

Effect of Specimen Thickness on Marshall Test Results

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ABSTRACT

A problem inherent in many standard test methods in materials engineering is the preparation of a standard test specimen. The Marshall test, ASTM D1559-76, "Standard Test Method for Resistance to Plastic Flow of Bituminous Mixtures Using Marshall Apparatus," is subject to variability introduced by nonstandard specimens. The Marshall test allows the testing of standard sized specimens prepared under laboratory conditions and cored specimens of varying thickness. This study investigated the effects of variations in specimen size, specifically specimen height, on Marshall stability and flow. To determine the adequacy of accepted correction methods, the observed variability introduced by nonstandard specimen heights was compared with the accepted correction method. Recommendations concerning the correction of stability and flow values resulting from nonstandard specimens are presented.

A problem inherent in many standard test methods in materials engineering is the preparation of a representative or standard specimen. The Marshall test, ASTM D 1559-76, "Standard Test Method for Resistance to Plastic Flow of Bituminous Mixtures Using Marshall Apparatus," is subject to variability introduced by nonstandard specimens. The Marshall test allows the testing of specimens prepared under laboratory conditions and also cored specimens. Laboratory specimens are to be 2.5 ± 0.05 in. in height. The ASTM test procedure does not allow the use of specimens that are outside the height tolerance and does not provide a correction method for out-of-tolerance specimens.

The height of cored specimens is dependent on the thickness of the pavement from which the core is removed. The ASTM test procedure allows the use of specimens from 1.0 to 3.0 in. For stability, a "correlation ratio" is multiplied with the raw stability reading to arrive at a corrected stability value. There is no correlation ratio for flow.

Many state and federal agencies have developed Marshall test procedures tailored to agency needs that are patterned after the ASTM test procedure. A large number, recognizing the difficulty of achieving laboratory specimens within the required tolerance in the construction quality control and quality assurance (QC/QA) environment, have relaxed the tolerance for the height of laboratory specimens and implemented a correction procedure. In most cases the correction procedure for laboratory specimens is identical to the ASTM correction procedure for stability on cores.

This study investigated the effects of variations in specimen height on Marshall stability and flow. The experimental procedure of this study relied on the testing of laboratory-prepared specimens. The test procedure did not include cored specimens. To evaluate the effect of specimen thickness on Marshall test results it is necessary to consider a range of different specimen thicknesses for the same asphalt concrete mixture. These conditions could not be met for cored specimens because there is no way to ensure that cores taken from the pavement will have a standard reference core that is 2.5 in. thick and other cores that vary above and below this thickness. Furthermore, the use of existing pavements would have introduced more unknown variables into the procedure. In designing the experimental

procedure it was decided that the procedure should duplicate as much as possible the conditions found in construction QC/QA testing. It was believed that this approach would yield the greatest benefit because QC/QA is making the greatest use of the correction procedure.

The variability of stability and flow in response to variations in specimen height observed in this experiment is compared with the theoretical variability consistent with the established procedures. Recommendations concerning the correction of stability and flow values resulting from nonstandard laboratory and cored specimens are presented.

CURRENT CORRECTION PROCEDURES

In current practice, corrections for nonstandard specimen height are used for stability. The correction procedure entails the multiplication of the observed stability value by a "correlation ratio" that is related to the specimen height. Correlation ratios are published in tabular form and appear in virtually all literature detailing the Marshall test. Table 1 is an example of such a table from ASTM D-1559. Correction procedures for flow do not appear in the literature.

Stability correlation ratios had their origin in a 1948 developmental report by the U.S. Army Corps of Engineers Waterways Experiment Station (1). However, this document contains only an explanation of the correction procedure and does not explain the development of the correlation ratios or the basis for their use. It also dismisses the possibility of a significant height effect on flow by stating, "A correction factor for flow is not necessary" (1). There is no evidence to support the assertion. A literature search failed to uncover the basis for the stability correlation ratios or the apparent dismissal of a significant height effect on flow.

An analysis of the stability correlation ratios as a function of specimen height was conducted. This was done in an effort to discern the physical relationship between specimen height and stability that corresponds to the published correlation ratios. A plot of the correlation ratios as a function of specimen height revealed a curve that could only be explained by a complex equation. This is shown in Figure 1.

TABLE 1 Stability Correlation Ratios from ASTM D-1559

Volume of Specimen (cm ³)	Height		Correlation Ratio
	(mm)	(in.)	
302-316	38.1	1 1/2	2.78
317-328	39.7	1 9/16	2.50
329-340	41.3	1 5/8	2.27
341-353	42.9	1 11/16	2.08
354-367	44.4	1 3/4	1.92
368-379	46.0	1 13/16	1.79
380-392	47.6	1 7/8	1.67
393-405	49.2	1 15/16	1.56
406-420	50.8	2	1.47
421-431	52.4	2 1/16	1.39
432-443	54.0	2 1/8	1.32
444-456	55.6	2 3/16	1.25
457-470	57.2	2 1/4	1.19
471-482	58.7	2 5/16	1.14
483-495	60.3	2 3/8	1.09
496-508	61.9	2 7/16	1.04
509-522	63.5	2 1/2	1.00
523-535	64.0	2 9/16	0.96
536-546	65.1	2 5/8	0.93
547-559	66.7	2 11/16	0.89
560-573	68.3	2 3/4	0.86
574-585	71.4	2 13/16	0.83
586-598	73.0	2 7/8	0.81
599-610	74.6	2 15/16	0.78
611-625	76.2	3	0.76

