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## Bearing Capacity of Eccentrically Loaded Continuous Foundations on Layered Sand

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### ABSTRACT

Laboratory model test results on the ultimate bearing capacity of continuous rough foundations resting on a layered sand are presented. For this study, the top layer of sand is a dense sand that is underlain by a loose sand at a limited depth. The eccentricity ratio for load application has been varied from zero to 0.25. The laboratory model test results have been compared with the theory presented by Meyerhof and Hanna, which has been modified to take into account the effective area concept for eccentrically loaded foundations. The agreement between the theory and the model test results is satisfactory up to an eccentricity ratio of 0.25.

The bearing capacity of shallow foundations has been the subject of intense study for the past 40 years since the pioneering work of Terzaghi (1). Most of these studies are related to foundations resting on homogeneous soil layers extending to great depths. However, the published literature on the bearing capacity of shallow foundations on layered soils is relatively scarce (2-8). Meyerhof and Hanna (5) have

more recently published a generalized ultimate bearing capacity theory for shallow foundations on layered soils subjected to inclined loading.

At this time, a survey of literature indicates that experimental works relating to the ultimate bearing capacity of eccentrically loaded foundations on layered sands have not yet been attempted. The purpose of this paper is to present some recent laboratory model test results for the bearing capacity of an eccentrically loaded continuous foundation resting on a dense sand layer underlain by a loose sand extending to a great depth.

### THEORETICAL SOLUTION FOR CENTRALLY LOADED CONTINUOUS FOUNDATION

To evaluate the ultimate bearing capacity of a continuous foundation resting on a stronger sand layer (unit weight =  $\gamma_1$  and angle of friction =  $\phi_1$ ) underlain by a weaker sand layer (unit weight =  $\gamma_2$  and angle of friction =  $\phi_2$ ), Meyerhof and Hanna (5) proposed a failure mechanism according to which a punching shear failure takes place in the top stronger sand layer, followed by a typical bearing capacity failure in the weaker soil layer located below the stronger soil. This is shown in Figure 1. According to this mechanism,

$$q_u = q_u(2) + \gamma_1 z^2 [1 + D_f/z] K_s \tan \phi_1 / B - \gamma_1 z \leq q_u(1) \quad (1)$$

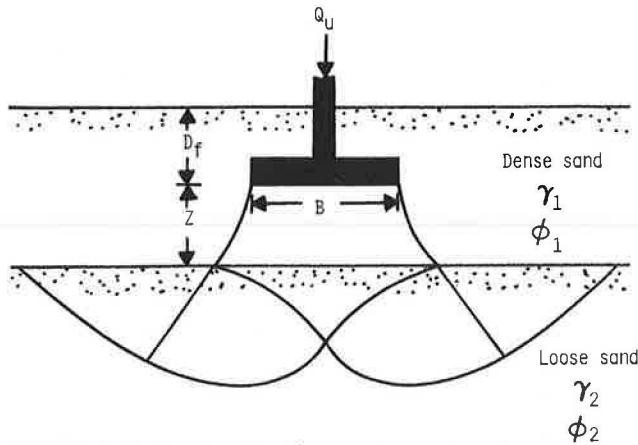


FIGURE 1 Failure in soil under a continuous foundation.

where

$K_s$  = the punching shear coefficient;  
 $Z$  = the distance between the bottom of the foundation and the top of the weaker soil layer;  
 $D_f$  = the depth of the foundation;  
 $q_u(2)$  = the ultimate bearing capacity of the lower weak sand layer

$$= \gamma_1(D_f + Z)N_q(2) + 1/2\gamma_2BN_{\gamma}(2) \quad (2)$$

$q_u(1)$  = ultimate bearing capacity of the foundation resting on the stronger sand layer extending to a great depth

$$= \gamma_1 D_f N_q(1) + 1/2\gamma_1 B N_{\gamma}(1) \quad (3)$$

where

$N_q(1), N_{\gamma}(1)$  = bearing capacity factors corresponding to the soil friction angle of the stronger sand (i.e.,  $\phi_1$ ); and  
 $N_q(2), N_{\gamma}(2)$  = bearing capacity factors corresponding to the soil friction angle of the weaker sand (i.e.,  $\phi_2$ ).

Trial calculations by Meyerhof and Hanna (5) using the logarithmic spiral method in layered sand have shown that the values of the punching shear coefficient ( $K_s$ ) are a function of the soil friction angle  $\phi_1$  and also the ratio of  $\gamma_2 N_{\gamma}(2)/\gamma_1 N_{\gamma}(1)$ .

