ABSTRACT

Field measurements of vertical deflections and track modulus under static loading conditions were obtained at four revenue service track locations. The track sections contained both concrete and wood ties. The measurements were made before and after a scheduled track maintenance operation and were repeated after the surfacing to determine the effects of maintenance on the vertical track response. Differences between the pre- and postmaintenance results were found and are discussed in terms of physical state of the ballast and the characteristics of the layered track systems. The resilient foundation properties of the field sites were obtained from both field and laboratory measurements. Predictions of vertical track response at the sites were then made using a three-dimensional, nonlinear, elastic, multilayer track analysis program. The predicted values of vertical track response were in general agreement with the measured values, although some differences were evident. The major variables affecting track modulus are identified and conclusions are presented on the usefulness of vertical track modulus as a measure of track performance.

DESCRIPTION OF SITES

Four revenue service track locations were selected as test areas for this project. The revenue service sites included three locations that contained concrete cross ties and a control section with wood cross ties. The wood tie control section and one concrete tie section are located in Leeds near Streator, Illinois, in the north-central section of the state. The remaining two concrete tie test sections are located at Aberdeen, Maryland, and Lorraine, Virginia. A detailed description of each of these four sites is given elsewhere (1).

The concrete tie test section at Leeds is an 800-ft-long portion of tangent track owned by the Atchison, Topeka, and Santa Fe (ATSF) Railroad. It is built on a fill embankment approximately 12 ft high. The ballast in this section is slag, and the ties are hardwood at 19.5-in. nominal center-to-center spacing. The Leeds concrete tie test section is contiguous to the wood tie section on the same track. This concrete tie section contains granite ballast, and both RT-7S and Costain Conforce CC244C ties at 24-in. center-to-center spacing. The total test section length is 800 ft, but the tests and measurements were confined to the track section having the Costain ties. Pandrol spring-clip fasteners are used on the Costain-type ties.

The Lorraine, Virginia, concrete tie test installation is owned by the Chesie system and is located in the western Richmond suburb of Lorraine, on the north bank floodplain of the James River. The test section is on a single main-line track built on an embankment about 7 ft high on the south side. This Lorraine concrete tie test section contains both CC224 ties and RT7-7S ties. The test section is in the middle of a 3-degree curve, with a 3-in. super-elevation on the outside rail. The ballast is predominantly gneiss and limestone. The ties are located at 25-in. center-to-center spacings.

The Aberdeen, Maryland, site is on one of three parallel main-line electrified tracks, owned by Consolidated Rail Corporation (Conrail), that carry traffic between Baltimore and New York. The ballast in the test section is a traprock. The track is tangent with RT7-552 concrete cross ties at 24-in. center-to-center spacings.

TRACK MODULUS FORMULATION

The use of track modulus for assessing track performance is common in the railroad industry. The theoretical formulation of track modulus is based on the assumption that the rail acts like a beam continuously supported on an elastic foundation. Track modulus \( [u] \) is defined as the supporting force per unit length of rail per unit deflection in the track system. A diagram of the assumed conditions for the formulation of track modulus is shown in Figure 1.

The differential equation describing the deflection, due to an applied vertical load, of a uniformly supported rail on a linear elastic foundation is given by

\[
EI(d^2y/dx^2) = q = -u\delta
\]  

where

\[
EI = \text{rail bending stiffness (units = } \text{kN-m})
\]

\[
\delta = \text{vertical rail deflection (units = } \text{m})
\]
The solution to Equation 1 is given by

\[ \delta = \left( \frac{8P/2u}{\sin \delta} \right) \left( \cos \delta + \sin \delta \right) \]

where \( P \) is the applied load and

\[ \beta = \left( \frac{u}{4EI} \right)^{1/4} \quad (\text{units} = 1/L) \]

The maximum rail deflection occurs at \( x = 0 \), and is given by

\[ \delta_x = \frac{8P/2u}{\sin \delta} \]

The equation for maximum rail deflection can be rearranged by substituting \( \delta \) from Equation 1 and solving for \( u \). The resulting equation for track modulus is

\[ u = \frac{1}{4 \left( \frac{P}{2u} \right)^2 \cdot \frac{1}{EI\delta^3}} \]

An important difference between the actual track support and the idealized formulation of a rail on an elastic support is that the rail load is actually applied to the foundation through discrete supports, which are the ties, not through support distributed along the track foundation. Another difference, for concrete tie track systems, is the inclusion of flexible tie pads between the rails and the tie rail seats. Even though these differences exist between the actual track structure and the theoretical formulation, the track modulus has historically been used as a measure of track quality. Further details on the historical development and interpretation of track modulus can be found elsewhere (2,3).

