A Proposed Track Performance Index for Control of Freight Car Harmonic Roll Response

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ABSTRACT

Analytical and experimental study results of high center of gravity freight car response to a range of track cross-level deviations are presented. Based on the criteria of excessive car body roll or excessive wheel lift, boundaries between safe and unsafe track cross-level conditions are established. These studies and industry experience indicate that although isolated low joints producing crosslevel deviations as large as 4 in. can be safely traversed, at the critical harmonic roll speed, a continuous series of 0.75-in. low joints will produce an unacceptable roll response for high center of gravity cars resulting in a potential derailment. The results also indicate that a 400-ft length of track is sufficient for the harmonic roll resonance to build to a critical amplitude. The cross-level variation conditions that form the boundaries between safe and unsafe harmonic roll response are reviewed. The results are then combined in a heuristically developed performance index that is intended to identify potentially unsafe track conditions without rejecting an excessive amount of track that does not have the potential for producing harmonic roll derailment. Analyses have been conducted on track geometry measurements of selected Class 2 and Class 3 track to evaluate the statistics of the proposed cross-level index. The results indicate that the index does successfully identify potential harmonic roll situations while permitting occasional large cross-level deviations that may be undesirable but are not unsafe. Illustrations of the response of the index to selected measured track geometry conditions are included.

As noted in a survey conducted by the Government Industry Program on Track Train Dynamics (<u>1</u>), the harmonic roll response of freight cars to periodically recurring cross-level variations on jointed track having one-half staggered rail lengths is of major concern to the railroad industry, in terms of both safety and damage to equipment and lading. This phenomenon of harmonic roll, often referred to as freight car rocking or rock and roll has been known to exist for many years, dating back to the 1920s (<u>2</u>,<u>3</u>). The harmonic roll problem is a highly nonlinear resonance condition typically occurring in the 10 to 25 mph speed range.

Figure 1 illustrates a freight car in the normal and rocking positions. In the normal (nonrocking) position, the car body moves with the bolster. For the more severe rocking case, the car body partially separates from the bolster and rocking occurs about the side bearing. A typical roll amplitude versus speed response characteristic for a freight car responding to periodic cross-level variations is shown in Figure 2. As the car speed is increased, the roll amplitude follows the lower branch (A) of the curve until it reaches a critical speed where a sudden jump in roll amplitude occurs accompanied by violent oscillations. As speed increases further, the oscillation decreases in amplitude. However, if the track section is entered at a decreasing speed, the response follows the upper branch of the curve (B) with an increase in roll amplitude as speed decreases. The roll responses encountered at decreasing speeds are higher than those obtained for increasing speeds.

The new generation of larger freight cars (70- to 100-ton range) with high center of gravity (c.g.) has increased the frequency of the harmonic roll problem. Although significant efforts are currently being made by the equipment supply industry to pro-

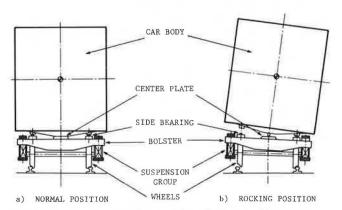


FIGURE 1 A typical freight car and truck illustrated in a transverse plane.

duce devices that control harmonic roll through the use of new truck and car design, it will be some time before the current fleet is replaced. It is therefore necessary to establish limits on track geometry variations to assure safe operation of the existing fleet. The simulation studies described here, along with industry experience, indicate that although isolated low joints producing cross-level deviations as large as 4 in. can be safely traversed, at the critical harmonic roll speed a continuous series of consecutive 0.75-in. low joints will produce an unacceptable roll response for high c.g. cars, resulting in a potential derailment. The results also indicate that a 400-ft length of track is sufficient for the harmonic roll resonance to build to a critical amplitude.

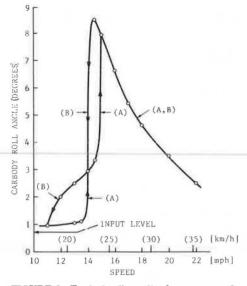


FIGURE 2 Typical roll amplitude versus speed response characteristic for freight car responding to periodic cross-level variations.

