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Computer-Assisted Random Sampling

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ABSTRACT

Many state transportation agencies use statistical quality assurance specifications to govern construction work. A vital step in the application of these and other types of specifications is the selection of random samples to obtain a valid estimate of the quality received. Random-sampling procedures are often tedious and time consuming but can be considerably simplified with computer assistance, either by using special forms generated by computer or by working directly at an interactive terminal. Examples of several applications are presented.

Of the various theoretical conditions on which statistical acceptance procedures are based, the assumption of random sampling is one of the most important. Only when all vestiges of personal bias are removed can the laws of statistical probability be relied on to function properly.

Random sampling is often defined as a manner of sampling that allows every member of the population (lot) to have an equal opportunity of appearing in the sample. This condition holds in the case of stratified random sampling for which the lot is divided into as many equal-sized sublots (strata) as there are samples to be drawn. A single random sample is then obtained from each sublot.

A more fundamental method of random sampling, sometimes called simple random sampling, allows every possible subset of the required sample size to have an equal chance of being selected. This is a less restrictive definition but it has some practical drawbacks that will be discussed shortly.

A variation of conventional stratified random sampling, discrete stratified random sampling, has also been found to be useful. With this type of sampling, discrete units (such as truckloads of material) are divided into subgroups and a random sample is chosen from each. Examples of this approach will also be given.

SIMPLE RANDOM SAMPLING

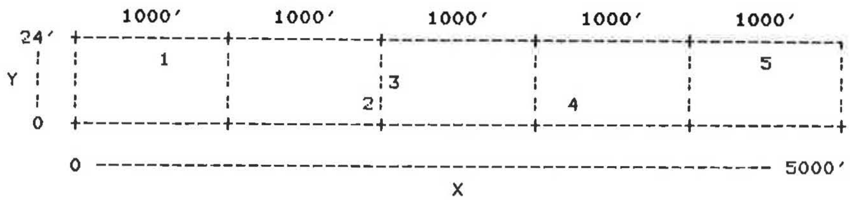
The least restrictive definition of random sampling is that of simple random sampling (1) for which all possible subsets of the required number of sample units are equally likely to be selected. However, a drawback of this type of sampling is that the sample locations occasionally tend to be clustered. For example, if a quarter mile of pavement were defined as a lot from which five thickness cores were to be obtained, it would be possible with simple random sampling for all five cores to be located in the first 100 ft of pavement. Although this sample would be statistically valid, neither the highway agency nor the contractor would believe that it adequately represented the lot. As a result, most agencies employ stratified random plans that force the sample locations to be spread more uniformly throughout the work.

STRATIFIED RANDOM SAMPLING

Stratified sampling plans for highway construction items are designed to avoid the clustering problem and tend to be quite similar. First, most plans divide the lot into equal-sized sublots on the basis of area, weight, or other appropriate measure. Then, within each sublot, provisions are made to select a single random sample. A typical example of this approach is shown in Figure 1. The uniform random numbers between 0 and 1 are obtained from standard tables or may be generated by computer.

In practice, some agencies carry this method one step further. In sampling bituminous concrete, for example, it may be more convenient to sample directly from the appropriate trucks than to wait until after the material has been placed. In this case, the random locations in Figure 1 are used to determine which trucks are to be sampled. This is normally done in advance on the basis of known total quantities and truck capacities.

(NUMERALS 1 - 5 INDICATE SAMPLING LOCATIONS)



DETERMINATION OF RANDOM X COORDINATES

SAMPLE NUMBER	RANDOM NUMBER	MULTIPLICATION TERM (SUBLOT LENGTH)	ADDITION TERM (CUMULATIVE LENGTH TO THIS SUBLOT)	X
1	0.603	× 1000	+ 0	= 603
2	0.992	× 1000	+ 1000	= 1992
3	0.086	× 1000	+ 2000	= 2086
4	0.214	× 1000	+ 3000	= 3214
5	0.551	× 1000	+ 4000	= 4551

DETERMINATION OF RANDOM Y COORDINATES

SAMPLE NUMBER	RANDOM NUMBER	MULTIPLICATION TERM (PAVEMENT WIDTH)	Y
1	0.750	× 24	= 18
2	0.286	× 24	= 7
3	0.542	× 24	= 13
4	0.081	× 24	= 2
5	0.877	× 24	= 21

FIGURE 1 Basic stratified random-sampling procedure applied to highway pavement.

This procedure has a minor flaw that can become more pronounced when the total lot size is small. If the random-sampling locations for two successive sublots both happen to fall close to the boundary separating the sublots, as in the case of samples 2 and 3 in Figure 1, they may both occur within the same truckload of material. When this happens, it is theoretically correct to take two samples from the same truck. However, because the material within a single truck is believed to be relatively homogeneous, this would provide little additional information about the quality of the lot. Consequently, most highway agencies require an alternate approach such as sampling the truck immediately preceding or following the truck that would have been double sampled. A provision such as this is a slight departure from truly random sampling but is considered by most practitioners to have little effect on the resultant quality estimates. However, it is possible to devise a truly random method for sampling directly from trucks as described in the next section.

DISCRETE STRATIFIED RANDOM SAMPLING

The stratified sampling plans just discussed divide the total quantity of the product into an appropriate number of equal-sized sublots and require that a single random sample be obtained from each. Not only is it desirable to develop an equivalent procedure for products that are measured in discrete units but, in many cases, such a procedure will prove to be useful for continuous products that are produced or delivered in discrete units such as batches or truckloads.

The objective is to develop a discrete stratified plan that performs like the continuous stratified plans (i.e., a plan that spreads the samples throughout the discrete population while allowing each member of the population an equal opportunity of being included in the sample). This is a simple task whenever the sublot size divides into the total lot an integral number of times but, unfortunately, this usually is not the case. For example, if 12 truckloads of concrete were scheduled for a particular structure and a sample size of six were desired, it would be a simple matter to randomly sample one of every two trucks arriving at the job site. However, if the scheduled number of trucks were 11, 13, or some other number not exactly divisible by six, the solution would be somewhat more involved.

The development of a method to achieve the desired result was published recently (2). Subsequently, a number of modifications were made to improve the computational efficiency of the procedure. The most recent version is shown in Figures 2 and 3, which, for actual use, are printed back to back on single sheets of paper. This avoids the need for a separate table of random numbers and provides single-sheet documentation of the random selection

ROUTE ______ SECTION _______ DATE ____!____!____ LOT ______

DESCRIPTION EXAMPLE: 22 TRUCKS OF CONCRETE SCHEDULED

LOT SIZE = 22 TRUCKS (MAXIMUM 50)

SAMPLE SIZE = 6 (IF LOT SIZE IS LESS THAN OR EQUAL TO SIX, OMIT RANDOM SELECTION PROCEDURE AND SAMPLE ALL TRUCKS IN LOT.)

RANDOM STARTING VALUE LESS THAN OR EQUAL TO LOT SIZE - - →

SUBGROUP SIZE	ADD PREVIOUS SUBGROUP SIZE TO PREVIOUS ENTRY TO GET NEXT ENTRY	RANDOM NUMBERS LESS THAN OR EQUAL TO SUBGROUP SIZE	TRUCKS TO BE SAMPLED: SUM OF PREVIOUS TWO COLUMNS -- SUBTRACT LOT SIZE FROM VALUES EXCEEDING LOT SIZE
3	17	2	19
3	20	3	23 - 22 = 1
4	23	4	27 - 22 = 5
4	27	2	29 - 22 = 7
4	31	2	33 - 22 = 11
4	35	3	38 - 22 = 16

LOT SIZE	SUBGROUPING	LOT SIZE	SUBGROUPING	LOT SIZE	SUBGROUPING
		21	3 3 3 4 4 4	36	6 6 6 6 6 6
7	1 1 1 1 1 2	22	3 3 4 4 4 4	37	6 6 6 6 6 7
8	1 1 1 1 2 2	23	3 4 4 4 4 4	38	6 6 6 6 7 7
9	1 1 1 2 2 2	24	4 4 4 4 4 4	39	6 6 6 7 7 7
10	1 1 2 2 2 2	25	4 4 4 4 4 5	40	6 6 7 7 7 7
11	1 2 2 2 2 2	26	4 4 4 4 5 5	41	6 7 7 7 7 7
12	2 2 2 2 2 2	27	4 4 4 5 5 5	42	7 7 7 7 7 7
13	2 2 2 2 2 3	28	4 4 5 5 5 5	43	7 7 7 7 7 8
14	2 2 2 2 3 3	29	4 5 5 5 5 5	44	7 7 7 7 8 8
15	2 2 2 3 3 3	30	5 5 5 5 5 5	45	7 7 7 8 8 8
16	2 2 3 3 3 3	31	5 5 5 5 5 6	46	7 7 8 8 8 8
17	2 3 3 3 3 3	32	5 5 5 5 6 6	47	7 8 8 8 8 8
18	3 3 3 3 3 3	33	5 5 5 6 6 6	48	8 8 8 8 8 8
19	3 3 3 3 3 4	34	5 5 6 6 6 6	49	8 8 8 8 8 9
20	3 3 3 3 4 4	35	5 6 6 6 6 6	50	8 8 8 8 9 9

FIGURE 2 Typical worksheet outlining steps for selecting stratified random sample of six items from a discrete population of as many as 50 items.

process for each lot. The worksheet is essentially self-explanatory and can be completed in only a minute or two without the use of a hand calculator.

