Modeling of Granular Materials in Pavements

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ABSTRACT

The problem of theoretical modeling of granular materials in pavements is considered; a previously published technique and associated materials data are used. A detailed stress-resilient strain model was used in a finite element configuration that is based on a secant modulus approach. A parametric theoretical study involving 56 different pavement structures with two granular materials provided extensive data on the in situ stress conditions in unbound layers and their equivalent stiffnesses. The incidence of failure elements is discussed and the conclusion is drawn that the simple K-θ nonlinear model and linear elastic layered systems are inadequate for computing stresses within the granular layer. Arbitrary adjustments to computed stresses that indicate apparent failure or tensile conditions are unnecessary when an accurate material model and associated computational techniques are used. The concept of a fixed modular ratio between a granular layer and a subgrade was found to be inappropriate because a particular granular material has an essentially constant equivalent stiffness. Linear elastic layered system computer programs can be used to determine critical design parameters when the granular layer stiffness is chosen on the basis of results from detailed nonlinear analysis.

In the design of new roads and in the expanding field of pavement structural evaluation, there is a continuing need for an adequate means of modeling unbound granular layers. The problem is not new; the nonlinear elastic properties of granular materials have long been appreciated and a number of techniques have been used to take these into account in structural analysis. These methods have included an iterative approach using linear elastic layered systems, first outlined by Monismith et al. (1) and application of the finite element method (2). This latter technique has been used as a basis for developing nomographic procedures for pavement design (3).

The majority of the work done in this field has used the so-called K-θ model to describe the nonlinear elastic characteristics of granular materials. This model was developed from repeated load triaxial test results and is of the form:

$$M_r = K_1\theta^{K_2} \qquad (1)$$

where

- M_r = resilient modulus, which is the repeated deviator stress divided by the axial resilient strain;
- θ = peak value of the sum of the principal stresses; and
- K_1, K_2 = material constants.

Brown and Pappin (4) have described a more detailed model for granular materials, which has wider applicability, and have discussed the limitations of the K-θ model. They also presented a computational procedure to incorporate their model in a finite element package, known as SENOL, to analyze pavement structures. The use of their procedure has also been illustrated (5).

The SENOL computer program has since been used to analyze a wide range of pavements and the results have thrown some additional light on the in situ behavior of granular materials. Use of the K-θ model has also been further investigated to establish its limitations. Because finite element analysis is still regarded as essentially a research tool in pavement engineering, SENOL has also been used to calibrate simpler analysis techniques based on linear elastic layered systems. The limitations of these have also been established.

GRANULAR MATERIAL MODELS

The resilient strain model described by Brown and Pappin (4,6) was developed from a comprehensive set of repeated load triaxial test data. The strains were expressed in terms of resilient shear and volumetric components leading to stress-dependent shear and bulk moduli. Stresses were expressed in terms of the invariants, mean normal effective stress ($p' = \theta/3$) and deviator stress (q). This model is referred to as the "Contour Model" because it is best illustrated, as in Figure 1, by use of strain contours in p'-q stress space.

Two materials are considered in this paper. The first, Model A, is a well-graded crushed limestone, and the second, Model B, is a uniformly graded material from the same source. They were selected to represent good- and poor-quality material in terms of stiffness. Details of both models have been presented by Brown and Pappin (4).

Coefficients in the corresponding K-θ relationships for these two materials are as follows:

Model A: K_1 = 8634 kPa, K_2 = 0.69

Model B: K_1 = 19 454 kPa, K_2 = 0.5

The contour models cannot be expressed as succinctly as this, so reference should be made to Brown and Pappin (4,5) for full details. Figure 2 shows a typical stress pulse in p'-q space for an element of granular material in a pavement. Point A represents overburden pressure and Point B is the peak stress that occurs when the wheel load is immediately above the element. The contour model was developed from a large number of stress paths such as AB covering the stress space of Figure 2 but limited to peak values of q/p = 1.67. This was done to avoid the development of significant permanent

