New Method of Simulating Layered Systems of Unbound Granular Material

W. O. YANDELL

ABSTRACT

The stress-strain analysis of multilayered pavements is becoming more precise--commencing with elastic solutions and developing through the more complex finite element analyses. The mechanolattice has been reasonably successful in predicting pavement performance because it takes relative plastic behavior into account. Recently, an option was developed that enables the mechanolattice to simulate any unbound granular layers in pavement. The principles of operation of the unbound simulation are described and its effects demonstrated using Sections 2 and 9 of the Pennsylvania State Test Track. The effects predicted with the "all bound" assumption are contrasted with those in which unbound layers are simulated as unbound. It is pointed out that the differences are functions of modulus ratios and the magnitude of relative plastic behavior. Many of the effects are not dependent on the absence of creep or relaxation.

Most modern pavement design and rehabilitation systems have used the theory of linearized elasticity to carry out the structural assessment of multilayered flexible pavement (1-2). Period of loading or temperature, or both, have sometimes been taken into account to simulate so-called visio-elasticity. The Council for Scientific and Industrial Research in Australia has produced investigatory stress-strain analyses such as CIRCLY and PAVAN that consider such things as cross-anisotropic materials and stress-dependent elastic moduli.

The author, recognizing the need to consider the plastic components as well as the elastic components of load-deformation behavior, developed the mechanolattice stress-strain analysis for multilayered elasto-plastic pavements (3-10). He used this to investigate the effects of the build-up of residual stresses and strains. However, the original version, like other techniques, was only suitable for bound road materials. Because some layers or the subgrade, or both, consist of unbound granular material an option was built into the mechanolattice package to enable selected layers to be treated as unbound and incapable of resisting tensile stresses.

A simple demonstration of one form of differing behavior resulting from lower layers being bound or unbound is shown diagramatically in Figure 1. A line load is applied to two-layer systems supported on rigid foundations. The upper layers are elastic and the lower are elasto-plastic. When the line loads are released there are two different outcomes. Figure 1(a), in which the lower layer is considered bound, shows a permanent deformation of the upper elastic layer with residual tension in the bottom surface and residual compression in the top. In contrast, Figure 1(b), in which the lower layer is considered unbound, shows that the elastic layer is able to spring up leaving a space under it but has no residual stress. Also, this form of behavior would obtain if both layers were bound but not bound to each other. This is of course a gross oversimplification when considering the greater realism of the mechanolattice analysis.

The mechanolattice multilayered analysis for bound material, followed by the unbound granular simulation option, is briefly described.

Comparisons are made between field observations and bound and unbound based predictions. Sections 2

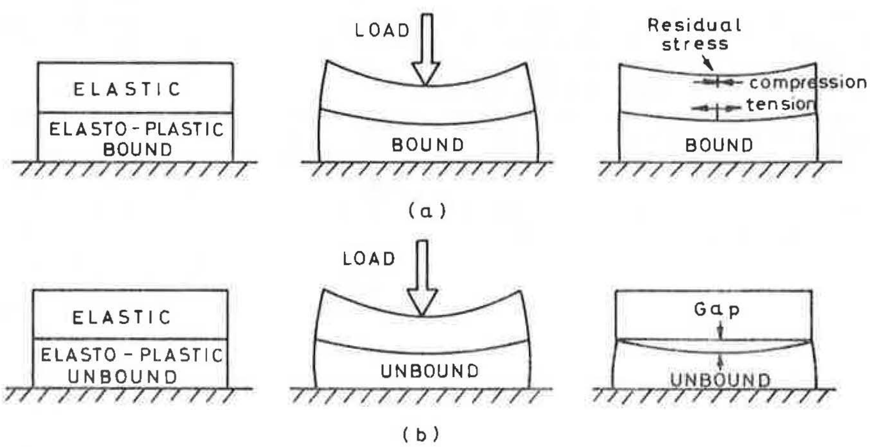
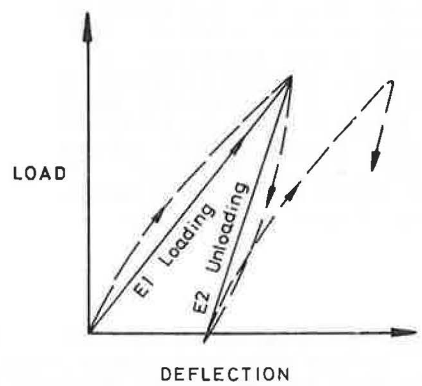


FIGURE 1 Differing behavior with bound and with unbound lower layer.

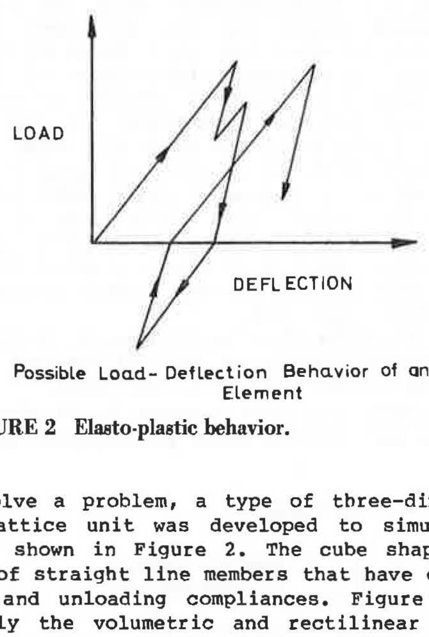
and 9 of the Pennsylvania State Test Track are used for comparison purposes.