Further consideration of the problem led to an analysis of the inverse of the correlation ratios as a function of height. A plot of this function showed that it is generally linear (Figure 2). A linear regression procedure performed over the range of 1.0 to 3.0 in. yielded a regression line with a coefficient of determination (r^2) of 0.992 and a significant ($\alpha = 0.05$ level) regression coefficient of 0.568.

A discontinuity in the slope of the function was observed at a height of 1.5 in. Because the region below 2.0 in. has little potential application, it was not considered in the experimental phases of this study. A linear regression analysis of the inverse correlation ratios over the range from 2.0 to 3.0 in. created a regression line with a slope of 0.6405 and an intercept of -0.6016. The coefficient of determination for this relationship was 0.9998, which denotes a nearly perfect correlation. These parameters were used to define the physical relationship between height and the published stability correlation ratio table. It therefore appears that the original "correlation ratios" were based on the inverse of specimen height.

An analysis of the laboratory procedure used to prepare specimens led to the conclusion that it was appropriate to limit the investigation to specimen heights of between 2.0 and 3.0 in. This provision

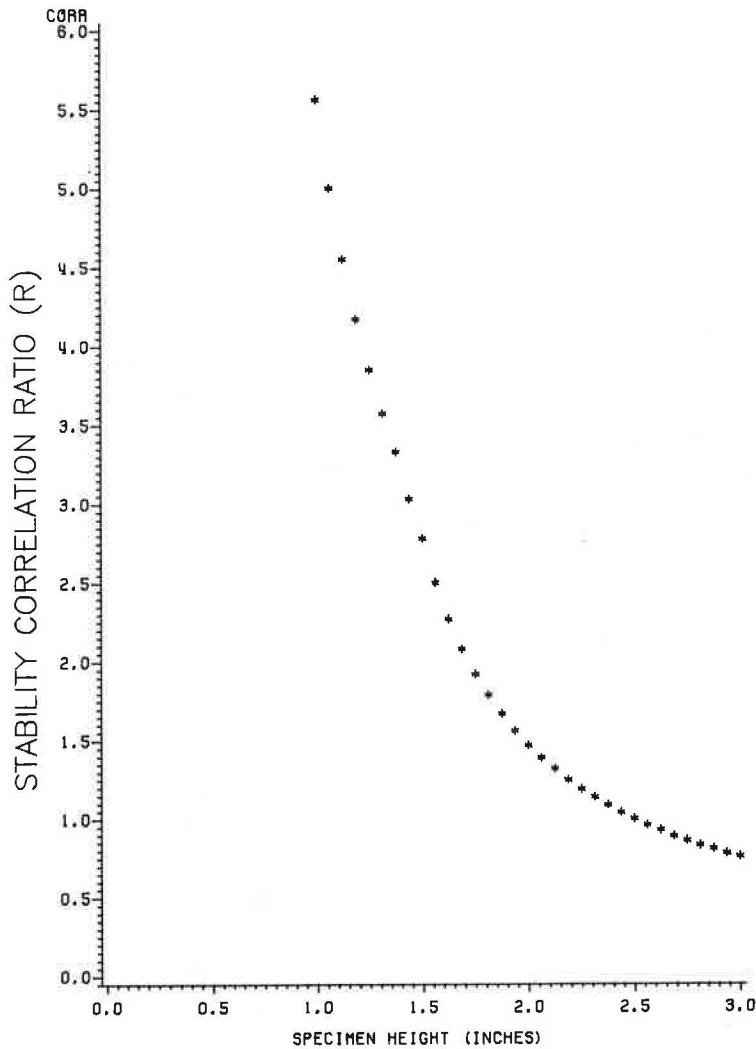


FIGURE 1 Stability correlation ratios as published in ASTM D-1559.