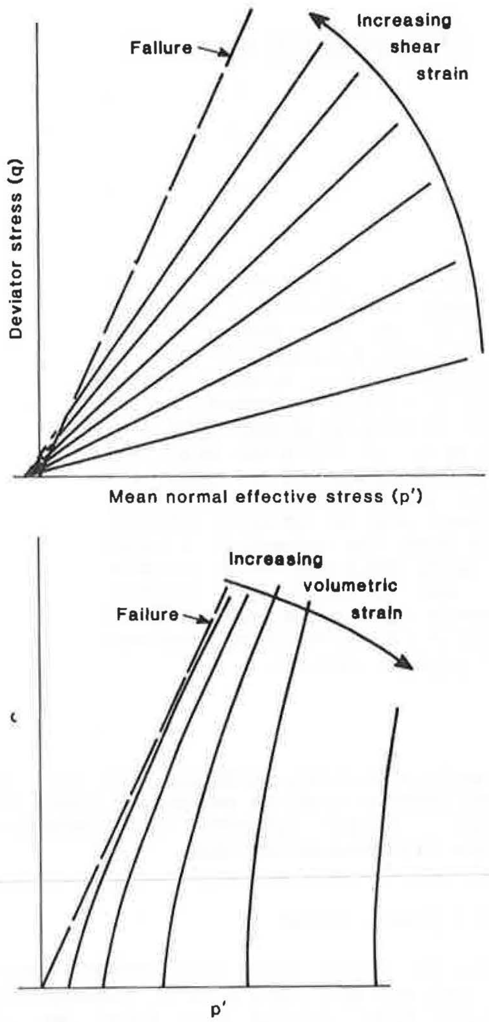

FIGURE 1 Contour model in p'-q stress space.

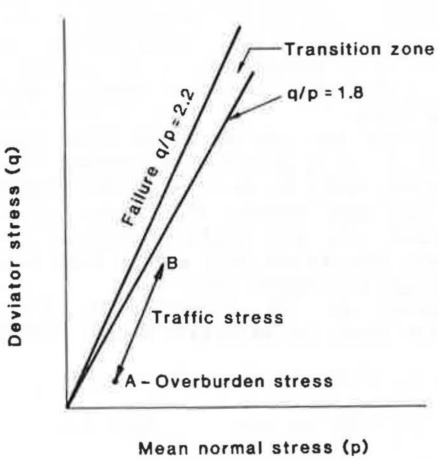

FIGURE 2 Typical in situ stress path due to wheel loading.

strains that occur when peak stresses probe close to the failure line; q/p = 2.2 for this material (both well graded and single sized). Some tests involving these high stress ratios showed that the basic resilient strain model was capable of extrapolation into this zone.

The K-θ model was developed from tests involving constant confining stress and deviator stress pulsed from zero to a peak value. The stress paths in Figure 2 for such tests would involve Point A being on the p' axis (equal to the confining stress at that point) and the slope of AB being 3. The parameter θ in Equation 1 is the value corresponding to Point B in Figure 2.

Hence the contour model is a more exact representation of the material behavior and is better able to predict stress conditions in a pavement in which a wide range of stress paths is possible.

In the computations that were performed during this investigation, both the asphalt and the subgrade layers in each pavement were assumed to be linearly elastic. When linear elasticity is applied to materials the basic characteristics of which are either nonlinear or viscoelastic, the term "elastic stiffness" is used in place of Young's modulus in this paper.

COMPUTATIONAL PROCEDURES

The finite element program SENOL was designed specifically to apply the contour model for granular materials to pavement analysis. Details of the computational procedure have been published by Brown and Pappin (4,5) but will be summarized here.

The starting point for analysis in a particular element is the overburden stress. This is used to establish the initial values of bulk and shear modulus. The effects of wheel load are then computed by applying it in 10 equal steps and, finally, iterating until satisfactory convergence is obtained. A significant feature of the procedure is that it is based on the secant modulus at each step (i.e., both stress and strain values are relative to zero).

When a convergent solution has been obtained, the program computes an equivalent Young's modulus and Poisson's ratio on the basis of the application of traffic loading alone. This is essentially a chord modulus and is of interest in calibrating simpler linear elastic procedures such as BISTRO (7,pp. 34-35) or the Chevron (H. Warren and W.L. Dieckman, Numerical Computation of Stresses and Strains in a Multiple-Layer Asphalt System, unpublished internal report, Chevron Research Corporation, 1963) layered system programs. These deal only with stresses induced by wheel loading and, because all layers are assumed to be linearly elastic, require an equivalent single value of Young's modulus for the granular layer. The SENOL program was used to determine appropriate values.