#### SOLUTION FOR ECCENTRICALLY LOADED CONTINUOUS FOUNDATION

In their work Meyerhof and Hanna (5) have suggested that for an eccentrically loaded foundation, the concept of "effective area method" (9) can be used in conjunction with Equation 1 to calculate the ultimate bearing capacity. For a continuous foundation with a load eccentricity  $e$ , the effective width  $B'$  can be given as  $B' = B - 2e$ . Thus, using the effective area method

$$q_u = Q_u/BL = B'/B[q_u'(2) + \gamma_1 Z^2(1 + D_f/Z) \times (K_s \tan \phi_1)/B' - \gamma_1 Z] \leq [q_u(1)]B'/B \quad (4)$$

where

$Q_u$  = ultimate load on the eccentrically loaded foundation;  
 $L$  = length of the foundation;

$$q_u'(2) = \gamma_1(D_f + Z)N_q(2) + 1/2\gamma_2 B' N_{\gamma}(2); \quad (5)$$

$$q_u'(1) = \gamma_1 D_f N_q(1) + 1/2\gamma_1 B' N_{\gamma}(1). \quad (6)$$

Although this was suggested by Meyerhof and Hanna (5), experimental verifications were not presented. The purpose of this paper is to present some laboratory model tests to verify Equation 4.

#### LABORATORY MODEL TESTS

The model tests were conducted in a sandbox measuring 1.53 m x 0.305 m x 0.93 m. The sides of the box were heavily braced to avoid lateral yielding.

Sand used for the model tests has a 100-percent passing rate for No. 10 U.S. sieve, a 74-percent passing rate for No. 40 U.S. sieve, and a 0 percent passing rate for No. 200 U.S. sieve. In conducting the tests, the model test box was filled with sand in layers of 50.8 mm by using a raining technique through a No. 10 sieve. For the loose sand, the average height of drop of sand (for each 50.8-mm-thick layer) by raining was 178 mm, and for the dense sand layers, the average height of raining was 915 mm. The dry unit weights of compaction achieved by this procedure were 15.25 kN/m<sup>3</sup> for loose sand and 17.06 kN/m<sup>3</sup> for dense sand layers with relative densities of 27 percent and 81 percent, respectively. The angles of friction of the loose sand ( $\phi_2$ ) and the dense sand ( $\phi_1$ ) at these relative densities of compaction were determined by direct shear tests to be 36 and 43 degrees, respectively.

The model foundation was 305 mm long and 101.6 mm wide, and was made out of a steel plate having a thickness of 9.53 mm. Coarse sandpaper was glued to the bottom of the model foundation to make it rough. Three circular grooves were made on the model foundation with eccentricities  $e = 0$  mm, 12.7 mm, and 25.4 mm.

For the laboratory tests, the model foundation was placed centrally on the sand in the box at the desired depth of embedment ( $D_f$ ). The box was placed inside a steel loading frame. For application of load to the foundation, a steel ball was placed on a circular groove (depending on the desired eccentricity) made on the top of the model foundation. Load on the steel ball, and hence the foundation, was applied by means of a hydraulic jack through a cylindrical shaft with a diameter of 36 mm. The bottom of the steel shaft had a circular groove to fit into the steel ball. A schematic diagram of the load application mechanism used in the laboratory is shown in Figure 2. The loads and the corresponding deflections along the line of load application were recorded by means of a proving ring and a dial gauge.

The sequence of the model tests conducted in the laboratory is summarized in Table 1. For each laboratory test, the load-displacement graphs were constructed to determine the ultimate loads at failure,  $Q_u$ , using the procedure suggested by Vesic (10). The ultimate loads as determined from the model tests are shown in Figure 3.

#### Determination of $N_{\gamma}(1)$ , $N_{\gamma}(2)$ , and $N_q(2)$

The experimental value of  $N_{\gamma}(1)$  can be calculated from the laboratory results of Test 1, which has been conducted in a homogeneous dense sand with  $D_f = 0$  and  $e = 0$ . For this case,

$$N_{\gamma}(1) = Q_u/[(BL)(1/2\gamma_1 B)] \quad (7)$$

Substitution of proper values in Equation 7 yields a value of  $N_{\gamma}(1)$  to be equal to 196.6. Vesic (10),

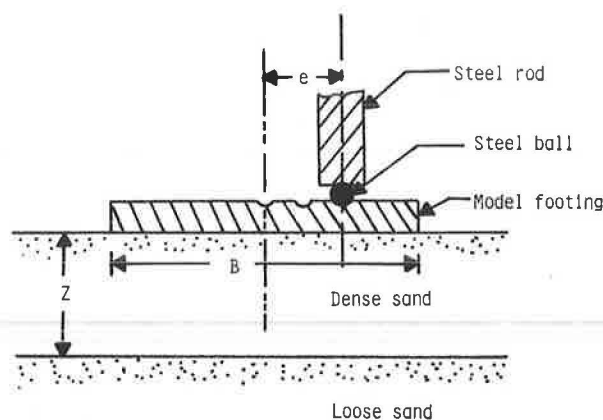


FIGURE 2 Schematic diagram of the load application mechanism to the foundation.