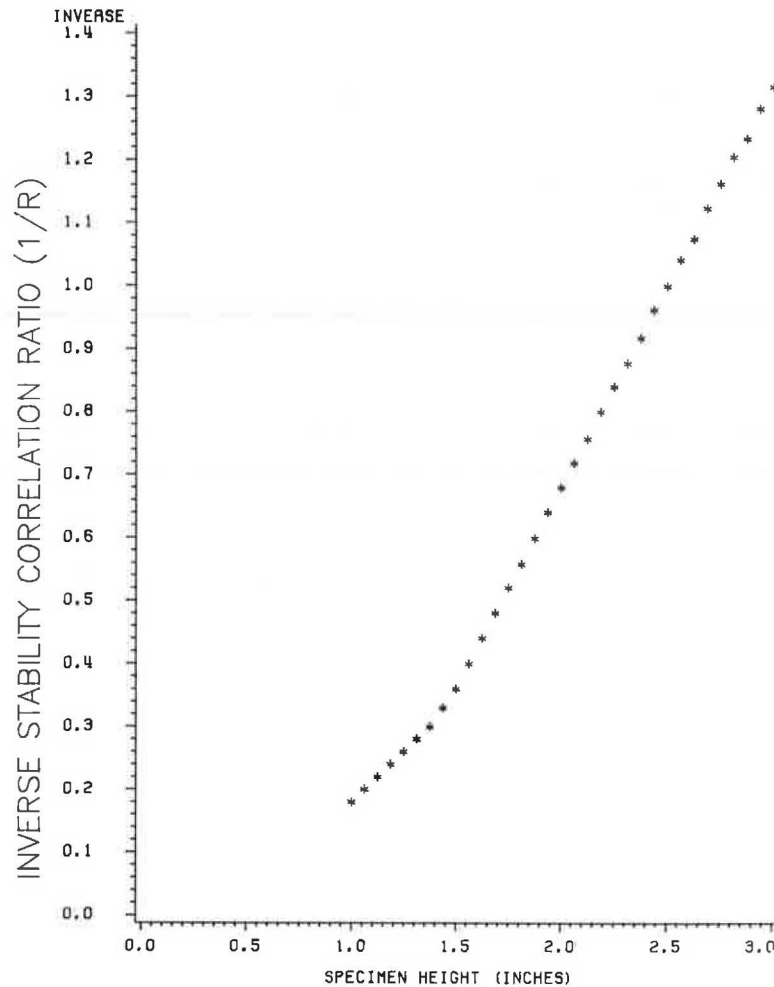


FIGURE 2 Plot of inverses of the stability correlation ratios.

for a one-half-inch tolerance around the standard was considered to be a generous allowance for the properly trained technician conducting QC/QA tests in a field laboratory. In addition, the limitations of the height of the compaction mold and the testing head width dictated that 3.0 in. be the largest allowable specimen height.

EXPERIMENTAL PROCEDURE

Specimens with heights varying from 2.0 to 3.0 in. were prepared under control conditions in accordance with standard procedures. The specimens were tested and Marshall properties were recorded. A linear regression analysis with stability and flow as dependent responses to specimen height was conducted. This produced regression lines that could be used to estimate the effect that variations in specimen height had on stability and flow readings for the mixture under study. The experiment was conducted five times on different mixtures. For each experiment, approximately 33 specimens of varying heights were prepared and tested.

The asphaltic concrete samples were obtained from commercial hot mix plants located in proximity to the laboratory facility. The material was sampled from a truck immediately after loading in accordance with ASTM D-979-74, "Sampling Bituminous Paving Mixtures." The total sample weighed approximately 100 lb and had a temperature of approximately 310°F. The

material was transported to the laboratory where it was placed in a heated oven. The material was maintained at a temperature of 280°F in an oven until it was removed and placed in the compaction mold. The asphaltic concrete was sampled from material that was produced as a surface or binder course for a South Carolina Department of Highways and Public Transportation (SCDHPT) contract subject to statistical quality control (4). Job mix formulas for the mixes are given in Table 2. The material consisted of crushed granite coarse aggregate with natural or manufactured sand and an AC-20 asphalt cement.

The compaction temperature of the mix was 250°F. This temperature was selected to be consistent with two procedures that are widely used at the national level. These procedures are MILSTD-620A (2) of the Department of Defense and the Federal Aviation Administration Eastern Region laboratory procedure (3).

A mechanical hammer was used to prepare the specimens. Fifty blows were applied to each face of all specimens. The same hammer was used throughout the experiment.

For each mix, specimens of varying target heights were prepared in random order. This was done in an effort to minimize potential effects of inconsistencies in the mix and the possible effects of time on the experiment. This randomization should minimize the possibility of introducing bias.

The determination of bulk specific gravity, stability, and flow was accomplished in accordance with ASTM D-1559 and ASTM D-2726-73, "Standard Test

TABLE 2 Job Mix Formulas for the Five Mixes Tested

Mix	Asphalt Content (%)	Percentage Passing by Weight							
		3/4 in.	1/2 in.	3/8 in.	No. 4	No. 8	No. 30	No. 100	No. 200
1	6.2	100	98	94	68	54	33	13	6
2	5.7	100	98	93	62	49	31	13	6
3	6.2	100	98	93	71	58	35	13	6
4	5.2	95	70		40	32			
5	5.0	100	99	80	39	29			

Method for Bulk Specific Gravity of Compacted Bituminous Mixtures Using Saturated Surface Dry Specimens." Specimen heights were calculated from volumetric data. A Marshall test press with an automatic load and deformation recorder was used for all testing.

EXPERIMENTAL RESULTS

Two separate analyses were conducted in the research. Results of analyses of the stability and flow results are presented next.

Analysis of Stability Results

A significant ($\alpha = 0.05$ level) correlation between specimen height and stability was observed to exist for each of the mixes tested. A slope and an intercept describing a linear regression line were calculated for each mix. These estimates and the coefficient of determination (r^2), a measure of correlation, are given in Table 3. A t-test of each of the regression coefficients showed them to be statistically significant at the 5 percent level (i.e., $\alpha = 0.05$).