MECHANOLATTICE ANALYSIS

The mechanolattice analysis has been described fully elsewhere (5-10). When it is applied to multilayered roads each layer is considered to be elasto-plastic. Figure 2(a) shows by broken lines the repeated loading on a load deflection plot for a hypothetical triaxial test. It will be observed that the residual deflections accumulate as repeated loading continues. To simplify computation the load-unload curves are simulated by straight lines. Possible load deflection behavior is shown in Figure 2(b).



(a) Simplification of Elasto-plastic Hysteresis Loop



(b) Possible Load-Deflection Behavior of an Element

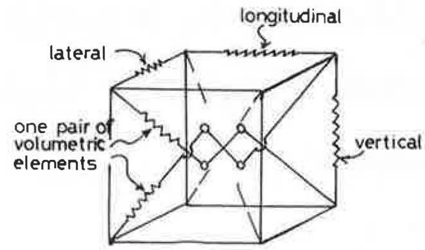
FIGURE 2 Elasto-plastic behavior.

To solve a problem, a type of three-dimensional mechanolattice unit was developed to simulate the behavior shown in Figure 2. The cube shaped units consist of straight line members that have different loading and unloading compliances. Figure 3 shows separately the volumetric and rectilinear elements in one view [Figure 3(a)] and the shear elements in the other [Figure 3(b)].

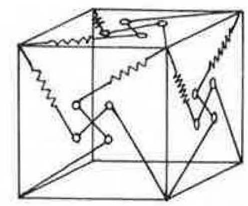
About 3,300 of these units are joined to simulate Section 9 of the Pennsylvania State Test Track as shown in Figure 4.

SEQUENTIAL TECHNIQUE FOR SOLUTION

Figure 5 shows a longitudinal section of the simulated pavement through the load. The units on the extreme left side, shown by broken lines, represent the initial conditions before a particular wheel pass. Elastic theory is used for predicting the



a. Volumetric and Rectalinear Elements



b. Shear Elements

FIGURE 3 Three-dimensional mechanolattice unit.

shape of each unit as it arrives at the simulating region from the residual no-load condition well forward of the "present" load. The consequent change in unit shape will cause the elements to change in length and therefore change their element load also. Similar things happen when the "wall" of units moves another place closer to the load. Thus, as the sequential movement of the wall of units from left to right--toward, under, and away from the load--takes place, the load-deflection history of each element is followed mathematically. This is done by calculating changes in length and changes in load with the aid of the stiffness factors. A permanent inventory of element loads is kept up to date.

Figure 5 also shows the sequential loading of a typical element of a unit as the load traverses from right to left--the three-dimensional problem is solved by imagining that the pavement structure moves from left to right with the wheel load considered to be fixed in position. For example, an element of units 1, 2, 3, and 4 has already been subjected to a loading history from previous wheel passes and as a result there is a residual stress state represented by point "a" of the inset figure of Figure 5. As the unit moves relative to the wheel from position A to position B the element becomes subject to a load level represented by point "b." Thus, as the load completely traverses the pavement, the loading of the element follows the path of a, b, c, d, e, f, and g, thus leading to a residual load. Similar behavior occurs in the other 27 elements of a unit as it moves toward, under, and away from the wheel load.

The computer program performs a similar, though more complex, task after each cycle of element length-load calculation in which the forces at each joint emanating from their attached elements are resolved into vertical, longitudinal, and lateral components. The joint is then moved in a damped manner in the direction of the unbalanced forces. The calculation damping factor is proportional to the largest force that is instantaneously out of balance at any free joint. The process is continued until all out-of-balance forces of free joints become insignificant. For this problem, between 1,500 and 2,000 computation cycles are needed. After convergence and after stresses have been calculated, the

