

Characterization of Granular Material

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ABSTRACT

Evaluation of the resilient modulus of granular material as used in design and structural evaluation of flexible pavements is addressed. It is shown that the well-known equation relating the modulus to the sum of principal stresses does not properly describe granular material behavior: the predicted response is not compatible with laboratory test results that show a strong dependence of the modulus on the stress ratio from which the equation is derived. A general law that includes the effect of shear strains is shown to be in good agreement with test results. The response of nonlinear materials is sensitive to their state of stress during loading. A literature review covers case histories of full-scale retaining wall models with compacted backfill, where residual stresses induced by compaction were measured. A proposed theory based on limit equilibrium is reported to give good estimates of residual stresses. It is found that granular base and subbase materials, compacted with heavy rollers, may develop relatively high residual horizontal stresses. The general law for characterizing granular materials is used with different postulated residual stresses in pavement analyses. The results obtained appear to be in good agreement with all aspects of granular material behavior, provided that a residual stress of the order of 1 to 2 psi is assumed to be induced by compaction.

Design and structural evaluation of flexible pavements are currently based on the layered elastic theory. This approach offers the possibility of a rational solution of the problem. The success of this approach depends on the accuracy with and manner in which material properties are evaluated and used in the analysis. The existence of nonlinear stress-strain characteristics in granular materials and soils has been well known for many years. Experimental data show that the response of these materials and especially that of the granular materials depends strongly on the prevailing state of stress. Therefore knowledge of the correct in situ stress conditions is vital for the rational approach to design and structural evaluation of flexible pavements.

In the last decade most research in the field of characterization of granular subbase and base layers has involved repetitive loading tests and has concentrated on developing nonlinear stress-strain models. The simple model, which is widely used and relates the resilient modulus to the sum of principal stresses, appears to have serious limitations. More complex and sophisticated models expressed in terms of shear and volumetric stress-strain relationships lead to quite low and unrealistic moduli when used in pavement analyses. Neither the material model nor the analysis is at fault: it appears that the state of stress prevailing in the field is not reproduced in the analysis.

This discussion suggests that residual compressive stresses induced by compaction during construction of the pavement or during repeated traffic loadings, or both, may be the cause of the incompatibility between common engineering values and the results of analyses of the sophisticated model. The existence of residual stresses in compacted materials is well supported in the literature by laboratory tests and full-scale retaining wall model tests with backfill compacted with equipment similar to that used in highway construction. If these residual stresses are taken into account in analysis as initial conditions, the state of stress that prevails

in the field is reproduced and the granular material moduli are realistically evaluated.

A literature review of the characterization of granular material and an estimation of residual stresses induced by compaction are presented. An analysis of a layered system is included to illustrate the effect of residual stresses on the moduli of granular material and to compare the results with values used in existing pavement design procedures.

LITERATURE REVIEW OF CHARACTERIZATION OF GRANULAR MATERIAL

Two different approaches exist for estimating the granular material properties used in the analysis of pavement systems. In the empirical or semiempirical one, the resilient modulus is related to layer thickness and underlying subgrade modulus (1) as well as to material type (2) or asphalt concrete modulus of elasticity (3). The values obtained from these procedures are similar and appear realistic from an engineering and phenomenological point of view. In the last decade research has concentrated on the rational approach in which the behavior of granular material is described by nonlinear stress-strain characteristics. Three different relationships are currently implemented: (a) the well-established one, relating the resilient modulus to the bulk stress (4-6); (b) the hyperbolic law proposed by Kondner (7) for static loading and extended by several researchers (8,9); and (c) the fundamental one (10-12) relating the bulk and shear moduli to octahedral stresses and stress path.

The first model is expressed as

$$MR = k_1 \sigma^k \quad (1)$$

where MR is the resilient modulus, $\sigma = \sigma_1 + \sigma_2 + \sigma_3$ is the sum of principal stresses, and k_1 and k_2 are regression coefficients derived from laboratory test results. Equation 1 has been implemented in

various computer programs (5,6) using iterative computation schemes.

Figure 1a presents the results of tests conducted on a dense graded aggregate (13,14). Lines of equal confining pressures (σ_3) have been added to indicate the tendency of variation of MR as a function of σ_3 . The relationship given by Equation 1 is also drawn.