In applying the K-θ model to pavement analysis using the finite element method, the peak stress (overburden plus traffic) is calculated using an assumed initial value. The value of θ at this peak stress is then computed and the corresponding resilient modulus is determined from Equation 1. This is regarded as Young's modulus and is combined with an appropriate value of Poisson's ratio, usually a constant value, for proceeding with the calculation. The inadequacies of the K-θ model lead to some elements exceeding failure conditions and these are arbitrarily adjusted to bring the stress condition down to an acceptable level (2). Arbitrary adjustments of this kind are also used in those approaches that adopt a "tension correction" for elements in a granular layer.

These adjustments are not necessary when using SENOL with the contour model because elements approaching failure are automatically assigned low stiffnesses in accordance with the greater detail of this model. Nonetheless, some elements do have final stress conditions just above failure. This is a consequence of the slight shortcomings of the contour model for stress conditions close to failure.

Computations were performed using the K-θ model as well as the contour model for the two materials noted previously. In applying the K-θ model, the same basic computational procedure was followed as for the contour model; that is, a secant modulus approach with the load applied in stages as described previously. In this case, secant values of E were determined at each stage of loading and a constant value of Poisson's ratio (0.3) was adopted throughout. In addition, linear elastic solutions were obtained for several structures using the BISTRO (7) computer program.

The results from these various computations were used to study the following points:

1. Stress conditions in the granular layer and the incidence of failure in particular elements,
2. Comparison of Models A and B for the granular layer,
3. Equivalent values of Young's modulus,
4. Comparison of critical parameters computed using SENOL and BISTRO, and
5. Assessment of the K-θ model.

DETAILS OF PAVEMENT STRUCTURES

A parameter study was conducted using the SENOL computer program and it involved computations on 56 pavement structures definitions of which are given in Table 1. Each structure consisted of a linear elastic asphalt layer, a nonlinear granular layer, and a linear elastic subgrade. Table 1 gives the combinations of stiffnesses and thicknesses that were used. Granular material Model A was adopted for all 56 cases. In addition, six cases, numbered 3 to 8 in Table 1, were analyzed using Model B and eight cases (1 to 8 in the table) were also analyzed using the BISTRO linear elastic procedure. Comparisons of the contour and K-θ models were made using structures numbered 3 to 8. A summary of the eight structures that were examined in detail is given in Table 2. In each case a 40-kN wheel load having a contact pressure of 500 kPa was used.

For the 56 structures that were analyzed, five solutions did not converge, some indicated elements at or slightly above failure ($q/p' > 2.2$), others included elements in the zone just below failure ($2.2 < q/p' < 1.8$), and all elements in the remainder were in the region of lower stress levels within which the contour model has greatest validity. These various categories are identified in Table 1, which shows a trend from the weakest (nonconvergent) structures, through the intermediate areas, to those strong pavements with the lowest peak stress ratios.

The significance of a nonconvergent solution is that a large number of elements within the granular layer are at failure. The general implication of this is that significant permanent deformations are likely to develop in such a structure. Shaw (8) has shown that the parameter that determines the tendency for permanent strain to accumulate under repeated loading is the minimum horizontal distance (value of p') between the end of the stress path and the failure line (see Figure 2).

Figure 3 shows, in more detail, the incidence of failure elements in Structures 1 and 2 of those investigated in detail. The elements with stress ratios in the transition zone just below failure are also shown. In both these cases the stress conditions in the granular layer are generally high and such pavements are unlikely to have long lives.