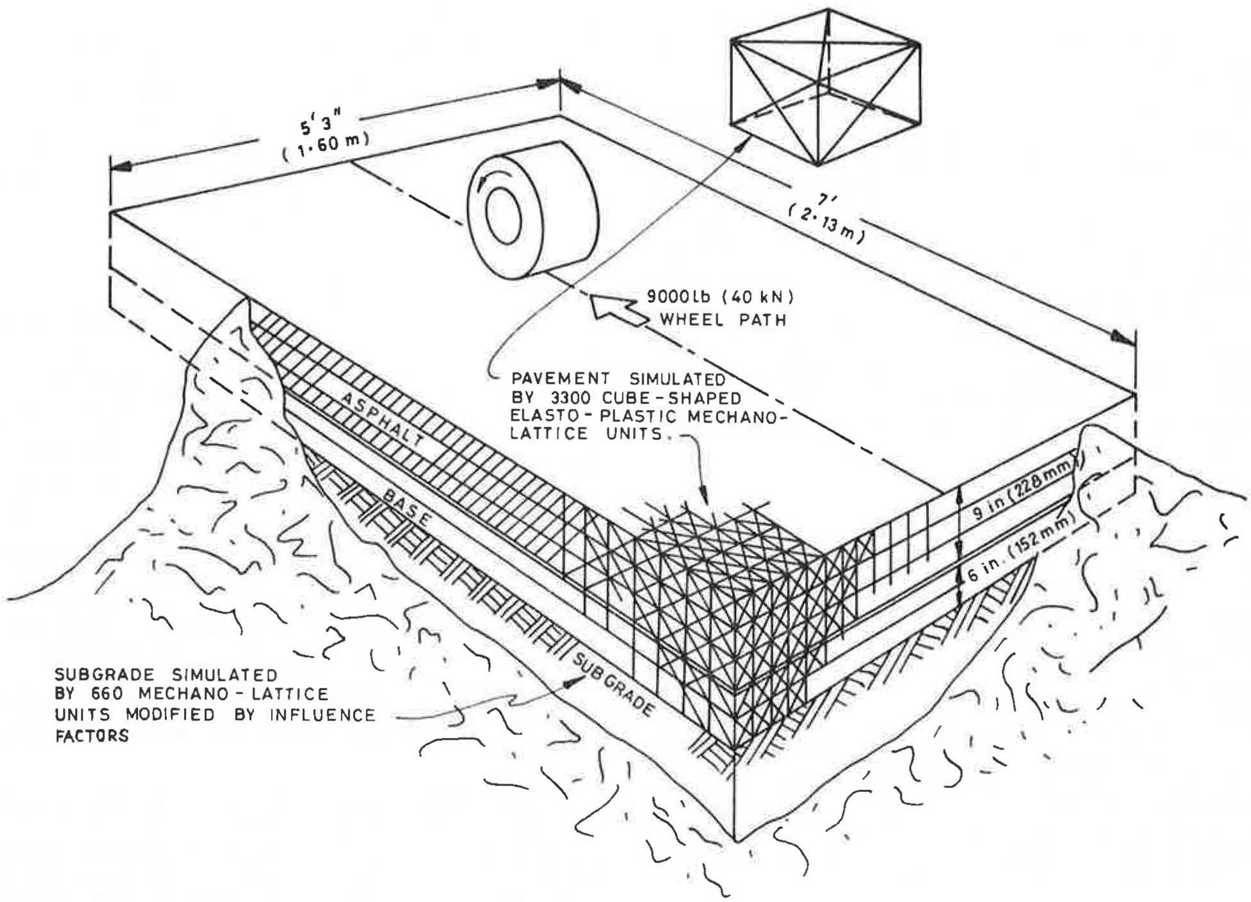


FIGURE 4 Simulation of pavements by assemblies of cube-shaped mechanolattice units.

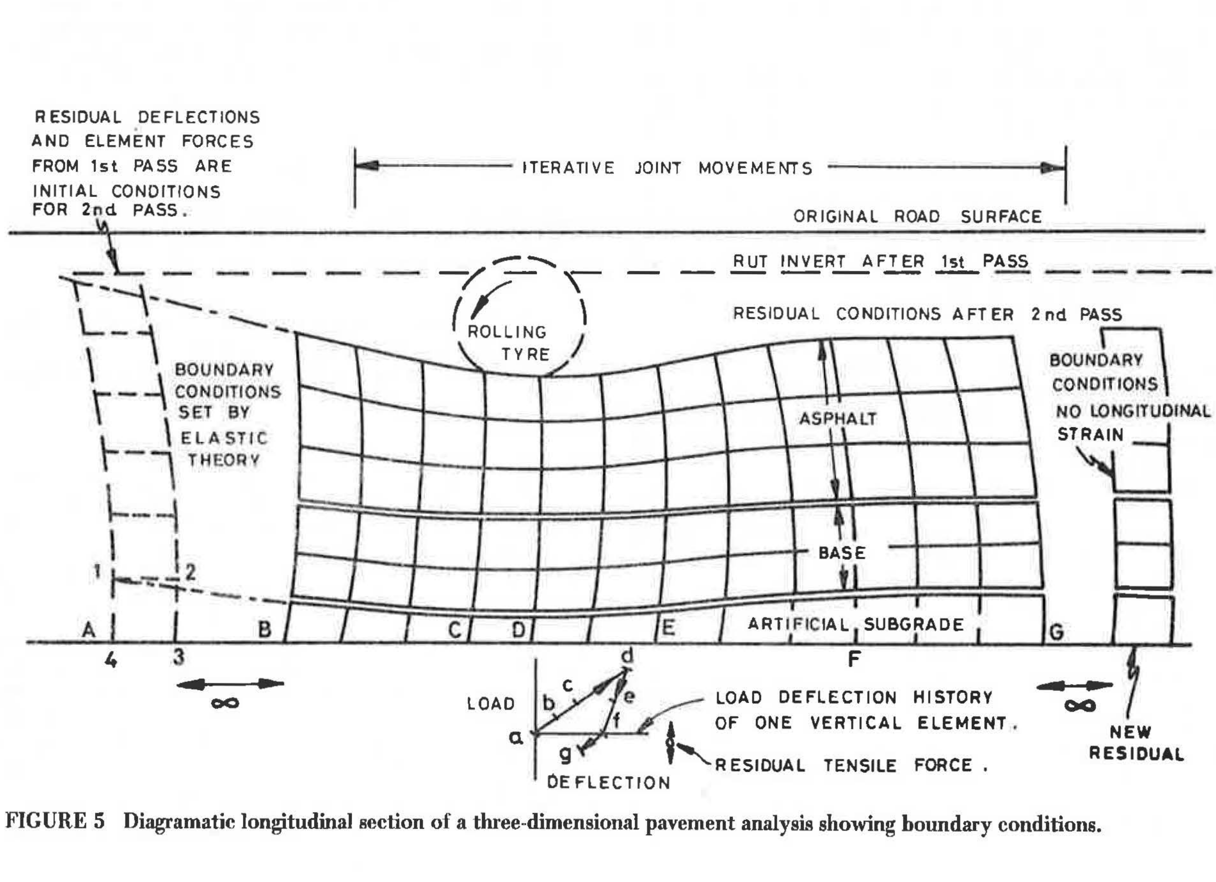


FIGURE 5 Diagramatic longitudinal section of a three-dimensional pavement analysis showing boundary conditions.

wall of units on the right of Figure 5 (in the residual condition) is used as initial conditions for the next simulated wheel pass and the process is repeated.