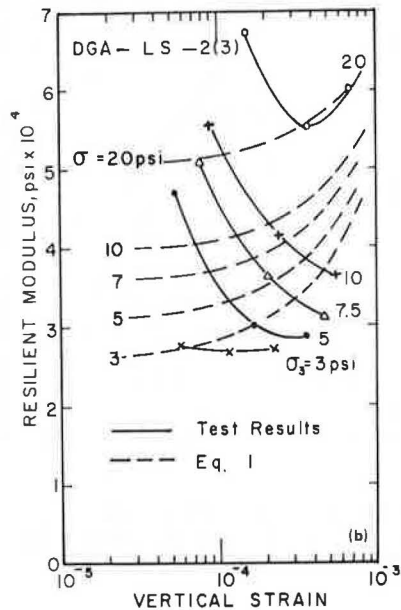
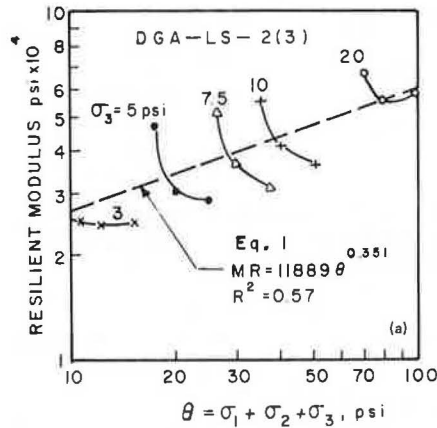


FIGURE 1 Test results and predicted behavior using Equation 1 for a dense graded aggregate (13, 14).

It is seen that Equation 1 fits the results quite well. However, reversed trends of resilient modulus at constant confining pressures are noticed. Figure 1b shows the same test results of the resilient modulus as a function of the resilient vertical strain (ϵ_a). Included, for comparison, is the following predicted relation derived from Equation 1:

$$\begin{aligned} \epsilon_a &= (\sigma_d / MR) = [(\theta - 3\sigma_3) / MR] \\ &= \{[(MR/k_1)^{1/k_2} - 3\sigma_3] / MR\} \end{aligned} \quad (2)$$

where $\sigma_d = \sigma_1 - \sigma_3$ is the repeated vertical stress. It is seen from Figure 1b that the predicted resilient modulus at constant confining pressure increases monotonically as the vertical strain increases.

The general form of the hyperbolic law is

$$1/E = (\epsilon_a / \sigma_d) = a + b\epsilon_a \quad (3)$$

where E is the elastic modulus, ϵ_a is the axial strain, and a and b are regression coefficients that correspond to the inverse of the initial tangent modulus of elasticity (at $\sigma_d = 0$) and stress at ultimate failure, respectively. Equation 3 corresponds to a nonlinear stress-strain relationship with E decreasing gradually as ϵ_a and σ_d increase. The initial modulus function resembles Equation 1 in which the sum of principal stresses is replaced by the confining pressure. Equation 3 has been widely used for characterizing subgrade soils in the static loading mode and for permanent deformation under repeated loads (8,9). It has also been suggested for resilient deformation of granular material, using only the initial modulus term (15, p.57). That is,

$$MR = k_5 p_a (\sigma_3 / p_a)^{k_6} \quad (4)$$

where p_a is atmospheric pressure and k_5 and k_6 are regression coefficients. In Equation 3 the modulus decreases as the vertical strain increases, in contradiction with the trend predicted by Equation 1.

Brown and Pappin (10,12) developed a nonlinear stress-strain relationship that is capable of taking into account effective, mean, and deviatoric stress and stress path dependence. The material properties are expressed in terms of bulk and shear moduli. The expressions derived are quite complex and may be used only in finite element analysis. It is interesting to study the predicted resilient modulus-vertical strain relationship at constant confining pressures, corresponding to the conventional triaxial test. Figure 2 shows the computed relations for the crushed limestone tested and those obtained using the Brown and Pappin model. It is seen that as the vertical strain increases, the modulus decreases first and increases afterwards at vertical strains greater than 2×10^{-4} . For comparison, the predicted modulus calculated using Equation 1 is also shown in

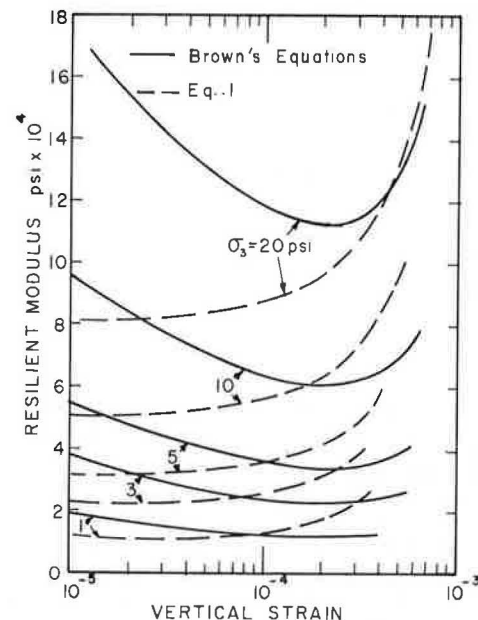


FIGURE 2 Comparison of predicted behavior of crushed stone using Brown's equations and Equation 1.