TABLE 3 Summary of the Linear Regression Analysis of Stability and Calculation of Stability Correction Line Parameters for Each of the Five Mixes Tested

Mix	Regression Line			S_{std}^d	Correction Line		
	N^a	a^b	b^c		a/S_{std}	b/S_{std}	$(r^2)^e$
1	34	-1454.1	1689.0	2768	-0.525	0.610	0.662
2	41	-791.2	1389.1	2681	-0.296	0.518	0.627
3	23	-193.3	731.7	1636	-0.118	0.447	0.537
4	33	-312.2	1090.2	2413	-0.129	0.452	0.548
5	35	-727.2	1460.3	2924	-0.249	0.499	0.646

^a Number of specimens.

^b Intercept of regression line.

^c Slope of regression line.

^d Stability of 2.50-in. specimen of the regression line.

^e Coefficient of determination.

The regression lines for stability were converted to "correction lines" corresponding to the line of inverse correlation ratios that was derived from the published correction method. This was accomplished by dividing the intercept and slope of the regression line by the standard stability (S_{std}) of a 2.5-in. specimen. The standard value used was the value predicted by the regression equation at a height of 2.5 in. By performing this operation, a function was defined that had as its ordinate the ratio of the stability at any specimen height to the stability of a standard 2.5-in. specimen. This can be demonstrated by dividing both sides of the equation for the regression line by S_{std} , such that

$$S_x/S_{std} = (a/S_{std}) + (b/S_{std})x \quad (1)$$

where

S_x = stability value at any height x ,
 S_{std} = stability of a 2.5-in. specimen,
 a = regression intercept,
 b = regression slope, and
 x = specimen height.

Correction line parameters that resulted from this operation are given in Table 3. It should be noted that each of the correction line slope estimates was less than the slope parameter of the published method.

To estimate the parameters of a stability correction line with acceptable precision and confidence, it was desirable to combine the data of all five tests into a single linear regression model. Therefore it was necessary to standardize the stability readings of the separate mixes to permit the pooling of data. To this end, a stability ratio (SR) was calculated for each observation by dividing the observed stability such that

$$SR = S_x/S_{std} \quad (2)$$

A prerequisite for combining the results of different tests into one regression model is a condition of homogeneity of regression lines. Before pooling the stability ratios of the five tests into one regression model, a test for homogeneity was performed to determine if the regression lines of any of the mixes differed significantly from those of the other mixes. The test for homogeneity consisted of an analysis of covariance. The analysis of covariance model indicated that there were no significant differences in the regression lines of the separate mixes at the 5 percent level of significance.

When the attempt to establish that significant heterogeneity existed within the correction line results of the five experiments failed, all of the observed stability ratios of the separate experiments were pooled and a regression analysis was performed. The regression analysis resulted in the calculation of statistically significant estimates for the regression parameters. Using 166 observations, an intercept of -0.2707 and a slope of 0.5082 were calculated. The coefficient of determination (r^2) was 0.607. A plot of the combined stability ratio data is shown in Figure 3. This figure illustrates the correction line that has the ratio of the stability at any specimen height to the standard stability as its ordinate.

The experimental correction line was compared with the correction line that was representative of the published correction methods. A t-test for homogeneity of the experimental regression coefficient and the slope derived from the published method affirmed that a significant difference existed at the 5 percent level. This disparity is shown in Figure 4.

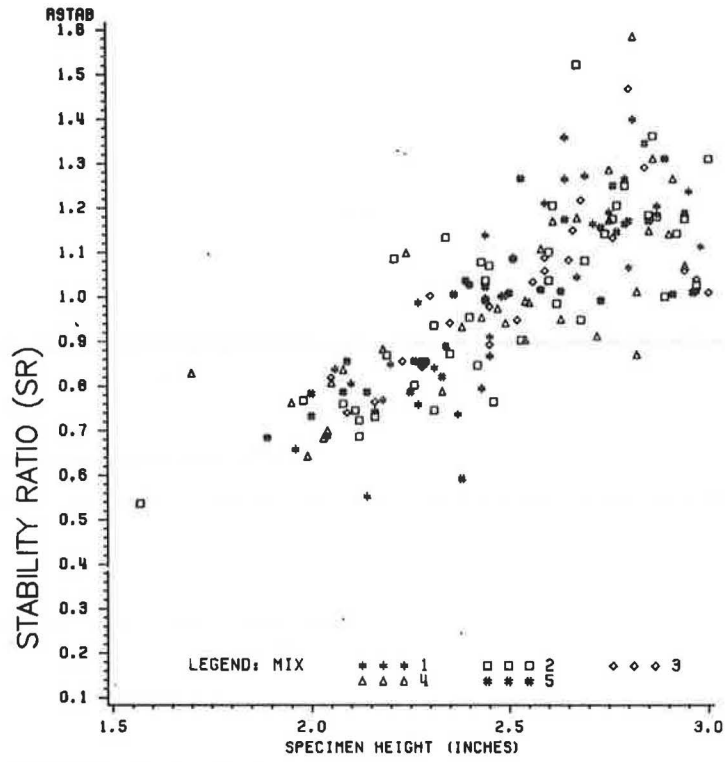


FIGURE 3 Combined stability ratio data of the five mixes tested with the resulting regression line and 95 percent confidence limits.

