Modulus to Model Pavement Responses. In Transportation Research Record 810, TRB, National Research Council, Washington, D.C., 1981, pp. 1-9.
17. S.F. Brown. Discussion. Proc., Fifth International Conference on the Structural Design of Asphalt Pavements, Delft, The Netherlands, 1982, Vol. 2.
18. A.F. Stock and S.F. Brown. Nonlinear Characterization of Granular Materials for Asphalt Pavement Design. In Transportation Research Record 755, TRB, National Research Council, Washington, D.C., 1980, pp. 14-20.
19. G.F. Sowers, A.D. Robb, C.H. Mullis, and A.J. Glen. The Residual Lateral Pressures Produced by Compacting Soils. Proc., 4th International Conference on Soil Mechanics and Foundation Engineering, 1957, pp. 243-247.
20. D.R. Carder, R.G. Pocock, and R.T. Murray. Experimental Retaining Wall Facility--Lateral Stress Measurements with Sand Backfill. TRRL Laboratory Report 766. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, 1977.
21. D.R. Carder, R.T. Murray, and J.V. Krawczyk. Earth Pressures Against an Experimental Retaining Wall Backfilled with Silty Clay. TRRL Laboratory Report 946. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, 1980.
22. B. Broms. Lateral Earth Pressures Due to Compaction of Cohesionless Soils. Proc., 4th Budapest Conference on Soil Mechanics and Foundation Engineering, 1971, pp. 373-383.
23. T.S. Ingold. The Effects of Compaction on Retaining Walls. Geotechnique, Vol. 4, 1979, pp. 265-283.
24. T.S. Ingold. Lateral Earth Pressures--A Reconsideration. Ground Engineering Journal, May 1980, pp. 39-43.

Publication of this paper sponsored by Committee on Mechanics of Earth Masses and Layered Systems.

Failure Criteria and Lateral Stresses in Track Foundations

HARRY E. STEWART, ERNEST T. SELIG, and GILLIAN M. NORMAN-GREGORY

ABSTRACT

In conventional track systems the ties are supported by an unbound ballast layer underlain by subballast or the subgrade, or both. Analytical models used to estimate stresses and deflections of these multilayer systems predict that the lower portions of the unbound layer will develop significant incremental tensile stresses. Their magnitude is such that, when combined with the normally expected geostatic stresses, failure of the ballast is predicted. Some analytical models have incorporated failure criteria for the unbound layer as a means of limiting these stresses to permissible values. A discussion of these approaches is presented along with the implications that predicted failure would have for permanent deformation prediction methodologies. An alternative method based on residual lateral stresses in the ballast is presented. A description is given of a laboratory box testing device used to measure residual lateral stresses. Experimental results are shown that indicate that relatively large residual stresses, due to repeated applications of loading, can develop in ballast. The effects of combining the initial lateral stresses in the unbound layer with the incremental tensile stresses predicted by continuum or finite element models are discussed. Particular attention is given to the effects that these residual stresses have on prediction of permanent deformation.

The deformation analysis of conventional track structures requires characterization of the overall track system. This system includes the rails and ties that are supported by a foundation. The foundation consists of a ballast layer that is underlain by a subballast layer, in some cases, followed by a subgrade. This foundation is a multilayer system wherein a relatively stiff ballast layer lies above a softer subballast or subgrade. The stiffness of each layer is generally represented by its resilient Young's modulus (E_Y).

There is a need to improve the modeling of these systems with emphasis on the geotechnical properties of the foundation layers. Methods of practice may make use of simplified approaches for estimating vertical stresses on the subgrade. Although the American Railway Engineering Association (AREA) Manual of Practice does not contain a method for predicting foundation stresses, a procedure such as that described by Talbot in the 1920s may be used. These simplified procedures do not address the multilayer nature of the foundation, nor do they address the internal stresses in the ballast that would be needed to predict permanent deformations of that layer.

Analytical models (1,2) are used to obtain estimates of the stresses and deflections in these multilayer systems under specified loading conditions. These results are then used for design applications by predicting either allowable resilient deflections or permanent deformation. These models, based on either multilayer elastic theory or finite element methods, predict that loading will cause significant incremental horizontal tensile stresses to develop in the lower portions of the ballast layer. When these stress states at the base of the ballast layer are combined with the normally expected geostatic stresses, the resulting total horizontal stresses are usually still tensile and thus a failure condition is predicted.