The foregoing techniques can be used to simulate the behavior of other materials such as elastic, perfectly plastic, and nonplastic energy absorbing material subject to gross deformation (11). The mechanolattice analysis is a rigorous technique that preserves equilibrium and has strain compatibility although some of the boundary conditions do involve approximations.

The stiffness factors of the elements are calculated by frame analysis. The loading moduli are determined from creep compliance tests (12). The unloading moduli [Figure 2(a)] are calculation expedients to set the relative plastic behavior. The plastic behavior is determined from repeated load triaxial tests (12).

UNBOUND OPTION

Although the mechanolattice technique may offend some because it does not employ classical mathematics, finite difference, or finite element techniques, it is rigorous, preserving both equilibrium and continuity. It also has a great advantage in its adaptability to a wide range of material property simulations with a minimum of extra computational effort. Thus the simulation of unbound granular material behavior is relatively simple.

The simulation will allow the material to crack when subject to tensile stress. Recompression will not start until the crack has fully closed. The incidence of cracking in a particular direction depends on the following preconditions:

1. The forces in the volume diagonals [Figure 3(a)] become tensile or the sum of the lengths of those four diagonals becomes greater than that obtained in the initial condition, or both (Figure 5). Then and only then will the forces in those diagonals be assumed equal to zero.
2. Also then and only then will the simultaneous occurrence of a tensile force in a horizontal, vertical, or lateral element lead to a crack opening in that direction.
3. Also, no increments of force can be added at the passage of an increment of time (moving from left to right in Figure 5) when condition "a" occurs cojointly with that element being longer at that instant than at the initial condition (left side in Figure 5). This ensures that recompression does not occur until the crack closes.

This logic is shown diagrammatically in Figure 6. The author has taken the liberty of assuming that shear stress can still be resisted when small cracks occur in that plane in this simulation of granular unbound material.

A pavement is analyzed by first solving the sequential multilayer problem assuming all layers are bound. The unbound criteria for base and subgrade are then invoked and an additional 1,000 iterations are made to achieve convergence once again. The cost per wheel pass is $87 on a central digital computer 76; the time required is 15 min. Cost of a full life prediction would be about $250.

Because the author is not aware of solutions to this type of problem by established finite element analysis, comparisons with it are difficult. However, finite element solutions to, for example, nonlinear problems of similar size are partly iterative and would take about 5 min. on a similar machine. Convergence of the mechanolattice is being improved. Finite element analysis has the advantage of being more widely used and understood and the disadvantage of being less adaptable to pavement problems and, so far, not being able to solve this type of problem. When the loading and unloading moduli are made equal the mechanolattice gives close agreement with other elastic analyses. The analysis has been used successfully for predicting the rutting and horizontal flow behavior of an indoor test track (7), the phenomenonological investigation of asphaltic concrete (A/C) cracking (13), and the behavior of two test roads in Sydney (14,15).

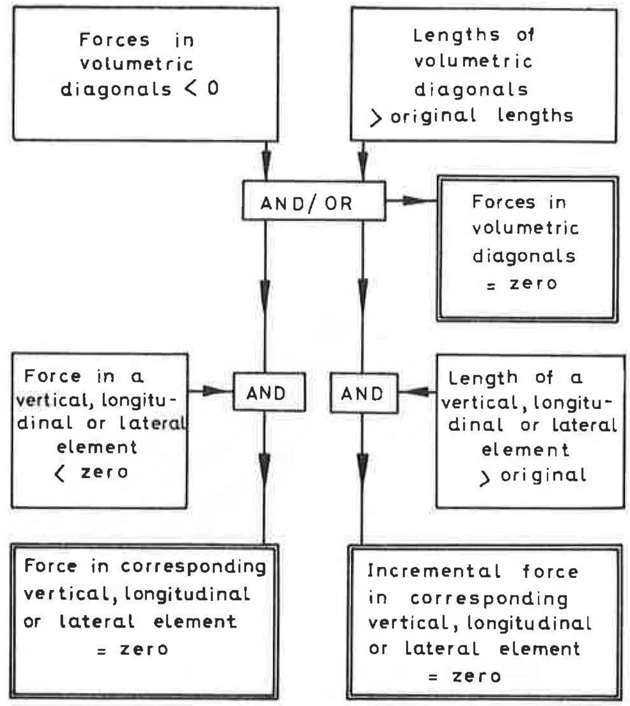


FIGURE 6 Flow diagram for unbound granular material option.

EFFECT OF TREATING SOME LAYERS AS UNBOUND

Rutting and fatigue cracking behavior are compared for the two analyses of Sections 2 and 9 of the Pennsylvania Test Track as follows:

1. All three materials are considered by the mechanolattice analysis to be bound.
2. The subbase and subgrade are considered unbound.