Selections from the random number tables are made manually and must be done in such a way that no obvious bias is introduced. A procedure frequently suggested for tables of this type is to gaze obliquely away from the page while touching the point of a pencil to the body of the table. The number touched by the tip of the pencil becomes the random selection. It is believed that this method is sufficiently random for most practical purposes although, if necessary, more sophisticated procedures can be devised.

The worksheet presented in Figures 2 and 3 is designed to select a stratified random sample of six items from a population of as many as 50 items and was developed for use with New Jersey's statistical specification for portland cement concrete. Figures 4 and 5 show another procedure designed to select a sample of five from a population of as many as 100 items. Other plans for different sample sizes or maximum population sizes can be patterned after these examples.

OTHER STRATIFIED RANDOM SAMPLING APPLICATIONS

Using this same general approach, it is a simple matter to construct worksheets for various other sampling applications. For example, if it were desired to base the sampling procedure in Figure 4 directly on tonnage rather than discrete truckloads, the procedure shown in Figure 6 could be used. With this procedure, the stratification and sampling locations are computed on the basis of tonnage and then converted to truck locations as the final step. However, this procedure does have one limitation. Whereas the procedure in Figure 4 can accommodate any lot size up to 100 trucks, the procedure in Figure 6 is suitable only for a lot size of 1,500 tons.

Figures 7 and 8 show still another procedure de-

46	40	37	38	9	7	11	49	15	26	42	24	11	27	44	34	18	46	32	14	47	5	26	3	37	33	29	43	34	22
12	41	9	5	16	42	10	37	2	50	23	31	20	31	9	44	42	23	28	16	10	40	15	48	14	6	31	35	12	48
8	3	24	35	45	28	32	15	8	24	44	23	36	6	22	35	39	20	1	21	10	16	49	33	42	29	3	27	1	18
27	28	9	39	25	34	25	4	49	6	8	19	38	38	33	1	43	41	50	45	13	45	20	20	21	17	2	12	34	13
22	44	17	47	42	38	35	46	33	13	24	35	31	47	20	43	28	18	30	36	36	21	31	37	34	29	4	50	11	26
46	34	4	10	25	13	44	23	29	30	44	1	34	34	23	8	20	10	16	13	39	25	18	39	10	33	43	30	45	49
7	26	29	38	25	19	31	23	6	42	41	29	41	1	7	40	8	37	37	20	15	3	17	5	49	1	7	2	36	29
32	43	7	4	28	18	47	3	33	31	35	28	9	46	15	14	32	12	7	10	48	30	24	16	30	26	39	39	30	30
11	21	17	2	48	49	42	13	32	37	39	49	22	47	5	28	6	45	19	21	45	27	5	33	42	47	40	11	35	8
47	14	46	5	41	41	26	3	44	22	3	16	45	8	45	41	50	25	6	47	1	38	2	9	48	18	24	1	19	19
11	40	17	48	48	40	5	17	5	11	36	23	44	38	15	21	25	10	21	8	43	11	32	50	21	17	3	22	22	13
7	50	41	21	14	39	18	27	27	38	6	40	7	41	15	19	40	46	22	4	19	2	12	24	27	14	9	36	20	17
35	6	28	26	19	32	33	27	26	29	33	15	14	22	47	50	14	49	50	4	43	12	2	49	9	6	1	28	30	36
25	25	36	43	24	18	32	16	17	48	12	10	30	24	39	19	4	34	29	12	14	36	4	13	32	3	42	45	2	9
38	40	23	46	7	50	4	27	35	37	20	26	2	16	48	31	16	18	23	43	37	44	12	5	46	11	31	8	15	13

2	1	4	1	6	1	3	7	5	3	4	9	2	3	3	5	3	3	9	9	3	6	2	4	8	3	8	5	9	6	5	8	3	2	9	6	1	1	6	7	8	2	6	9	2
3	7	7	1	7	5	5	5	3	3	4	1	1	7	7	3	6	2	8	9	5	9	6	6	2	6	7	8	1	2	7	2	9	9	7	4	9	3	9	2	7	2	6	4	6
4	8	4	6	3	3	7	6	1	2	6	4	3	5	5	8	5	7	4	1	8	2	4	5	3	7	5	3	8	6	2	4	3	6	7	3	7	7	4	2	4	3	5	1	8
8	8	2	2	9	7	5	1	8	3	3	1	7	1	8	8	8	7	8	5	4	9	6	2	9	4	5	1	4	5	5	9	4	6	8	9	7	1	9	8	1	1	1	1	2
4	7	7	1	1	2	4	2	5	2	5	6	6	3	7	4	8	8	6	6	1	4	4	9	6	4	3	7	5	3	1	7	9	8	8	5	7	2	4	8	3	9	4	7	1
1	8	4	9	5	2	6	2	7	4	4	9	1	9	9	8	6	6	7	4	8	2	3	3	4	5	8	9	5	6	2	7	7	2	2	9	2	1	3	7	9	8	9	4	8
4	5	4	8	1	2	8	4	1	5	8	6	2	6	9	6	8	9	5	8	1	1	7	5	1	7	5	9	9	3	3	8	2	1	9	2	5	4	2	8	6	4	9	6	5
6	7	9	2	5	4	4	4	1	9	1	9	3	7	2	9	3	1	7	3	6	6	8	2	5	3	5	2	6	6	3	6	5	7	7	3	5	3	1	9	6	1	5	5	8

6	9	8	5	5	4	9	5	6	3	2	2	3	4	7	3	5	2	1	2	4	8	9	4	5	8	4	6	1	3	4	1	8	4	2	9	2	5	3	2	4	4	7	1	1
1	3	8	8	1	8	1	9	8	5	5	7	6	3	4	5	5	4	3	3	2	2	6	8	7	6	7	9	7	9	9	2	3	8	9	6	3	1	9	3	5	3	7	8	6
6	3	7	3	5	2	4	9	5	8	7	9	5	8	3	9	8	4	7	9	5	7	4	6	2	8	4	2	9	4	7	1	9	9	5	7	9	5	3	2	3	8	1	1	7
6	8	5	2	1	6	4	6	9	3	6	9	8	6	5	2	7	3	4	3	3	9	6	5	9	6	3	5	4	2	5	2	7	1	5	3	9	2	6	5	9	8	1	1	7
4	6	3	4	7	9	8	2	4	4	6	1	8	2	9	8	7	6	5	2	7	6	7	1	1	7	8	3	3	9	8	7	8	8	1	9	2	7	5	4	2	8	5	5	1
3	5	1	6	5	1	6	8	7	5	6	5	7	3	4	7	9	7	6	2	6	9	2	6	1	7	4	9	3	4	4	7	3	8	9	6	2	5	7	4	7	6	3	1	4
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6	1	8	8	8	1	7	2	7	1	6	1	2	4	4	4	9	1	4	4	2	4	4	3	3	6	9	2	4	9	5	4	7	7	4	7	6	7	1	5	7	5	9	7	7
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5	1	6	4	8	2	8	8	9	9	4	7	3	5	2	8	2	9	9	2	5	1	2	8	4	8	6	6	6	5	3	9	5	4	5	8	1	5	4	9	6	5	5	9	9

5	8	4	6	1	2	4	5	6	7	6	6	8	9	4	5	5	7	8	4	6	1	1	8	5	2	9	9	5	8	2	2	4	7	8	5	8	3	9	9	3	7	1	3	8
1	3	8	5	5	1	1	6	5	1	2	9	6	6	8	6	3	9	4	5	4	3	1	3	2	1	4	7	7	7	4	4	2	2	5	5	7	9	2	3	6	3	9	9	3
2	3	1	6	7	6	4	2	7	6	4	7	1	2	8	8	6	6	3	5	4	9	4	5	1	6	1	5	1	5	8	6	4	4	3	7	6	4	1	5	1	8	2	9	2
7	1	5	7	5	6	7	7	6	2	9	3	8	1	6	6	4	7	7	8	1	2	3	2	3	7	5	1	4	4	8	9	4	3	4	9	9	8	8	1	3	8	6	4	8
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9	3	3	7	7	6	5	1	3	3	7	4	3	5	9	5	4	9	6	4	6	9	2	8	1	1	8	5	3	2	6	7	4	5	9	8	2	8	1	2	6	4	4	4	7

6	8	5	8	2	6	5	4	1	8	7	5	1	7	5	6	2	2	3	9	1	1	8	9	3	9	3	8	3	4	2	9	1	8	8	9	8	6	5	1	9	3	7	9	4
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3	3	9	6	6	3	9	6	6	1	2	8	6	5	8	2	3	8	7	6	5	4	8	1	9	4	1	5	3	1	7	1	8	8	5	9	7	1	2	2	4	8	5	4	5
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5	3	7	7	9	4	5	7	1	7	2	4	5	9	5	8	7	3	7	3	7	2	1	4	1	7	9	4	9	3	3	1	6	6	8	5	9	4	6	5	2	7	5	4	4

FIGURE 3 Random number tables printed on back of worksheet shown in Figure 2.

