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## System and Route Optimization Model for Minimizing Urban Transit Operating Deficits

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### ABSTRACT

U.S. transit operators are faced with escalating operating deficits along with growing opposition to the increase in taxes required to offset them. This financial situation has created an immediate need to restructure inefficient and underproductive transit operations. In response to this need, this study developed an analytical framework to help control transit operating deficits. A bi-level optimization model based on nonlinear programming was developed at system and route levels of detail. The model postulates that transit operators could reach a feasible solution for minimizing operating deficits through modifications of current fare and service policies. The model has an economic framework (through the specification of appropriate cost and revenue functions) and solves for optimality through system supply-demand equilibrium. Solutions of the optimization model will provide transit operators with specific operating guidelines for minimizing deficits subject to resource and policy constraints. The nonlinear optimization model is solved using a large-scale (sparse matrix) successive linear programming algorithm. The model was implemented on a micro-computer and was tested with a real-world application to establish its practicality and usefulness.

The financial status of most urban transit properties in the United States is at best bleak. During the past decade, total operating deficits rose more than \$4.5 billion, and the problem is likely to get worse. The underlying causes of operating deficits are escalation in transit operating costs, rapid

service expansion, and operators' decisions to reduce fare levels. In the past, deficits have been met primarily by government subsidies, with a significant share coming from federal sources. However, as part of the Reagan administration's Program for Economic Recovery, federal operating assistance to

local transit properties may be scheduled to be phased out. With the expected subsidy reduction and escalating operating costs, transit operators will obviously be faced with serious financial difficulties.

If the transit system is to be a viable element in urban transportation, it is imperative that the operators establish and maintain a self-reliant operating budget. In considering ways to operate without federal assistance, a first response of most transit operators is to alter fare or service policies, or both. Results from the American Public Transit Association survey indicate that 89 percent of the nation's operators will raise fares and 67 percent will reduce service (1). This trend has been substantiated by the results of another national survey (2). Although an increase in fare may be instrumental in improving transit financing, it must be made without unduly suppressing ridership, because this could actually result in decreasing the overall fare-box revenue. Along with an increase in fares, reductions in operating costs through service cuts may also be necessary to decrease deficits. However, the public demand for transit travel is usually more sensitive to the quality of service than to the level of fare (3).

The net effect is that a reduction in transit service will have a greater negative impact on ridership and hence on revenues than will an increase in fare. Further, transit fare and service variations leading to adjustments in operating costs and revenues should be considered as an interactive process. Although fare and service structures lead to a level of fare-box revenue, a targeted level of revenue can also dictate fare and service policies. Revenue increases resulting from fare hikes, for example, could lead to a demand for a commensurate improvement in the quality of service. This may result in an increase in costs and hence deficits, necessitating a further increase in fares to keep the deficit at the same level. Thus, fare, service costs, revenues, and deficits interact dynamically and sequentially.

Many transit operators have also indicated that the federal subsidy loss will be partly recovered by increased operational efficiency (4). The transit industry is being encouraged to become more productive, not only because of diminishing federal subsidies but also because of the overall economic conditions of the operators. However, this is a difficult task because transit policy making has many components and the problem of reducing operating deficits has many dimensions. Economic, social, and political factors all bear on fare and service decisions. The complexity of the problem stems not only from its magnitude but from the diversity of its parts. At present, there remains a scarcity of comprehensive, yet easy-to-use, procedures that model the realities of transit pricing and operation to aid operators in reducing deficits through fare and service modifications. The literature review on transit planning and optimization aids included in Kour (5) and Yu and Lall (6) bears out this statement.

#### STUDY OBJECTIVE

The principal objective of this study was to develop an optimization model to minimize or control transit operating deficits by manipulating current fare and service policies, instead of by capital-intensive system changes. The model, which is fully responsive to the typical environment of the urban transit system, can be used as an effective management and planning tool. Emphasis was placed on the practical-

ity of the approach, so the data required are either readily available from or easily assembled by transit operators. The model can be simply applied by using a variety of microcomputers with an interactive preprogrammed package. The model is able to assess the impacts of and develop a strategy for the implementation of various fare and service policies, subject to resource limitations and policy constraints for different situations in which transit services are provided.

The optimization model was developed with the following specific objectives:

1. Generality of application (independent of route configuration and temporal period of application);
2. Focus on minor system modifications (i.e., fare, service frequency, stop spacing) of an existing system;
3. Satisfaction of transit goals specified at the system level through modifications implemented at the route level (i.e., treatment of each route individually and simultaneously in a systemwide context); and
4. Accurate representation of costs, service options, relationship of demand to fare and service, interactions between transit operation components and between supply and demand, and physical and social constraints on system operation.

#### MODEL FORMULATION

The complexities of transit operation and the need to provide solutions at a relevant level of detail necessitate the consideration of two hierarchical levels of analysis: the system as a whole and the individual route. The reason for this system and route structure is the need to provide solutions to the overall systemwide problem that can be implemented at the route level. The model structure is shown in Figure 1. The optimization model attempts to minimize systemwide operating deficit using route fare and service characteristics as decision variables. Transit operating deficits are total operating costs minus total fare-box revenues. Fare-box management and service cost control are inextricably intertwined. Optimal solutions to these two problems are not independent.

#### Objective Function

The objective function represents system operating deficits and is expressed in terms of a set of model variables and input parameters. Model variables are partitioned into two subsets: decision variables and relational variables. The latter are quantities defined as functions of the former and are used for a concise model presentation. Decision variables are defined for fare and service options over which transit operators have control. The optimal solution that leads to a minimum operating deficit is obtained by iteratively selecting values for these variables. Interaction between the costs of providing services and the revenue generated by these services determines the optimal state in a supply-demand equilibrium framework.

The objective function is formulated as the difference between system operating cost, defined as the sum of individual route operating costs, and system revenue, defined as the sum of individual route revenues. The objective function is stated as

$$\text{Minimize } D = \sum_{i=1}^I (C_i - R_i) \quad (1)$$

where

- $D$  = total system operating deficit,  
 $C_i$  = operating cost of route  $i$ ,  
 $R_i$  = fare-box revenue of route  $i$ , and  
 $I$  = total number of routes in the system.