Methods have been used to limit these stresses to allowable values by incorporating either special conditions for the failed zones or specific failure criteria for the unbound ballast layer. A discussion of these methods will be presented, along with the implications that predicted failure would have for permanent deformation prediction methodologies.

An alternative method is based on residual lateral stresses in ballast. These residual lateral stresses affect the unloaded or geostatic stress state and thus influence the total stress states in the loaded condition. A description will be given of a laboratory testing device used to observe residual lateral stresses, along with experimental results that indicate that relatively large residual lateral stresses can develop in ballast as a result of repeated loadings.

The effects of combining these unloaded stresses with the predicted incremental tensile stresses will be discussed. Attention will be directed toward the implications that these residual stresses have for predictions of permanent deformation.

STRESS STATES

The unloaded vertical stresses (σ_{vo}) at depths within the ballast layer are determined by the weight of the overlying track structure (rails and ties) and the unit weight of the ballast. The unloaded lateral stresses (σ_{ho}) are typically defined as $\sigma_{ho} = K_o \sigma_{vo}$, where K_o is the coefficient of lateral earth pressure at rest. For the purposes of this paper it is assumed that the ballast is clean and free draining so that any effects of pore pressure can be neglected. The unloaded stress state at the base of the ballast is entirely compressive, as shown in Figure 1a. When surface loads are applied to the multilayer system wherein the modulus of the ballast layer is greater than that of the underlying subballast or subgrade layer, incremental compressive vertical and tensile lateral stresses will be analytically predicted near the base of the ballast layer, as shown in Figure 1b.

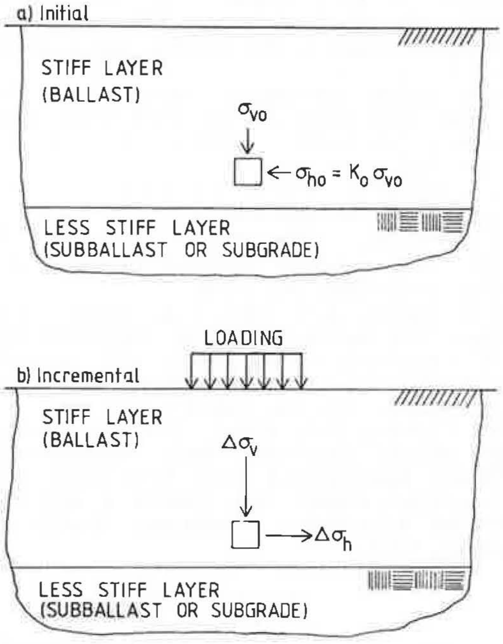

FIGURE 1 Initial and incremental stresses at the base of a layered system.

The stress states in the loaded condition are determined by adding the initial to the incremental stress states such that $\sigma_v = \sigma_{vo} + \Delta\sigma_v$ and $\sigma_h = \sigma_{ho} + \Delta\sigma_h$. Beneath the loaded area, at all depths, the vertical stresses are always compressive. The incremental horizontal stresses are compressive near the top of the stiff upper layer and become tensile in the lower regions. The transition depth from incremental compressive to incremental tensile horizontal stresses is affected by the depth of the ballast layer and the relative stiffness of the ballast compared to the underlying layer.

Many analyses in geotechnical engineering require assuming a value of K_o because of a lack of either laboratory or field data. Typical values of K_o might be in the range of 0.5 to 1.0 for granular materials such as ballast. The loaded stress states at the bottom of the ballast layer, for a K_o value of 1.0 and the incremental tensile stresses shown in Figure 1, are shown in Figure 2. This Mohr circle stress state is not allowable because it exceeds the Mohr-Coulomb failure envelope. Thus, near the base