Based on the criteria of excessive car body roll or excessive wheel lift, boundaries between sale and unsafe track cross-level conditions have been established by the simulation studies described here. The cross-level variation conditions that form these boundaries are combined into a heuristically developed performance index or cross-level index (CLI), which is intended to identify potentially unsafe track conditions without rejecting an excessive amount of track that lacks the potential for harmonic roll derailment. The statistics of this proposed cross-level index are evaluated from analyses of track geometry measurements of Class 2 and Class 3 track. The results indicate that the index does successfully identify potential harmonic roll sections while permitting occasional large cross-level deviations that may be undesirable but are not unsafe. This paper includes illustrations of the response of the index to selected measured track geometry characteristics.

MODELING TECHNIQUES

The analytical model used in this study is an approximation of the flexible vehicle model used by Tse ($\underline{4}$) in a study of freight car rocking ($\underline{5}$). The model simulates freight car dynamic response based on numerical integration of the equations of motion, including nonlinearities related to springs, side bearing clearances, friction snubbers, and kinematic constraints. The freight car rocking model has been validated by comparison with field test data from tests conducted at the Transportation Test Center ($\underline{6}$).

As a rail vehicle undergoes harmonic roll, six car body roll configurations are possible, depending on the degree of rocking as shown in Figure 3. The first configuration (C-0) is the no-roll static condition. The smallest degree of roll will produce configuration C-1 where the centerplate surfaces remain in contact as the car body and truck bolster roll together. At larger car body roll angles, there is partial centerplate separation in which rocking takes place on the centerplate (C-2). Further rotation will result in both centerplate and side bearing rocking together (C-3). In configuration C-4,

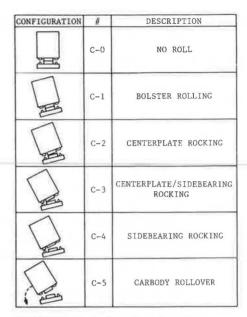


FIGURE 3 Car body roll configurations.

there is full centerplate separation with rocking taking place about the side bearing. In the final configuration (C-5), the car body rolls off. The analytical model used in this study is a 5 degree of freedom nonlinear rail car model capable of representing all the foregoing roll configurations. The model of the rail car in a transverse plane is shown in Figure 4 along with the bolster and truck suspensions. The orientation of the rail car model traveling over a track with cross-level variation is shown in Figure 5.

The vertical rail profile is based on a five term exponential decaying series of the form

$$i=2$$

$$z(x) = \sum_{i=2}^{n} A_{i}e - |x - (2i + 1)L/2|/a \qquad (1)$$

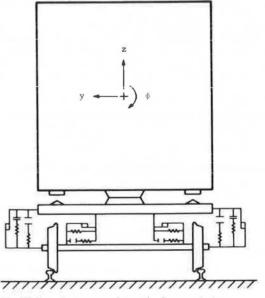


FIGURE 4 Rail car model with observer facing the direction of forward motion.

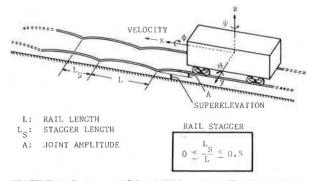


FIGURE 5 Rail car model at initial position (T = 0) traveling over a track with cross-level variation.

where the A_i are selected to produce the appropriate joint amplitude, L is the rail length, and a is the inverse decay rate. This relationship is based on a study of measured track geometry data and characterizes bolted rail with one-half staggered joints (7). Based on this relationship, the shape of the vertical rail profile for a 1-in. joint amplitude is shown in Figure 6. Further discussion of this exponential series, as well as the effect of summing a greater number of terms, is given by Corbin (7). Because the vertical rail amplitudes can be individually specified, it is possible to produce discrete (single, double, etc.) or continuous perturbations. Superelevation is included in the track geometry by adding independent terms to the left and right rail functions, so that either rail can serve as the high rail. To obtain variable rail stagger, the relative longitudinal position of the right and left rails is a variable that can be specified to produce any stagger from 0 (no stagger) to 0.5 (full stagger).