Figure 2. It is seen that the two predicted relations are quite different.

Discussion

These results summarize the state of the art in the field of granular material characterization for pavement design. The model developed by Brown and his colleagues appears to be the most promising. Its application to realistic conditions in pavements has been criticized for being based on triaxial tests where two principal stresses are equal. It is this author's opinion that its refinement in the full three-dimensional state of stress and strain will not lead to drastic changes in the values of the predicted modulus.

Figure 2 shows clearly that, for low vertical strain values, the modulus decreases as the vertical strain increases. The increase in the modulus values occurs at maximum to minimum principal stress ratios (σ_1/σ_3) of more than 2 to 3. In the model, the bulk modulus increases monotonically as the ratio of deviatoric to mean stress (q/p) increases. This behavior is well known in dense granular material as dilation. Therefore the increase of the resilient modulus in repetitive loading could be attributed to dilation effects and to the accumulation of permanent shear strain.

According to Figure 2, Equation 1 fails to describe the descending branch of the relationship and predicts a quite sharp ascending branch. It should be noted that, according to May and Witczak (16), Equation 1 neglects the effect of shear strain and is therefore applicable only in the range of low strain values. The complete expression for Equation 1 should read:

$$MR = k_1 \theta k^2 f(\epsilon_a) \quad (5)$$

where $f(\epsilon_a)$ is the correction function that decreases as ϵ_a increases. Equation 5 is similar to that used in earthquake analyses: the function is given at discrete values of the shear strain, and the computer program performs the required interpolation.

It has been suggested that this function be approximated as follows:

$$MR = k_1 \theta^{k_2} \epsilon_a^{k_3} \quad (6a)$$

or

$$MR = k_1 \theta^{k_2} \sigma_d^{k_4} \quad (6b)$$

with

$$\epsilon_a > 10^{-5}$$

$$\sigma_d > 0.1\sigma_3$$

Test results (13,14) are used with multiple regression analyses to derive k_1 , k_2 , and k_4 material parameters. Figures 3-5 show comparisons of measured and predicted moduli using Equation 6b. (For the crushed limestone in Figure 3, the "measured" values were computed using the Brown and Pappin model.) It is seen that the prediction is quite good.

It is worth mentioning that the hyperbolic law (Equation 3) fails to describe the effect of dilation and accumulation of permanent shear strains. However, under the load where the vertical strain is between 1 to 5 x 10⁻⁴, the modulus could be approximated by Equation 4, which states that the modulus depends on the confining pressure only.

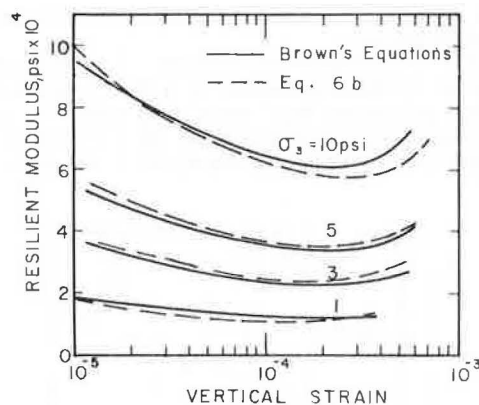


FIGURE 3 Comparison of predicted behavior of crushed stone using Brown's equations and Equation 6b.

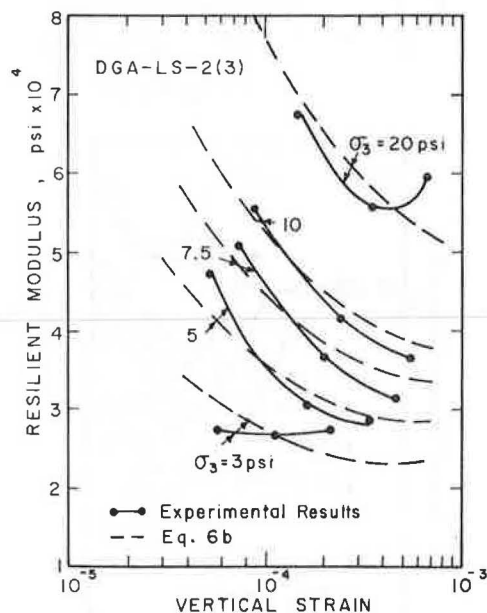


FIGURE 4 Comparison of test results and predicted behavior using Equation 6b for a dense graded aggregate (13,14).