ROUTE _______ SECTION ________ DATE ____!____!____ LOT _______

DESCRIPTION EXAMPLE: BITUMINOUS CONCRETE LOT = 1500 TONS
1500 TONS/20 TONS PER TRUCK = 75 TRUCKS

LOT SIZE = 75 TRUCKS (MAXIMUM 100)

SAMPLE SIZE = 5 (IF LOT SIZE IS LESS THAN OR EQUAL TO FIVE, OMIT RANDOM SELECTION PROCEDURE AND SAMPLE ALL TRUCKS IN LOT.)

RANDOM STARTING VALUE LESS THAN OR EQUAL TO LOT SIZE - - -

SUBGROUP SIZE	ADD PREVIOUS SUBGROUP SIZE TO PREVIOUS ENTRY TO GET NEXT ENTRY	RANDOM NUMBERS LESS THAN OR EQUAL TO SUBGROUP SIZE	TRUCKS TO BE SAMPLED: SUM OF PREVIOUS TWO COLUMNS -- SUBTRACT LOT SIZE FROM VALUES EXCEEDING LOT SIZE
15	45	7	52
15	60	8	68
15	75	11	86 - 75 = 11
15	90	14	104 - 75 = 29
15	105	10	115 - 75 = 40

SUBGROUP SIZES FOR SPECIFIC LOT SIZES

Lot						Lot						Lot						Lot					
1:	1					26:	5	5	5	5	6	51:	10	10	10	10	11	76:	15	15	15	15	16
2:	1	1				27:	5	5	5	6	6	52:	10	10	10	11	11	77:	15	15	15	16	16
3:	1	1	1			28:	5	5	6	6	6	53:	10	10	11	11	11	78:	15	15	16	16	16
4:	1	1	1	1		29:	5	6	6	6	6	54:	10	11	11	11	11	79:	15	16	16	16	16
5:	1	1	1	1	1	30:	6	6	6	6	6	55:	11	11	11	11	11	80:	16	16	16	16	16
6:	1	1	1	1	2	31:	6	6	6	6	7	56:	11	11	11	11	12	81:	16	16	16	16	17
7:	1	1	1	2	2	32:	6	6	6	7	7	57:	11	11	11	12	12	82:	16	16	16	17	17
8:	1	1	2	2	2	33:	6	6	7	7	7	58:	11	11	12	12	12	83:	16	16	17	17	17
9:	1	2	2	2	2	34:	6	7	7	7	7	59:	11	12	12	12	12	84:	16	17	17	17	17
10:	2	2	2	2	2	35:	7	7	7	7	7	60:	12	12	12	12	12	85:	17	17	17	17	17
11:	2	2	2	2	3	36:	7	7	7	7	8	61:	12	12	12	12	13	86:	17	17	17	17	18
12:	2	2	2	3	3	37:	7	7	7	8	8	62:	12	12	12	13	13	87:	17	17	17	18	18
13:	2	2	3	3	3	38:	7	7	8	8	8	63:	12	12	13	13	13	88:	17	17	18	18	18
14:	2	3	3	3	3	39:	7	8	8	8	8	64:	12	13	13	13	13	89:	17	18	18	18	18
15:	3	3	3	3	3	40:	8	8	8	8	8	65:	13	13	13	13	13	90:	18	18	18	18	18
16:	3	3	3	3	4	41:	8	8	8	8	9	66:	13	13	13	13	14	91:	18	18	18	18	19
17:	3	3	3	4	4	42:	8	8	8	9	9	67:	13	13	13	14	14	92:	18	18	18	19	19
18:	3	3	4	4	4	43:	8	8	9	9	9	68:	13	13	14	14	14	93:	18	18	19	19	19
19:	3	4	4	4	4	44:	8	9	9	9	9	69:	13	14	14	14	14	94:	18	19	19	19	19
20:	4	4	4	4	4	45:	9	9	9	9	9	70:	14	14	14	14	14	95:	19	19	19	19	19
21:	4	4	4	4	5	46:	9	9	9	9	10	71:	14	14	14	14	15	96:	19	19	19	19	20
22:	4	4	4	5	5	47:	9	9	9	10	10	72:	14	14	14	15	15	97:	19	19	19	20	20
23:	4	4	5	5	5	48:	9	9	10	10	10	73:	14	14	15	15	15	98:	19	19	20	20	20
24:	4	5	5	5	5	49:	9	10	10	10	10	74:	14	15	15	15	15	99:	19	20	20	20	20
25:	5	5	5	5	5	50:	10	10	10	10	10	75:	15	15	15	15	15	100:	20	20	20	20	20

FIGURE 4 Typical worksheet outlining steps for selecting stratified random sample of five items from a discrete population of as many as 100 items.

signed to determine pavement-coring locations in a stratified random manner. Because of the large range of random numbers required for this application, it was not practical to construct the tables in the same form as those for the previous procedures. Consequently, true randomness has been compromised to a small extent. For the table in Figure 7, a total of 2,280 single digits (228 each of the digits 0 through 9) have been randomly scrambled. [A Fortran subroutine developed to perform this operation is contained in a recent publication (3).] Three-digit numbers can then be obtained horizontally, vertically, or diagonally from any location in this table. Although it would be a formidable task to check if all possible three-digit random numbers are equally represented by this table, the manner of construction and use is sufficiently random that it is believed to be extremely unlikely that any significant degree of bias exists. The other two tables in Figure 8 will exhibit no bias if sampling with replacement is used (allowing identical selections to occur; that is, same number and same location in the table). However, because the tables contain large quantities of each number that might be chosen, little bias will occur even if sampling with replacement is not strictly practiced. (This relatively minor problem is completely avoided in the previous examples because only a single selection is made from each section of the various tables that are used.)

The procedure shown in Figure 7 also permits the user to randomly select the lanes from which the cores will be taken. Alternatively, the lane for the first sublot could be selected at random with the subsequent lanes following in some predetermined order.

The prominent role played by the computer in the development of these procedures is quite evident.

SELECT RANDOM STARTING VALUE LESS THAN OR EQUAL TO LOT SIZE FROM THIS TABLE

35	91	24	27	12	46	72	14	2	76	91	71	68	75	55	100	57	60	33	16	60	85	10	51	69	73	89	25	82	62
44	71	63	23	41	90	74	34	12	72	52	92	40	45	84	96	8	33	86	80	43	43	3	3	76	39	15	14	11	45
65	95	31	32	92	7	50	76	57	84	62	29	77	97	49	41	39	79	93	61	38	13	16	66	9	85	21	83	82	40
29	81	33	8	26	13	81	49	87	2	80	11	84	38	28	97	47	74	40	30	52	28	35	53	13	55	89	44	71	88
95	98	58	32	6	36	60	69	93	54	68	61	78	66	62	92	85	9	77	33	64	40	21	87	59	18	19	72	73	70
1	48	46	13	12	53	39	55	14	6	63	81	99	10	90	59	53	51	22	65	42	67	31	23	30	2	77	5	2	87
34	43	88	50	4	75	83	38	11	20	25	95	75	62	14	82	19	38	68	92	20	9	19	69	70	36	9	16	96	9
98	8	32	17	87	88	33	42	30	88	67	8	18	24	70	52	20	99	26	86	55	41	89	28	65	47	73	78	76	41
78	52	26	94	1	42	15	79	24	48	3	53	56	87	30	46	1	51	20	66	19	31	96	55	60	64	99	95	91	58
51	94	18	22	21	70	19	52	75	58	69	63	39	85	53	11	23	90	92	13	23	54	74	93	27	21	27	69	62	36
86	63	89	44	15	36	11	17	29	33	4	29	66	22	54	48	35	27	16	14	96	57	41	1	99	4	24	82	43	51
61	47	17	99	93	67	52	88	82	56	7	25	97	42	10	92	78	12	59	65	32	71	50	80	79	35	99	94	63	12
59	36	49	55	27	40	94	37	4	71	30	85	85	83	37	98	50	80	20	57	12	34	17	66	34	28	40	21	95	15
32	50	86	88	61	77	2	79	7	5	23	4	81	75	43	76	42	94	46	74	28	19	68	22	77	57	15	56	86	83
39	3	95	56	25	17	23	9	45	16	51	59	100	84	47	22	100	7	46	98	89	97	37	64	18	27	84	87	83	70
72	64	91	60	16	35	61	34	84	45	6	6	29	65	90	35	7	59	45	25	73	60	6	78	13	90	84	61	24	75
67	73	98	97	66	49	98	50	91	38	100	5	68	81	56	37	58	17	8	56	64	71	18	48	44	96	5	8	10	6
44	24	3	26	38	2	49	58	26	28	74	5	15	54	14	62	93	4	32	31	67	47	90	57	81	46	79	100	58	34
83	89	7	47	1	53	31	48	54	44	72	93	80	48	41	36	76	80	69	21	78	10	96	1	37	68	91	39	22	54
26	42	3	29	100	65	43	77	70	20	31	49	18	94	67	72	97	86	25	5	74	73	82	10	45	79	37	30	63	11