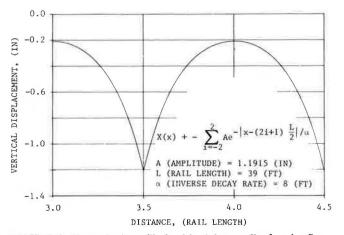


FIGURE 6 Vertical rail profile for 1-in. joint amplitude using five exponential terms.

SIMULATION RESULTS

Three types of track cross-level geometries were studied: cross-level variation on one-half staggered level track (typically encountered in jointed rails on tangent track), cross-level variation on one-half staggered superelevated track (to acquire insight into curving behavior), and cross-level variation at different staggers (situations for a rail stagger other than one-half). The two important safety performance measurements for rail car response are peak car body roll angle and maximum wheel lift. In this paper, the safety criteria chosen were a threshold value of 5 degrees (10 degrees peak-to-peak) for peak car body roll angle and 0.50-in. maximum allow-able wheel lift.

Results are presented for a 100-ton loaded hopper car with truck center spacing of 39.5 ft. Response for a 70-ton loaded box car is presented by Lee and Weinstock (8). The results presented here are for typical representative loaded cars having the nominal parameters given by Tse (4). More severe responses result with reduced snubber friction. Variations in truck design parameters, such as side bearing clearance and spacing, have a small effect on response characteristics. More significant improvements are indicated by the use of supplemental devices such as centerplate extension pads and hydraulic snubbers. The rail cars studied here, however, are typical of those in current service. It is expected that these cars will not be replaced or fully retrofitted in the immediate future. One-half staggered track is assumed and the car enters the cross-level variations from level track with zero initial conditions.

Peak car body roll angle and maximum wheel lift are shown in Figures 7 and 8, respectively, as a function of track cross-level amplitude. As the cross-level amplitude increases for a given speed,

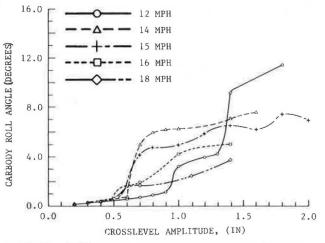


FIGURE 7 Peak car body roll angle response to periodic crosslevel variations.

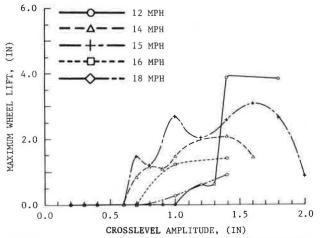


FIGURE 8 Maximum wheel lift response to periodic cross-level variations.

the peak car body roll angle increases almost linearly until a cross-level amplitude is reached, where a dramatic change in response occurs. At this critical amplitude, the peak car body roll angle continues increasing, but at a faster rate with increasing cross-level amplitude. The critical amplitude at which the response characteristic changes is a function of speed. For a 15 mph case, the critical cross-level amplitude is about 0.6 in. The 5-degree threshold value for peak car body roll angle is reached at 1 in. of cross-level amplitude. The trend for the maximum wheel lift is similar to the peak car body roll except that below the critical amplitudes there is no wheel lift, as shown in Figure 8. The 0.50-in. maximum allowable wheel lift is reached at a cross-level amplitude of 0.52 in. for a rail car at 15 mph.

A convenient display for defining the number of repetitions of a track cross-level variation amplitude that can be tolerated is the relation between the cross-level amplitude and the distance a rail car can travel before a derailment threshold is reached. The data points plotted in Figure 9 are the joint amplitudes that will produce a 0.50-in. wheel lift as a function of the length of track having consecutive low joints for a 70-ton boxcar and a 100-ton hopper car while traveling at the critical speed. Similar results have been constructed in terms of the threshold car body roll angle. For safe operation, a given track cross-level amplitude must not cause the vehicle to exceed these threshold values for a specified distance of travel.

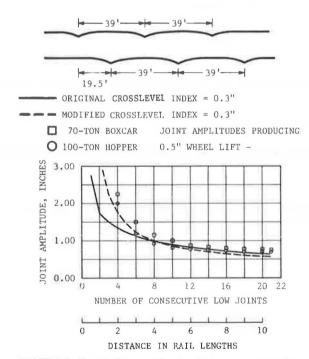


FIGURE 9 Permissible cross-level variations versus number of consecutive low joints.