Structures 3 and 4 (Table 2) had some elements in

TABLE 1 Modular Ratios Between Granular Layer and Subgrade

		h_1 (mm)		
h_2 (mm)	E_3 (MPa)	50	100	200
Asphalt Stiffness = 4 GPa				
200	20			
	30			2.5^{+}
	50			1.5^{6}
	70			1.5
450	20			4.0
	30			3.5
	50		2.5^{2*}	2.0
	70		2.0*	1.5
700	20			6.0
	30		5.0*	4.0^{5}
	50		3.0*	2.5
	70			
Asphalt Stiffness = 7 GPa				
200	20			3.5
	30			2.5
	50		2.0^{+}	1.5
	70	2.5*	1.5^{+4}	1.5
450	20		5.5*	5.0
	30		3.5^{+}	3.0
	50	2.0*	2.0^{+}	2.0
	70	2.5*	1.5^{+}	
700	20		5.5^{+3}	5.0
	30	6.0*	3.5^{+}	4.0
	50	3.5*	2.5^{+}	
	70			
Asphalt Stiffness = 12 GPa				
200	20		3.5*	3.0
	30	NC	2.5^{+}	
	50	NC	2.0	
	70	NC	1.5^{8}	
450	20	NC	4.5^{7*}	
	30	5.5*	3.5^{+}	
	50	3.5*	2.0	
	70	2.5*		
700	20	NC	7.5^{+}	
	30	6.0^{1*}		
	50	4.0*		
	70			

Note: h_1 = asphalt thickness, h_2 = granular thickness, E_3 = subgrade stiffness, * = some failure elements, NC = noncovergence—general failure, and + = elements close to failure; superscripts 1 to 8 refer to pavement numbers in Table 2.

the transition zone, and the remaining structures had all elements at stress ratios less than 1.8. The relative potential performance of the granular layer in six cases is reflected by the pavement lives given in Table 2. These were calculated using the pavement evaluation techniques developed by Brunton and Brown (9) and relate to British conditions. In all cases, except Pavement 4, the potential failure mechanism was fatigue cracking of the asphalt. In Pavement 4 it was excessive rutting. The elastic stiffness of the granular material used in this evaluation, which is based on linear elastic analysis, was derived from the SENOL computations. This point is dealt with in the next section.

EQUIVALENT STIFFNESSES FOR THE GRANULAR LAYER

Values of Young's modulus and Poisson's ratio are computed in the SENOL program on the basis of the stresses and resilient strains resulting from traffic loading alone. This "chord modulus" is printed out for each element together with the corresponding Poisson's ratio.

The variation of these parameters through each of the 56 structures that were analyzed allows conclusions to be drawn about the equivalent Young's mod-

TABLE 2 Details of Pavement Structures Investigated in Detail

		Asphalt		Granular			
Pavement No.	Life (msa)[a]	Stiffness (GPa)	Thickness (mm)	Thickness (mm)	Nonlinear Model	Subgrade Stiffness (MPa)	BISTRO Calculation
1	0.5	12	50	700	A	30	No
2	0.1	4	100	450	A	50	No
3	0.5	7	100	700	A,B, K-θ	20	Yes
4	0.3	7	100	200	A,B, K-θ	70	Yes
5	2.3	4	200	700	A,B, K-θ	30	Yes
6	1.3	4	200	200	A,B, K-θ	50	Yes
7		12	100	450	A,B, K-θ	20	Yes
8		12	100	200	A,B, K-θ	70	Yes

[a]msa = millions of standard (80 kN) axles.