SELECT RANDOM NUMBERS LESS THAN OR EQUAL TO SUBGROUP SIZE FROM THIS TABLE

5	1	2	12	8	10	4	4	14	19	1	11	6	15	8	10	20	8	1	2	2	1	18	6	3	18	4	20	5	3	15	20	4	16	18	20	10	19	8	4
15	10	6	11	17	14	17	6	16	6	16	8	19	3	2	17	14	19	19	15	13	14	13	13	4	17	6	9	11	20	16	10	5	18	12	5	2	8	13	4
5	3	18	17	9	5	2	12	7	5	6	5	1	17	6	15	9	9	14	2	14	17	6	20	13	3	18	9	8	1	9	19	10	15	17	17	18	5	16	7
18	9	20	11	1	19	17	19	15	9	14	12	6	13	7	9	16	4	20	2	9	17	16	6	11	20	18	6	18	19	12	7	4	11	5	14	4	10	7	4
18	18	4	17	8	8	10	2	4	20	13	11	5	8	12	7	15	19	7	15	12	15	14	20	13	18	10	5	16	6	12	19	10	8	13	9	9	6	12	14
18	6	11	14	9	19	12	19	5	5	9	18	18	1	2	9	8	15	17	11	10	14	3	3	2	11	12	2	17	16	15	5	12	19	8	14	13	7	8	7
10	5	17	9	12	5	13	20	5	9	6	3	13	20	1	16	10	2	15	12	9	9	18	3	10	14	16	15	16	15	11	7	2	4	14	3	1	1	10	15
10	1	17	10	10	14	18	13	2	15	6	2	16	18	8	9	5	2	3	3	10	7	11	20	17	7	4	3	6	8	7	1	13	8	3	4	8	19	12	12
9	10	4	2	19	20	14	1	19	6	5	3	6	7	4	3	10	11	13	3	16	1	14	19	11	1	13	14	13	2	2	2	15	15	7	14	15	16	16	10
11	8	1	19	7	16	11	14	15	12	4	17	3	16	2	3	3	6	7	3	12	9	17	12	11	11	7	20	8	20	16	11	18	11	13	14	11	8	14	1
4	13	7	19	20	4	6	19	13	11	2	5	16	14	12	17	12	1	7	1	13	13	3	19	4	14	9	6	5	18	20	12	8	19	1	1	11	18	17	17
3	7	1	17	8	16	7	15	5	7	7	4	18	20	17	19	4	8	16	12	20	20	13	10	13	11	2	16	15	12	18	15	6	9	10	1	20	3	5	20
18	19	10	19	7	1	5	5	3	20	17	19	1	12	13	9	16	12	2	11	8	6	2	20	4	17	1	10	17	17	19	2	7	2	1	2	9	9	15	20
20	1	19	18	9	15	9	6	4	5	17	14	15	7	9	20	16	16	14	9	8	5	5	1	10	12	3	16	6	7	9	19	15	6	15	19	10	17	18	5
7	7	5	7	1	15	10	12	20	9	17	13	3	4	12	10	4	11	2	2	13	12	11	8	3	10	14	4	8	2	16	10	8	14	18	11	4	1	7	14
1	15	13	10	14	4	14	17	5	18	4	15	8	1	9	6	17	4	6	9	15	16	12	12	16	6	7	8	11	11	4	14	13	18	20	6	4	13	3	11
13	9	11	17	5	3	7	9	18	11	5	13	4	13	16	8	2	19	2	10	15	6	4	6	5	11	3	5	10	16	3	5	19	15	5	16	19	17	18	8
16	3	17	3	2	15	2	6	10	14	9	11	8	17	2	5	17	2	14	12	18	11	16	5	20	3	3	13	15	7	18	14	3	10	16	3	14	16	1	18
16	15	12	16	7	6	10	19	19	12	7	6	11	20	20	8	2	7	17	16	8	7	19	12	10	15	8	19	20	6	3	20	18	14	11	17	14	12	3	14
19	8	12	7	10	15	17	3	7	6	16	16	14	12	3	1	19	8	10	1	20	10	4	18	8	11	7	15	14	12	18	2	4	15	9	18	15	5	13	9
5	9	13	1	1	8	15	5	2	12	15	20	4	16	2	2	3	20	12	18	18	5	13	12	2	19	15	13	14	19	4	6	14	6	6	6	20	7	7	3
8	1	8	13	3	17	14	17	11	1	18	17	1	12	19	14	3	1	7	4	13	2	15	8	18	4	11	2	16	3	17	1	5	8	12	11	13	1	20	12
20	19	9	18	11	4	6	16	7	13	10	10	2	13	20	11	9	18	18	12	11	17	13	9	5	20	11	14	4	19	3	14	7	13	20	13	12	1	10	15
10	6	4	4	2	9	11	18	9	5	5	8	16	8	19	6	6	17	11	19	7	14	17	10	19	13	9	20	10	4	8	13	1	16	1	20	6	20	9	18
8	4	1	9	11	19	5	5	7	8	15	2	14	10	13	19	19	10	11	18	2	17	18	19	6	14	7	1	1	16	10	9	1	1	12	17	2	18	19	2
14	19	13	5	12	18	17	19	4	19	8	2	15	8	7	17	12	7	15	18	15	18	17	20	16	11	1	15	20	5	9	16	5	17	7	1	4	2	14	11
3	16	16	18	4	1	7	4	3	18	15	3	16	3	5	13	5	1	7	4	5	12	18	13	14	20	8	8	4	3	13	7	1	15	12	11	11	16	2	5
4	19	1	6	13	2	17	4	9	17	17	16	4	10	2	17	16	6	6	12	11	20	10	3	11	5	5	6	5	4	12	4	12	10	17	9	16	13	3	16
10	20	14	9	18	2	3	13	19	7	16	15	14	13	4	3	3	5	3	8	7	5	14	6	7	10	8	8	20	9	10	7	10	17	8	18	10	4	2	13
20	1	19	19	15	20	7	2	18	15	12	10	11	2	14	8	18	5	10	13	6	10	4	2	20	6	10	10	11	14	12	1	19	16	15	18	12	8	1	7
15	5	3	13	2	13	8	13	1	19	8	12	5	17	19	9	20	11	17	8	5	15	14	12	16	6	5	3	2	1	3	2	11	13	18	1	6	2	17	16
5	14	13	13	10	6	16	2	7	4	5	9	20	6	1	7	1	12	15	15	19	4	7	6	11	1	4	12	18	12	4	16	17	14	19	8	14	9	15	3
11	6	9	1	3	6	3	9	18	15	1	2	6	2	10	15	12	17	19	7	12	1	14	16	2	6	11	9	9	9	15	20	15	19	9	8	17	8	13	17
20	5	9	8	14	14	7	20	3	20	7	8	15	10	6	16	14	15	13	16	11	11	6	2	10	5	7	11	8	9	11	20	18	19	4	20	4	18	16	6
20	17	10	6	13	19	12	12	3	11	17	1	8	4	2	19	16	19	12	4	15	8	17	18	9	10	20	14	12	13	17	20	13	19	14	11	18	6	3	20
14	12	7	18	3	20	14	11	20	10	9	10	13	17	18	8	7	9	3	3	20	16	6	12	11	9	14	13	6	18	9	14	15	5	4	9	11	3	16	7
1	14	7	15	4	7	6	19	2	20	12	9	8	1	13	12	15	15	6	12	14	2	10	17	15	17	4	1	20	17	8	6	13	15	17	3	11	13	9	18
13	12	1	9	18	16	13	5	9	4	7	8	17	15	2	2	5	7	4	18	11	20	19	6	19	20	7	17	5	14	2	10	17	16	11	16	11	17	8	2
14	6	7	2	9	19	14	19	15	9	8	11	19	17	7	2	5	9	20	5	6	16	19	6	10	20	11	10	4	1	2	5	7	4	13	5	17	3	18	13
7	5	3	7	5	16	13	2	18	14	3	11	1	16	20	17	18	8	19	1	20	4	7	12	18	12	7	18	2	12	19	7	4	19	14	14	9	20	18	10
13	14	5	18	3	3	2	11	1	7	13	2	2	1	14	17	3	14	2	15	7	4	10	16	15	11	13	13	1	10	5	2	10	20	3	11	16	9	3	19
19	17	11	14	16	11	12	17	6	11	13	18	20	16	5	10	1	14	6	3	3	1	8	4	2	1	14	3	11	10	20	11	3	17	13	5	18	17	7	6
9	12	4	14	15	16	17	17	10	10	20	9	12	4	12	15	11	13	13	4	18	8	15	8	4	15	3	5	15	12	6	6	15	20	3	12	10	4	6	9
15	16	6	18	6	4	5	13	10	6	16	8	9	19	3	19	10	11	9	10	11	9	7	8	19	8	20	8	15	19	4	2	1	15	14	10	10	16	19	17
20	13	12	18	17	15	5	9	6	16	19	5	16	15	16	17	9	6	11	12	3	8	4	12	3	6	19	2	16	1	9	4	10	19	1	3	19	1	11	2
10	20	14	4	14	16	12	3	8	7	11	2	13	5	5	17	2	7	19	12	6	10	16	1	7	3	2	12	4	5	19	20	4	10	8	13	6	1	18	5
8	8	12	8	9	19	8	16	1	8	14	6	13	3	1	15	11	20	15	6	12	5	1	18	6	5	3	12	11	18	1	20	16	2	9	9	8	14	16	10
17	10	3	9	20	15	13	13	18	18	14	7	7	13	7	14	20	7	9	16	15	11	12	20	18	9	14	8	17	3	10	20	8	4	18	1	4	12	18	14
10	9	13	5	12	11	10	2	1	19	14	12	13	13	5	7	6	6	11	20	6	15	3	9	7	6	10	3	5	16	16	8	8	20	4	20	20	20	16	16
11	9	5	10	20	14	17	12	20	8	1	12	2	5	6	7	17	3	5	7	13	1	11	5	8	18	16	18	15	3	20	6	3	6	11	15	7	3	10	6
2	17	8	20	8	14	15	12	6	18	11	12	12	18	16	5	11	19	7	3	11	12	18	10	10	5	20	2	13	10	14	10	4	13	9	5	3	18	6	19
18	13	15	16	8	5	6	3	15	19	11	3	11	12	16	18	6	1	1	5	19	11	18	20	18	8	2	1	9	8	4	7	19	16	19	19	11	9	2	4
18	9	14	12	4	9	19	14	15	3	13	14	13	17	8	2	1	18	17	14	16	2	3	9	6	11	1	7	17	16	20	4	13	2	3	15	1	4	2	17
16	10	4	14	20	18	9	12	16	10	5	1	4	13	9	15	2	17	11	14	12	16	8	17	13	4	14	13	13	10	18	11	16	6	1	19	7	4	15	16
15	6	4	13	15	20	7	14	17	13	13	20	19	17	17	13	8	7	12	12	15	17	1	2	12	16	8	3	7	9	2	3	17	15	13	5	14	3	10	17
3	11	15	2	9	5	1	17	2	8	20	6	1	4	4	6	20	1	1	10	8	2	19	19	4	4	7	9	10	11	19	9	19	10	6	6	5	4	15	2
18	16	18	17	2	19	11	10	13	8	16	14	7	16	19	5	12	19	13	17	8	14	14	2	2	18	9	20	17	11	6	10	18	1	17	5	10	5	4	3
9	15	7	5	19	19	9	8	2	14	7	18	20	5	2	9	8	4	5	12	19	15	1	7	12	6	13	4	7	20	3	1	10	12	18	16	8	13	19	9
4	18	18	17	12	8	7	14	19	4	15	6	10	12	13	14	3	7	1	16	8	20	10	18	9	5	8	16	15	3	20	12	15	9	14	7	10	18	3	1
11	15	11	6	14	15	9	17	14	16	6	14	7	8	20	4	12	17	11	11	7	9	2	3	17	19	2	14	4	3	7	1	15	11	5	1	1	20	10	12