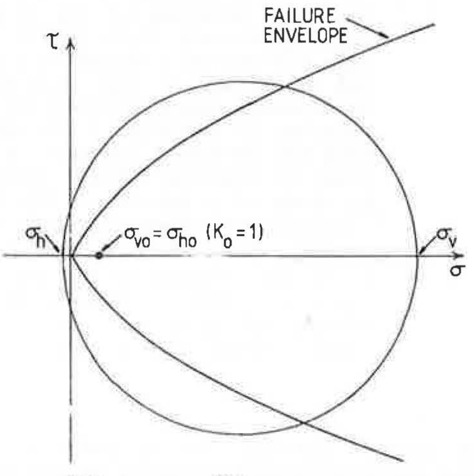

FIGURE 2 Loaded stress state in the tension zone for low K_o conditions.

of the ballast layer, the incremental tensile stresses combined with the geostatic conditions described previously would indicate a condition of static failure. Because the loaded stress state shown in Figure 2 cannot occur, differences must exist between the predicted and the actual stress states in the "tension" zone.

PREVIOUS APPROACHES

Various methods have been proposed to circumvent the problem of predicted failure. One method is to subdivide the upper granular layer into many small sublayers about 1 in. thick (3). However, tensile stresses will still develop in the lower portions of at least some of the sublayers, or possibly in an entire sublayer. To determine the stress state for the stress-dependent resilient modulus for each of the sublayers, a middepth location is often used. This location may be above the predicted tension zones for some sublayers, but the predicted tensile stresses still may exist at middepth for lower sublayers. In such cases a lower modulus is arbitrarily assumed for the sublayer. This approach will cause the newly computed resilient moduli to decrease with depth and cause more gradual transitions in modular ratios, which reduces the tendency for tensile stresses. However, this method still does not address the actual material behavior and may increase the computer time necessary to obtain the solutions.

Another approach is to lower the modulus of the granular material in those areas where predicted failure occurs (1). This provides for a more gradual transition between the stiffer ballast layer and the softer underlying subballast or subgrade, but it is a compensating scheme that is not consistent with the material response.

Figure 3 is a representation of a triaxial test result in which the sample is loaded to a failure or near-failure state and then unloaded and reloaded several times. During the primary loading, the tangent modulus decreases as the sample approaches failure. When the material is unloaded the resilient portion of the strain is recovered and some permanent strain remains. On reloading, the soil is stiffer than during primary loading. As the soil stresses approach the failure level, followed by another unloading, additional permanent strain develops while the resilient strain is recovered. Even though the tangent modulus may be low near the failure state, the resilient modulus remains high.

The appropriate soil modulus for the ballast layer in a track system, or for any layer in a system subjected to repeated loadings, is the resilient modulus, shown in Figure 3, that remains high even when the material is repeatedly stressed to levels at or near static failure. Imposing an arbitrarily low modulus in those zones predicted to be in a state of static failure is equivalent to using a near-failure tangent modulus. This is not representative of the material's resilient response under the predicted stress state. The resilient modulus should remain high. The use of low moduli may eliminate the tension zone predictions, but such moduli are uncharacteristic of actual material behavior and will overpredict elastic deformation.

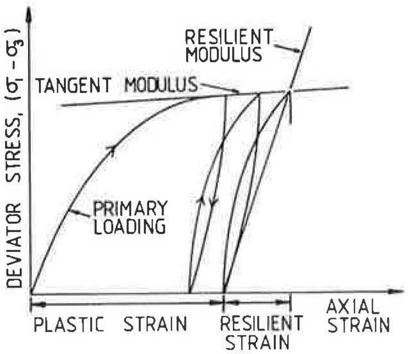

FIGURE 3 Representation of a stress-strain curve from a repeated load triaxial test to near-failure conditions.

The Mohr-Coulomb failure criterion has been used directly to modify the stresses predicted at the base of the granular layers in a multilayer system (4). To do this, Mohr-Coulomb parameters c and ϕ are first assigned to the granular materials. Presumably, the parameters c and ϕ may be taken as the effective stress parameters for a granular material, and the inclusion of the cohesion term (c) is to account for the curvature of the Mohr-Coulomb failure envelope and not to be interpreted as true material cohesion.