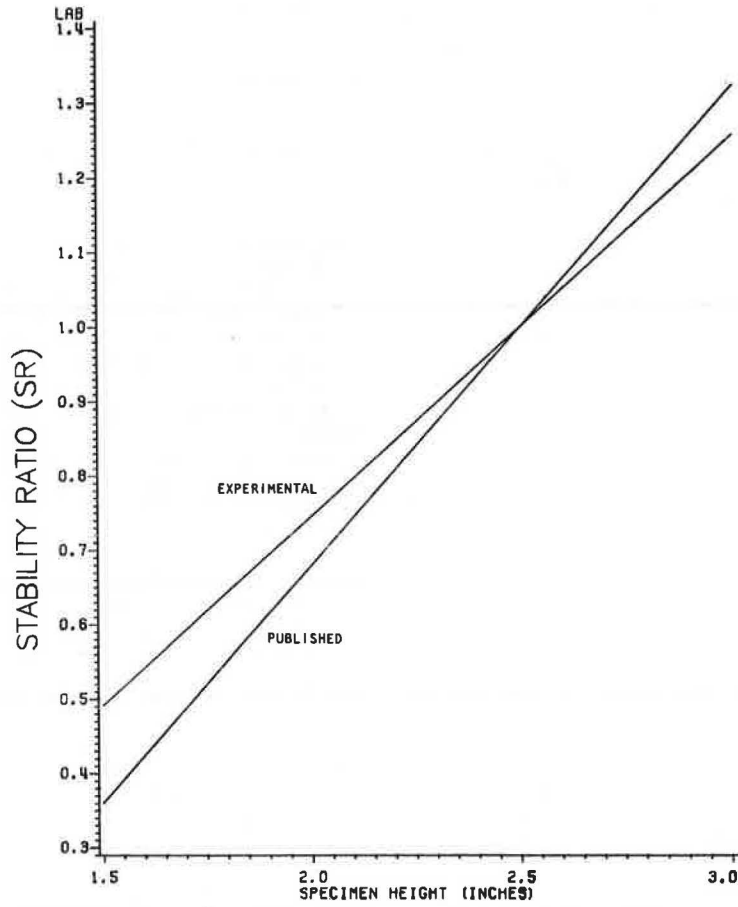


FIGURE 4 Comparison of the stability correction line of the published method with the experimentally derived correction line.

Analysis of Flow Results

The analysis of flow results parallels the analysis of stability results. Each of the mixes studied demonstrated that a strong relationship between Marshall flow and specimen height existed. This is emphasized by the high coefficient of determination (r^2) and the observation of statistically significant ($\alpha = 0.05$ level) regression coefficients. These estimates are given in Table 4.

As was done for stability, the flow regression lines were converted to correction lines by dividing the regression parameters by the standard flow value of each mix (F_{std}). The correction line parameter

estimates that resulted from this operation are given in Table 4.

Flow ratios (FRs), the ratio of the observed flow value (F_x) to the appropriate standard flow (F_{std}), were calculated for all observations such that

$$FR = F_x / F_{std} \tag{3}$$

An analysis of covariance test for homogeneity of the regression lines of flow ratios of the separate experiments was conducted. From this analysis it was concluded that no statistically significant difference existed between the regression lines of any of the mixes at the 5 percent level of significance.

Because significant heterogeneity within the regression lines of the flow ratios of each of the five mixes was not identified, the flow data from all of the mixes were combined into a single linear regression model. This procedure resulted in the calculation of statistically significant estimates ($\alpha = 0.05$ level) for the regression line parameters. Using all 166 observations, an intercept of -0.0576 and a slope of 0.4230 were calculated with a coefficient of determination of 0.765. This regression line corresponds to a flow correction line. The ordinates of this function equal the ratio of the flow at any specimen height to the flow of a standard specimen. Figure 5 shows the flow ratio observations and the resulting regression line. Figure 6 shows the difference between this experimental correction line and a correction line corresponding to published methods--specifically, no correction.

TABLE 4 Summary of the Linear Regression Analysis of Flow and Calculation of Flow Correction Line Parameters for Each of the Five Mixes Tested

Mix	Regression Line			F_{std}^d	Correction Line		
	N^a	a^b	b^c		a/F_{std}	b/F_{std}	$(r^2)^e$
1	34	-0.355	5.297	12.89	-0.028	0.411	0.817
2	41	0.458	4.694	12.08	0.038	0.385	0.792
3	23	0.076	3.694	9.31	0.008	0.397	0.596
4	33	-0.820	5.200	12.18	-0.067	0.426	0.768
5	35	-3.148	6.523	13.16	-0.239	0.496	0.812

^a Number of specimens.
^b Intercept of regression line.
^c Slope of regression line.
^d Flow of 2.50-in. specimen on the regression line.
^e Coefficient of determination.