Section 9 is obviously not a representative case because its asphaltic concrete layer acts with less plasticity than its subbase and subgrade and the difference in predicted effects between the bound and the unbound assumption is great. In contrast, Section 2 of the same test track has all layers acting with closer plasticity and the treating of some layers as unbound instead of bound has little effect on predicted fatigue and rutting. Plastic behavior for a particular layer and material may be represented here by the residual deflection shown in Figure 2(a). Such behavior is a function of the repeated stress and of the loading and the unloading modulii. In Section 2 these residual strains are of a similar magnitude for each layer whereas in Section 9 they are largest for the lower layers.

Rutting

Figure 7 shows by half cross section a comparison of rutting after a few standard axle passes predicted by (a) assuming bound material (full lines) and (b) assuming the subbase and subgrade of Section 9 are unbound (broken lines).

It will be noted that the absolute rutting (or settlement) and straight-edge rutting is less at the surface when the unbound option is used. But rutting at the top of the subgrade and subbase is much greater with the unbound option, leaving an increasing horizontal gap between the asphaltic concrete and the subbase as each wheel passes. The gap, which was due to the asphaltic concrete behaving less plastically than the subbase and subgrade, could not occur when all layers were assumed bound in themselves and to each other. The asphaltic concrete then held the subbase and subgrade up and suffered greater permanent rutting itself. Horizontal gaps or cracks similar to these were observed by the National Institute of Transport and Road Research in South Africa (C.R. Freeme, NITRR, personal communication, 1984).

Figure 8 shows a comparison between rut prediction and rut measurements for up to 1.5 x 10^6 standard axle passes. It will be noted that the straight-edge rutting for the all-bound cases is seven times as great as for the unbound case whereas the absolute rutting is only one and one-half times as great. This is because the unbound subbase and subgrade had not the tensile strength to maintain the residual curvature in the asphaltic concrete layer. Figure 9 shows a similar comparison for Section 2. In contrast to the behavior in Section 9 there is little difference between rutting predictions in the bound and unbound cases. This is due to the more uniform plasticity between layers.

Fatigue Life

A comparison of lateral stresses in half cross sections under the traveling wheel load in Section 9 is

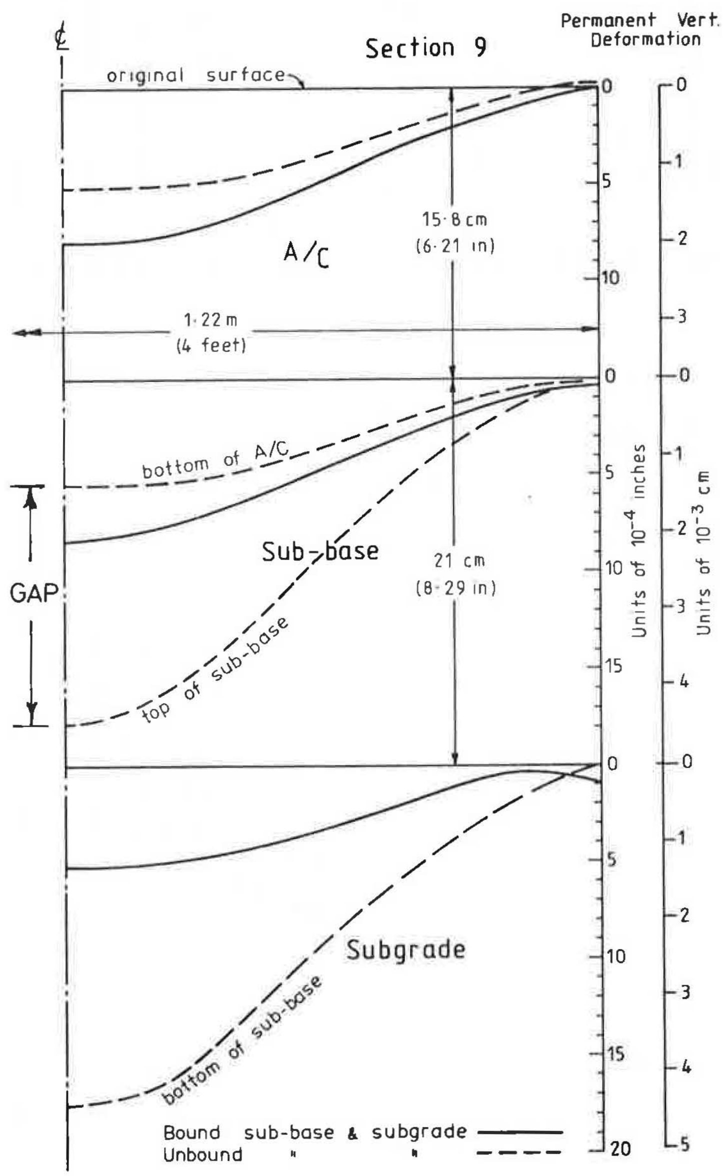


FIGURE 7 Rutting cross sections of Section 9 with bound and unbound subbase and subgrades.