This function can be applied to any independent time-of-day (i.e., peak, off-peak, weekend) operation and for any length of the total planning period (i.e., month, season, year). In computing operating deficit, the planning period usually refers to 1 year because the operator's budget outlay is typically on an annual basis.

In the following sections the formulation of the operating cost function and of the operating revenue function as components of the objective function will be briefly presented.

#### Operating Cost Function

The operating cost function is formulated using a cost allocation procedure designed during this study. This procedure assigns all relevant variable costs to four resources: vehicle-hours, vehicle-miles, peak vehicles, and stops. The cost of vehicle-hours relates to labor costs, whereas the cost of vehicle-miles reflects vehicle operation costs. Vehicle and stop costs are confined to the local share of capital depreciation because both are subsidized through federal capital grants. This assignment procedure is a significant departure from the traditional one. Only those variable costs that vary directly with minor system modifications are taken into consideration. All fixed costs and some of the operating

costs that do not vary with minor service changes are not included because they are not optimizable. A detailed discussion of the assignment procedure can be found elsewhere (6).

The operating cost function at the route level can be expressed as

$$C = c_h H + c_m M + c_v V + c_y Y \quad (2)$$

where

- $C$  = total operating cost of a route,  
 $c_h$  = unit cost of vehicle-hours,  
 $c_m$  = unit cost of vehicle-miles,  
 $c_v$  = unit cost of vehicles,  
 $c_y$  = unit cost of stops,  
 $H$  = vehicle-hours operated for the route,  
 $M$  = vehicle-miles operated for the route,  
 $V$  = peak vehicles needed for the route, and  
 $Y$  = total number of stops on the route.

All route costs are summed to obtain the total system cost. The unit costs are assumed to be constant for the range of system modifications and the duration of planning period considered. The unit cost of each resource is derived by dividing the total system cost allocated to a resource by the total use of that resource.

The four resources for each route are then presented in terms of decision variables (frequency of service and stop spacing), relational variable (vehicle operating speed), and other input parameters as follows:

$$H = \lambda a(1 + L)n/u \quad (3)$$

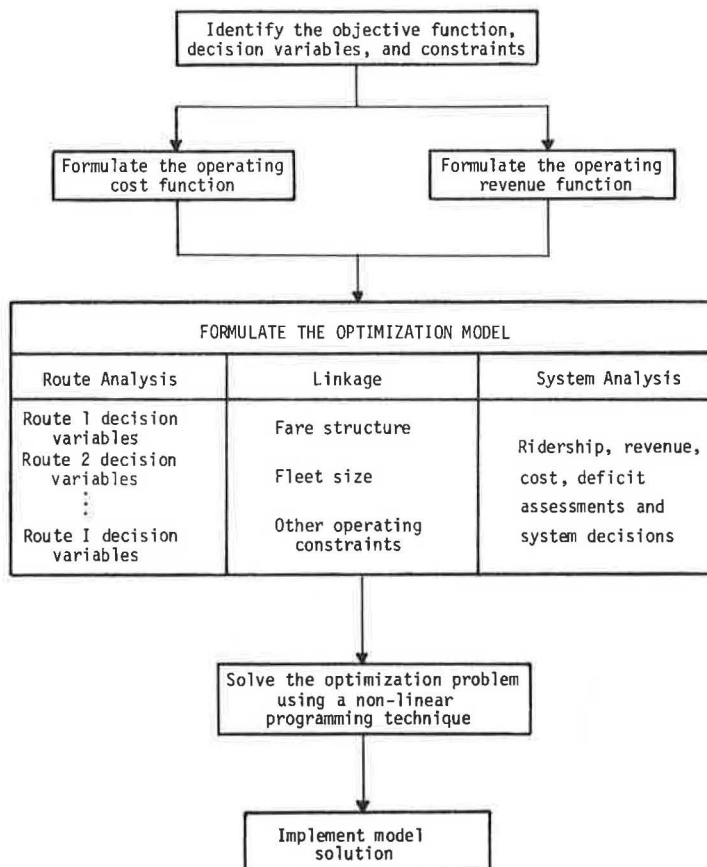


FIGURE 1 Developmental framework of the optimization model.

$$M = \lambda a n \quad (4)$$

$$V = \lambda (1 + P_1) n / u \quad (5)$$

$$Y = \lambda y \quad (6)$$

where

- $\lambda$  = round-trip route length in miles,
- $a$  = service hours per operating period,
- $L$  = layover time factor as a fraction of round-trip travel time,
- $n$  = frequency of service per hour,
- $u$  = average vehicle operating speed in miles per hour (mph),
- $P_1$  = additional vehicle factor as a fraction of vehicles operated on the route, and
- $y$  = number of stops per route mile.

Equation 3 states that the annual vehicle-hours operated on a route are given as the product of round-trip travel time (round-trip length divided by operating speed), annual operating hours, and frequency of service per hour. The amount of time spent on each round trip includes the layover time required. Equation 4 states that the number of annual vehicle-miles of a route is given by the round-trip route length multiplied by the number of round trips during the operating period. Equation 5 states that the number of peak vehicles used on a route is given by the product of round-trip time and frequency of service, divided by the average operating speed, plus additional vehicles expected to be on the maintenance schedule and needed to meet other requirements. Equation 6 states that the total number of stops on a route is equal to the product of round-trip route length and number of stops per mile on that route.

The average operating speed (relative variable) for a route can be expressed in terms of decision variables and input parameters as follows:

$$u = \lambda / \{ [(\lambda - d\lambda y)/r] + [(d\lambda y)/V_a] + [(c\lambda y)/3600] + (bQ/3600a) \} \quad (7)$$

where

- $r$  = vehicle peak running speed between stops on the route (mph),
- $V_a$  = average vehicle speed during acceleration and deceleration (mph),
- $d$  = average distance traveled during acceleration and deceleration per stop (miles),
- $c$  = vehicle clearance time per stop (sec),
- $b$  = boarding and alighting time per rider at a stop (sec), and
- $Q$  = ridership of an operating period for the route.