Although consecutive low joints of 0.75-in. amplitude produce wheel lift and car rocking that would likely lead to derailment, consecutive low joints of 0.50 in. can be sustained indefir'ely. It was also found that for track segments with repeated low joints of 0.75-in. amplitude or greater, the results were strongly initial-condition dependent. For example, entering the zone at a decreasing speed would produce a different response than entering at an increasing speed. However, for consecutive low joints of less than 0.50 in., the results were not significantly influenced by initial conditions. If the track perturbations are limited to four consecutive low joints, then a joint amplitude of 1 in. can be sustained. As the number of consecutive low joints is reduced, a larger track cross-level amplitude can be safely sustained.

Response for the 100-ton covered hopper car was also obtained for continuous track cross-level variation on one-half-staggered track while operating at a speed corresponding to a 3-in. unbalance. This situation was simulated by running the computer program by Platin (5) on tangent track with a superelevation of 3 in. The car body roll angles, relative to the initial static car body roll angle (3.54 degrees), are only slightly higher than the corresponding values on level track. For wheel lift on track producing a 3-in. unbalance, the peak values are 23 percent lower than peaks in wheel lift occurring on level track.

The potential effects of track curvature on the roll response and wheel lift were investigated by Blader and Mealy (9). Their studies indicated that the roll response and wheel lift characteristics were not strongly related to track curvature. However, with track curvature, the wheel lift tendency was accompanied by a tendency toward wheel climb on the high rail.

In terms of overall dynamic response, a 3-in. Superelevation is no worse than the results obtained on level track ($\underline{8}$). The effect of variation in rail stagger is studied by using a track input of onethird and one-fourth rail stagger. As the rail stagger is reduced, there is an increase in the joint amplitude required to maintain a constant crosslevel amplitude. Comparison of peak car body roll angle and maximum wheel lift with similar results for one-half rail stagger indicate that varying the rail stagger has little effect on vehicle response at the corresponding cross-level amplitudes ($\underline{8}$).

TRACK SAFETY PERFORMANCE INDEX

Initial Formulation of Cross-Level Index

The requirements for a useful safety performance index are to develop a specification for identifying track with a sequence of cross-level variations that may produce unsafe response, without rejecting an excessive amount of track that does not have the potential for harmonic roll derailment. It must agree with simulation results and industry experience pertaining to derailment. For example, although isolated low joints as large as 4 in. can be safely traversed, a continuous series of consecutive 0.75-in. low joints will produce an unacceptable roll response of high c.g. cars traveling at the critical harmonic roll speed and result in a potential derailment. Simulation results also indicate that a 400-ft length of track is sufficient for the harmonic roll resonance to build to a critical amplitude. Conversely, existing transient roll oscillations are usually dissipated entirely in less than 10 rail lengths.

A root mean square (RMS) deviation in cross-level greater than 400 ft of track is used as the basic criterion in the track performance index. To prevent the index from being triggered by normal superelevation in curves or by the variation in superelevation associated with spirals, the calculations are based on the deviations of cross-level, $\delta(x)$, from a 100-ft moving average of cross-level, Z(x), as defined by

$$\delta_{1}(X) = Z(X) - \frac{1}{100} \int Z(V) dV$$
(2)
X-50

where

The initial formulation of the cross-level index is then

$$CLI(x) = \begin{pmatrix} x+200 \\ 1/400 & \int \delta_1^2 & (v) \, dv \end{pmatrix}^{1/2}$$
(3)
x-200

A property of this RMS cross-level deviation is that it acts to filter out those cross-level variations at wavelengths that are longer than the lengths of the moving averages. The intent of this filtering process is to exclude those cross-level variations that do not contribute to harmonic roll response, such as superelevation in curves. In order to quantify this filtering effect, a study was conducted of the effect of averaging length on the cross-level deviation for a sinusoidal cross-level variation with variable wavelength. Figure 10 is a plot of cross-level deviation against wavelength, with averaging length as a parameter. Wavelengths exceeding the averaging lengths are attenuated and filtered, whereas wavelengths that are shorter than the averaging lengths pass through.