**SELECTION OF PARAMETERS FOR PREDICTIONS**

The GEOTRACK model (4,5) was used to determine track deflections for predicting values of track modulus for the revenue field sites. To do this, the track structural properties and foundation characteristics of each site had to be chosen. Table 1 gives the structural properties representing each of the sites. The subgrade layer properties used for the GEOTRACK analyses were chosen on the basis of the results shown in Figures 2 and 3. The subgrade values shown in Figures 2 and 3 were derived from repeated load triaxial tests (6) and additional correlations made between cone and standard penetration data (7). Layer divisions for the subgrades were made where there appeared to be significant changes in the measured resilient properties. The average resilient modulus for each layer was used as the representative value for the layer. The moduli for the subgrade layers were held constant because stress-state-dependent relationships were not available for the subgrade.

A shear stress-resilient strain formulation (8) was used to characterize the stress-dependent ballast properties for these sites. The moduli \( (E_u) \) of the ballast from the final iteration of the

**TABLE 1 Track Structural Properties for Field Sites**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Leeds Wood</th>
<th>Concrete CC244</th>
<th>Aberdeen CC244</th>
<th>Lorraine CC7-SS2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tie type</td>
<td>Hardwood CC244</td>
<td>RT7-SS2</td>
<td>CC244</td>
<td></td>
</tr>
<tr>
<td>Tie spacing (in.)</td>
<td>19.5</td>
<td>24.0</td>
<td>24.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Tie length (in.)</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
</tr>
<tr>
<td>Tie bottom width (in.)</td>
<td>9.00</td>
<td>10.75</td>
<td>10.75</td>
<td>10.75</td>
</tr>
<tr>
<td>Tie bending stiffness ( [E_L/\text{in.}^2 \times 10^6] )</td>
<td>386</td>
<td>1740</td>
<td>1740</td>
<td>1740</td>
</tr>
<tr>
<td>Rail section</td>
<td>136RE</td>
<td>136RE</td>
<td>140RE</td>
<td>122RE</td>
</tr>
<tr>
<td>Rail bending stiffness ( [E_L/\text{in.}^2 \times 10^6] )</td>
<td>2742.6</td>
<td>2742.6</td>
<td>2138.6</td>
<td></td>
</tr>
<tr>
<td>Rail fastener type</td>
<td>Cut spikes</td>
<td>Pandrol</td>
<td>Pandrol</td>
<td>Pandrol</td>
</tr>
<tr>
<td>Rail fastener or pad stiffness ( \text{lb/in.}^2 \times 10^6 )</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

**FIGURE 1 Assumed conditions for beam theory formulation of track modulus.**

**FIGURE 2 Resilient modulus versus depth for Leeds wood and concrete tie sections.**
Table 2. Ballast Moduli Determined for Track Modulus Predictions

<table>
<thead>
<tr>
<th>Site</th>
<th>Thickness of Ballast Layer (in.)</th>
<th>Resilient Modulus (E_r) in ps</th>
<th>Wheel Load (Kips)</th>
<th>Estimated Standard Deviations (KIPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leeds Wood</td>
<td>9</td>
<td>5,300</td>
<td>6</td>
<td>3,000</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>10,000</td>
<td>12</td>
<td>6,500</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>8,700</td>
<td>15</td>
<td>8,500</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>10,800</td>
<td>20</td>
<td>10,800</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>9,500</td>
<td>30</td>
<td>9,500</td>
</tr>
<tr>
<td>Concrete</td>
<td>7</td>
<td>7,800</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>6,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>8,700</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>10</td>
<td>10,800</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>9,500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The measured averages and standard deviations for the pre- and postmaintenance load-deflection curves are given in Figures 4-7 for the Leeds wood and concrete, Lorraine, and Aberdeen sites, respectively. The pretreatment curve for the Leeds concrete section (Figure 5) was based on measurements taken after about 1 month of traffic had passed in situ of immediately before maintenance because a surfacing operation had taken place just before the initial site visit. Although both pre- and postmaintenance values for the Aberdeen site were recorded, only the postmaintenance results were reported by BCL.