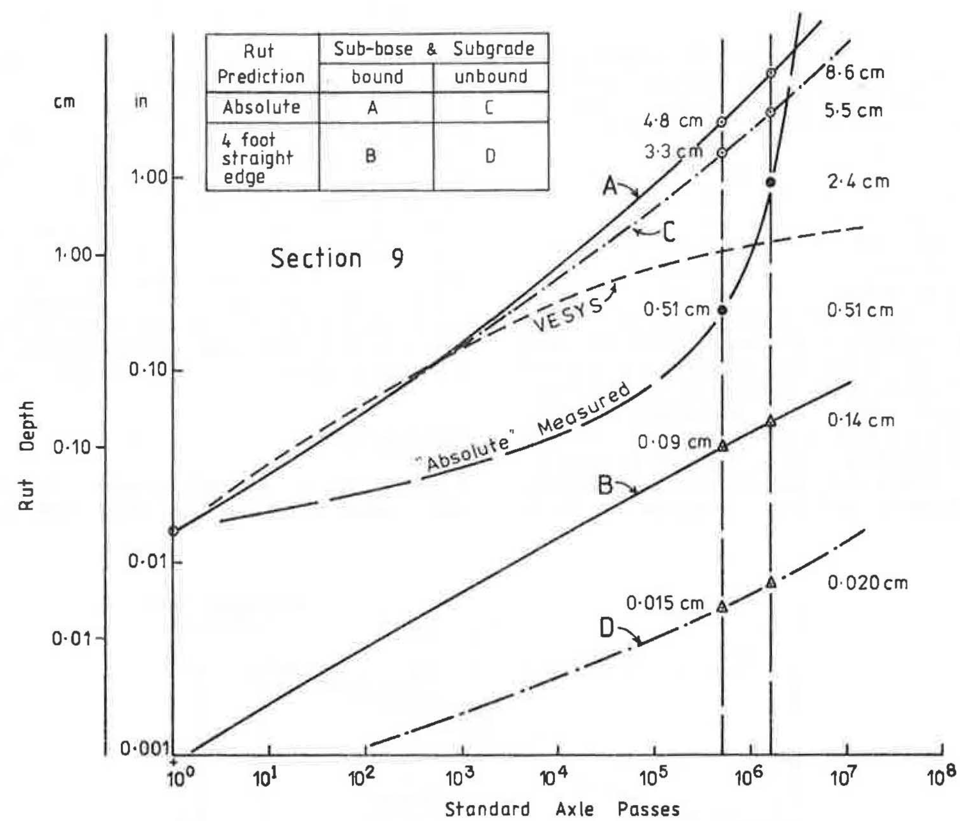


FIGURE 8 Comparison of VESYS- and mechanolattice-predicted rutting with measured rutting in Section 9.

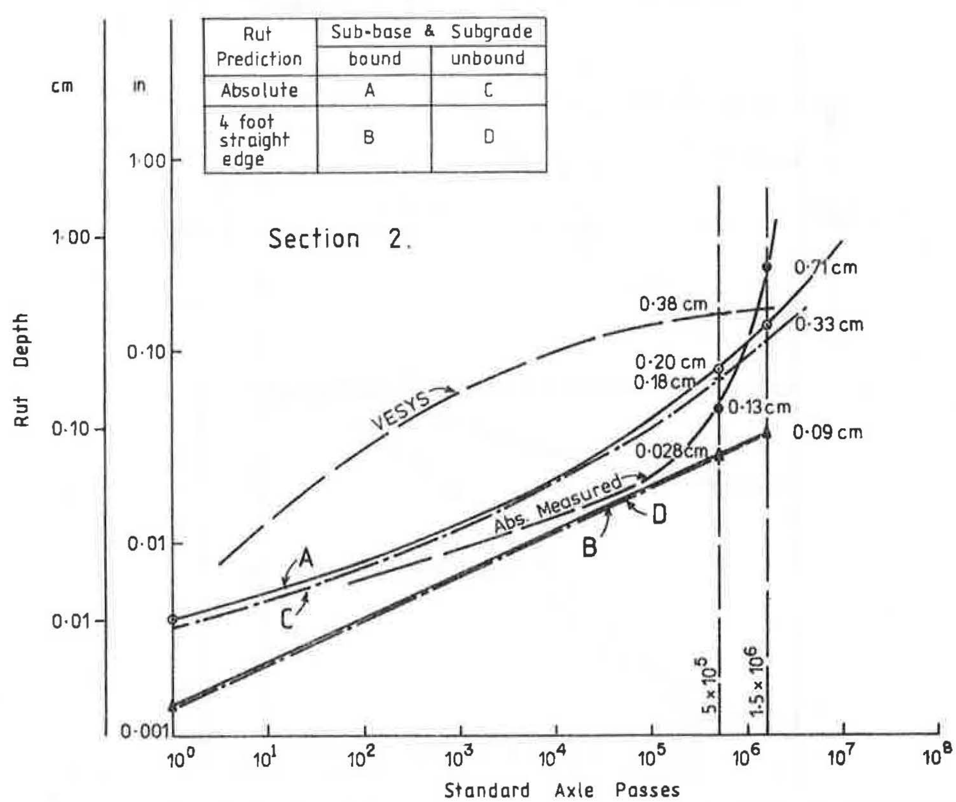


FIGURE 9 Comparison of VESYS- and mechanolattice-predicted rutting with measured rutting in Section 2.

shown in Figure 10 for the third standard axle pass. It will be noted, in the bound case, that tensile lateral stresses occur at the bottom of the subbase under the wheel path. However, as expected, no tensile lateral stresses exist in the subbase for the assumed unbound case. This means the subbase has no beam action to distribute the load laterally so the asphaltic layer has a greater imposed bending moment, increasing the bottom fiber tensile stress from 77 psi to 88 psi.