Application to Pavement Analyses

The three models presented have all been used to predict deflections, stresses, and strains in pavements. The first model is currently included in a pavement design method (6). Chou (15) conducted a comparative study with different stress-strain relations for granular materials and subgrade soils. He included Equations 1 and 4 for the granular materials and used finite element analyses.

The computed deflection at the pavement surface using Equation 4 was greater than that obtained using Equation 1. Chou (15,p.57) stated that "when tensile stresses were developed at the bottom of granular layers, the elastic modulus reduced drastically as the load increment increased," while "the use of Equation 1 can greatly increase the elastic moduli of granular materials." Because Equation 4 is inapplicable when tensile minimum principal stress develops, it has been abandoned, and Equation 1 has become well established. Its use has been improved by the addition of the Mohr-Coulomb failure law in

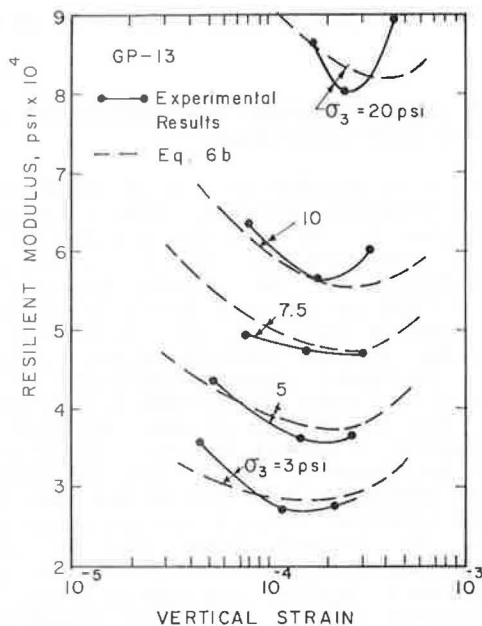


FIGURE 5 Comparison of test results and predicted behavior using Equation 6b for a bank gravel (13,14).

order to keep the stress values inside the space delimited by the failure law envelope.

Recently Witczak (Use of NDT Deflection Data to Estimate In-Situ Material Moduli, working paper presented to TRB Task Force A2T56--Nondestructive Evaluation of Airfield Pavements, Fredericksburg, Virginia, 1981) compared the computed pavement response using Equation 1 and surface deflection measurements. He found that "an adjustment must be made to the values of any unbound granular material." May and Witczak (16) suggested that the adjustment factor for getting an effective resilient modulus is a function of the shear strain induced by the surface loading. Smith and Witczak (14) compared the computation results using Equation 1 and those of the empirical approach (1-3). They found that (a) for high-quality base materials, the results are in quite good agreement, especially for $k_1 = 8000$, and (b) for poorer quality materials, the comparison is not as good. It should be noted that these analyses indicate that the elastic modulus of the granular material is essentially independent of its thickness. Moreover, the computation results presented later in this paper show that the modulus increases slightly with increasing applied load.

Brown and Pappin (11,12) presented measured and predicted stresses and strains in an experimental section. Large discrepancies were reported. For example, the measured vertical strain near the bottom of the crushed stone layer was about 500 microstrains whereas the predicted one was about three times larger. The measured radial and tangential strains were lower than the predicted ones by a factor of two.

The results of analyses conducted using the accurate model for characterizing granular material and the finite element method have led Brown (17, p.56) to assign quite low moduli to the subbase and base layers. Suggested design values are about 100 MPa (15 ksi) and 40 MPa (6 ksi) for well-graded crushed rock and poorer material, respectively. These values are not realistic.

In the light of these results, it seems that the characterization of granular material is incomplete at present. The lack of agreement is frequently at-

tributed to the axisymmetric stressing conditions of the triaxial test that do not correspond to the actual conditions in pavements. This criticism is basically correct; however, it is believed that improvement of the model cannot lead to drastic changes in the values of the moduli. Therefore it is suggested that residual stresses induced during compaction of the material be taken into account in order to be closer to the real conditions existing in pavements.

It should be noted that Stock and Brown (18) recognized the effect of compaction equipment on the granular material. They adopted coefficients of earth pressure at rest from the overconsolidation theory, which give relatively high horizontal stresses. The existence of the residual stresses and their magnitude are discussed hereafter.