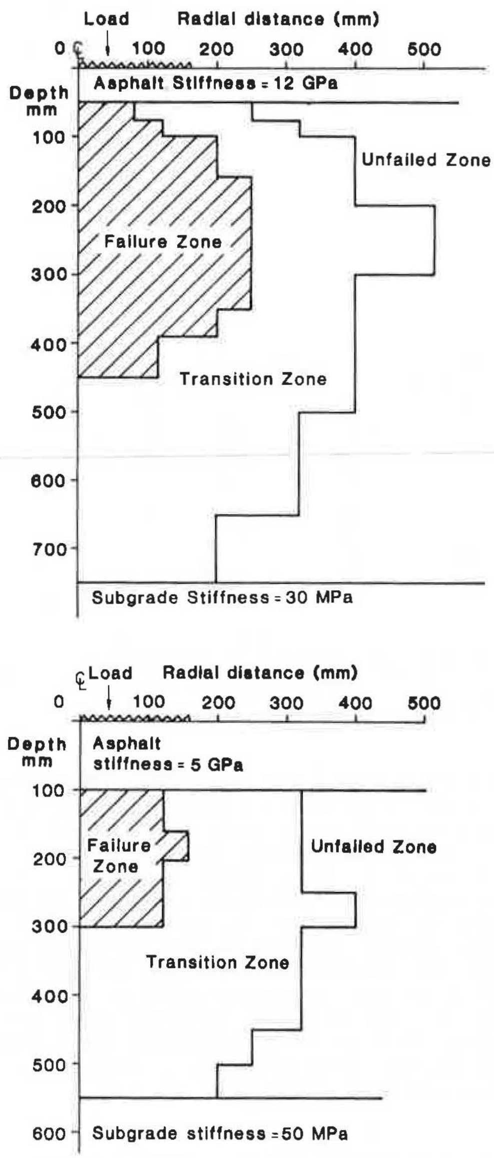

FIGURE 3 Incidence of failure elements in Pavements 1 and 2.

ulus, or stiffness, that the granular material mobilizes in situ. This information is particularly useful for allowing selection of the appropriate single value of granular material stiffness for use with linear elastic layered system programs such as BISTRO (7) and the Chevron program. Alternatively, it could form a basis for subdividing the granular layer if this were more appropriate.

In the past the approach to defining stiffness for a granular layer has been to use a certain value of modular ratio between this layer and the subgrade. Values in the range 1.5 to 5 have been generally adopted with 2 the most common. This approach implies that the stiffness of a particular granular material adjusts itself in situ in response to the stiffness of the support and the consequent stress conditions.

The SENOL data for all 51 of the structures, for which solutions were obtained, were studied and mean values of modular ratio, based on the computed chord moduli, were extracted. These are given in Table 1 from which it will be seen that they range from 1.5 to 7.5, a spread similar to that reported from in situ vibration testing (10). However, it will be noted that the high ratios were for the soft subgrade and vice versa, implying that the actual stiffness of the granular layer does not vary greatly.

To produce reliable values of these deduced equivalent stiffnesses for the granular layer, only those structures with peak stress ratios below 1.8 (well below failure) were considered. This reduced the number of relevant solutions to 22 as can be seen from Table 1.

Because a value of chord modulus is computed for each element, variations within the structure were studied. Within a radius of 350 mm from the load centerline the variation of this parameter and the associated Poisson's ratio were quite small. Figure 4 illustrates this point for Pavements 5 and 6 of those analyzed in detail (Table 2). The shaded zones cover the range of values up to a radius of 350 mm and results are shown for both granular material Models A and B. There is a general trend for stiffness to increase slightly with depth in each case. However, this variation is sufficiently modest to consider a single equivalent value of stiffness for the layer as a whole, when contemplating linear elastic layered system calculations.

For Model A, the mean equivalent stiffness for the 22 structures under consideration varied from 60 to 125 MPa. These values are small in relation to the stiffnesses of the asphalt layers (4 to 12 GPa). It was, therefore, considered appropriate to use a single value of 100 MPa in pavement design calculations based on linear elasticity and involving good quality granular subbases. This was adopted for the Nottingham analytical design procedure (9).

For Model B, representing poorer quality material, only six pavements were studied (3 to 8 in Table 2) with stress levels well below failure. The range of mean stiffnesses was 35 to 50 MPa and a mean value of 40 MPa is suggested for routine design. For both models, Poisson's ratio was 0.3 to 0.4, the former value having been adopted for design.