With the cross-level index set at a limiting value of 0.3, Figure 9 shows the amplitude of crosslevel variation that would be permitted for a specified number of low joints within a 400-ft length of track. The specification implied by establishing a limit of 0.3 for the CLI is compared with the simulation results for a 100-ton and 70-ton car obtained for the same number of low joints encountered consecutively. This index value of 0.3 will permit consecutive cross-level variations up to 0.625 in. and represents a good fit to the simulation results for eight or more consecutive low joints (four rail lengths). For a smaller number of low joints, the index is somewhat conservative in comparison with the simulation results. However, current practice

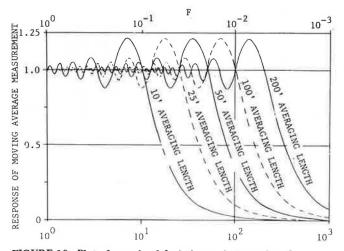


FIGURE 10 Plot of cross-level deviation against wavelength.

does not permit individual cross-level deviations greater than 2 in. for Class 2 track, so that the cross-level index with a level of 0.3 in. appears to be a fair representation of current accepted practice.

Although the cross-level index, as initially defined, has the advantage of simplicity of formulation, further evaluation indicated some drawbacks in selectivity. For example, the index was applied on a pilot basis to records of track geometry accumulated by the Federal Railroad Administration (FRA) to examine its effectiveness as a track safety index. As shown in Figure 11, the index could be triggered by large single events or large track warp. In reviewing the records it was found that two out of three times the cross-level index value of 0.3 was exceeded; there was either a periodic cross-level variation capable of inducing harmonic roll, or some other track situation likely to result in an unsafe condition. However, in one out of three exceedances, a clearly unsafe condition could not be identified although maintenance was definitely desirable.

Although this error rate might be acceptable for a maintenance criterion, it was believed that the error was too large to permit the index, as originally formulated, to be used to define safety. Other evaluations of the applicability of CLI indicated that in some cases, the index was triggered by 78-ft wavelength cross-level variations of a smaller level

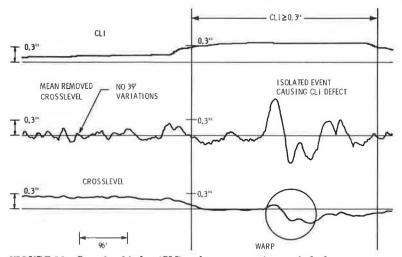


FIGURE 11 Cross-level index (CLI) and warp exception: spiral of curve.

than would be expected to cause harmonic roll problems.

To obtain further insight into wavelength effects, a study was conducted of the variation of CLI with wavelength due to a 1-in. amplitude sinusoidal cross-level variation. As shown in Figure 12, CLI has a fairly flat response for wavelengths up to 100 ft then falls off somewhat gradually at longer wavelengths, making it essentially a cross-level energy measure. It also has peak response at about a 78-ft wavelength, which is responsible for the sensitivity to warp and the longer wavelengths as previously noted.

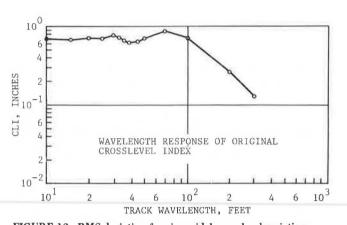


FIGURE 12 RMS deviation for sinusoidal cross-level variations, 1-in. amplitude.

Formulation of Modified Cross-Level Index

Modification to the cross-level index should permit the index to distinguish between randomly located low joints and successive low joints. A succession of six low joints occurring consecutively produces a more severe condition than three pair of low joints distributed with one pair at the start of the 400-ft track segment, one pair in the center, and the final pair at the end of the track. For a given amplitude, the first scenario would permit a significant roll to build up, whereas the second scenario would produce a response that would not be significantly worse than a single low joint pair. Therefore, additional terms were added to the cross-level index to represent the correlation of the cross-level variation with the cross-level at one and two rail lengths away. This produces a heavier weighting to periodic events than to events occurring several rail lengths away. The additional terms Q1, Q2, and 03 are defined as follows:

$$Q1(x) = 1/400 \int \delta_2^2(v) dv$$

x-200

$$x+200-(L/2)$$

$$y_{2}(x) = 1/(400 - L) \int \delta_{2}[v + (L/2)]$$

$$x-200+(L/2)$$

$$x \delta_{2}[v - (L/2)]dv$$

$$x+200-L Q3(x) = 1/(400 - 2L) \int \delta_2(v + L) \delta_2(v - L) dv$$
(4)
$$x-200+L$$

where L is rail length in feet (usually 39 ft). In addition, the averaging length was modified to 40 ft

to eliminate some of the undesirable amplification effects of the 100-ft moving average for wavelengths longer than 39 ft:

$$\delta_{2}(\mathbf{x}) = Z(\mathbf{x}) - \frac{1}{40} \int Z(\mathbf{v}) d\mathbf{v}$$
(5)
x-20

The final requirement in the formulation of the modified cross-level index was the proper weighting of the values of Q1, Q2, and Q3 to optimize predictions of derailment for a wide range of track wavelength. Initially, equal weighting of 1/3 was given to each correlation value, Q, producing the modified cross-level index with equal weighting CLIM_E:

$$CLIM_{\rm E} = [1/3 (Q1 + Q2 + Q3)]^{1/2}$$
 (6)

As shown in Figure 9, for consecutive low joints on 39-ft rail, $CLIM_E$ produced a much better agreement with the simulation results and was definitely less conservative than the original CLI. It should be noted, however, that with this equal weighting, the modified index requires a knowledge of the rail length in order to be applied. In a real-time inspection of track this information might be ambiguous, because of the use of mixed rail lengths or welded rail. To obtain more insight into the effect of rail length, the behavior of $CLIM_E$ with track cross-level variation wavelength was examined, as shown in Figure 13. The modified index with equal weighting and L set at 39 ft is sharply tuned to

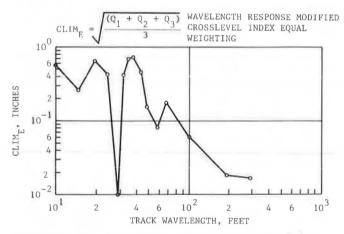


FIGURE 13 Behavior of CLIM_E with track cross-level variations.

39-ft wavelength effects, but sharply attenuates effects at rail lengths other than 39 ft, including some shorter rail lengths that are still in common use. At some of these shorter rail lengths, the index would not be conservative if L was fixed at 39 ft in the index computation, and it might permit the existence of potentially unsafe conditions. Consequently, a modified cross-level index with unequal weighting $CLIM_{II}$ was formulated,

$$CLIM_{TI} = (0.601 + 0.302 + 0.103)^{1/2}$$
(7)

with L set at 39 ft, which produces the wavelength characteristics shown in Figure 14. This weighting produces an emphasis of the 39-ft rail length as well as an adequate response to other rail lengths in common use. Comparison with simulation results indicates that $CLIM_{\rm U}$ provides good agreement with estimated safe limits.

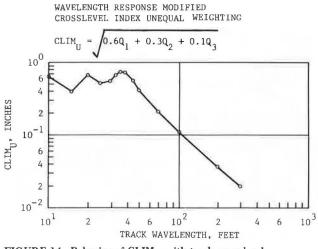
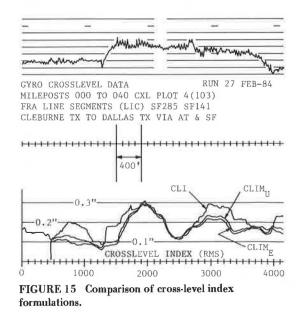


FIGURE 14 Behavior of \mbox{CLIM}_U with track cross-level variations.

STATISTICS OF CROSS-LEVEL INDEX

The behavior of the cross-level indices on selected track geometry records of track classified as Class 2 and Class 3 track has been studied by Ensco Incorporated under contract to the FRA. The study considered 223 miles of Class 2 and 361 miles of Class 3 track geometry data. All three index formulations successfully identify cross-level situations that are capable of producing harmonic roll response. However, the modified indices are more selective and do not respond to the large single exceedances that would trigger the original index. Comparison of the indices with typical field data containing significant harmonic cross-level variations is shown in Figure 15.