It is widely recognized in the transit industry that the costs associated with providing service during peak, off-peak, and weekend periods might differ substantially due to the quantity and quality of service required. To account for this temporal variation, a procedure based on Cherwony's and Mundle's peak-base model (7) was employed to derive unit cost adjustment factors for each of the three periods of service. In support of using such a relatively simple costing procedure, a comparative study recently performed by Carter, Mundle, and McCollom on various costing procedures (8) found that the increased sensitivity and complexity of the more detailed procedures did not increase relative model accuracy for minor service modifications. In addition, information required by the peak-base model

was found to be more easily obtainable from the transit operator.

The following equation, based on the peak-base model, was used to calculate the unit cost adjustment factors for temporal variations:

$$A_p = (H_p^d / H_p^v) \left( \sum_p H_p^v / \sum_p H_p^d \right) \quad (8)$$

where

- $p$  = peak, off-peak, weekend, or night period index;
- $A_p$  = temporal variation adjustment factor for period  $p$ ;
- $H_p^d$  = driver pay-hours for period  $p$ ; and
- $H_p^v$  = revenue vehicle-hours for period  $p$ .

The adjustment factors are multiplied by the basic vehicle-hour unit cost to achieve separate unit costs for different service periods. The factors are applied only to the vehicle-hour unit cost because the major percentage of temporal variation in cost results from variability in driver pay-hours and corresponding benefits, which are the main input in calculating the basic vehicle-hour unit cost, for various periods of operation.

In calculating the total system operating cost using the cost functions for different operating periods, the vehicle cost and stop cost would be repeatedly counted. To avoid this cost repetition, unit cost weighting factors are devised to rationally distribute the peak vehicle cost and the stop cost among the different operating periods. The peak vehicle cost is basically the capital depreciation of the vehicles, which in turn is a function of vehicle mileage. Thus the weighting factors are determined by

$$W_p^v = M_p / \sum_p M_p \quad (9)$$

where  $W_p^v$  is weighting factor of vehicle cost for period  $p$  and  $M_p$  is vehicle-miles generated during period  $p$ . The vehicle unit cost weighting factors are then normalized so that the sum of the factors equals one (i.e.,  $\sum W_p^v = 1$ ).

The unit cost weighting factors ( $W_p^y$ ) for the stop costs are determined using a similar procedure. Instead of using revenue vehicle-miles, the relative number of revenue vehicle-hours for different operating periods of stop utilized are used. The stop cost weighting factor is therefore obtained by

$$W_p^y = H_p / \sum_p H_p \quad (10)$$

where  $W_p^y$  is weighting factor of stop cost for period  $p$  and  $H_p$  is time-sharing stop utilization during period  $p$ . Again, the factors  $W_p^y$  are normalized and applied to the unit stop cost in each time-of-day cost equation so that  $\sum W_p^y = 1$ .

Incorporating Equations 3 through 9 and 8 through 10, the final total cost function for a route is obtained as follows:

$$C = c_h A_t \lambda a (1 + L) n / u + c_m \lambda a n + c_v W_p^v \lambda (1 + P_1) n / u + c_y W_p^y \lambda y \quad (11)$$

### Operating Revenue Function

The revenue function is formulated by examining the effects of service and fare structure modifications on ridership and hence on fare-box revenue. The fare-box revenue on a particular route is computed as the product of the weighted average fare and the total ridership. The demand function is specified in terms of model variables through elasticity considerations.

The average fare for a route is determined by

$$F = F_C \sum_j w_j e_j \quad (12)$$

where

- $F$  = average fare for a route;
- $F_C$  = base cash fare;
- $w_j$  = fraction of riders on the route using method of payment  $j$ ; and
- $e_j$  = discount rate (i.e., the ratio of fare paid by the  $j$ th method to base cash fare).

In addition to the level of fare, the average trip time of riders is selected as a crucial measure of the quality of transit service and its impact on rider demand responsiveness. Although other measures of service quality (e.g., reliability) were considered, total trip time was believed to be of greatest concern to riders, and it lends itself most easily to the quantitative treatment required for inclusion in the model. Travel time is divided into in-vehicle time and out-of-vehicle time.

Ridership response is modeled with a "shrinkage-ratio" elasticity formulation. Although other formulations may have somewhat greater theoretical validity, the paucity of data precludes their use in practice. It was thought that a shrinkage-ratio formulation would provide adequate accuracy over the constrained range of response modeled. The route revenue function for each route is represented as

$$R = F_C^0 \left\{ 1 + \sum_j a_j w_j [(F_C - F_C^0)/F_C^0] + \beta_1 [(t_1 - t_1^0)/t_1^0] + \beta_2 [(t_2 - t_2^0)/t_2^0] \right\} \quad (13)$$

where

- $R$  = route fare-box revenue,
- $F_C^0$  = existing route ridership,
- $a_j$  = fare elasticity of rider group  $j$ ,
- $\beta_1$  = in-vehicle time elasticity,
- $\beta_2$  = out-of-vehicle time elasticity,
- $t_1$  = average in-vehicle time, and
- $t_2$  = average out-of-vehicle time.

The superscript  $o$  represents a value for existing conditions (i.e., input).

The average rider in-vehicle time on a travel route is defined by

$$t_1 = L_a / u \quad (14)$$

where  $L_a$  is the average round-trip length for a rider on the route.

In the framework of the optimization model it is assumed that the average trip lengths ( $L_a$ ) are not affected by minor system modifications during the course of optimization. This implies that origin-destination characteristics of riders on a given route are relatively stable.

Out-of-vehicle time has two components: walking time and waiting time. The former is the time spent

from the rider's origin or destination to a stop and the latter is the time spent waiting for a vehicle after arriving at a stop.