A small amount of slack may have been present in the track structure as indicated by an initial break in the curves as shown in Figures 4-7. This initial slack was assumed to have been eliminated after about a 6-kip load was applied. To remove this effect, the track modulus values were calculated for the 6- to 30-kip load range.

The variabilities of the track modulus measurements were estimated using the mean track deflections from the averaged load-deflection curves, and the deflections at ±1 standard deviation at the 6- and 30-kip load levels shown in Figures 4-7. However, the actual variabilities of the track modulus measurements were probably greater than the estimates determined in this manner. For the purposes of this paper, the standard deviations of the track modulus values will refer to the limits calculated on the basis of the standard deviations of the rail deflections. It must be noted that these are not the true standard deviations, and these values of standard deviation are not symmetrical about the mean values.

To determine track modulus with GEOTRACK, deflections were calculated for the single-axle solution with loads of 6 and 30 kips. The difference in loads and the difference in deflections were substituted into Equation 5 to obtain track modulus.

The measured values of track modulus and the estimated standard deviations for all of the revenue field sites are given in Tables 3 and 4 and shown in Figure 8, along with the predicted values based on the GEOTRACK analyses. Several items on Figure 8 deserve attention. First, there were no significant changes in the track modulus values as a result of the surfacing operations. However, the pretreatment values did appear to be less variable than the posttreatment values. This variation is also apparent in the average load-deflection curves (Figures 4-7) where the scatter about the mean is visibly larger for the postmaintenance values. That the average measured values were greater in the Leeds wood and Lorraine sections after maintenance is probably not statistically significant because the estimated standard deviations all overlap.

Another observation from Figure 8 is that there did not appear to be a correlation between height of raise and postmaintenance track modulus. Raises of 1.5 to 2 in. were given to the Leeds and Lorraine sections and only about 0.1 in. to the Aberdeen section. In spite of this, the pre- and postmaintenance values for any one test section were approximately equal, and the Aberdeen value was between the Lorraine and Leeds values.

An explanation of the increased variability of the postmaintenance modulus values compared to the premaintenance values could be that the surfacing decreased the uniformity of track support conditions.
between the locations. One purpose of track maintenance is to improve the overall track surface by smoothing out longitudinal differential track deformations. Varying amounts of raise must be applied beneath the ties to achieve a uniform surface. Variations in the actual raises applied beneath the individual ties could cause local differences in the amount of ballast disturbance, hence variations in physical state of the ballast from one tie to another.

Part of the difference in absolute magnitude of the average field track modulus measurements can be explained in terms of the differences in the track substructures. A parametric study using the GEOTRACK model (5) indicated that track modulus increased as ballast depth increased. The Leeds wood section had only about 9 in. of ballast beneath the tie, whereas the Leeds concrete section contained about 14 in. The Aberdeen site had 20 in. of ballast, and the
ballast depth at Lorraine was estimated to be 28 in. below the tie. Although the influence of site location cannot be separated from the results, this trend of increasing track modulus with increasing ballast depth for the field sites was confirmed by the field measurements as shown in Figure 9.

**SUBGRADE EFFECTS**

The track modulus is also influenced by the subgrade characteristics. The GEOTRACK model indicates that the compression of the ballast layer accounts for about 10 to 20 percent of the total vertical deflection of the track structure. The remainder of the total deflection is due to the compression of the subgrade materials. Furthermore, 25 to 40 percent of the subgrade deformation indicated by GEOTRACK occurs below a depth of about 10 ft, even though the stresses below this depth are low (5).

The Lorraine test section was found to have the greatest depth of ballast-type material and the stiffest subgrade. Correspondingly, the Lorraine section had the highest values of measured and predicted track modulus. As can be seen in Figure 9, the predicted value of track modulus was higher than the average measured values but well within the estimated standard deviations.

The Aberdeen postmaintenance value was lower than the Lorraine value because of a combination of reduced ballast thickness and lower overall subgrade stiffness. The predicted track modulus for the Aberdeen site was in close agreement with the measured values.

For the Leeds sites, the predicted values of track modulus were higher than the field values. However, the field values appear to be unusually low. The lower ballast thickness at the Leeds sites can account for some of the differences between the Leeds sites and the other two field sites, but these differences in ballast layer thicknesses were not enough to cause the low values measured at both Leeds sites.