FIGURE 5 Random number tables printed on back of worksheet shown in Figure 4.

ROUTE ________ SECTION ________ DATE ____/____/____ LOT ______

DESCRIPTION _EXAMPLE: BITUMINOUS CONCRETE LOT = 1500 TONS_

SUBLOT	FIRST TON OF THIS SUBLOT	RANDOM NUMBERS FROM 0 - 299	SAMPLING LOCATIONS: TONS -- SUM OF PREVIOUS TWO COLUMNS	SAMPLING LOCATIONS: TRUCKS
1	1	136	137	7
2	301	53	354	18
3	601	269	870	44
4	901	83	984	50
5	1201	141	1342	68

022120102	591736011968996078485977231252	103962145841559152080625809197
112220200	403839563537419092829588642446	179738930679921607736126132332
201022000	796348590612212532502941502840	730007335064810791629428226790
021110021	786183968760466398058840264610	640929858075544583744876587716
012101212	325301535976643171131579470037	194859225886323670324571445865
121020111	122257639419888070574731407524	336282100474998616833491445051
011100221	977105733400082107958755744056	262046768452767308210005805545
122200211	802709981095783275325252495529	550137815710766539617235910728
010201002	488422181374513666663117688661	319219154129847699512885177978
220020102	546408843856684536434190607330	205332766723288694664319812706
011201121	913451106376990232647810164297	420434309955687101743858440312
120012221	940239389201258277112939829457	497604049638839259490323431869
122122100	327006905542420349932568959294	505973927691752325106419721475
101022101	347250601320946634848981731678	849486187547893486624695944933
002202011	678811097070512789089852520176	580746403097464624081742237955
010211120	405481932886232773979044467494	738030181812836691660990035900
021000211	964130382657080311566358521564	281015758419387324952385068263
202022121	332618692483570152796531175171	827062237614185320130177692551
020211201	320276753551097977252106962065	128567011035285454221772809552
102011211	885149346754930647653953569169	676945673069204133458990692959
100221100	722402644414347800626909495800	353442966178338059994805979987
220012011	681379838821811170136148999310	244138821727929713464112666230
222002212	883560446381291258789248774273	108308448388533107163374117708
011012200	876122497324155681555200030337	856020705621664554130089754276
002022000	037110535034725284488455989908	374163198523489100743739820817
210110211	180604922861255610572237063850	644751985079199235174907624456
021221012	042843069121127798667669915447	929443073150604554464223369056
102120122	76920995396318393020711744283	096159229697811823581082681273
101110022	137108088592128666874475195556	073687478630538683080469775875
010221120	333964601677493182547232053794	549122915281610765034216822305

FIGURE 6 Stratified random-sampling procedure based on fixed lot size of 1,500 tons.

Not only are the worksheets themselves printed by computer, the specially constructed random number tables are generated by computer programs written specifically for this purpose.

INVESTIGATION OF POTENTIAL BIAS

Because most sampling procedures in common use depart from perfect randomness to some extent, it was deemed worthwhile to empirically check for any bias that might be introduced. The results of such an investigation should provide useful guidance in the development of practical sampling plans for many applications.

For practical purposes, it was decided to limit the investigation to three basic types of sampling procedures:

1. Simple random sampling for which all possible subsets of the required sample size are equally likely to be chosen,
2. Conventional stratified random sampling for which the item to be sampled is divided into equal-sized sublots and a single random sample is taken from each, and
3. Modified stratified random sampling that is subject to the additional restriction that no two sampling locations may be closer together than some specified minimum distance.

An investigation such as this can most easily be accomplished with the aid of computer simulation (4). First, an array of randomly generated population values is created. As an added measure of realism, the program provides the capability of making successive population values correlated to any specified degree. The three sampling plans are then applied repeatedly a great many times and the sample values are compared to the true population parameters to assess how well each plan performs.

