where \( d \) is the depth of seepage in the flownet, and all depths in the model will be l/N that of the prototype, then

\[
\frac{q_m}{q_p} = \frac{1}{N}
\]

The times, \( t \), for seepage to occur between two geometrically similar points in the model and prototype will be

\[
\frac{t_m}{t_p} = \frac{1}{N^2}
\]

solely as a result of the decrease in all distances in the model.

Centrifuge Models

Laut (1) noted that in models experiencing laminar flow at 1 g, subjected to an increase in self-weight of N times but still maintaining laminar flow, an increase in velocity will occur, such that

\[
\frac{v_m}{v_{1g}} = N
\]

This was interpreted by Pokrovsky and Fyodorov (2) as an increase in Darcy’s coefficient of permeability, \( k \), because \( k \) is directly proportional to the unit weight of the permeant, which is normally water in the geotechnical context. Such an increase in self-weight, however, will have no effect on the shape of the flownet, which for laminar flow is dependent on the problem geometry only, provided the geometry of the boundary conditions, including the upstream and downstream water levels, remains unchanged. This increase in velocity will mean that the time for flow to occur between two points in the centrifuge model, which correspond geometrically to two points in a prototype of the same soil, but with all dimensions N times larger, will be reduced such that

\[
\frac{t_m}{t_{1g}} = \frac{1}{N^2}
\]

as a combined result of the velocity increase, and the reduction in model dimensions. This increase in \( v_m \) by a factor of N also will mean that

\[
q_m = q_p
\]
Pokrovsky and Fyodorov (3) noted that according to the basic laws of kinematics, the time for mass soil movements to occur in the model will be less than the time for a similar movement in the prototype by a ratio

$$\frac{t_m}{t_p} = \frac{1}{N}$$

(7)

This ratio is in conflict with the ratio of times for laminar seepage in Equation 5. Although in many cases the time for seepage and the time for mass soil movement need not be modeled in similarity, particularly in soils with very low permeability, there may be instances where this is an important requirement. According to Hazen’s observation, for granular soils where $D_{10}$ is greater than 0.1 mm, less than 3 mm, and the coefficient of uniformity is less than 5, $k_{so} = N$. This presents an opportunity to bring these two time scales into similarity by reducing $D_{10}$:

$$\frac{D_{10m}/D_{10p}}{1/N}$$

(8)

which, in turn, will reduce soil permeability and seepage velocity, so that at 1 g

$$k_m = k_p/N$$

(9)

and at Ng

$$v_m = v_p$$

(10)

$$\frac{t_m}{t_p} = 1/N$$

(11)

$$\frac{q_m}{q_p} = 1/N$$

(12)

This technique, commonly applied in modeling, will only be allowable from the geotechnical point of view if there are no significant changes to the density, cohesion, and friction of the soil. It will not affect the laminar flownet configuration provided the problem geometry (including the boundary water levels) remains unchanged.

**Turbulent Flow**

When seepage emerges from the soil, or when runoff or overtopping occurs, turbulent flow usually prevails, and the Chezy equation will now govern steady surface flow:

$$V = \frac{(8g/f)^{1/2} R^{1/2} S_r^{1/2}}{Q}$$

(13)

where

- $g$ = earth’s gravitational acceleration;
- $f$ = a dimensionless coefficient of roughness;
- $R$ = the hydraulic mean radius and is equal to the depth of flow, $d$, when the channel width is much greater than the depth; and
- $S_r$ = the slope of the energy line, which for steady uniform flow is equal to the soil slope inclination, $S$.