RESIDUAL STRESSES INDUCED BY COMPACTION

Compaction of granular materials in pavements is required to provide sufficient strength and stability (i.e., to resist shear failure and permanent deformation under applied repetitive loads). The effect of compaction on material characterization has in the past been taken into account in analyses only through the effect of density and degree of saturation on the material property. However, because the material is highly nonlinear and undergoes large shear deformation, compaction creates a stress history for the finished material. In other words, even in a newly constructed pavement, the initial stress condition is not stress free, as is generally assumed.

Compaction produces a stable layer that will not deform under further rolling of the compaction equipment. This condition suggests that the finished material is confined. The hypothesis that residual compressive stresses are induced by compaction is well supported by laboratory test results (19) and full-scale retaining wall models with compacted backfill (20, p.18; 21, p.21). The analysis evaluating the induced lateral pressures can be only approximate because large deformations are involved and the stress path in each material element is quite complex. The simple method proposed by Broms (22) and Ingold (23,24) appears to predict quite well the compaction-induced lateral earth pressure. This method is based on

1. Classical earth pressure theory that defines the extreme limits for the lateral pressures that can be developed in a soil mass.

2. The assumption that, under loading, the vertical stress increases and the horizontal stress remains unchanged until limit equilibrium is reached; then both the vertical and the horizontal stresses increase according to the limit corresponding to the active state case. That is,

$$\sigma_h = K_a \sigma_v \quad (7)$$

where σ_v and σ_h are the vertical and horizontal stresses, respectively, and K_a is the coefficient of active lateral earth pressure. Therefore, horizontal compression develops in the soil mass.

3. The assumption that, under unloading, the vertical stress decreases and the horizontal stress remains unchanged until limit equilibrium is reached; then both vertical and horizontal stresses decrease, according to the limit corresponding to the passive state case. That is,

$$\sigma_h = K_p \sigma_v \quad (8)$$

where K_p is the coefficient of passive lateral earth pressure. The final state of stress is reached when the vertical stress is equal to the overburden.

4. Vertical stress under the roller is determined using line loading condition and semi-infinite elastic mass (Boussinesq case).

Figure 6 shows the method of analysis for a vibratory compactor applying 3 ton/ft (100 kN/m) and for a well-graded subbase or base material with $c = 0.7$ psi (5 kN/m²) and $\phi = 45$ degrees. The vertical stress of 61 psi (420 kN/m²) is determined at a 6-in. (0.15-m) depth corresponding to the bottom of the layer being compacted. It is seen that, under 0.44 psi (3 kN/m²) overburden pressure, a maximum residual horizontal stress of about 6 psi (40 kN/m²) is expected to be induced. This example may be viewed as representative of granular materials in pavement because the cohesion, the angle of internal friction, and the load magnitude are typical of present pavement construction technology.

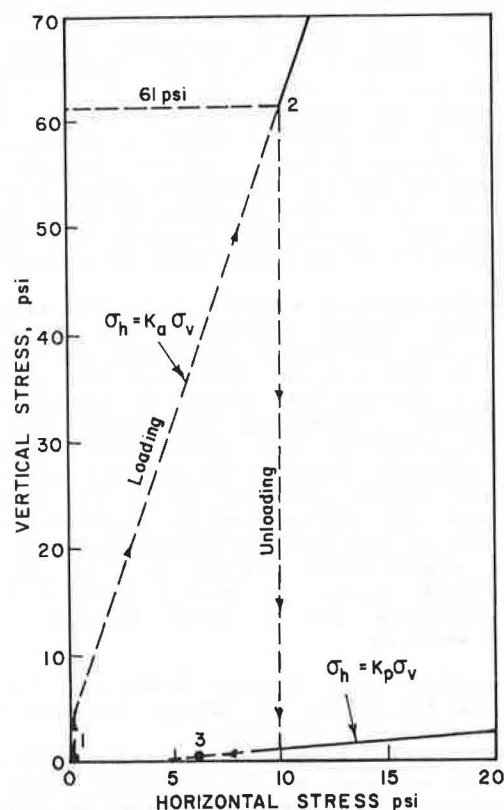


FIGURE 6 Schematic representation of stress path during compaction.

It is worth mentioning that, according to the method of analysis, the residual stress depends on the material properties and on the compacting load. For a cohesionless material, the envelope for the passive case (lower envelope in Figure 3) intersects the overburden line at quite low horizontal stress compared to cohesive material. Furthermore, to develop residual stresses, sufficient load must be applied to reach the active state condition.