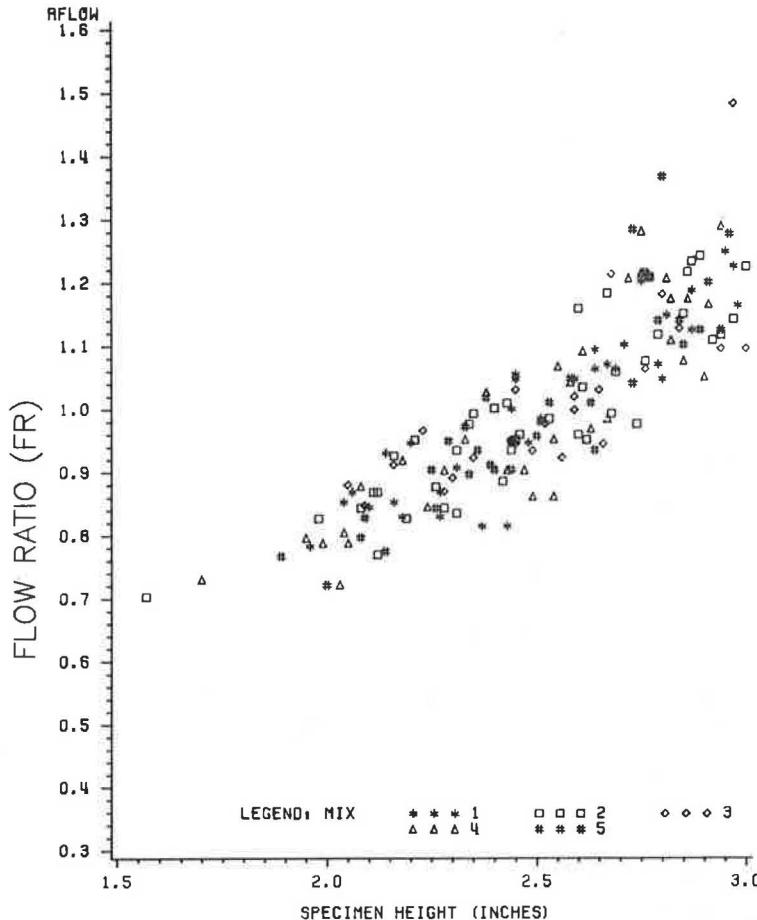


FIGURE 5 Combined flow ratio data of the five mixes tested with the resulting regression line and 95 percent confidence limits.

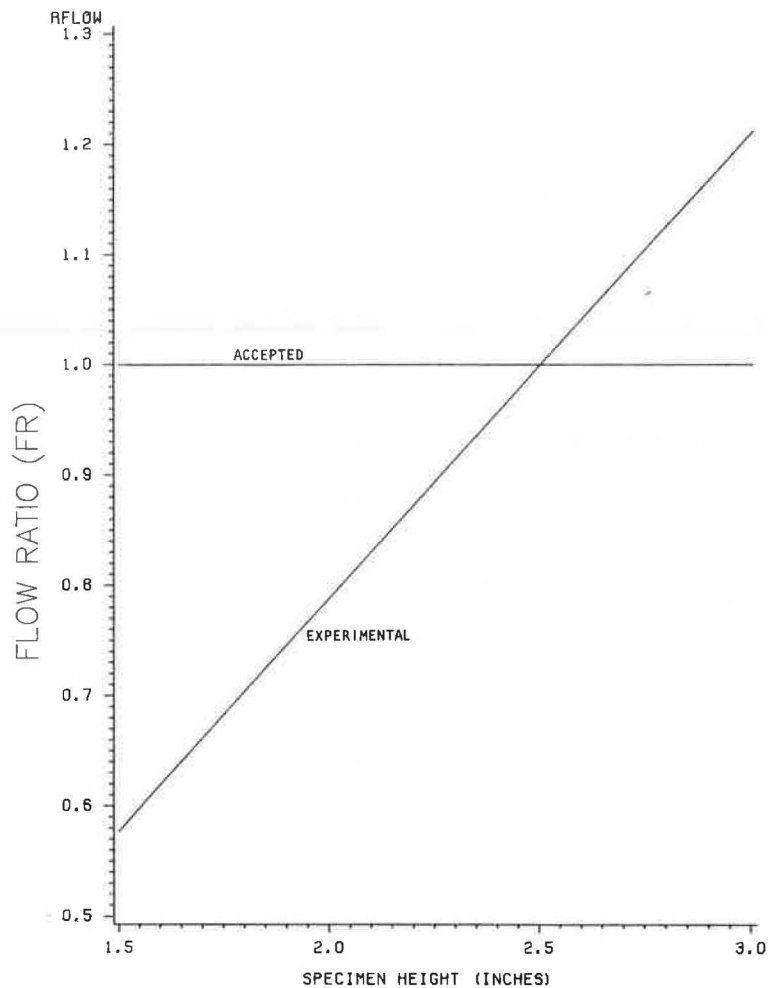


FIGURE 6 Comparison of accepted flow correction line (no correction) with the experimentally derived correction line.