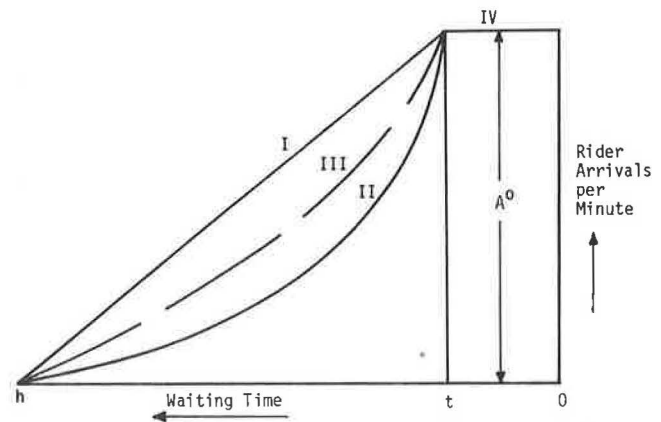
Average walking time is obtained by dividing the average walking distance by the average walking speed of the riders (normally 3 mph). Determination of average walking distance is based on the assumption that potential riders are uniformly distributed in the neighborhood of a stop and that there is a maximum walking distance ( $W_m$ ) beyond which no riders are attracted. The average walking time derived by this study is given as

$$t_w = \{1 + 2W_m y(n_s + m_s + 1) + 2[W_1 n_s(n_s + 1) + W_2 m_s(m_s + 1)]\} / [4y(1 + n_s + m_s)V_s] \quad (15)$$

where

- $n_s$  = average number of blocks walked parallel to a route =  $2 \times \text{int}(W_m/W_1)$ ,
- $m_s$  = average number of blocks walked perpendicular to a route =  $2 \times \text{int}(1/2yW_2)$ ,
- $W_1$  = average block length along the route,
- $W_2$  = average block length perpendicular to the route,
- $V_s$  = average walking speed, and
- $\text{int}$  = integer operator.

Estimation of average waiting time is based on the assumption that the rider arrival rate is uniform during a final waiting time interval ( $t$ ) and is a mixture of an exponential and triangular distributions during the early waiting time (from the time of departure of the previous vehicle to the start of the final arrival period) if the headway ( $h$ ) is greater than  $t$ . An exponential arrival rate distribution implies the response of well-informed and knowledgeable riders served by a reliable transit system; a triangular distribution implies riders who are misinformed or not well aware of service schedules. Figure 2 shows the concept of waiting times as



- I = Triangular distribution for arrival rate
- II = Exponential distribution for arrival rate
- III = Mixed (I & II) distribution for arrival rate
- IV = Uniform arrival distribution

FIGURE 2 Function of waiting time distribution.

defined. If no information on riders' awareness is available, the average waiting time is

$$t_s = 1/2(t/3) + [1200/(tn^2 + 60n)] + \{[n(t^2 + 2t + 2)e^{-t} - (1200n + 2n^2 + 3600)e^{-60/n}] \div [2n^2(t + 1)e^{-t} - 2(n^2 + 60n)e^{-60/n}]\} \quad (16)$$



The total out-of-vehicle time ( $t_2$ ) is then given by  $t_w + t_s$ . The detailed derivation of  $t_w$  and  $t_s$  can be found in the final research report (6).

The complete specification of the revenue function for a route can be expressed as follows:

$$R = (F_C \sum_j w_j e_j) Q^0 \left( 1 + \sum_j a_j w_j [(F_C - F_C^0)/F_C^0] + \beta_1 (u^0/u - 1) + \beta_2 [(t_w + t_s)/(t_w^0 + t_s^0)] - 1 \right) \quad (17)$$

### Constraints and Bounds

The objective function is minimized subject to a set of existing resource and policy constraints. Explicit constraints are specified to (a) limit system peak vehicle use to a ratio of the existing fleet size, (b) limit peak ridership per vehicle to maximum vehicle loading capacity for each route, and (c) constrain total system ridership to a desired ratio of total existing ridership.

### Constraints

#### System Fleet Size

The peak number of vehicles used for the system should not exceed a specified fleet size ( $N_f^u$ ) and should have a value of at least  $N_f^l$ .

$$N_f^l \leq (1 + P_1) \sum_i l_i n_i / u_i \leq N_f^u \quad (18)$$

Superscripts  $u$  and  $l$  represent upper and lower limits and subscript  $i$  refers to the  $i$ th route.

#### Permissible Vehicle Loading

The number of riders on any vehicle should not exceed the capacity of the vehicle ( $L_p$ ). The constraint applies to each route.

$$P_2 P_3 (Q_i / a_i u_i) \leq L_p \quad (19)$$

where  $P_2$  is ratio of peak load in major flow direction to average round-trip loading and  $P_3$  is average vehicle occupancy factor between major loading points in major flow direction.

#### Total System Ridership

Total system ridership should remain above some fraction ( $q$ ) of total existing ridership ( $Q_t^0$ ):

$$\sum_i Q_i \geq q Q_t^0 \quad (20)$$

### Bounds

Upper and lower bounds (superscripts  $u$  and  $l$ ) are placed on each decision variable ( $x$ ) where  $x$  includes  $F_C$ ,  $n$ , and  $y$  for all routes:

$$x^l \leq x \leq x^u \quad (21)$$

### SOLUTION TECHNIQUE

As indicated previously, the optimization model is based on a nonlinear programming technique. The non-

linear programming problem formulated is solved using successive linear programming. The algorithm used is one developed by Palacios-Gomez et al. (9) and Lasdon and Kim (10).

The general statement of the nonlinear programming problem is

$$\begin{aligned} &\text{Minimize } f(x) \\ &\text{subject to} \\ &g^l \leq g(x) \leq g^u \\ &x^l \leq x \leq x^u \end{aligned}$$

where

$x$  = a vector of decision variables;  
 $f(x)$  = the objective function;  
 $g(x)$  = a vector of constraints, with  $g^l$  and  $g^u$  vectors of constraint for lower and upper bounds, respectively; and  
 $x^l$  and  $x^u$  = vectors of lower and upper bounds on the variables, respectively.