Lateral earth pressures of 2 psi (15 kN/m²) and 6 psi (40 kN/m²) have been measured for cohesionless and cohesive materials, respectively, compacted behind retaining walls and bridge abutments (23,24). The values of the measured residual stresses were

found to be in good agreement with this method of analysis.

ANALYSIS OF LAYERED SYSTEMS AND INVESTIGATION OF BEHAVIOR OF GRANULAR MATERIAL

The behavior of granular material is investigated by analyzing different pavements, using a finite element program for axisymmetric problems. Constant moduli of elasticity are assigned to the asphalt concrete layer and to the subgrade. Equations 1 and 6b are used for the resilient moduli of the base and subbase granular materials. With Equation 1, only the geostatic stresses (vertical stress = overburden pressure; horizontal stress = coefficient of earth pressure at rest times the vertical stress) are considered. When Equation 6b is used, different additional residual horizontal stresses (induced by compaction) are assumed to exist in granular layers. It should be mentioned that, because the intermediate and minor principal stresses are not equal in the pavement, the deviator stress (σ_d) in Equation 6b was replaced by the octahedral shear stress.

The pavements analyzed are made of 4 in. of asphalt concrete with a modulus of elasticity of 500,000 psi, 6 in. of dense graded aggregate base (see Figure 4), 8 or 14 in. of gravel subbase (see Figure 5) in Pavements I and II, respectively. Two subgrade moduli were included in the analysis, 4,500 and 15,000 psi. Poisson's ratios of 0.4 were assumed for all materials. Because granular material behavior is described by a nonlinear law and the finite element program used treats the two dimensional (axisymmetric) case, only a single wheel load is applied at the pavement surface. A contact area with a constant radius of 6 in. and uniformly distributed contact pressures were assumed.

Figure 7 shows the distribution with depth of the computed moduli close to the axis of the wheel load when applying a contact pressure of 70 psi. The results shown correspond to Pavement I, and, using Equation 6b with different residual stresses and Equation 1, it is seen that (a) as the residual stress increases, the computed modulus increases and (b) the modulus computed using Equation 1 (without the additional residual stresses) appears to correspond to a residual stress of about 2 psi in the

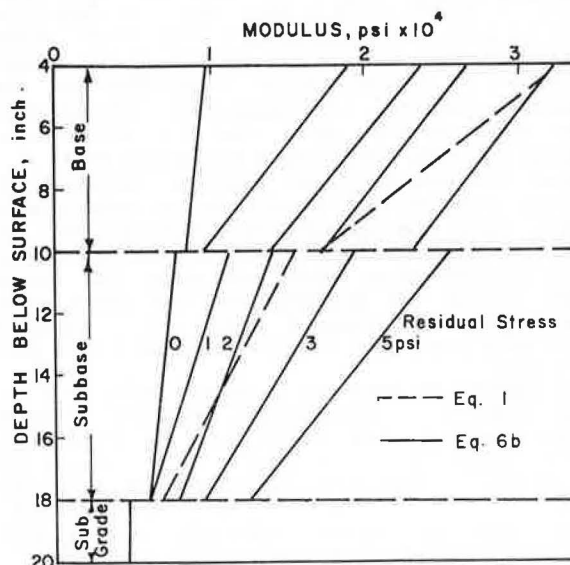


FIGURE 7 Distribution with depth of base and subbase moduli under the load.

subbase and of more than 3 psi in the base (with Equation 6b).

Figures 8 and 9 show the distribution along the radial distance of the computed moduli at middepth of the base and subbase layers for subgrade moduli of 4,500 and 15,000 psi, respectively. It is seen that: (a) The distributions obtained using Equations 1 and 6b are different. With Equation 1, the modulus is higher under the load than far from it. With Equation 6b, the modulus is in some cases higher and in other cases lower under the load than far from it. The distribution depends on the subgrade modulus that controls the sum of principal stresses and deviatoric stresses, especially under the load. (b) The modulus varies with the radial distance. Therefore the commonly used layered analyses with constant moduli for the granular layers must be viewed as approximate.

To compare the results of analyses with the empirical approach, an equivalent modulus of the granular material (base and subbase) was computed. It was derived on the basis of equal deflections at the center of the circular wheel load.