For the 223 miles of Class 2 geometry data studied, there were 243 locations at which the CLI index threshold of 0.3 was exceeded, representing 5.7 percent of the length of track. Of these 243 exceedances, 75 could be clearly associated with harmonic cross-level deviations, 62 were associated with exceedances to current FRA track warp standards, about 20 represented single cross-level deviations in excess of 2 in., and the remainder were produced by long wavelength effects.

For the 361 miles of Class 3 geometry data, there were 206 locations at which the CLI index threshold of 0.3 was exceeded, representing 1.2 percent of the length of track. Of these 206 exceedances, 18 were clearly associated with harmonic cross-level activity, 144 represented exceedances to current FRA track warp standards, about 10 represented single cross-level deviations in excess of 2 in., and the remainder were produced by long wavelength effects.

For the 223 miles of Class 2 data, the modified cross-level index $CLIM_U$ threshold of 0.3 was exceeded at about 42 locations, representing about 0.9 percent of the track length. For the Class 3 data, the CLIM_U index of 0.3 was exceeded at 18 locations, representing about 0.13 percent of the track length. Each of the locations identified by exceedances of the CLIM_U threshold of 0.3 could be identified with a high level of harmonic cross-level deviation activity.

The results of the analyses of the statistics of the cross-level index indicate that it is successful at identifying locations of cross-level variations that are capable of producing harmonic roll derailment. It is, however, sensitive to other track cross-level deviations that are not likely to produce harmonic roll and may not require as high a level of maintenance priority. It is therefore recommended that the CLI index be used as a maintenance tool to identify track segments with high crosslevel activity that should be given special attention in maintenance planning.

The modified index $CLIM_U$ has been found to be a highly selective identifier of track segments capable of producing harmonic roll derailment. It is hoped that this index will be adopted by the industry as a safety specification for improving control of harmonic roll.

PILOT APPLICATION OF CROSS-LEVEL INDEX

To facilitate measurement of cross-level deviations and computation of the cross-level index, the Transportation Systems Center (TSC) has developed a portable, self-contained cross-level measurement system that is mounted on the end of a locomotive axle. This system includes an environmentally rugged navigation grade rate integrating gyroscope that measures the roll angle of the locomotive axle. The signals from the gyroscope are transmitted to a minicomputer that calculates the cross-level and cross-level indices on a continuous basis. The computed data are displayed on a chart recorder and are recorded on a cassette tape recorder for use in later analysis. This instrumentation package is shown in Figure 16.

This system permits the measurement of crosslevel and computation of the cross-level index during normally scheduled runs without requiring special measurement cars or interfering with normal train operating schedules. Use of the locomotive axle results in loaded track geometry measurements with vertical loads that are comparable to those of loaded hopper cars.

The system has been applied to recent surveys of track condition on track owned by the following railroads: Sante Fe, Burlington Northern, Kansas City Southern, Boston and Maine, and the Alaskan railroad.

In addition, several railroads have been augmenting their current track geometry data collection and maintenance and safety programs with computation and

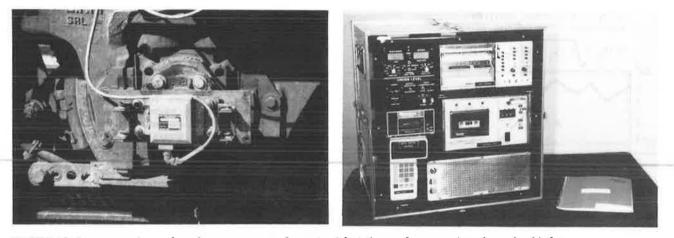


FIGURE 16 Instrumentation package for measurement of cross-level deviations and computation of cross-level index.

evaluation of the cross-level indices. The Chessie System has integrated the CLI index algorithm into its geometry car, and the Norfolk and Southern, Boston and Maine, and Atchison Topeka & Sante Fe railroads have been active in processing and monitoring CLI and CLIM_U data.

These activities are expected to provide the basis for comparing the performance of the crosslevel indices with the judgment and operational experience of railroad personnel under actual operating conditions. The results reported at this point tend to provide additional confidence in the use of the CLI as a tool for maintenance surveys of track cross-level and the use of CLIM_U as an indicator of potentially unsafe track.

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