The constraint set is assumed to be composed of a mix of purely linear and nonlinear constraints. The vector of variables is also partitioned into two subsets: linear and nonlinear. The nonlinear constraints are then transferred to the objective function using penalty weights specified by the user.

The optimization problem can then be stated as

$$\begin{aligned} &\text{Minimize } f(x) + W g(x) \\ &\text{subject to} \\ &b_1 < \sum_j a_{ij} x_{jl} < b_2 \\ &x^l < x < x^u \end{aligned}$$

where

$W$  = penalty weights,  
 $a_{ij}$  = coefficients of the  $j$ th linear variable in the  $i$ th linear constraint,  
 $x_{jl}$  = linear variables, and  
 $b_1$  and  $b_2$  = bounds on the linear constraints.

The problem is then linearized by evaluating the nonlinear objective function using a Taylor series approximation at a current solution. The resulting linear problem is solved using a standard linear programming (LP) algorithm designed for large sparse matrices. The LP solution is then used to compute the feasibility of the nonlinear constraints. If the solution to the nonlinear constraints is infeasible, Newton's method is used to find the closest feasible point. A new LP iteration is initiated at this stage and the process is repeated. For a purely nonlinear problem, the algorithm behaves in the same manner as the well-known gradient projection algorithm. A number of criteria are used for termination of the iterative scheme. These include (a) satisfaction of the Kuhn-Tucker conditions, (b) cycling between iterations, (c) slow rate of improvement of objective function, and (d) slow rate of change in penalty functions at an infeasible point.

The implementation of the standard LP package is transparent to transit operators; no mathematical sophistication on their part is required. Input can be provided from data files or interactively with a matrix-generating program that provides data prompts and input instructions.

### MODEL COMPUTERIZATION

An interactive, user-friendly, machine-independent, modular structure was adopted for the computer im-

plementation of the optimization model. All the computer programs are developed as a modular package with three basic components--a preprocessor, an optimization process, and a postprocessor. Each of these packages is basically independent allowing for ease of modification. The programs are written in ANSI standard FORTRAN 77 for portability and are fully interactive with the user with respect to data input, result output, and help displays. Backup files for all data entered into the programs are automatically provided, with a label for each piece of data. Provisions have also been made for data entry using data files. The data saved in the backup files may be edited and submitted as a data file.

#### MODEL APPLICATION--CASE STUDY

The optimization model was applied to a medium-sized transit system operated by the Utah Transit Authority (UTA) to demonstrate its real-world usefulness and practicality. Fiscal year 1983 data were used for this case study. UTA serves an area covering approximately 200 square miles and encompassing two main urbanized areas, Salt Lake City and Ogden. During the study year the population of the service area was estimated to be about 910,000. UTA employment was a total of 745, of which 410 were hired as bus drivers for a fleet of about 400 vehicles. There are 89, 69, and 60 routes for regular peak, off-peak, and weekend services, respectively. UTA received total operating subsidies amounting to more than \$21 million in 1983 with more than \$4.6 million coming from federal sources and earned fare-box revenues covering only 21 percent of total operating expenses.

In applying the model the bulk of the data was obtained from UTA, mostly from their 1983 Section 15 annual report (11). However, because the Section 15 report provides systemwide data only, route-level data were obtained or derived from UTA's surveys, schedules, monthly passenger count summaries, and technical study reports, as well as personal interviews and special studies conducted by the authors with the help of UTA.

The total expenses incurred and resources provided, along with the unit costs calculated for four resources, are given in Table 1. These values are

TABLE 1 System Cost and Resource Totals and Unit Costs for UTA Regular Services

Resource	Expenses Assigned (\$) (A)	Resource Provided (B)	Unit Cost (\$) (A ÷ B)
Vehicle-hours	7,742,483	533,564	$c_h = 14.51$
Vehicle-miles	7,014,497	8,461,880	$c_m = 0.83$
Peak vehicles	231,815	361	$c_v = 642.15$
Stops	55,750	11,150	$c_y = 5.00$

operating statistics during peak, off-peak, and weekend service periods. The basic unit costs were then subjected to the temporal and weighting factor adjustments discussed previously. All adjustment factors and the final unit costs for each of the three service periods are given in Table 2.

The revenue function for UTA was estimated by using the values of those parameters shown in Equation 17. A summary of these parameter values is given in Table 3. All parameter values vary by route and are not presented in this paper.

TABLE 3 Selected Input Parameters for UTA<sup>a</sup>

Parameter	Peak	Off-Peak	Weekend
Cash fare, $F_c$ (\$)	0.50	0.40	0.40
Methods of payment, $j$	5	5	5
Discount rate, $e_j$ (percentage of cash fare)			
Student	0.6337	0.6337	0.6337
Adult pass	0.8770	0.8770	0.8770
Special group pass	0.4385	0.4385	0.4385
Commuter pass	1.0719	1.0719	1.0719
Fare elasticity, $\alpha_j$			
Cash	-0.33	-0.43	-0.43
Student pass	-0.44	-0.54	-0.54
Adult pass	-0.32	-0.42	-0.42
Special group pass	-0.35	-0.45	-0.45
Commuter pass	-0.11	-0.21	-0.21
In-vehicle time elasticity, $\beta_1$	-0.52	-0.12	-0.12
Out-of-vehicle time elasticity, $\beta_2$	-0.59	-0.51	-0.51
Final waiting time interval, $t$ (min)	10	10	10
Average street block length, $W_1$ and $W_2$ (mile)	0.1	0.1	0.1
Rider walking speed, $W_s$ (mph)	3	3	3
Maximum walking distance, $W_m$ (mile)	0.25	0.25	0.25
Rider awareness factor, $w_t$	0.5	0.5	0.5

<sup>a</sup>From UTA and the literature.