This coefficient of roughness, $f$, may be proportional to $(Re^{1/4}, \log_{10}(Re), \log_{10}(d/D_{10}))$, where $Re$ is the Reynolds number, depending on whether the flow is hydraulically smooth, rough, or turbulent ($d$), but relatively large variations in values of $Re$, $d$, and $D_{10}$ will have very small effects on velocity, so that $f$ may be considered constant. In terms of flow rate, for steady uniform flow, which overtopping or runoff may be idealized as, $q = Vd$, and according to Equation 13,

$$q = \frac{(8g/f)^{1/2} Q^{1/2} S_r^{1/2}}{Q}$$

(14)

The correctness of modeling surface flow (in instances where gravity is important) is measured by two dimensionless parameters: the Reynolds number, $Re$, which defines the nature of the flow as laminar, turbulent, or transitional, and the Froude number, $F$. These parameters are defined as

$$Re = \frac{Vd}{\nu}$$

(15)

and

$$F = \frac{V^2}{gd}$$

(16)

1 g Models

In a conventional 1 g model, which is geometrically similar but has dimensions $1/N$ that of the prototype, hydraulic similitude will be maintained if $F_m = F_p$, and if the values of $Re_m$ and $Re_p$ indicate the same flow regime by falling into the same rather wide range of values of $Re$ associated with each regime. Equation 16 requires that for that 1 g model

$$\frac{V_m}{V_p} = \frac{1/N^{1/2}}{1}$

(17)

If the rates of discharge in model and prototype are dictated by Equation 2, as they would be if the source of the water were laminar seepage, then according to Equation 17,

$$\frac{d_m/d_p}{1/N^{1/2}}$$

(18)

and from Equation 13,

$$\frac{V_m/V_p}{1/N^{1/2}}$$

(19)

Times for surface flow between geometrically similar points in model and prototype will then be related as

$$\frac{t_m}{t_p} = 1/N^{1/2}$$

(20)

This relationship is in conflict with the scaling of time for seepage flow in Equation 3. Only in a full scale model with $V_m = V_p$ in seepage and surface flow will those two time ratios be equal.

If, instead, a requirement that $\frac{d_m}{d_p} = 1/N$ for surface flow were imposed on this 1 g model, and the Froude numbers in model and prototype were made equal, then the velocities of surface flow in model and prototype would be related as

$$\frac{V_m}{V_p} = 1/N^{1/2}$$

(21)

$$\frac{t_m}{t_p} = 1/N^{1/2}$$

(22)

and

$$\frac{q_m}{q_p} = 1/N^{1/2}$$

(23)

This will only be acceptable in instances where overflow rather than emerging seepage will dominate surface flow, and the implications with respect to erosion will be discussed later.

**Centrifuge Models**

Examine a centrifuge model, again geometrically similar, but with soil altered according to Equations 8 and 9, and with model surface flow rate controlled...
in keeping with Equation 12. For the Froude numbers to be equal
\[ \frac{d_m^2}{d_p^2} = \frac{d_m^2}{d_p^2} \]
from which
\[ \frac{d_m}{d_p} = \frac{1}{N} \]
for either emerging seepage or overflow depths. According to Equation 13
\[ V_m = V_p \]
and
\[ \frac{t_m}{t_p} = \frac{1}{N} \]
which is identical to Equation 11 for scaling times for seepage events. Unlike the conventional 1 g model, then, the centrifuge model can satisfy simultaneously the requirements for hydraulic similitude and for scaling time of laminar seepage and turbulent surface flow, provided the values for Re indicate the same flow regime, with the understanding that
\[ \frac{Re_m}{Re_p} = \frac{1}{N} \] (28)

These equations have been developed for steady uniform flow, where \( S_f = S \), which is more characteristic of overflow conditions but Goodings (5) demonstrated that these equations will also be correct for steady nonuniform flow, which describes the nature of emerging seepage.

**Imitation of Erosion**

Emerging seepage, overflow, or runoff may cause soil erosion. Shields (6) examined initiation of erosion in cohesionless soils in essentially horizontal beds, which is relevant to riverbed erosion. He defined the boundary shear stress of the moving water, \( \tau_o \), to be
\[ \tau_o = \rho_w g d S_f \]
and the critical shear stress, \( \tau_c \), which must be exerted on the particles for dislodgment of a particle to occur, is
\[ \tau_c = F_s (\rho_s - \rho_w) D \]
where
- \( F_s \) is an entrainment function, also written as \( 1/\phi \),
- \( \rho_s \) is the density of the soil solid,
- \( \rho_w \) is the density of water, and
- \( D \) is the representative particle size.

Shields found that when \( \tau_o \geq \tau_c \), erosion was initiated.