TABLE 1 Sequence of Laboratory Model Tests

Test No.	Type of sand	$D_f/B$	$Z/B^*$	$e/B$
1	Dense	0	$\infty$	0
2		0	$\infty$	0.125
3		0	$\infty$	0.25
4	Loose	0	0	0
5		0	0	0.125
6		0	0	0.25
7	Loose	1	0	0
8	Dense over loose	0	0.5	0
9		0	1.0	0
10		0	1.5	0
11		0	2.0	0
12		0	2.5	0
13	Dense over loose	0	0.5	0.125
14		0	0.75	0.125
15		0	1.0	0.125
16		0	1.25	0.125
17		0	2.0	0.125
18	Dense over loose	0	0.5	0.25
19		0	0.75	0.25
20		0	1.0	0.25
21		0	1.65	0.25
22		0	2.0	0.25

$Z/B = \infty$  means uniform homogeneous dense sand  
 $Z/B = 0$  means uniform homogeneous loose sand

after careful evaluation of the bearing capacity factors, has recommended that  $N_\gamma$  can be estimated according to the work of Caquot and Kerisel (11), which can be approximated by the expression

$$N_\gamma = 2[e^{\pi \tan \phi} \tan^2(45 + \phi/2) + 1] \tan \phi \quad (8)$$

The value of the friction angle for dense sand as determined experimentally is 43 degrees. With this friction angle, Equation 8 yields a value of 186.54. This is in fairly satisfactory agreement with the experimental value.

In a similar manner, the experimental value of  $N_{\gamma(2)}$  can be calculated from the results of Test 4, which has been conducted on a homogeneous loose

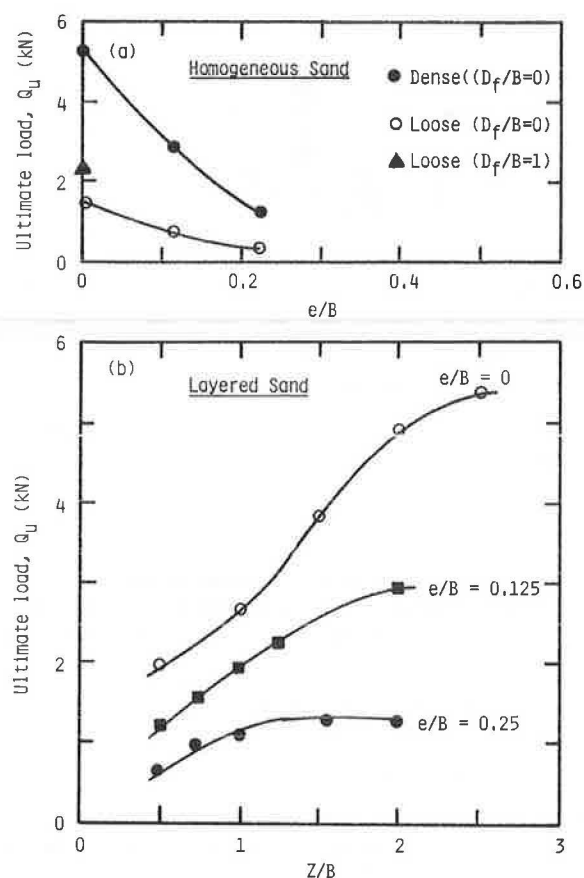


FIGURE 3 Experimental ultimate load ( $Q_u$ ) variation: (a) homogeneous sand and (b) layered sand.

sand with  $D_f = 0$  and  $e = 0$ . This can be done by using the equation

$$N_{\gamma(2)} = Q_u / [(BL)(1/2\gamma_2 B)] \quad (9)$$

Substitution of proper values into Equation 9 yields  $N_{\gamma(2)} = 57.17$ . For loose sand that has a friction angle of 36 degrees, the theoretical value from Equation 8 would give  $N_{\gamma(2)} = 56.31$ . The experimental value of  $N_{q(2)}$  can be obtained from the results of Tests 4 and 7 as

$$N_{q(2)} = [Q_u(D_f/B=1) - Q_u(D_f/B=0)] / [(BL)(\gamma_2 D_f)] \quad (10)$$

Substitution of proper experimental values in the right side of Equation 10 gives  $N_{q(2)} = 20.59$ . This experimental value of  $N_{q(2)}$  agrees well with the theory presented by Vesic (12) for the condition of local shear failure, according to which

$$N_q = [e^{(3.8\phi \tan \phi)}] \tan^2(45 + \phi/2) \quad (11)$$

The theoretical value of  $N_q$  for  $\phi = 36$  degrees, as obtained by using Equation 11, is 21.83.