The model was independently applied to peak, off-peak, and weekend periods of operation. The overall deficits for optimal conditions could be compared with the actual deficits incurred for the study year 1983. Comparisons of actual and estimated quantities for existing conditions revealed insignificant differences (less than 3 percent error for all service periods), indicating that the model performs well as a forecasting tool.

The computerized model produces an extensive amount of information at the system and route levels, such as the amounts of revenue, cost, deficits, and resources used for existing and optimal conditions. The model also produces recommended service and fare policy changes to achieve the goal of minimizing operating deficits. In addition, values of system and route performance indicators are provided. Performance indicators are formulated to represent a variety of perspectives on transit system performance. A sample computer output for system-level and route-level results is shown in Figures 3 and 4, respectively.

TABLE 2 Adjustment Factors and Final Unit Costs for UTA

Period	Vehicle-Hour Unit Cost Adjustment Factor ( $A_p$ )	Peak Vehicle Cost Weighting Factor ( $W_p^V$ )	Stop Cost Weighting Factor ( $W_s^C$ )	Vehicle-Hour Unit Cost ( $c_h$ )	Vehicle-Mile Unit Cost ( $c_m$ )	Peak Vehicle Unit Cost ( $c_v$ )	Stop Unit Cost ( $c_y$ )
Peak	1.067	0.353	0.450	15.48	0.83	226.04	2.25
Off-peak	0.962	0.491	0.419	13.96	0.83	315.30	2.10
Weekend	0.955	0.157	0.313	13.86	0.83	100.82	0.66

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## TEST SLC AND OGDEN FOR APRIL 12 1984 PEAK HOUR

SYSTEM LEVEL RESULTS			
CATEGORY	EXISTING	OPTIMAL	%CHANGE
TOTAL OPER. DEFICIT (\$)	3181855.85	2501739.40	-21.37
TOTAL OPER. REVENUE (\$)	2382920.49	2561575.36	7.50
CASH FARE	.50	.54	8.00
AVERAGE FARE	.35	.38	8.57
ANNUAL RIDERSHIP	6833147.70	6805441.91	-.41
TOTAL OPER. COST (\$)	5564776.34	5063314.76	-9.01
TOTAL REV.VEH. HOURS	191417.00	171084.41	-10.62
COST OF REV.VEH. HOURS(\$)	3035181.25	2712779.97	-10.62
TOTAL REV.VEH. MILES	2942509.00	2735465.27	-7.04
COST OF REV.VEH. MILES(\$)	2439200.38	2267570.95	-7.04
FLEET UTILIZED	158.99	141.67	-10.38
TOTAL FLEET SIZE	361.00	321.72	-10.88
COST OF REV.VEH. (\$)	60619.23	54023.46	-10.88
TOTAL NUMBER OF STOPS	11217.97	10903.34	-2.80
AV. STOP SPACING/MILE	5.48	4.95	-9.67
TOTAL COST OF STOPS (\$)	29775.48	28940.37	-2.80
AVERAGE FREQUENCY/HR.	1.53	1.40	-8.50
PERFORMANCE INDICATORS			
REV.VEH. MILES/REV.VEH.	8150.99	8502.60	4.31
REV.VEH. HOURS/OPER.COST	.03	.03	.00
REV.VEH.MI./REV.VEH.HRS	15.37	15.99	4.03
RIDERS/REV.VEH. HOUR	35.70	39.78	11.43
REVENUE/REV.VEH. HOUR	12.45	14.97	20.24
RIDERS/OPER.COST	1.23	1.34	8.94
OPER.REVENUE/OPER.COST	.43	.51	18.60
IN VEH TIME (MIN/MI)	3.08	3.08	.00
AV. WALKING TIME (MIN)	4.65	4.60	-1.08
AV. WAITING TIME (MIN)	9.81	9.71	-1.02
AV OUT OF VEH TIME (MIN)	14.46	14.31	-1.04
AV OPER SPEED (MPH)	19.50	19.46	-.21
OPER.COST/RIDER	.81	.74	-8.64
OPER.DEFICIT/RIDER	.47	.37	-21.28
REV.VEH. HOURS/REV.VEH.	530.24	531.78	.29
REVENUE/REV.VEH. MILE	.81	.94	16.05
RIDERS/PEAK REV.VEH.	44.06	48.08	9.12
RIDERS/REV.VEH. MILE	2.32	2.49	7.33
REV.VEH. MILES/OPER.COST	.53	.54	1.89

FIGURE 3 Sample computer printout of system-level results.

System-Level Study Results

Using the cost and revenue input data, the optimization model was independently applied to peak, off-peak, and weekend periods of service. The system cost, revenue, and deficit totals are obtained by aggregating the statistics for each of the three periods of operation.

Because ridership is a major indicator of social benefit and is a crucial performance measure with respect to system productivity, it was meaningful to conduct a parametric sensitivity analysis to see how ridership levels influenced deficit totals. For this case study, optimal solutions were obtained for five levels (80, 90, 100, 110, and 120 percent) of present ridership to illustrate the interactive effects between cost, revenue, deficits, and ridership. Thus the relative change in ridership can be estimated for various expected deficit levels, and, conversely, future deficits can be estimated using targeted ridership levels. In addition, deficits,

costs, and revenues, the key factors of interest to all transit operators, were examined together with the underlying causes affecting any forecast changes in ridership, service, and fare levels.