DISCUSSION

The results of this study can be divided into two areas, those that relate to the Marshall stability readings and those that relate to the Marshall flow values.

Stability Correction Procedure

The results of this study indicate a high correlation between specimen height and Marshall stability readings. This finding supports the concept of linear adjustment that is presented in published testing procedures. However, the table of correlation ratios that is presented in published testing procedures is not consistent with the experimental results of this study. The application of the published correction method to each of the mixes tested would have yielded inaccurate estimates. Table 5 gives correlation ratios derived from the experimental correction line (Figure 4). These factors differ significantly from the accepted values.

To illustrate the magnitude of the difference, a comparison of the published stability correlation ratio with the observed ratio at a height of 2.0 in. is useful. The published correlation ratio is 1.47 (Table 5). The experimental correlation ratio is the inverse of 0.746, the ordinate of the stability correction line at a 2.0-in. height (Figure 4). This

TABLE 5 Stability Correlation Ratios Based on Experimental Results

Volume (cm ³)	Height		Experimental Correlation Ratio	Published Correlation Ratio
	(mm)	(in.)		
406-420	50.8	2	1.34	1.47
421-431	52.4	2 1/16	1.29	1.39
432-443	54.0	2 1/8	1.23	1.32
444-456	55.6	2 3/16	1.19	1.25
457-470	57.2	2 1/4	1.15	1.19
471-482	58.7	2 5/16	1.10	1.14
483-495	60.3	2 3/8	1.07	1.09
496-508	61.9	2 7/16	1.03	1.04
509-522	63.5	2 1/2	1.00	1.00
523-535	64.0	2 9/16	0.97	0.96
536-546	65.1	2 5/8	0.94	0.93
547-559	66.7	2 11/16	0.91	0.89
560-573	68.3	2 3/4	0.89	0.86
574-585	71.4	2 13/16	0.86	0.83
586-598	73.0	2 7/8	0.84	0.81
599-610	74.6	2 15/16	0.82	0.78
611-625	76.2	3	0.80	0.76

value, recorded in Table 5, is 1.34. The ratio of the published correlation ratio to the experimental ratio is 1.097. Therefore, given a stability reading at 2.0 in., the standard stability value obtained from the published method will exceed the value obtained by using the experimental ratio by 9.7 percent. At 3.0 in., the ratio of the published corre-

lation ratio to the experimental ratio is 0.95. Given a stability reading at a height of 3.0 in., the standard value obtained using the published method will be 95 percent of the value obtained using the experimental ratio, for a 5 percent difference. Inaccuracies of this magnitude may not reflect a true appraisal of the quality of the tested material.

This analysis of stability results leads to the conclusion that although a correction method based on a linear stability response is appropriate for the material tested, the slope of the published correction line is too great, resulting in inaccurate estimates. As demonstrated, this problem is crucial for specimen heights of between 2.0 and 2.5 in. because it results in an overestimate of the standard stability value. This suggests that the correction line slope should be adjusted to more accurately reflect the stability response to variation in specimen height.

Flow Correction Procedure

The flow test results of this study confirm a high correlation between specimen height and Marshall flow readings. This finding suggests that it is appropriate to use a correction method similar to that used for stability to adjust for variations in specimen height. The current practice of ignoring variations in specimen height prevents the accurate appraisal of the Marshall flow for specimens other than those 2.5 in. in height.

Experimental correlation ratios were derived from the flow correction line (Figure 6) and are given in Table 6. For a 2.0-in. specimen, the correlation

TABLE 6 Flow Correlation Ratios Based on Experimental Results

Volume (cm ³)	Height		Correction Factor	Experimental Correlation Ratio
	(mm)	(in.)		
406-420	50.8	2	0.788	1.27
421-431	52.4	2 1/16	0.814	1.23
432-443	54.0	2 1/8	0.841	1.19
444-456	55.6	2 3/16	0.868	1.15
457-470	57.2	2 1/4	0.894	1.12
471-482	58.7	2 5/16	0.921	1.09
483-495	60.3	2 3/8	0.947	1.06
496-508	61.9	2 7/16	0.974	1.03
509-522	63.5	2 1/2	1.000	1.00
523-535	64.0	2 9/16	1.026	7.97
536-546	65.1	2 5/8	1.053	7.095
547-559	66.7	2 11/16	1.079	0.93
560-573	68.3	2 3/4	1.106	0.90
574-585	71.4	2 13/16	1.132	0.88
586-598	73.0	2 7/8	1.159	0.86
599-610	74.6	2 15/16	1.185	0.84
611-625	76.2	3	1.211	0.83

ratio of 1.27, the inverse of the correction line ordinate at 2.0 in., indicates the need for a 27 percent increase in the flow reading from the test. At 3.0 in., the correlation ratio indicates the need for a 17 percent reduction in the reading obtained from the test. Further, the correction factors determined in the study corresponding to 2.0 and 3.0 in., 0.79 and 1.21, respectively, indicate that the actual flow readings taken at the extreme heights of 2.0 and 3.0 in. differ from the standard flow by 21 percent of the standard flow value. Variation by this amount requires the standardization of flow values to reflect a meaningful appraisal of the Marshall flow of the material.