#### Comparison of Theory with Experimental Results for Tests on Layered Soil

Figures 4 (a), (b), and (c) show the theoretical variation of  $q_u$  as obtained by using Equations 4, 5, and 6 (for  $e/B = 0, 0.125$ , and  $0.25$ ). It should be pointed out that in obtaining the preceding theoretical curves, Equation 8 was used to obtain  $N_{\gamma(1)}$  and  $N_{\gamma(2)}$ , and Equation 11 was used to obtain

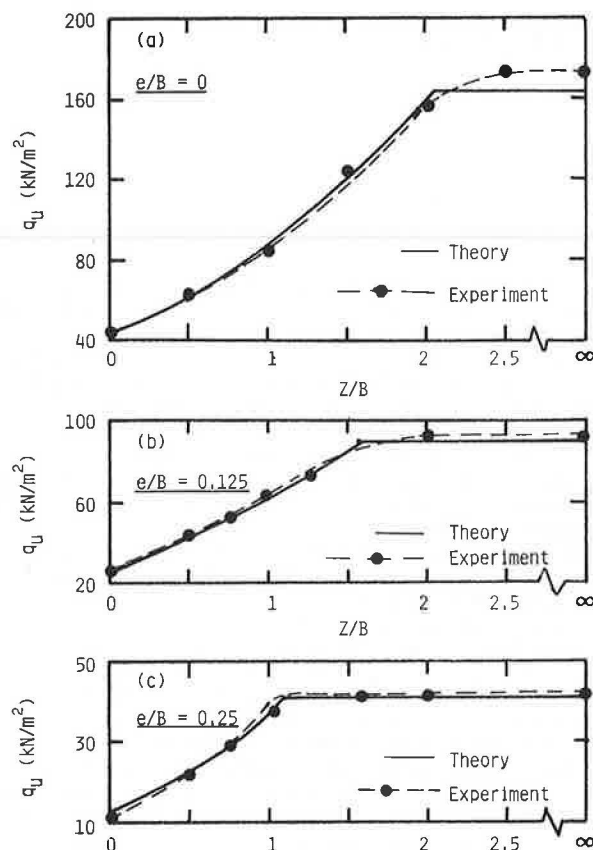


FIGURE 4 Comparison of experiment with theory—variation of  $q_u$  with  $Z/B$ .

$N_q(2)$  with  $\phi_1 = 43$  degrees and  $\phi_2 = 36$  degrees. The value of  $K_s$  was obtained by interpolation from the theory given by Meyerhof and Hanna (5).

To compare the theory with the experiment, the ultimate bearing capacity for all tests was calculated as

$$q_u = Q_u / BL \quad (12)$$

The values of  $Q_u$  for all tests are given in Figures 3 (a) and (b). Figures 4 (a), (b), and (c) also show the experimental variation of  $Q_u$  with  $Z/B$  for  $e/B = 0, 0.125$ , and  $0.25$ . A comparison of the experimental results with the theory indicates that the agreements are generally satisfactory for the laboratory tests reported here.

#### CONCLUSIONS

Laboratory model test results for the bearing capacity of surface foundations resting on a dense sand layer underlain by a loose sand layer have been presented. These model test results have been compared with the theory presented by Meyerhof and Hanna (5). On the basis of this study, the following conclusions can be drawn:

1. For foundations located over homogeneous soils, the experimental bearing capacity factor  $N_\gamma$  is in satisfactory agreement with the theoretical values obtained by Caquot and Kerisel (11).

2. The experimental bearing capacity factor  $N_q$  for local shear failure condition of a homogeneous loose soil located under a foundation is in satisfactory agreement with the theory presented by Vesic (12).

3. The effective area method of Meyerhof (9), when incorporated into the Meyerhof-Hanna theory (5), gives good results in predicting the ultimate bearing capacity of eccentrically loaded continuous foundations resting on a dense sand layer that is underlain by a loose sand layer. This is true up to an eccentricity ratio of  $e/B = 0.25$ .

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