Operating Deficits

The relationship between the optimal UTA deficits for the five levels of ridership and existing UTA deficits is shown in Figure 5. Point A in the figure represents the optimal level of deficit corresponding to no federal operating subsidies required. Achieving this indicated only a 5 percent decrease in ridership. However, most transit operators would like to reduce deficits while maintaining or increasing ridership. It can be seen from the figure that it is possible to keep ridership between existing and approximately 112 percent of existing levels for UTA while keeping the deficit at or below existing levels. Also, a reduction of 34 percent (about



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## STATISTICS FOR ROUTE 11

CATEGORY	EXISTING	OPTIMAL	%CHANGE
OPER. DEFICIT (\$)	71584.09	49479.40	-30.88
REVENUE (\$)	36665.56	38554.99	5.15
AVERAGE FARE (\$)	.35	.37	5.71
ANNUAL RIDERSHIP	105964.00	103170.80	-2.64
OPERATING COST (\$)	108249.65	88034.39	-18.67
REV.VEH. HOURS	3602.31	2818.78	-21.75
REV.VEH. MILES	59552.68	50602.96	-15.03
STOPS/MILE	6.90	5.53	-19.86
ROUTE FLEET SIZE	6.99	5.47	-21.75
FLEET UTILIZED	3.08	2.41	-21.75
FREQUENCY PER HOUR	1.77	1.50	-15.25
PERFORMANCE INDICATORS			
REV.VEH. MILES/REV.VEH	19340.99	21002.59	8.59
REV.VEH. HOURS/OPER.COST	.03	.03	.00
REV.VEH. MI./REV.VEH.HRS.	16.53	17.95	8.59
RIDERS/REV.VEH. HOUR	29.42	36.60	24.41
REVENUE/REV.VEH. HOUR	10.18	13.68	34.38
RIDERS/OPER.COST	.98	1.17	19.39
OPER.REVENUE/OPER.COST	.34	.44	29.41
IN VEH TIME (MINS/MILE)	3.24	2.99	-7.72
AV. WALKING TIME (MIN)	4.64	4.68	.86
AV. WAITING TIME (MIN)	8.80	9.76	10.91
AV. OUT OF VEH TIME(MIN)	13.45	14.44	9.09
OPERATING SPEED (MPH)	18.51	20.10	8.59
OPER.COST/RIDER	1.02	.85	-16.67
OPER.DEFICIT/RIDER	.68	.48	-29.41
REVENUE/REV.VEH. MILE	.62	.76	22.58
RIDERS/PEAK REV.VEH.	52.58	60.25	14.59
RIDERS/REV.VEH. MILE	1.78	2.04	14.61
REV.VEH. MILES/OPER.COST	.55	.57	3.64

FIGURE 4 Sample computer printout of route-level results.

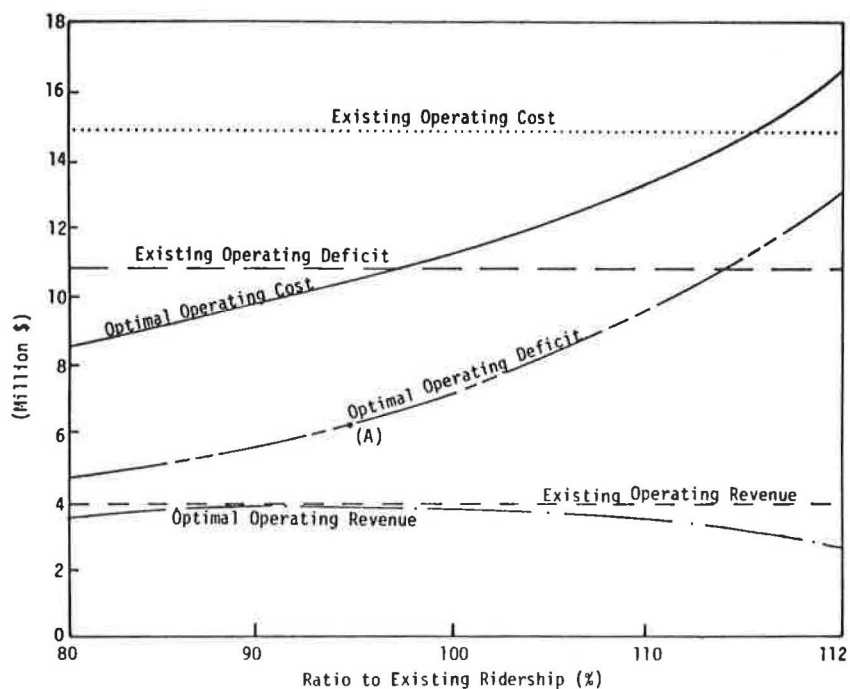


FIGURE 5 System costs, revenue, and deficits as a function of ridership.

\$3.7 million) of the total annual operating deficits could be achieved without decreasing ridership. The transit operator may focus on this operational range of increased ridership in optimizing the overall system. Also shown in the figure is that optimal operating deficits rise faster with ridership levels in excess of current levels than with lower ridership levels.

Figure 6 shows the variation in UTA optimal deficits as a function of time period of operation (peak, off-peak, weekend) and as a function of ridership level. It is interesting to note that, for all three operation periods, the percentage reduction in the optimal deficit is almost identical when a reduction in ridership from 80 to 90 percent of existing levels is considered. When the level of ridership is increased, it is observed that the most improvement in deficits occurs for weekend operation, followed by off-peak, and then peak. Increasing the level of ridership beyond existing levels leads to a much higher rate of deficit increase for peak than for off-peak and weekend operation. This observation is consistent with intuition and actual system observations. The justification is that the ridership level is the highest for peak, followed by off-peak and weekend operation, implying higher incremental costs for providing additional service. These in turn imply higher fares (marginal revenue) for system equilibrium and consequently reduced ridership increases resulting in increased operating deficits.

#### Operating Costs

Operating costs are reduced substantially from existing levels to produce operating deficit reduction. The main goal of the optimization model is to reduce deficits by increasing efficiency and productivity; thus the lowered operating cost resulting from increased efficiency and productivity is a key element in lowering total deficits.

The optimal costs with respect to various ridership levels for UTA are shown in Figure 5. As can be

seen in the figure, cost reduction provides the main contribution to the overall deficit reduction. Costs can be reduced by approximately \$4 million while maintaining the present ridership level. Ridership can be increased up to approximately 115 percent of existing ridership without increasing costs beyond the present level.

Optimal costs are achieved by modifying service policies to increase vehicle use, thereby reducing vehicle-hours, vehicle-miles, and number of vehicles required for service. Frequencies are reduced slightly leading to a decrease in the number of peak vehicles required. The number of stops per mile is also decreased leading to an increase in vehicle operating speed. These changes result in an overall increase in productivity and efficiency, which leads to reduced operating cost.