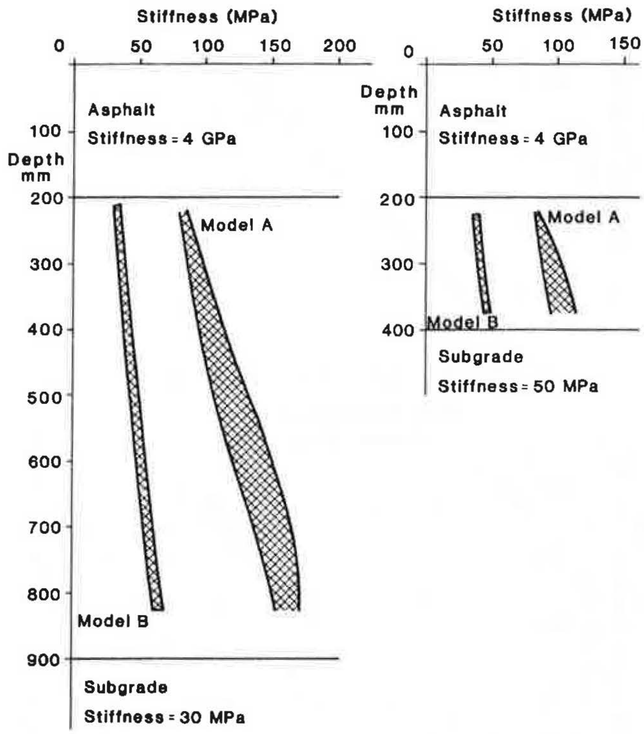

FIGURE 4 Variation of chord modulus (stiffness) through the granular layer.

EVALUATION OF LINEAR ELASTIC SOLUTION

Against the foregoing background, a number of calculations were carried out using the BISTRO computer program so that comparisons could be made with SENOL results for certain critical parameters. The parameters selected were tensile strain at the bottom of the asphalt layer (the fatigue cracking design criterion) and surface deflections at radii up to 350 mm as an indication of overall pavement response. Model A material was used and a mean stiffness of 100 MPa was adopted for the granular material in the BISTRO computations.

Figure 5 shows comparisons that generally indicate that the linear elastic layered system approach produces quite reasonable values for these two parameters, although surface deflections computed using BISTRO are somewhat high. There was no significant difference in the deflection comparisons at different radial positions.

In reality, not only is the granular layer nonlinear, so is the subgrade. A few calculations were conducted with a nonlinear elastic model for the subgrade (5) derived from work by Brown et al. (11). The results, based on a linear elastic subgrade, were compatible with those discussed in this paper.

STRESS CONDITIONS IN GRANULAR LAYERS

The foregoing section has shown that linear elastic layered system computations can determine critical design parameters when an appropriate equivalent stiffness is assigned to the granular layer. They are unlikely, however, to be able to reliably calculate stress conditions within the granular layer itself.

One of the particular problems in this connection is the tendency for tensile stresses to be apparent in granular layers when linear elastic assumptions are used. This point is illustrated in Figure 6 for

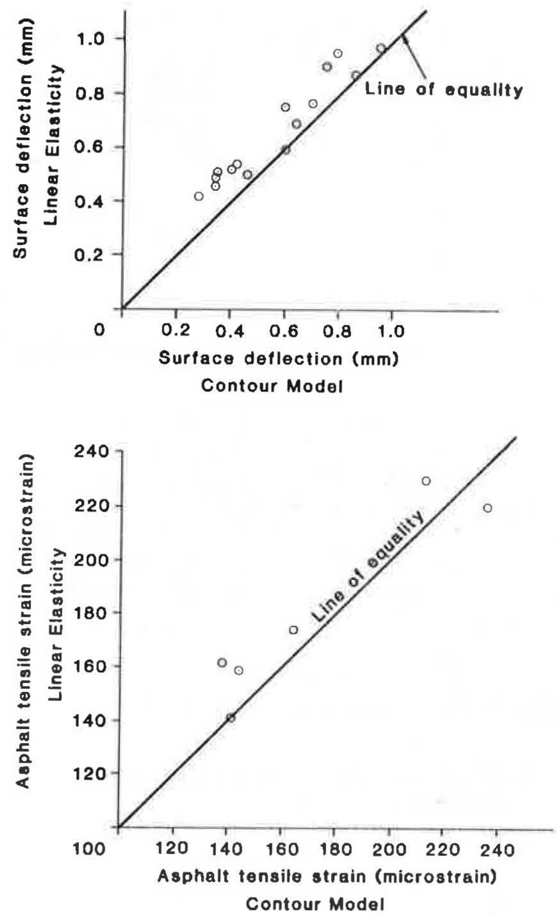

FIGURE 5 Comparison of results from SENOL and BISTRO computations.