Given the similarity between the subgrade stiffnesses at the Leeds sites and the Aberdeen site, closer agreement between the field measurements from these sites would be expected. It is possible that the embankment in the Leeds wood section resulted in reduced subgrade confinement and hence increased vertical deflections. This would result in lower values of track modulus. However, the concrete section at Leeds, which was built at grade, had comparably low track modulus values. Thus the embankment condition must not have been a major factor.

It would be necessary to reduce the subgrade stiffnesses that were used in the GEOTRACK analyses by at least 50 percent to match the field track modulus measurements at the Leeds test sections. However, no rational justification for making adjustments of these magnitudes could be found. If the soil moduli were overestimated at the Leeds sites, a similar systematic error should have occurred with the Lorraine and Aberdeen subgrades. Because the predicted Lorraine and Aberdeen values were in good agreement with the measurements, a similar adjustment to the subgrade moduli at those sites would shift the predicted values away from the measurements.

**SUPPORT CONDITIONS AND MAINTENANCE EFFECTS**

The measured and predicted values of track modulus are dependent on several factors, one of which is the support condition of the tie. Support conditions are a function of track settlement and maintenance.

![Figure 8](image-url)
Rail deflections under a particular applied load and, therefore, comparatively higher values of track modulus.

The interactions among variable physical states of ballast, tie support conditions, and structural factors such as tie stiffness and rail size make good predictions about track modulus uncertain. This is particularly true because the degree and type of maintenance disturbance and traffic history of a site can change the physical state of ballast in varying amounts. The scatter of the field measurements was such that there were no clear trends distinguishing the premaintenance track modulus values from the postmaintenance values. The predictions of track modulus using the GEOTRACK program are somewhat limited by the uniform layer property, the full contact representations, and the inability to represent the maintenance factors for the field sites. For these reasons variations between the measured and the predicted track modulus values for the sites can be expected because of the variations in ballast properties and support conditions that were affected by the maintenance operations.

The possible centerbinding and uniformity of support conditions beneath the tie bottom may not, however, be a significant factor contributing to the track modulus values, although the effects are physically rational. Differences between the bending stiffnesses of wood and concrete ties would also not result in large differences in track moduli. Because only 10 to 20 percent of the total track deformation is due to ballast compression, the subgrade deflections appear to be much more important. It has been shown that variations in tie stiffnesses (8) do not have a major effect on the vertical subgrade stresses beneath the rail seats. Thus, the subgrade contribution to track modulus should be about the same for wood and concrete ties. Also, dynamic measurements of resilient subgrade deflection (5) showed no significant difference between the deflections in wood and in concrete sections. This would indicate that, although the stress distributions and deformations in the ballast layer were affected by tie stiffness and possibly centerbinding, the subgrade responses were controlled mainly by the subgrade properties, with some effects of ballast layer thickness.

**SUMMARY**

The main purpose of this paper was to make available the experimental results of maintenance effects on track modulus and, by comparing the field measurements with predictions of track modulus for the revenue sites, to develop a further understanding of the factors affecting vertical track response.

Several observations were made on the basis of the results presented. The field measurements did not indicate that significant changes in the magnitude of track modulus could be attributed to maintenance. Also, the longitudinal variability of the track support was not improved as a result of the surfacing, as shown by the increased variability of the postmaintenance load-deflection curves.

The uniformity of ballast properties for under individual ties was improved by the raise and tamping operations, as was indicated by the plate load tests (9). The uniformity of contact between the ties and ballast surface, however, may not be improved. Uniformity of ballast properties and contact should lead to a lower track modulus, but because the ballast work is always related to the subgrade and the relative contribution of the ballast deformation to the overall track deflection is small, maintenance does not have a great effect on track modulus.
The major factors contributing to the magnitude of vertical track modulus were ballast depth and subgrade stiffness. Variations due to tie spacing or tie stiffness were not significant factors. Predictions of track vertical deflections made with GEOTRACK were generally in reasonable agreement with the field measurements. The use of track modulus alone for assessment of the quality of track may not be too helpful, especially if the assessment is based only on the average magnitude of track modulus. The variability of track modulus between tie locations is a direct measure of the longitudinal uniformity of the track, which is extremely important.

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