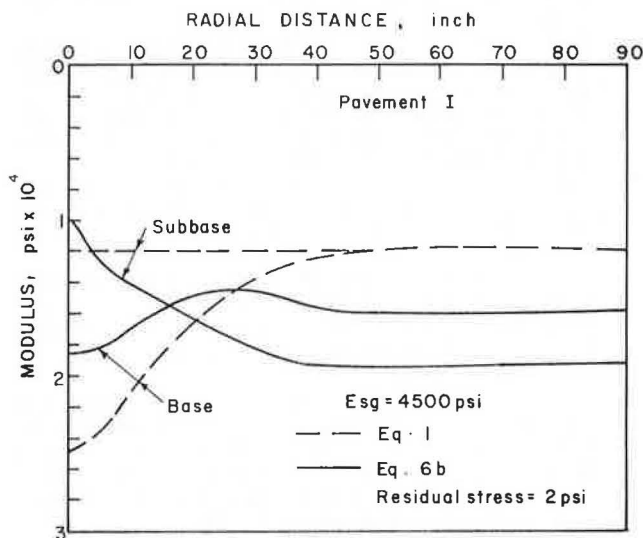


FIGURE 8 Distribution of base and subbase moduli versus radial distance (subgrade modulus 4,500 psi).

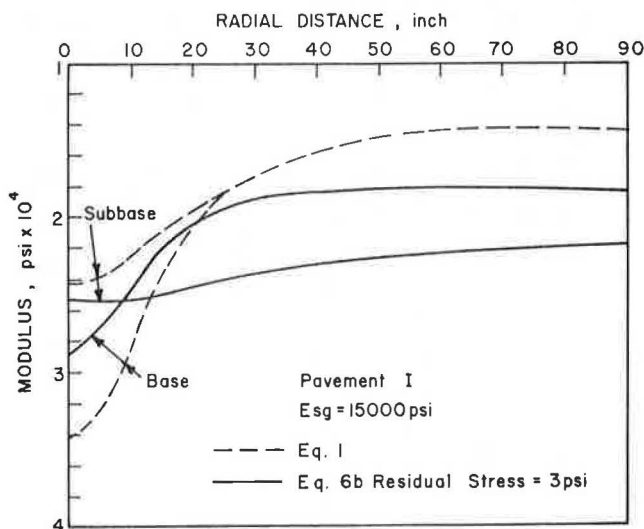


FIGURE 9 Distribution of base and subbase moduli versus radial distance (subgrade modulus 15,000 psi).

Figure 10 shows the results of computations for Pavement I and a subgrade modulus of 4,500 psi. Three load levels are included. It is seen that (a) the modulus according to Equation 6b increases with increasing residual stresses and with decreasing load level, (b) the modulus according to Equation 1 increases slightly with increasing load level, and (c) the range of empirical moduli of granular material corresponds to residual stresses of up to 3 psi.

Similar results are shown in Figure 11 for Pavement II. From Figures 10 and 11, it can be clearly seen that the equivalent modulus increases with increasing thickness of the granular material. The increase in the modulus is slightly more pronounced

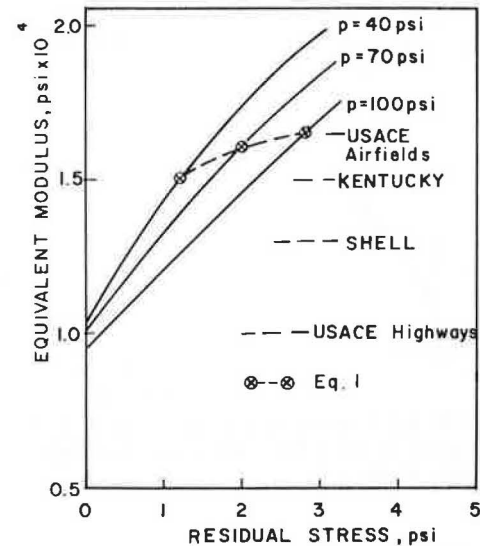


FIGURE 10 Equivalent modulus of granular material for Pavement I and subgrade modulus of 4,500 psi.

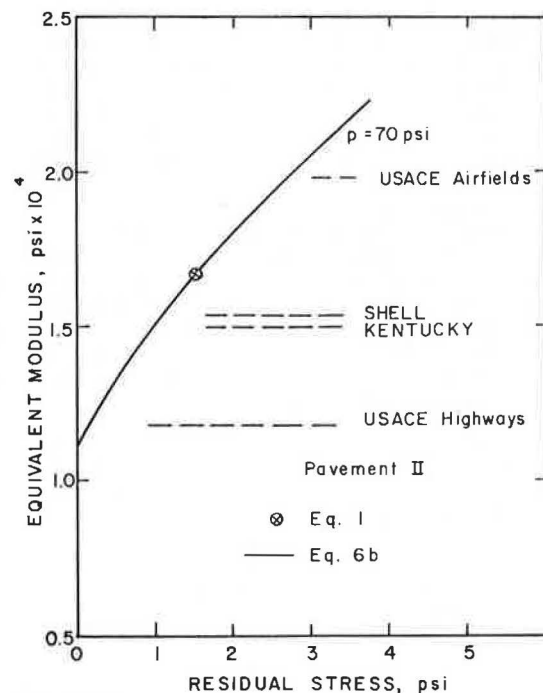


FIGURE 11 Equivalent modulus of granular material for Pavement II and subgrade modulus of 4,500 psi.