Pavements 3 and 4 (see Table 2), and the results from BISTRO calculations are compared with those from SENOL using Model A material. The left line of each pair represents the traffic-induced horizontal stress in the granular layer, and the right line shows the influence of the compressive overburden pressure. For linear elasticity, even when overburden is included, tensile stresses still result in the lower half of the layer. Similar analysis for Pavements 5 and 6, which were stronger, showed that these combined stresses can become compressive in favorable circumstances. However, by contrast, the SENOL results show compressive stresses in all these cases and in most others as well.

The incidence of tensile stress in a granular layer does generally imply a failure condition. However, failure is defined by the stress ratio (q/p'), which is influenced by vertical stresses as well as horizontal ones.

Tensile total stresses, in soil mechanics terms, may correspond to compressive effective stresses if the granular material is subject to negative pore pressure, which in general it will be. However, quantification of this pore pressure may not be easy.

Table 3 gives the peak q/p' ratios determined at the top and bottom of the granular layer for Pavements 3 to 6. The SENOL values range from 0.8 to 2.1, all below the failure condition of 2.2, and in only two cases are the BISTRO values below failure. These data, therefore, confirm the point that detailed study of stress conditions within granular layers cannot be undertaken using linear elastic theory.

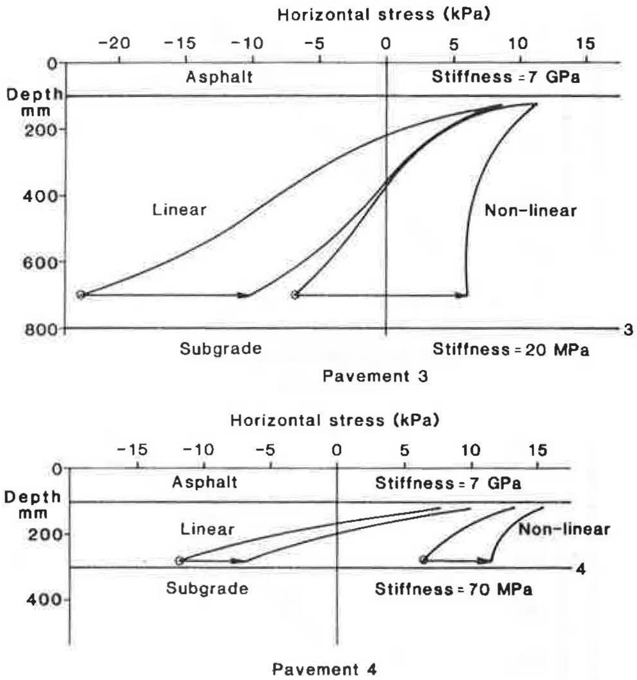

FIGURE 6 Horizontal stresses in granular layer.

TABLE 3 Stress Ratios at Top and Bottom of Granular Layer

Pavement No.	q/p SENOL/Contour Top	q/p SENOL/Contour Bottom	q/p BISTRO Top	q/p BISTRO Bottom
3	2.10	1.77	2.15	17.5
4	1.92	1.81	2.27	4.34
5	1.73	0.80	2.53	2.23
6	1.71	1.40	1.14	2.47

EVALUATION OF K-θ MODEL

The K-θ equation for the well-graded crushed limestone was used in the SENOL program with a secant modulus approach for Pavements 3 to 8 (Table 2). The maximum tensile strain in the asphalt and surface deflections up to a radius of 350 mm were extracted from the output for comparison with the contour model results. Figure 7 shows that the deflections compare favorably. However, the K-θ approach underpredicts the tensile strain and is less satisfactory than the linear elastic layered system solutions (Figure 5), which used a single value of stiffness for the granular layer.

Although the K-θ model may be of use in evaluating effects in other layers, the results showed that stress conditions in the granular layer are not correctly determined. This point is illustrated by Figure 8, which shows substantial numbers of failure elements in Pavements 5 and 6 that were analyzed using the K-θ approach, whereas no failure elements were predicted using the contour model.

CONCLUSIONS

1. A detailed study of the structural behavior of unbound granular materials in pavements requires an accurate stress-strain model to define nonlinear elastic response.