The procedure given by Raad and Figueroa (4) sets numerical limits on the major and minor principal stresses that can be developed at depths within the granular layer. These limiting stress states are based on principal stress ratios for active and passive failure, using the calculated values of vertical stress (σ_v) in each soil element as one of the principal stresses. These limiting principal stress states, for a given σ_v, are shown in Figure 4 as $(\sigma_1)_{max}$ and $(\sigma_3)_{min}$. The computed minor principal stress (σ_3) within a soil element is then compared with these limiting values and adjustments are made to the element stresses so that the modified stresses result in a Mohr circle, tangent to the failure envelope but having principal stresses between the $(\sigma_1)_{max}$ and $(\sigma_3)_{min}$ limits. This modified stress circle is shown as dashed line A in Figure 4. Alternatively, if σ_3 is negative, the element stress state is set at that represented by dashed line B in Figure 4. The modified stresses are then used in a stress-dependent resilient modulus formulation, generally one based on bulk stress, and the foundation stresses are reanalyzed. This method is said (4) to converge and satisfy equilibrium and boundary conditions. The result of this procedure is that any predicted tensile stresses are adjusted to nontensile values and a high resilient modulus is maintained. However, conditions of static failure still exist, which makes the method unreasonable for use in permanent deformation predictions.

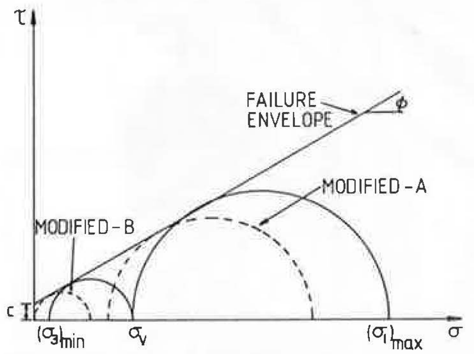

FIGURE 4 Mohr-Coulomb limiting and modified failure stress conditions.

Brown and Pappin (see their paper in this Record) have reviewed the results of the University of Nottingham research on layered system response. They use the program SENOL to evaluate layer behavior including the stresses developed in the unbound granular layer. Their soil model, termed "contour" model, is represented by families of strain contours on a deviator stress-mean normal stress plot. These plots were developed from extensive repeated load triaxial tests. This soil model is thus nonlinear and stress state dependent. The results show that the incremental horizontal stresses from the load given by SENOL in the lower part of the granular layer are tensile. However, when the geostatic stresses are added, assuming $K_o = 1$, the combined loaded stress states are compressive. Furthermore, the stress state in the lower part of the granular layer is below failure.

The elimination of the failure state in the approach by Brown and Pappin is believed to be a result of a more accurate soil model than that used in any other approaches.

RESIDUAL STRESSES

An examination of the horizontal stresses predicted from elastic multilayer theory led to the hypothesis that residual horizontal stresses are induced in ballast by repeated wheel loads. A laboratory study was undertaken to examine the nature and extent of these horizontal stresses. To do this a ballast box was constructed (Figure 5) with instrumented side and end panels to measure horizontal stress and a flexible bottom to represent the effect of subgrade conditions. The box was intended to simulate conditions in track near the rail seat and adjacent crib areas. The ballast box was 12 in. wide, 24 in. long, and 19 in. deep. Further details on construction of the box are given elsewhere (5).