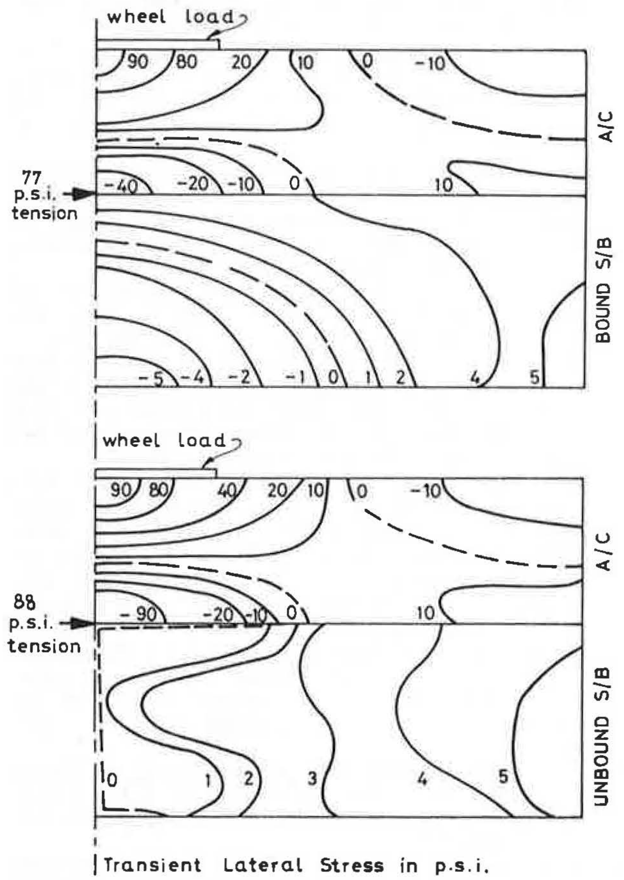


FIGURE 10 Half cross sections of transient lateral stress patterns in Section 9 under the moving load (a) with bound and (b) with unbound subbase and subgrade.

Figure 11 shows that part of the fatigue envelope chart near fatigue life end. Repeated $\log_{10}$ (lateral strain) is used as ordinate by convention. The strains are elastic equivalents to calculated stresses using the loading Young's modulus for the conversion. The radial stress at the bottom of the A/C increases with axle passes in Section 9 due to the accumulation of residual tension there, and the predicted life is thereby shortened. This is due to the A/C acting less plastically than the lower layers.

In Section 2 all layers acted with similar plasticity so small tensile residual stresses accumulated in the bottom of the A/C so the fatigue life was not shortened greatly (Figure 11). However, the fatigue life in the unbound case was shorter than in the bound case. This behavior is opposite to that of Section 9.

In Section 9, although the lateral tensile stress in the bottom of the asphaltic concrete is initially larger in the case of unbound lower layers, it increases at a lower rate because the accumulating residual tensions are less, which leads to a longer fatigue life as shown in Figure 11. This can be explained as follows: because the A/C is able to spring up after each wheel passes, it goes through greater ranges of stress and hence accumulates more residual tensile strain thus relieving accumulating tensile stresses. It will also be noted from Figure 11 that the size and shape of the contact patch have a great effect on fatigue life.

CONCLUSION

Any precision that the mechanolattice analysis may have could be partly due to its taking the elasto-plastic behavior of each material directly into account as well as to its simulating the loaded wheel as traveling in one direction. However, it was seen here that treating the unbound layers as unbound has brought this analysis closer to reality--as it would any analysis. It is ironic that having an