A typical run of this program is shown in Figure 9. The program was designed to check for bias in the

ROUTE -------- SECTION --------- DATE ----/----/---- LOT -------

DESCRIPTION EXAMPLE: CORING LOCATIONS

STATION 0 + 00 TO 37 + 50 NUMBER OF LANES IN LOT = 2

LOT LENGTH (FEET) = 3750 SUBLOT LENGTH (TO NEAREST FOOT) = 750

SUBLOT	STARTING STATIONS — ADD SUBLOT LENGTH TO PREVIOUS STATION TO GET NEXT STATION	RANDOM NUMBERS LESS THAN OR EQUAL TO SUBLOT LENGTH	LONGITUDINAL SAMPLING LOCATIONS — SUM OF PREVIOUS TWO COLUMNS	TRANSVERSE SAMPLING LOCATIONS: LANE	TRANSVERSE SAMPLING LOCATIONS: RANDOM DISTANCE FROM RIGHT EDGE
1	0+00	245	2+45	2	5
2	7+50	516	12+66	1	9
3	15+00	162	16+62	2	6
4	22+50	52	23+02	2	3
5	30+00	613	36+13	1	8

6692389598274071102635080045781784284564909692822361675211012753877493758761
0114120016637438586688570321913142076018189447092003212432535214323747579054
3072094049101309334420087129803836655568197068713986811777486036220856595280
8638409019806561449462242048749198387464779315460787198642926107928178743467
8202026599496057624573402131816479835116376219356315813344599926853076693812
3266987210192400749291842176287718759599652842053636224766737807250398734934
5002433754926157430317833641421051916352755857553199005441599960849968710536
9985152258340677445473957548383940926413366629181429639880974833858274437207
2160956163613747677480761377187796133235541814645524097799077723710564481448
7923128206851426666347195019400177949788021034289231670211393493731132811364
4606815180863139925516896210695133725940210301293912957105707773205363317576
1315987173337846021502955766569677565159027696238565059240295399141283715012
0807608960330720353198051325562422754250172578892052465349767952368982692957
5043697310370112929055466996231625936502863349048843564919846351440489950704
9251729632695401318111631087371621476318151047892227160489839844543791796851
9408512834495668020877842438556352695068935940305913287029256280939861244044
9528404201058601810467408268353785689167234293904830418893659009308814347898
3256846017666782837536958001315177527558999542905115144638045276624544939223
1130487948671767385787485652276153622719003014810682444000972455462726064408
8546984283264501458144751201775696826924855030818417873303071705717205690609
1023385963954370468925747254621430523331805365750113339364700341684699188753
5039212889555259668368285627223409897784808035778885920707285876726428995987
3077274632295042189113509201823870161548887047867639795051396323475574266321
9942384555513358019827291673998573044049622656131674317989046797950172362433
7145062782064274396903916229872343725179286738921425860527903700523256032269
5430083274806531469849279847607668827050207000926953453004029840049786960558
0427473148052121918401663449216391805131060372645104693596453805791057855055
4824318141463940274857109189704003019467886772292887888688684921346482113036
8377781414965351709599345679325018063230591145863417521253030956950902616806
6151761581627212436346917923142841351537896813018296611854825495422976245587

FIGURE 7 Stratified random-sampling procedure for selection of pavement-coring locations.

estimates of four commonly used statistical parameters: mean, standard deviation, variance, and percent defective. The average bias for 1,000 replications is given along with the one-tailed statistical significance level for each result. The significance levels for the bias of the estimates of the mean and the variance were computed in the customary manner using the Student's t-test and chi-square distributions, respectively. The significance levels for the bias of the estimates of the standard deviation and percent defective were computed from t-test values and, consequently, are only approximate.

It can be seen from the results in Figure 9 that all the biases are quite small and none achieved a significance level low enough to be attributable to anything other than chance. Numerous additional runs with a variety of different input values produced essentially the same results. Although far from an exhaustive study, this suggests that both conventional and modified stratified random sampling are equivalent to simple random sampling for most practical purposes.

Although it appears to have little effect, the requirement in modified stratified random sampling that all sampling locations be separated by some minimum distance should be used with caution. For the sample size of 5 used in the example in Figure 9, a minimum separation of 25 percent of the lot length would be totally unacceptable because it would exclude all sample combinations but one (samples taken at the ends and the quarter points of the lot). A rough rule of thumb might be to stay below the percentage given by 50/(sample size). For example, for a lot length of 1,000 ft and a sample size of 5, the required separation should be no larger than 50/5 = 10 percent or 100 ft. An applica-

4	3	4	3	2	2	2	3	1	4	4	1	3	3	3	1	2	4	1	3	4	4	1	2	1	1	1	4	3	1	3	2	4	4	3	1	4	3	1
2	4	2	2	4	3	1	2	4	1	3	3	3	4	4	3	1	1	1	4	1	1	4	4	1	2	1	3	3	4	4	2	3	2	4	3	3	2	4
3	4	3	1	1	4	4	4	4	4	4	3	4	2	3	1	3	3	2	4	4	4	1	4	3	3	4	3	2	3	1	1	3	2	1	3	2	4	2
4	2	4	2	2	1	2	3	4	2	1	4	3	3	1	1	4	2	2	4	4	4	4	4	1	2	2	1	1	1	3	3	3	1	3	3	3	1	2
2	2	1	1	2	1	4	1	2	4	1	3	2	4	3	2	3	1	3	2	4	2	2	4	2	4	2	1	3	1	4	3	2	4	3	2	3	1	3
2	2	1	1	1	2	1	2	2	2	2	2	3	3	3	2	1	1	2	1	3	3	1	3	2	3	2	3	2	3	1	1	2	3	3	4	3	4	3
2	4	2	2	4	3	2	1	4	1	1	2	2	2	4	3	2	1	4	1	1	4	2	2	1	4	2	2	1	1	2	2	3	4	4	1	1	1	4
1	3	4	1	4	2	2	3	2	3	4	1	2	3	2	3	4	4	3	1	3	4	3	4	4	4	2	2	3	1	3	4	2	2	2	4	2	2	3
2	1	1	1	1	4	2	4	3	1	3	2	1	1	4	1	3	1	2	4	4	4	1	4	2	1	3	1	3	2	2	4	2	2	4	2	4	3	2
1	4	1	3	1	3	2	4	2	3	4	1	4	3	1	3	4	4	2	4	2	1	1	3	2	2	1	3	3	1	1	3	3	1	4	4	1	3	1
4	1	2	2	1	4	2	2	2	4	3	1	3	1	1	1	3	3	1	3	1	2	2	1	1	3	4	2	3	2	1	1	4	1	4	2	4	4	2
2	4	2	2	4	3	3	1	3	1	3	4	3	2	1	1	2	4	1	3	4	1	1	3	1	3	4	4	2	3	4	1	2	3	1	3	4	4	4
3	3	2	4	4	3	1	2	2	1	4	3	4	4	1	3	3	2	4	4	4	1	1	2	1	3	3	1	2	3	4	3	3	3	4	3	3	1	3
3	3	1	3	2	2	3	2	1	2	3	3	4	4	3	2	4	2	4	1	2	2	2	3	3	3	4	3	3	3	4	3	4	1	3	4	2	1	4
2	3	4	3	4	1	4	4	1	1	3	4	1	2	2	2	4	2	4	1	3	2	2	2	2	4	2	1	4	1	4	4	4	1	1	2	4	1	3
2	1	1	2	1	2	3	3	4	3	1	1	3	1	2	4	4	2	2	1	1	4	4	2	4	1	2	1	2	4	4	3	2	1	2	3	3	3	1