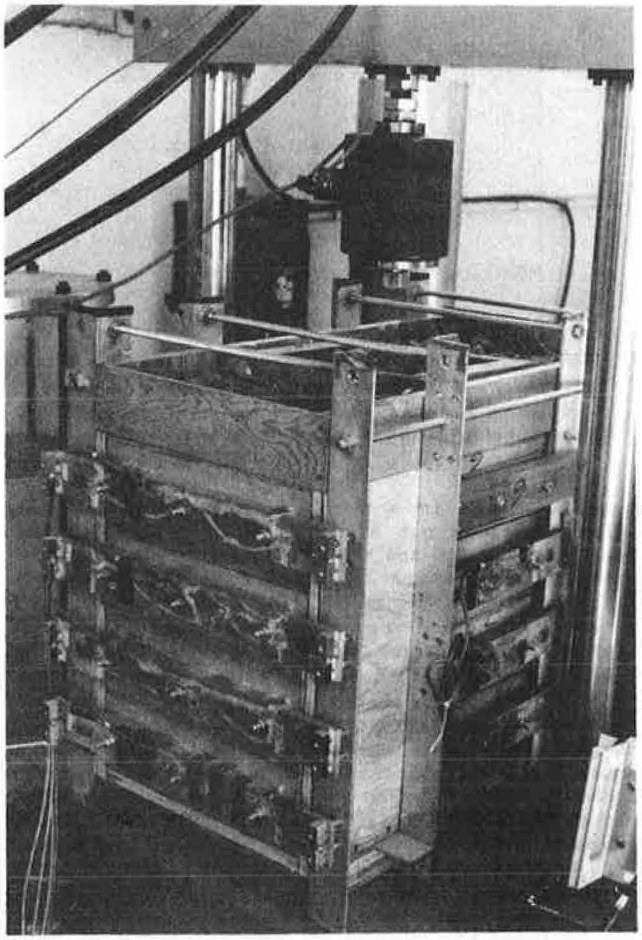

FIGURE 5 Ballast box apparatus.

The ballast material used in these tests was an angular traprock with an AREA No. 4 gradation. The loads were applied through a tie segment 9 in. wide by 11.5 in. long. The ballast depth below the tie segment was about 12 in. The maximum cyclic load was 4,000 lb, producing a tie contact pressure equivalent to that of a 32-kip wheel load from a train.

Typical measurements from the side panels of the box are shown in Figure 6. The results indicated that there was a rapid buildup of horizontal stresses during the first loading and that high residual stresses develop after the first unloading. Furthermore, the horizontal stresses in the loaded state decreased and the residual stresses increased up to about 100 cycles. After 100 cycles, the horizontal stresses tended to stabilize and the unloaded value tended to converge to the loaded value. This means that the horizontal stresses tended to become constant during the load cycle. The maximum horizontal stresses acting on the side panels occurred at about middepth of the ballast. The minimum horizontal stresses on the side panels occurred near the base of the ballast layer.

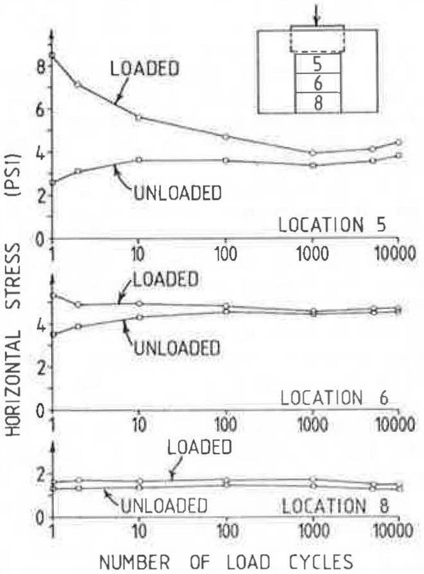

FIGURE 6 Horizontal stresses on side panels.

The measured horizontal stresses at the end panels are shown in Figure 7. Similar rapid buildup of the lateral residual stresses was observed at the ends of the box. The maximum lateral stresses again occurred about 6 in. above the base of the ballast layer.

Although the magnitude of the effects may be a function of the box boundary conditions, these tests clearly show that substantial horizontal residual stresses can develop in ballast. The measured residual stresses in the unloaded state were used to calculate values of K_o. Theoretically, the maximum residual horizontal stress must be limited to stresses at the passive failure condition. The ratio of the major principal stress at failure ($\bar{\sigma}_{1f}$) to the minor principal stress at failure ($\bar{\sigma}_{3f}$) at the passive

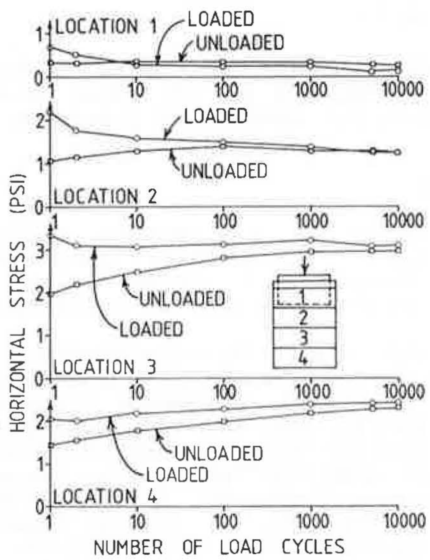

FIGURE 7 Horizontal stresses on end panels.