CONCLUSIONS

The experimental results presented in this study indicate a correlation between stability and specimen height. Because of this, the use of a stability correction procedure for nonstandard laboratory specimens is a sound and practical solution to the problem of specimen height variability. However, the use of ASTM D-1559 correlation ratios that are intended to correct the stability values of nonstandard cored specimens is not consistent with the observed stability-specimen height linear regression estimated in this study.

A correction line for stability was calculated on the basis of the data taken from five mixes. For each mix, it was observed that the accepted table of correlation ratios would yield inaccurate results because the accepted correction line slope, derived from ASTM D-1559 correlation ratios, was too steep. Table 5 gave a set of correlation ratios that are based on the correction line parameter estimates of this study.

Marshall flow was observed to exhibit a linear response to specimen height. Variations in flow readings over a range of specimen heights were large enough to require adjustment. These results would indicate that the development of a flow correction procedure for laboratory specimens would be beneficial. The observed correlation of flow and specimen height supports the development of a flow correction procedure similar to the previously discussed stability correction procedure. A correction line based on observation of the Marshall flow of five mixes was calculated. A table of correlation ratios (Table 6) derived from the correction line was presented.

The observations concerning the flow response to specimen height also have potential impact on the testing of cored specimens. The variability of flow in response to height variability was firmly established in laboratory-prepared specimens. It is logical to infer that the flow of cored specimens would vary similarly. Currently, there is no recognized flow correction procedure for cored specimens. It is probable that such a correction procedure, similar to the stability correction procedure, could be developed with further testing and regression analysis of cored specimens. It is important to point out that the results of this study weaken the assumption that flow is not subject to variability because of variations in specimen height. It would appear that the flow results of cored specimens that vary from the standard height should not be used without correction for height variation.

The experimental results presented in this study represent the testing of five asphaltic concrete mixtures produced within a single geographic area. This small sampling cannot be used as the sole basis of a definitive correction method applicable to the large number of materials that are available. However, the results of this study may be used as a basis for suggesting the implementation of a correction method that improves on current practices. The new method must conform more closely to the response of Marshall properties to variations in specimen height as observed in this and future experimental testing.

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Computer-Assisted Random Sampling

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ABSTRACT

Many state transportation agencies use statistical quality assurance specifications to govern construction work. A vital step in the application of these and other types of specifications is the selection of random samples to obtain a valid estimate of the quality received. Random-sampling procedures are often tedious and time consuming but can be considerably simplified with computer assistance, either by using special forms generated by computer or by working directly at an interactive terminal. Examples of several applications are presented.

Of the various theoretical conditions on which statistical acceptance procedures are based, the assumption of random sampling is one of the most important. Only when all vestiges of personal bias are removed can the laws of statistical probability be relied on to function properly.

Random sampling is often defined as a manner of sampling that allows every member of the population (lot) to have an equal opportunity of appearing in the sample. This condition holds in the case of stratified random sampling for which the lot is divided into as many equal-sized sublots (strata) as there are samples to be drawn. A single random sample is then obtained from each subplot.

A more fundamental method of random sampling, sometimes called simple random sampling, allows every possible subset of the required sample size to have an equal chance of being selected. This is a less restrictive definition but it has some practical drawbacks that will be discussed shortly.

A variation of conventional stratified random sampling, discrete stratified random sampling, has also been found to be useful. With this type of sampling, discrete units (such as truckloads of material) are divided into subgroups and a random sample is chosen from each. Examples of this approach will also be given.

SIMPLE RANDOM SAMPLING

The least restrictive definition of random sampling is that of simple random sampling (1) for which all possible subsets of the required number of sample units are equally likely to be selected. However, a drawback of this type of sampling is that the sample

locations occasionally tend to be clustered. For example, if a quarter mile of pavement were defined as a lot from which five thickness cores were to be obtained, it would be possible with simple random sampling for all five cores to be located in the first 100 ft of pavement. Although this sample would be statistically valid, neither the highway agency nor the contractor would believe that it adequately represented the lot. As a result, most agencies employ stratified random plans that force the sample locations to be spread more uniformly throughout the work.

STRATIFIED RANDOM SAMPLING

Stratified sampling plans for highway construction items are designed to avoid the clustering problem and tend to be quite similar. First, most plans divide the lot into equal-sized sublots on the basis of area, weight, or other appropriate measure. Then, within each subplot, provisions are made to select a single random sample. A typical example of this approach is shown in Figure 1. The uniform random numbers between 0 and 1 are obtained from standard tables or may be generated by computer.

In practice, some agencies carry this method one step further. In sampling bituminous concrete, for example, it may be more convenient to sample directly from the appropriate trucks than to wait until after the material has been placed. In this case, the random locations in Figure 1 are used to determine which trucks are to be sampled. This is normally done in advance on the basis of known total quantities and truck capacities.