12	7	5	8	7	10	11	9	11	1	8	5	6	8	8	10	6	8	12	6	10	5	3	9	2	11
1	8	6	7	2	2	10	8	11	1	1	5	6	10	1	12	6	7	7	12	6	4	9	12	4	6
11	9	2	3	3	5	8	3	12	10	7	4	2	7	1	2	8	3	9	6	7	8	4	7	10	4
10	3	8	4	7	7	11	1	1	3	11	4	9	8	7	2	2	9	8	11	10	11	6	6	12	6
4	11	10	4	3	3	8	12	6	5	12	8	6	4	7	5	9	11	4	8	3	10	12	4	10	2
10	3	10	5	10	2	1	1	11	4	8	2	11	6	1	6	12	5	5	8	12	10	11	8	11	10
4	3	12	2	5	8	5	10	9	8	5	7	2	11	8	7	5	6	8	4	5	5	6	3	9	7
11	5	4	9	7	3	6	12	4	3	8	1	3	7	12	12	2	12	5	9	4	2	12	9	10	7
1	10	1	2	3	10	9	5	5	3	4	2	5	8	5	1	11	3	9	6	8	11	7	2	7	9
2	1	5	3	9	7	11	3	9	11	6	3	2	8	11	11	8	1	2	8	2	6	1	9	8	10
1	4	7	5	2	7	9	5	11	3	9	12	10	1	7	1	12	4	7	7	4	7	3	7	11	9
3	8	2	8	4	6	4	9	3	1	12	9	12	7	2	11	11	11	1	9	5	9	2	11	7	10
2	2	8	10	5	6	9	8	6	7	5	1	3	1	12	3	9	11	4	2	7	4	1	6	6	6
11	11	6	12	7	4	3	2	8	11	12	12	10	2	12	4	11	5	11	1	7	7	1	11	10	12
1	3	11	4	10	10	8	11	7	4	4	1	4	11	9	12	11	2	4	4	1	7	2	10	10	7
3	9	1	6	4	8	9	11	10	4	12	7	8	11	11	8	11	6	9	10	10	4	8	3	3	11
4	12	12	5	11	11	1	9	6	6	12	3	10	1	5	6	8	3	3	10	11	3	8	4	7	10
6	1	4	3	10	12	9	10	9	1	1	8	5	2	1	7	7	6	3	11	11	3	8	7	10	2
7	10	1	2	4	2	9	10	5	7	10	12	8	3	5	9	5	8	12	6	6	7	1	1	2	11
1	5	9	11	9	3	4	9	10	9	5	8	5	9	11	9	4	12	6	11	4	8	11	11	12	9
12	12	1	5	11	9	6	10	4	3	6	12	5	6	9	1	2	5	10	12	1	4	4	6	7	2
12	5	6	2	4	10	7	8	6	5	12	10	6	6	10	7	12	1	2	3	7	1	9	8	1	7
7	1	4	5	8	12	12	3	1	11	3	5	3	4	6	3	3	9	11	4	9	6	4	2	1	7
8	4	4	11	1	9	12	2	1	9	7	12	6	6	10	4	11	9	11	12	5	4	1	2	12	5
10	2	10	10	2	11	12	8	5	8	4	5	11	9	11	10	2	7	3	12	5	5	8	9	5	7
6	8	7	11	2	7	4	5	4	7	12	12	4	9	12	10	12	9	11	6	11	5	7	7	8	9
2	1	1	2	1	11	5	12	11	11	10	5	12	1	11	7	7	3	1	5	6	10	1	6	3	7
8	4	9	12	8	8	6	12	5	2	7	12	9	6	6	4	9	3	1	3	3	2	9	12	9	1
1	5	3	9	5	5	2	12	9	7	6	12	11	10	2	10	7	6	8	1	3	3	12	7	4	12
4	4	4	9	12	10	6	12	10	2	3	1	9	8	1	7	6	3	8	4	9	12	6	7	6	4
9	5	10	2	11	2	6	8	7	5	12	10	11	11	3	10	5	8	2	6	6	4	8	10	5	2
6	3	12	3	10	9	8	6	9	3	4	3	3	7	2	10	2	5	3	4	10	6	1	2	4	8
9	5	3	7	1	3	5	8	11	2	10	3	4	11	7	1	6	10	10	1	5	11	1	8	4	1
12	4	5	8	9	2	12	12	1	5	12	12	4	5	3	7	6	6	3	5	1	2	4	3	11	12
10	11	7	8	7	8	10	9	10	8	11	4	8	2	10	3	12	3	4	6	2	1	12	7	3	4
12	3	7	7	8	5	8	4	6	3	2	9	2	1	10	10	1	9	7	8	3	2	9	1	2	6
9	3	2	3	4	10	10	5	5	8	5	5	1	11	6	7	1	12	5	2	1	11	10	10	3	8
9	4	8	6	12	8	3	11	5	2	8	4	7	12	6	3	11	2	9	1	4	8	9	5	2	9
9	8	7	2	9	10	5	3	7	1	3	5	2	10	8	2	7	12	2	6	1	10	2	4	8	5
5	11	2	3	9	2	5	3	6	6	9	9	9	12	2	1	6	12	10	10	7	7	5	4	4	8
10	12	1	1	8	9	11	10	6	2	1	6	6	1	4	2	4	7	9	3	5	9	5	8	1	10
12	2	6	12	6	11	2	9	10	8	2	3	1	3	4	5	12	11	11	5	6	10	7	6	6	2

FIGURE 8 Random number tables printed on back on worksheet shown in Figure 7.

tion of this procedure is presented in the next section.

INTERACTIVE SAMPLING PROGRAM

To further simplify the work of coring crews, a request was received to develop an interactive computer program to select random coring locations from areas of various shapes. The primary application is for straight or curved rectangular areas, such as main-line paving, but the program had to also have the capability of handling a variety of irregular shapes that might be encountered.

The input stage for the sampling of a rectangular area is shown in Figure 10. The initial entry enables the user to record project name, lot number, date, or any other identifying information. In response to the question, "is the area to be sampled rectangular?", the user has typed in "yes." For clarity, the starting and stopping stations have been entered in the form 45 + 67.89 but the program will also accept the form 4567.89. Finally, the total width and the number of stratified random samples to be taken are entered.

The output stage for this run is shown in Figure 11. The first item to be printed is the identifying information. The computer then prints the random generator seed number used to initiate the sampling sequence. (This would be needed if it were desired to conduct a complete check of the operations performed by the computer.) The stationing for the section to be sampled is printed next followed by the necessary intermediate results and random numbers so that the procedure can readily be verified, if desired. The station and offset for each sampling location are given to the nearest foot in the last two columns of the table.

In addition to the obvious longitudinal stratification, it can be observed from the offsets in the last column that the procedure also performs a

```
run biastest
EXECUTION BEGINS...

ENTER MINIMUM REQUIRED SEPARATION FOR MODIFIED STRATIFIED RANDOM
SAMPLING (PERCENT OF TOTAL LOT LENGTH)
?
5

ENTER CORRELATION COEFFICIENT FOR SUCCESSIVE POPULATION VALUES
?
0.5

ENTER POPULATION MEAN, STANDARD DEVIATION, AND PERCENT DEFECTIVE
?
10 1 10

ENTER SAMPLE SIZE, NUMBER OF REPLICATIONS, AND RANDOM GENERATOR
SEED NUMBER
?
5 1000 7654321

ACTUAL POPULATION CHARACTERISTICS
---------------------------------
   MEAN = 10.02
   STANDARD DEVIATION = 1.01
   VARIANCE = 1.02
   PERCENT DEFECTIVE =  9.87
   CORRELATION OF SUCCESSIVE VALUES =  0.51

RESULTS OBTAINED WITH SIMPLE RANDOM SAMPLING
--------------------------------------------
   MEAN =        10.03      BIAS =  0.01      SIGNIF. LEVEL = 0.281
   STD. DEV. =    1.02      BIAS =  0.01      SIGNIF. LEVEL = 0.170
   VARIANCE =     1.04      BIAS =  0.02      SIGNIF. LEVEL = 0.176
   PCT. DEF. =    9.91      BIAS =  0.04      SIGNIF. LEVEL = 0.449

RESULTS OBTAINED WITH CONVENTIONAL STRATIFIED RANDOM SAMPLING
------------------------------------------------------------
   MEAN =        10.01      BIAS = -0.01      SIGNIF. LEVEL = 0.349
   STD. DEV. =    1.01      BIAS = -0.00      SIGNIF. LEVEL = 0.477
   VARIANCE =     1.02      BIAS = -0.00      SIGNIF. LEVEL = 0.482
   PCT. DEF. =   10.17      BIAS =  0.30     .SIGNIF. LEVEL = 0.196

RESULTS OBTAINED WITH MODIFIED STRATIFIED RANDOM SAMPLING
--------------------------------------------------------
   MEAN =        10.02      BIAS =  0.00      SIGNIF. LEVEL = 0.438
   STD. DEV. =    1.01      BIAS =  0.01      SIGNIF. LEVEL = 0.306
   VARIANCE =     1.03      BIAS =  0.01      SIGNIF. LEVEL = 0.314
   PCT. DEF. =    9.95      BIAS =  0.08      SIGNIF. LEVEL = 0.407
```

FIGURE 9 Typical run of computer simulation program to test for bias.

```
run areasamp
EXECUTION BEGINS...

ENTER PROJECT IDENTIFICATION (MAXIMUM 50 CHARACTERS)
?
route 123 -- lot 45 -- 3/14/84

IS THE AREA TO BE SAMPLED RECTANGULAR
?
yes

ENTER STARTING STATION
?
45 + 67.89

ENTER STOPPING STATION
?
65 + 43.21

ENTER TOTAL WIDTH OF AREA TO BE SAMPLED
?
24

ENTER NUMBER OF STRATIFIED RANDOM SAMPLES TO BE TAKEN (MAXIMUM 30)
?
5
```

FIGURE 10 Input stage for random sampling from a rectangular area.