failure condition, for a strictly cohesionless material such as ballast, is defined as

$$\bar{\sigma}_{1f}/\bar{\sigma}_{3f} = (K_o)_{max} = K_p = [(1 + \sin \bar{\phi}) \div (1 - \sin \bar{\phi})] \qquad (1)$$

The calculated K_o values, based on the unit weight of the ballast, the static surcharge of the tie segment, and the box test side and end panel measurements, are shown in Figure 8. All are greater than unity except in the crib above the base of the tie segment. The extremely large values of K_o for the upper portion of the side zone would require a friction angle ($\bar{\phi}$) of about 56.5 degrees, which is unusually high. However, if the curved failure envelopes, the possible interlock of the large particles, and the low vertical stresses in this zone are considered, a friction angle of this magnitude is possible. Certainly, values of K_o = 6 would be possible because the required friction angle would be only about 45.5 degrees. Triaxial tests, run on a variety of ballast materials, have shown that friction angles within this range are possible for low effective confining pressures.

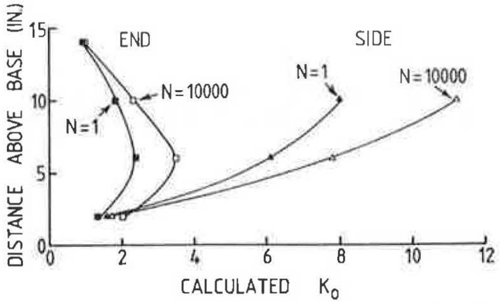

FIGURE 8 Variation of K_o as a function of depth from box experiments.

These experimental observations of the high residual lateral stresses and correspondingly high K_o values may be one of the main considerations necessary to account for what actually occurs in the lower portions of the granular base in a multilayer system.

IMPLICATIONS FOR PERMANENT DEFORMATIONS

Methods of analysis that make use of significantly reduced moduli and static failure criteria contain inconsistencies or make assumptions that are not compatible with observed material response. One main disagreement is that a condition of static failure cannot exist at the base of a granular layer such as the ballast in track. Static failure, in terms of the Mohr-Coulomb criterion, represents a limiting stress condition where the full strength of the material has been developed. Unlimited strains would result if such a stress state were developed. In track systems, such a failure condition near the base of the ballast would mean that excessive deformations would occur under a single load cycle. However, because the layered systems used in track and pavements do not fail under even one load cycle, let alone continuously fail under repeated load as would be evidenced by large, rapid surface deterioration, the use of failure criteria contains fundamental inconsistencies.

The loaded stress states in track are dependent on both the geostatic stresses and the incremental stresses imposed by the applied loading. The existence of high residual compressive stresses in the unloaded state could offset the predicted incremental tensile stresses. If these residual stresses are large enough, a nonfailure final stress state would result. This is shown in Figure 9 for the same magnitude of initial vertical stress (σ_{vo}) and incremental stresses ($\Delta\sigma_v$ and $\Delta\sigma_h$) as shown in Figure 2. The initial residual horizontal stress (σ_{ho}) is larger than σ_{vo} because the K_o value is greater than unity. The final vertical stress (σ_v) in Figure 9 is the same as that given in Figure 2, but the final horizontal stress (σ_h) is sufficient to reduce the maximum stress difference and thus maintain a nonfailure loaded stress state.

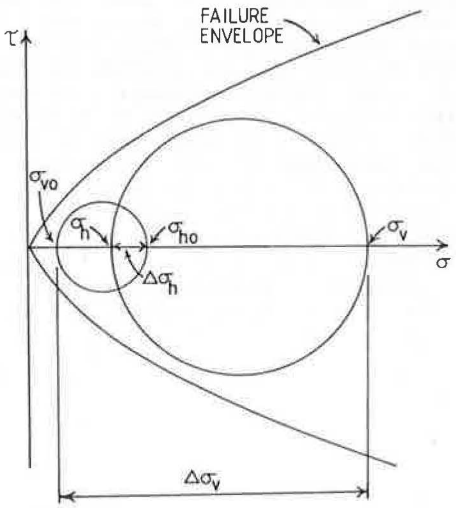

FIGURE 9 Loaded stress state in the tension zone for high K_o conditions.