with Equation 1. Figure 12 shows results of computations for a subgrade modulus of 15,000 psi and a contact pressure of 70 psi. It should be noted that, in this case of a strong subgrade, the empirical procedure predicts a modulus of the order of 40,000 psi, a value that is not reached even with 5 psi residual stresses.

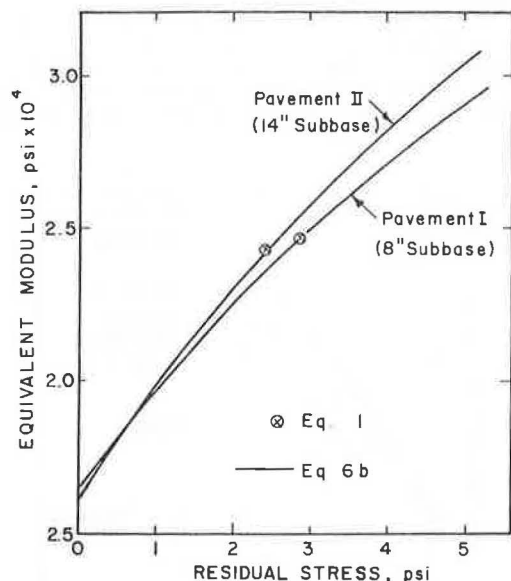


FIGURE 12 Comparison of equivalent moduli of granular material for Pavements I and II (subgrade modulus 15,000 psi).

SUMMARY AND CONCLUSIONS

It has been demonstrated that Equation 1 is not sufficient to describe the behavior of granular material in the test results from which it is derived. Although the modulus values obtained in pavement analyses are in the proper range, their variation with respect to load level and position (under and far from the load) does not appear to fit experimental results. Analyses with Equation 1 show that the granular modulus increases as the load level increases, a trend that is not, to the knowledge of this author, supported by field test results. It should be remembered that Equation 1 neglects the effect of shear strains.

The results of analyses using Equation 6b appear to be in good agreement with all aspects of granular material behavior, provided that a residual stress induced by compaction is postulated. The modulus increases with decreasing load level and increasing granular layer thickness. The residual stress required to develop values in the proper range, compared to empirical values, appears to be 1 to 2 psi. On full-scale retaining wall models, such values have been measured even with sandy backfill. It will therefore be not unrealistic to assume that dense subbase and base materials that comply to specifications will develop residual stresses of this order of magnitude.

Further research is required to better understand the material properties and construction techniques that govern the induction of residual stresses.

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Failure Criteria and Lateral Stresses in Track Foundations

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ABSTRACT

In conventional track systems the ties are supported by an unbound ballast layer underlain by subballast or the subgrade, or both. Analytical models used to estimate stresses and deflections of these multilayer systems predict that the lower portions of the unbound layer will develop significant incremental tensile stresses. Their magnitude is such that, when combined with the normally expected geostatic stresses, failure of the ballast is predicted. Some analytical models have incorporated failure criteria for the unbound layer as a means of limiting these stresses to permissible values. A discussion of these approaches is presented along with the implications that predicted failure would have for permanent deformation prediction methodologies. An alternative method based on residual lateral stresses in the ballast is presented. A description is given of a laboratory box testing device used to measure residual lateral stresses. Experimental results are shown that indicate that relatively large residual stresses, due to repeated applications of loading, can develop in ballast. The effects of combining the initial lateral stresses in the unbound layer with the incremental tensile stresses predicted by continuum or finite element models are discussed. Particular attention is given to the effects that these residual stresses have on prediction of permanent deformation.

The deformation analysis of conventional track structures requires characterization of the overall track system. This system includes the rails and ties that are supported by a foundation. The foundation consists of a ballast layer that is underlain by a subballast layer, in some cases, followed by a

subgrade. This foundation is a multilayer system wherein a relatively stiff ballast layer lies above a softer subballast or subgrade. The stiffness of each layer is generally represented by its resilient Young's modulus (E_r).

There is a need to improve the modeling of these