Several experimentalists concerned themselves with determining values for \( F_s \), where
\[ F_s = \rho_w d S (\rho_s - \rho_w - SF/g) D \] (31)
which they found to be a function of the particle Reynolds number, \( Re^* \), defined for steady uniform two-dimensional flow as
\[ Re^* = (\tau_o/\rho_w)^{1/2} D/\nu = (gdS)^{1/2} D/\nu \] (32)

The plot of the relationship between \( F_s \) and \( Re^* \) is called the Shields diagram, and is shown in Figure 1. There is some scatter in the development of this graph because packing will have some effect in protecting particles against the full boundary shear force of the flowing water, and the frictional resistance of particles will vary according to material properties. This curve, however, accounts for the effects of turbulent velocity fluctuations, up-lift, strong velocity gradients, and the different characteristics of laminar and turbulent flow. This function dominates the initiation of erosion, but it is noteworthy that the variation of \( F_s \) with \( Re^* \) is small, becoming negligible for turbulent flow over coarse particles when \( Re^* \) exceeds 500 (7).

In most rivers the slope of the bed is so flat that the assumption that it is horizontal is acceptable. In the case of a soil slope, however, the inclination may be too great to ignore. A steep incline makes particles less stable and the initiation of erosion easier. The effect of the slope is accommodated in the critical value of the entrainment function by a slope factor, so that the critical value of \( F_s \) is reduced to \( F_s' \), where
\[ F_s' = F_s [1/\cos \beta (\tan \phi - \tan \beta)] \] (33)

**Figure 1** Shields diagram [after Shields (6)].

In this equation, \( \beta \) is the slope angle and \( \phi \) is the angle of frictional resistance of the soil particle.

Emerging seepage will also act to destabilize particles, increasing erodibility. Oldenziel and Brink (8) used the work of Martin and Aral (9) to demonstrate that erosion will occur at a lower level of boundary shear stress in the presence of seepage. They redefined Equation 31 to include that effect:
\[ F_s = \rho_w d S (\rho_s - \rho_w - SF/g) D \] (34)
\( SF \) is the seepage force per unit area perpendicular to the slope, equal to
\[ SF = c \gamma (\text{dh/dy}) \]
where \( c \) is an empirical constant between 0.35 and 0.40, and \( dh/dy \) is the hydraulic gradient perpendicular to the slope (8).

**1 g Models**

In modeling erosion of cohesionless soils, similarity of the particle Reynolds numbers of the model and prototype is not critical, provided the values of the entrainment function, \( F_s \), are equal. In a
conventional 1 g model, erosion will occur at a depth of flow

\[ d > F_s (\rho_s - \rho_w) \frac{D}{S} \]  

(36)

If \( F_{sm} = F_s \) and if the depth of model flow is \( 1/N \) that of full scale flow, then for initiation of erosion to be observed at the same time in the model as in the prototype, every \( D \) in the model must be \( 1/N \) as large as in the prototype. In cases where steep slopes and emerging seepage may also be important factors, the prototype and model entrainment functions will remain equal provided the geometries of the two slopes and the flow nets are identical and the angles of particle friction are the same, so that the slope factors (Equation 33) and the seepage forces (Equation 34) are equal in model and prototype.

**Centrifuge Models**

These requirements for similarity must also be satisfied for the same model subjected to an increase in self-weight on the centrifuge, if the initiation of erosion is to be correctly modeled. Equation 36 applies equally to the centrifuge model, with the same proviso that if \( dm/dp = 1/N \), then all values of \( D \) in the model should be \( 1/N \) those in the prototype when \( F_{sm} = F_s \). In substituting Equation 36 into Equation 14,

\[ \eta_m > \left( \frac{8nF_p}{D} \right) \frac{V_m}{\left( \rho_s - \rho_w \right)} \frac{D}{S} \left( \frac{S}{D} \right)^{1/2} \]  

(37)

This equation emphasizes that to initiate erosion in a centrifuge model, the modeler has the option to either increase the surface flow, \( q_m \), holding the model at a constant \( N \), or to decrease the self-weight, \( N \), at a constant \( q_m \).