Considerable effort has been devoted, both in the United States and abroad, to predicting the permanent deformations that accumulate in track due to repeated loadings (6-8). The approaches taken in the United States make use of the stress path method by combining estimates of stresses in a layered track system with laboratory measurements of the inelastic soil response under representative stress conditions in a triaxial test. When zones in the track foundation are predicted to be in a state of failure, there is no way to predict the amount of permanent strain that can be expected to develop. When residual stresses are used to eliminate the nonallowable tensile lateral stresses such that the final stresses are permissible, as conceptually shown in Figure 9, the stress path approach can be used. The approach taken by Brown and Pappin (see their paper in this Record) showed that $K_o = 1$ was sufficient to eliminate the tensile stresses. Use of even higher K_o values such as observed in the ballast box will result in lower permanent strain predictions that could well prove to be even more realistic when combined with their estimated stress paths.

SUMMARY

The purpose of this paper has been to discuss methods of dealing with predicted failure zones in a layered system. An approach based on experimental measurements of residual lateral stresses is recommended.

The use of many sublayers and the selection of depth points above the predicted tension zone may, in some cases, give reasonable results, but this bypasses and does not fully address the problem. Adjustment of the soil modulus to a low value may reduce the tendency for incremental tensile lateral stresses to develop but does not represent the actual resilient modulus of the granular material even at a failure condition. Thus even the predicted elastic deformations should be unrealistic. The application of static failure criteria and the adjustment of the stresses to limit equilibrium conditions may result in satisfactory predictions of the resilient surface vertical deformation of a layered system. However, failed soil zones beneath the loaded areas would imply large plastic deformations, which is inconsistent with actual responses. The use of residual horizontal stresses, as observed in the ballast box experiments, does not eliminate the predicted incremental tensile stresses. Instead, these residual stresses compensate for the tensile stresses and result in nonfailure loaded stress states.

ACKNOWLEDGMENTS

This study was partly supported by the Association of American Railroads and the Federal Railroad Administration, U.S. Department of Transportation.

REFERENCES

1. S.D. Tayabji and M.R. Thompson. Finite Element Analysis of a Railway Track Support System. Report FRA-OR&D-76-257. Federal Railroad Administration, U.S. Department of Transportation, May 1976.
2. H.E. Stewart and E.T. Selig. Predicted and Measured Resilient Response of Track. Journal of the Geotechnical Engineering Division, ASCE, Vol. 108, No. GT11, Nov. 1982, pp. 1423-1442.
3. D.R. Luhr and B.F. McCullough. Structural Analysis of AASHO Road Test Flexible Pavements for Performance Evaluation. In Transportation Research Record 888, TRB, National Research Council, Washington, D.C., 1982, pp. 63-69.
4. L. Raad and J.L. Figueroa. Load Response of Transportation Support Systems. Journal of the Transportation Engineering Division, ASCE, Vol. 106, No. TE1, Jan. 1980, pp. 111-128.
5. G.M. Norman and E.T. Selig. Ballast Performance Evaluation with Box Tests. AREA Bulletin 692, Vol. 84, May 1983, pp. 207-239.
6. J.E.D. Alva-Hurtado. A Methodology to Predict the Elastic and Inelastic Behavior of Railroad Ballast. Ph.D. dissertation, Report OUR80-245D. Department of Civil Engineering, University of Massachusetts at Amherst, May 1980.
7. H.E. Stewart. The Prediction of Track Performance Under Dynamic Traffic Loading. Ph.D. dissertation, Report FRA82-292D. Department of Civil Engineering, University of Massachusetts at Amherst, May 1982.
8. M.J. Shenton. Ballast Deformation and Track Deterioration. Proc., Track Technology for the Next Decade, Institute of Civil Engineers, University of Nottingham, England, July 1984, pp. 175-187.

Publication of this paper sponsored by Committee on Mechanics of Earth Masses and Layered Systems.