#### Operating Revenue

In examining the total revenue produced for the various levels of ridership as shown in Figure 5, it is seen that the total UTA operating revenue remains relatively stable with respect to ridership change. As indicated previously, cost reduction provides the main contribution to deficit reduction for the UTA application.

UTA revenue remains close to existing levels from approximately 88 percent ridership through 100 percent ridership. Revenue decreases from the 100 percent ridership level to approximately 75 percent of existing revenue at 120 percent of existing ridership. This downward trend is brought about because, in order to attract more riders, not only must the cost associated with providing better service increase but fare levels should simultaneously decrease. Because of the fare elasticities given earlier, the fare levels decrease at a higher rate than ridership increases; thus total operating revenue is decreased. Drastic service cuts (cost reduction) lead to reduced ridership. To satisfy the ridership constraints, fares have to be cut substantially, leading to reduced fare-box revenue.

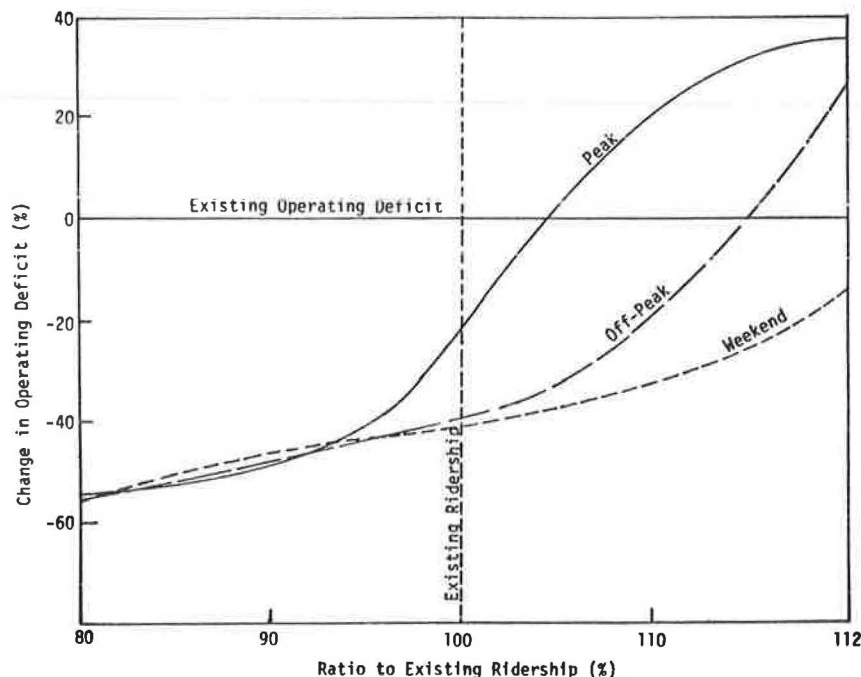


FIGURE 6 Percentage changes in UTA operating deficit versus ridership by time of day.

### Route-Level Study Results

The results presented in the preceding sections are based on a system-level optimization; therefore no individual route-level statistics are presented. As indicated previously, the optimization model was developed within a system and route context. Therefore, it is capable of optimizing the entire system at the route level of detail. Any fare and service policy changes recommended to bring about overall system deficit reductions must be implemented at the route level. The system may, however, be defined as comprising all routes, or as few as one route, of a given transit system. In this way the model can be employed to optimize route operations within the system context.

For the sake of brevity, not all details of the route-level analysis are presented. To illustrate model application at the route level, the results obtained for an example route, UTA Route 11, are presented in Figure 4. The economic and operating statistics representing peak-period service correspond to the case where existing ridership is maintained. It is seen that operating cost can be reduced by almost 19 percent and revenue increased more than 5 percent, leading to an overall deficit reduction of approximately 31 percent. The two main modifications responsible for the deficit reduction are the decrease in frequency per hour and the decrease in stops per mile. Frequency per hour, representing the average frequency for both directions, can be decreased 15 percent, and stops per mile can be reduced almost 20 percent. As a result of changes in frequency and in stops per mile, operating speed increased from 18.5 mph to 20.1 mph. On the basis of the modification of cash fare from \$0.50 to \$0.55 at the systemwide level, the average fare for Route 11 during the peak period should change from the existing \$0.35 to about \$0.37 per rider.

It should be noted that the results are theoretical. Slight modifications would have to be made in implementing the suggested modifications. For example, the optimal headway may be 28.7 min. In actual practice, a headway of 30 min would be used. The slight modifications necessary for application to the real-world will, however, change the deficit reduction only slightly.

### CONCLUSIONS

The model formulated by this study optimizes system deficits, subject to systemwide constraints, through minor individual-route service changes. Model results at the system and route level of detail represented by the decision variables (average fare, frequency of service, and number of stops per mile) are suitable for direct implementation by a transit operator. The optimal condition results in an overall increase in system and route efficiency and productivity, which leads to a reduction in transit operating deficits.

The model was developed in conjunction with continual input from a typical transit operator (UTA) and comprehensively incorporates most of the modeling consideration relevant to transit operators. It has been implemented as a portable, efficient, user-friendly, interactive computer routine.

The solution algorithm (standard LP) used for the nonlinear optimization program performed successfully and satisfactorily. The exploitation of efficient, commonly available, large, sparse-matrix-oriented linear programming solution algorithms makes the choice of standard LP particularly attrac-

tive. Convergence to optimal solutions was fairly rapid for the size of the problems solved.

The experience with model applications for UTA indicated that the data needed for the model can be readily assembled by a transit agency. The model results were meaningful, implementable, and intuitively consistent. For all applications of the model using UTA data, significant deficit reductions were achieved without major system modifications, loss of ridership, or undue fare increases.

In summary, the performance of the developed optimization model was judged to be good and representative of the type of model presented.

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