The initiation of erosion of cohesive soils from surface flow is a subject that has not received as much attention as erosion of cohesionless soils. With the exception of dispersive clays, the natural cohesion of the soil gives it much greater resistance to erosion than that predicted from the Shields diagram, although not to an extent that erosion is no longer a problem. A few authors have examined the problem, in an effort to establish some relationship between the soil properties and the boundary shear stress necessary to initiate erosion: Smerdon and Beasley (10) looked for relationships between boundary shear stress and plasticity index and the percentage of clay in a soil; Flaxman (11) attempted to draw some relationship between the product \( T_0 V \) and unconfined compressive strength, and was also concerned with the importance of permeability, although permeability may be more a useful indicator of particle size and soil structure than a significant property in itself; and several authors (for example, Grissinger and Aasmussen (12)) believed that chemical and environmental factors were most important. The conclusion that can be drawn from their work and others has indicated, however, that resistance to erosion of cohesive soils varies greatly, but that no critical shear stress relationship exists for those soils in the same way that one exists for cohesionless soils (7).

Such a conclusion makes comment on critical parameters for modeling erosion in cohesive soils more difficult. If plasticity index, percentage of clay, unconfined compressive strength, permeability, or chemical and environmental factors are important, it is apparent that the prototype soil must be used in the model and prepared to duplicate its in situ conditions in every respect. There exists a problem, however, in conventional 1 g modeling of the initiation of erosion in that \( T_0 \) in the model must equal that in the prototype to be correct. In a model with slopes of the energy line equal to that in the prototype, this will only occur if the depth of surface flow are also equal, which is to say, the model must be full scale. In the centrifuge model, however, the same obstacles do not exist: if the depth of flow in the model equals \( 1/N \) that in the prototype, then velocities and \( T_0 \) will be equal. Accordingly, the technique of implementing research on initiation of erosion in cohesive soils, erosion will be modeled correctly on a centrifuge model but not on a conventional 1 g reduced scale model.

**SEDIMENT TRANSPORT**

After the threshold of erosion is reached, sediment will be carried away by the water, both maintained in suspension by turbulence, and rolled along the slope surface as bedload. Henderson (13) pointed out that equations have been developed to predict the relative concentrations at different depths but that the absolute volumes of suspended solids and bedload solids have not been predicted using equations that consider the mechanisms of transport. A variety of empirical equations have been derived to fit observed values of \( q_s \), the rate of sediment discharge per unit width of flow, in river-beds of sand and gravel. The best known of these is Einstein's (14) equation in which the bed load function, \( \theta \), is a function of Shields's entrainment function:

\[ \theta = q_s / wD = f(F_s) \]  

(38)

where \( w \) is the fall velocity that Einstein defined as

\[ w = G \sqrt{g(D(G - 1))} \]  

(39)

and

\[ G = \sqrt[3]{2/3 + \left( \frac{16d}{\rho g D^2 (G - 1)} \right) - \left( \frac{32d}{\rho g D^2 D (G - 1)} \right)} \]  

(40)

in which \( G \) is the specific gravity of the soil particles. Later, he related \( \theta \) to total sediment load to include suspended load.

According to Einstein's work, correct modeling should be achieved if values of \( F_s \) are equal in model and prototype, which, as for the initiation of erosion, requires that all particles in the model be \( 1/N \) in size that of the prototype, both for conventional 1 g models and for centrifuge models. The effect of such a change in grain size will be very small in the case of \( G \), although it will be less in the centrifuge model than in the conventional 1 g model. Neglecting any change in the value of \( G \), fall velocity, \( w \), in the conventional 1 g model will be slower than in the prototype by a factor of \( N^{-2/3} \), because of the reduction in \( D \). The fact that particles will take longer to fall back into the bed will mean that for the same value of \( F_s \) in model and prototype, the rate of sediment discharge per same unit width, \( q_s \), should be much larger in the model, by a factor of \( N^{3/2} \). That is to say that erosion should proceed much more quickly, by a factor of \( N^{3/2} \). In the centrifuge model, the fall velocity will be the same as in the prototype, although for the same value of \( F_s \), \( q_s \) should be \( N \) times larger in the model as a result of the smaller grain size or erosion will occur \( N \) times faster in the model. In terms of relating the times for an erosion event in a model that has caused the erosion of the volume of soil, \( V_m' \), to a similar erosion event in the prototype, which has volume \( V_p' = N V_m' \), for a conventional 1 g model,
The relationships governing the laminar seepage, the turbulent surface flow, the initiation of erosion and the rate of sediment transport in conventional 1 g models and centrifuge models of cohesionless and cohesive soils have been examined. The overall conclusions are as follows:

1. Turbulent surface and laminar seepage flow can be modeled in similarity in the centrifuge if permeability is reduced by a factor of N. This similarity of flows cannot be achieved in a conventional 1 g model.

2. In modeling the process of erosion of a granular soil on the centrifuge, all conditions for similarity can be satisfied for overflow conditions if all model grain sizes are reduced by a factor of 1/N compared with that in the prototype although erosion will proceed much more quickly than any other model event when modeling erosion that arises predominantly from emerging seepage, however, two conflicting grain size requirements exist, Equations 8 and 40, which will be difficult to satisfy simultaneously. In conventional 1 g modeling of erosion of a granular soil, correct modeling is possible for overflow conditions if all model soil grains are reduced in size by a ratio of 1/N although erosion will proceed much more quickly. Erosion arising from emerging seepage will not be modeled correctly.

3. In centrifuge modeling of erosion of cohesive soil due to overflow, initiation of erosion is expected to be correctly modeled, but subsequent sediment transport is not modeled correctly, according to current theory. Erosion due to emerging seepage will not be modeled correctly but this is not normally a problem in cohesive soils of low permeability. Erosion of cohesive soils will not be modeled correctly at a reduced scale at 1 g with respect to either initiation of subsequent sediment transport.

4. One feature of modeling erosion peculiar to the centrifuge, is that erosion may be initiated either by increasing flow at a constant acceleration, N, or by decreasing N at a constant flow.

The centrifuge model, then, is at least as good as the conventional 1 g model for modeling erosion, and for modeling geotechnical events that depend simultaneously on replication of the prototype stress gradient and on undermining effects of erosion, the centrifuge model can be altered to be acceptable although it may be imperfect. Even with the limitations noted earlier, however, the centrifuge model still provides a better means of modeling such events than any other presently available physical technique.

REFERENCES

Comparison of Diametral and Triaxial Repeated Load Testing Techniques for Untreated Soils

JOSE R. MONTALVO, CHRIS A. BELL, and JAMES E. WILSON

ABSTRACT

The techniques involved in, and the results from, resilient modulus testing of subgrade soils typically found in Oregon are described in this paper. In addition, two methods of testing, the triaxial and diametral repeated load procedures, were investigated. Subgrade soils obtained from two projects were tested. One project was a new alignment construction project in the Willamette Valley (Salem Parkway) for which there were two distinct subgrade soils (AASHTO classifications A-7-6 and A-4); the other was an overlay project in central Oregon (US-97) with a pumiceous subgrade soil (AASHTO classification A-1-b). It was found that the diametral testing procedure was adequate for use with cohesive soils, typical of those occurring in the Willamette Valley, but it is not recommended for use with the noncohesive volcanic soils occurring in eastern Oregon. For such soils, the triaxial testing mode is recommended. The major advantage of the diametral test for treated materials is its simplicity compared to the triaxial test. However, the necessity to consider the effects of confining pressure for untreated soils diminishes this advantage, and with cohesionless soils, the test is no simpler than the triaxial test, which is preferable for modeling the in situ stress regime.

OBJECTIVES

The specific objectives of this study were to (a) compare the diametral and triaxial repeated load testing techniques for untreated soils, and (b) recommend procedures for routine use of the diametral test for soils evaluation and pavement design.

STUDY APPROACH

The results of a study to examine the use of two repeated load testing procedures, the diametral and triaxial devices, are presented. Soils typical of those occurring as subgrades in Oregon were selected for testing to achieve the objectives of this study. The soils used were obtained from Oregon highways, the Salem Parkway in the Willamette Valley, and the US-97 highway in central Oregon, and represent typi-