ROUTE 123 -- LOT 45 -- 3/14/84

RANDOM GENERATOR SEED NUMBER = 2643629

STATION 45 + 67.89 TO 65 + 43.21

TOTAL LENGTH = 1975.32
SUBLOT LENGTH = 395.06
TOTAL WIDTH = 24.00

SAMPLE	SUBLOT BOUNDARIES			RANDOM NUMBERS		SAMPLING LOCATIONS	
				STATION	OFFSET	STATION	OFFSET
1	45 + 67.89	TO	49 + 62.95	0.10560	0.8442	46 + 10	20
2	49 + 62.95	TO	53 + 58.01	0.22488	0.1147	50 + 52	3
3	53 + 58.01	TO	57 + 53.08	0.34415	0.5120	54 + 94	12
4	57 + 53.08	TO	61 + 48.14	0.78576	0.4492	60 + 64	11
5	61 + 48.14	TO	65 + 43.21	0.48775	0.8578	63 + 41	21

FIGURE 11 Output stage for random sampling from a rectangular area.

transverse stratification. The total width is divided into quarters, the quarters are alternated in a random manner, and random transverse locations are selected within each quarter. Unlike the procedure shown in Figure 7, the transverse stratification is based directly on width rather than on the number of traffic lanes. This approach is more generally applicable for the various lane and shoulder configurations that might be encountered.

Although it is invisible to the user, the procedure contains another useful refinement that can readily be incorporated when the computations are performed by computer. Conventional stratified random sampling will occasionally produce sampling locations that are quite close together, such as points 2 and 3 in Figure 1. This is undesirable from a practical standpoint because a second measurement made at nearly the same location usually provides little additional information about the population being sampled. This difficulty can be overcome by the use of modified stratified random sampling, discussed in the previous section, which requires that all sampling locations be separated by a specified minimum distance. This is accomplished by completely discarding any unsuitable combination of locations and repeating the entire selection procedure. The program has been designed to do this and will default to the conventional procedure only if the computation time becomes excessive.

Sampling from irregularly shaped areas posed a somewhat more challenging problem. A method had to be found to uniquely define a wide variety of shapes and the computer program had to be capable of recognizing the shapes and dealing with them properly. As with the procedure for rectangular areas, it was still necessary that each unit of area have an equal chance of being sampled. Finally, some form of stratification was required to avoid the possibility that the samples might be clustered together in a relatively small area.

It was decided to limit the shapes to those having straight sides (or those whose sides can be closely approximated by straight lines). By so doing, it is possible to uniquely define the shape by listing the coordinates (station and offset) of its vertices in either clockwise or counterclockwise order. Another limitation is that the procedure is not designed to handle shapes that contain holes (areas within the figure that are not part of the item to be sampled). Holes can be accommodated either (a) by rerunning the program if any of the sample locations happen to fall within a hole or (b) by breaking the area into two or more parts that do not contain holes.

The procedure for sampling an irregularly shaped area is conceptually quite simple. First, using the coordinates of the vertices, the maximum width and height of the figure are determined. On the basis of these overall dimensions, a suitable grid size is chosen and a grid system is superimposed on the figure. Next, the grid system is scanned and all points falling within the figure are counted and their coordinates are stored in memory. Finally, the discrete stratified random-sampling procedure described earlier is applied to the population of internal grid points to determine the coordinates of the sampling locations. Like the procedure for rectangular areas, the separation between all sampling locations is checked and, if any are too close together, the complete sampling sequence is repeated.

The procedure for determining whether any particular grid point lies within the area to be sampled is tedious but relatively fast when performed by computer. From each individual grid point, the vertices of the figure are scanned in clockwise or counterclockwise order. The algebraic sum of the central angles swept out by this process is computed using the law of cosines. If the point lies within the figure, the total angle will be 360 degrees; otherwise it will be zero.

The input stage for this application is shown in Figure 12. This time, in response to the question, "is the area to be sampled rectangular?", the user has entered "no." This causes a different series of input instructions to be printed out asking for the number of vertices, their coordinates, and the sample size. Here again, the coordinates have been entered in the form 35 + 00, 41 but the program will accept several variations of this.

Figure 13 shows the output stage for this application. As before, the first items to be printed out are the lot identification and the random generator seed number. Next are the coordinates of the vertices and other pertinent information so that, if desired, the various computations can be checked. Like the previous example for a rectangular area, the station and offset for each sampling location are given to the nearest foot in the last two columns of the table. A plot of the random sampling locations selected for this area is shown in Figure 14.

```
run areasamp
EXECUTION BEGINS...

ENTER PROJECT IDENTIFICATION (MAXIMUM 50 CHARACTERS)
?
route 234 -- lot 56 -- 3/30/84

IS THE AREA TO BE SAMPLED RECTANGULAR
?
no

ENTER NUMBER OF VERTICES (3 - 30)
?
4

ENTER STATION AND OFFSET FOR EACH VERTEX ON A SEPARATE LINE
(ENTER POINTS IN EITHER CLOCKWISE OR COUNTERCLOCKWISE ORDER)
?
35 + 00, 41
?
35 + 06, 9
?
35 + 20, 5
?
35 + 40, 45

ENTER NUMBER OF RANDOM SAMPLES TO BE TAKEN (MAXIMUM 30)
?
5
```

FIGURE 12 Input stage for random sampling from a nonrectangular area.

```
ROUTE 234 -- LOT 56 -- 3/30/84
------------------------------

RANDOM GENERATOR SEED NUMBER =      8697447

COORDINATES OF VERTICES
-----------------------
  STATION         OFFSET
  -------         ------
 35 +  0.0         41.00
 35 +  6.00         9.00
 35 + 20.00         5.00
 35 + 40.00        45.00

NUMBER OF INTERNAL GRID POINTS = 243          INITIAL GRID POINT
                                                 RANDOM NUMBER = 0.2284
GRID INCREMENTS                                  GRID NUMBER =  56
   STATION =  2.00                               STATION =    35 + 25.00
   OFFSET  =  2.00                               OFFSET =    18.00
```

SAMPLE	SUBLOT BOUNDARIES (GRID NUMBERS)	RANDOM NUMBER	GRID NUMBER	SAMPLING LOCATIONS STATION	SAMPLING LOCATIONS OFFSET
1	56 - 103	0.3893	74	35 + 15	22
2	104 - 151	0.7508	140	35 + 7	32
3	152 - 200	0.9472	198	35 + 21	38
4	201 - 6	0.7438	237	35 + 35	42
5	7 - 55	0.1489	14	35 + 17	10

FIGURE 13 Output stage for random sampling from a nonrectangular area.

IMPLEMENTATION

At the time of this writing, the worksheet shown in Figure 2 and the interactive program have both received extensive use. Field personnel have experienced little difficulty in learning the new procedures and report a considerable savings in both time and effort. The worksheet has enabled field inspectors to quickly and easily determine which trucks arriving at the job site are to be sampled. With the aid of the interactive program, a series of core-sampling calculations that used to require a full day can now be done effortlessly in less than an hour.

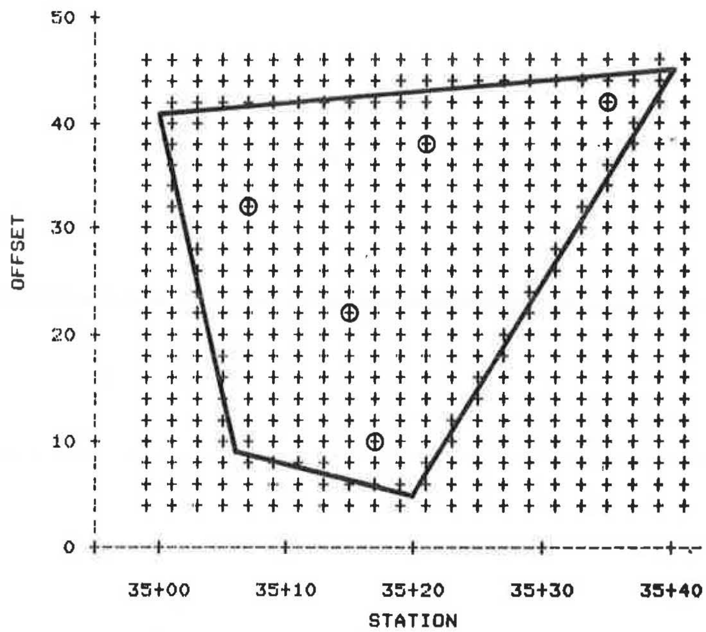

FIGURE 14 Plot of random-sampling locations obtained in Figure 13.

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