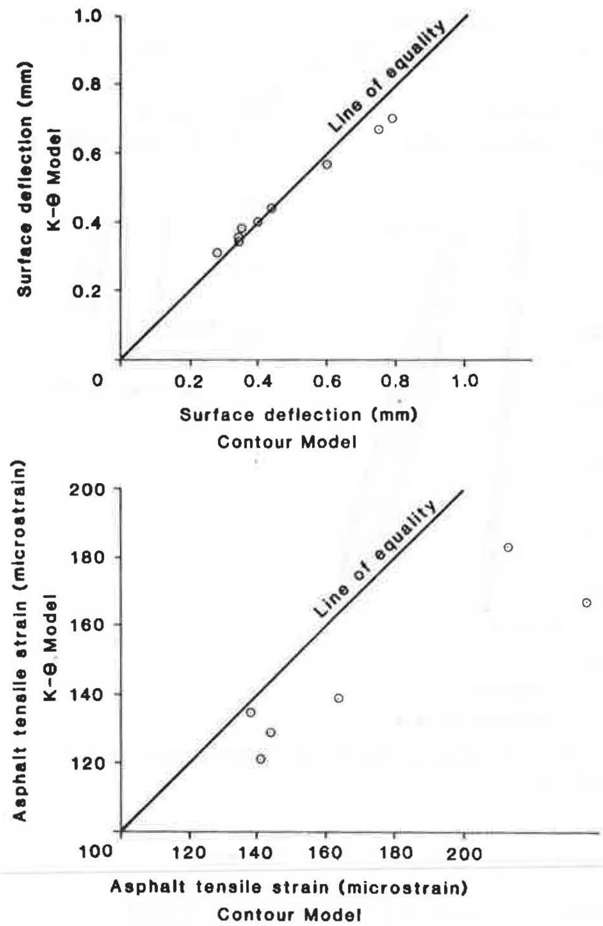

FIGURE 7 Comparison of results from contour and K-θ models.

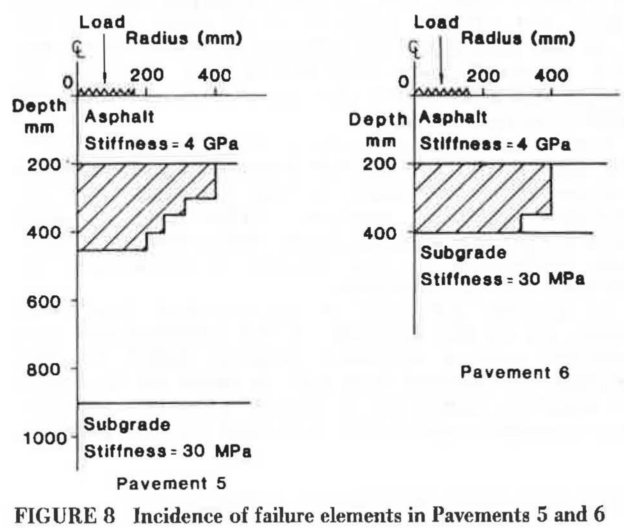

FIGURE 8 Incidence of failure elements in Pavements 5 and 6 using the K-θ model.

2. The contour model published by Brown and Pappin and the associated SENOL finite element computer program allow this to be done, but improved modeling is still desirable.

3. The SENOL program and the contour model allow equivalent elastic stiffnesses for granular layers to be determined for use in layered system analysis.

4. The concept of a fixed modular ratio between

a granular layer and the subgrade appears inappropriate because a single value of stiffness, dependent on the granular material, may be used in linear elastic analysis to determine effects in other layers of the structure for pavement design and evaluation purposes.

5. A well-graded crushed limestone base has an equivalent stiffness of 100 MPa, whereas a poorly graded material has a stiffness of only 40 MPa for the range of conditions investigated.

6. Linear elastic layered system programs can determine surface deflections and maximum asphalt tensile strains to an acceptable accuracy for design when the correct equivalent stiffness is assigned to the granular layer.

7. The finite element method incorporating the K-θ model can be used to determine surface deflections and asphalt tensile strains but is unable to determine the stress conditions within the granular layer.

8. Conclusions 6 and 7 suggest that the simplest approach to design calculations for surface deflection or asphalt tensile strain involves the use of linear elastic layered systems, provided the correct equivalent stiffness is defined from detailed nonlinear finite element analysis.

9. Design computations involving deformation or failure within the granular layer require a detailed model and finite element analysis.

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