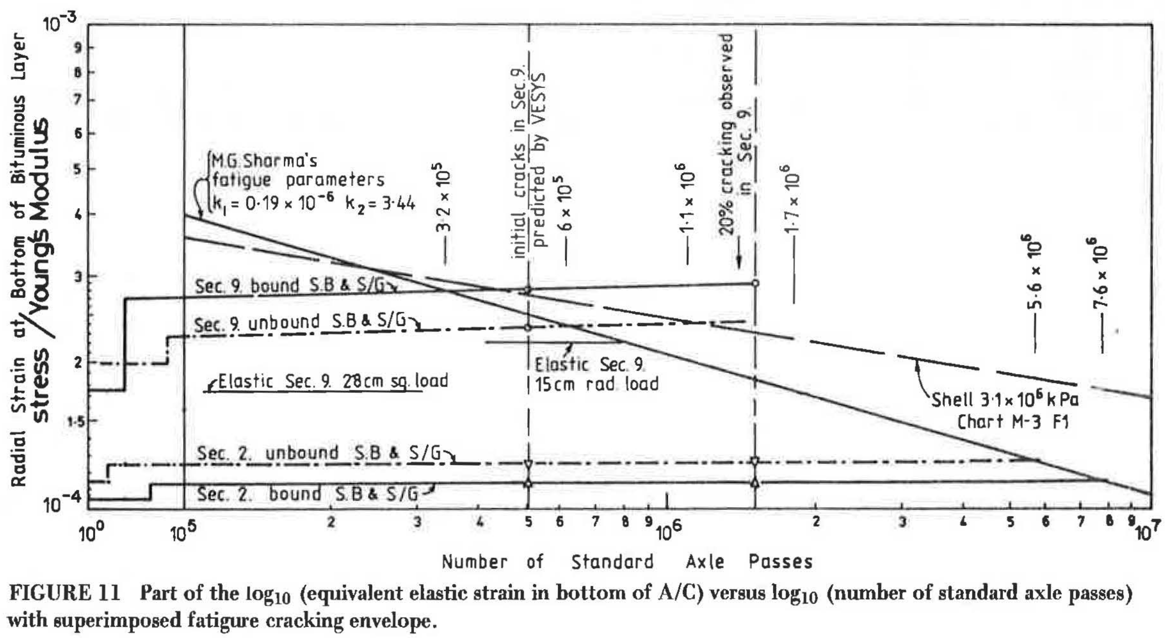


FIGURE 11 Part of the $\log_{10}$ (equivalent elastic strain in bottom of A/C) versus $\log_{10}$ (number of standard axle passes) with superimposed fatigue cracking envelope.

unbound base and subgrade in certain cases apparently extends the fatigue life of the asphaltic concrete by giving it a greater plastic behavior. However, other possible effects of these greater strain cycle ranges are unknown.

The effect of introducing the unbound option varies with the relative plastic behavior between the layers and with the modular ratios. In contrast to those of Section 9 the behavior predictions of Section 2 of Pennsylvania State Test Track were relatively insensitive to the unbound option.

It should be noted that the unusual behavior predicted in Section 9 with the unbound option of the mechanolattice analysis does not depend for its validity on the absence of creep or stress relaxation.

The computer programs used here and user manuals should be generally available by early 1985.

ACKNOWLEDGMENTS

Appreciation is extended to the Australian Research Grants Committee for the financial support of this work and to I.K. Lee for his encouragement.

REFERENCES

1. A.I.M. Claessen et al. Asphalt Pavement Design--The Shell Method. Proc., 4th International Conference on Structural Design of Asphalt Pavements, Ann Arbor, Mich., 1977.
2. W.J. Kenis. A Design Method for Flexible Pavements Using the VESYS Structural Sub-System. Proc., 4th International Conference on Structural Design of Asphalt Pavements, Ann Arbor, Mich., Vol. 1, 1977, pp. 101-118.
3. W.O. Yandell. The Prediction of the Behaviour of Elastoplastic Roads During Repeated Rolling Using the Mechano-Lattice Analog. In Highway Research Record 374, HRB, National Research Council, Washington, D.C., 1971, pp. 29-41.
4. W.O. Yandell and R.L. Lytton. Residual Stresses Due to Travelling Loads and Reflection Cracking. Report FHWATX79-207-6. Texas Transportation Institute and Texas Department of Highways and Public Transportation, College Station, Texas, June 1979.
5. W.O. Yandell and R.L. Lytton. The Effect of Residual Stress and Strain Build-Up in a Flexible Pavement by Repeated Rolling of a Tyre. Report RF4087-1. Texas Transportation Institute, College Station; American Trucking Associations, Alexandria, Va., Oct. 1979.
6. W.O. Yandell. Residual Stresses and Strains and Fatigue Cracking. Journal of the Transportation Engineering Division, ASCE, Vol. 108, No. TE1, Jan. 1982, pp. 103-116.
7. W.O. Yandell. Measurement and Prediction of Forward Movement and Rutting in Pavements under Repetitive Wheel Loads. In Transportation Research Record 888, TRB, National Research Council, Washington, D.C., 1982, pp. 77-84.
8. W.O. Yandell. Possible Effect of Relative Plastic Behavior on Pavement Life. In Transportation Research Record 930, TRB, National Research Council, Washington, D.C., 1983, pp. 86-90.
9. W.O. Yandell. The Use of the Mechano-Lattice Analysis to Investigate Relative Plastic Behavior. Proc., International Conference on Constitutive Laws for Engineering Materials, University of Arizona, Tucson, Jan. 1983.
10. W.O. Yandell. Mechano-Lattice Prediction of Pavement Performance. Proc., CAPSA, Capetown, South Africa, March 12-16, 1984, pp. 201-215.
11. W.O. Yandell. The Measurement of Surface Texture of Stones with Particular Regard to the Effect on the Frictional Properties of Road Surfaces. Ph.D. dissertation. University of New South Wales, Sydney, Australia, 1970.
12. M.G. Sharna, W.J. Kenis, T.D. Larson, and W.L. Gamling. Evaluation of Flexible Pavement Methodology Based upon Field Observations at Pennsylvania State University Test Tract. Proc., 4th International Conference on Structural Design of Asphalt Pavements, Ann Arbor, Mich., Vol. 1, 1977, pp. 158-174.
13. F. Hugo and T.W. Kennedy. New Design Considerations for Improved Asphalt Pavement Life. Proc., 4th Conference on Asphalt Pavements for Southern Africa, Vol. 1, March 1984, pp. 100-114.
14. P. Clarke. Report on the Road Trial at Church Lane, Prospect, Between 1979 and 1982. M.E.S. thesis. University of New South Wales, Sydney, Australia, 1982.
15. R.B. Smith. Comparison of Predicted Performance of Two Full Scale Crushed Rock Pavements using Mechano-Lattice Analysis with Field Performance During Initial Service Life. M.E.S. thesis. University of New South Wales, Sydney, Australia, 1984.

Publication of this paper sponsored by Committee on Mechanics of Earth Masses and Layered Systems.