Computer-Assisted Random Sampling

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ABSTRACT

Many state transportation agencies use statistical quality assurance specifications to govern construction work. A vital step in the application of these and other types of specifications is the selection of random samples to obtain a valid estimate of the quality received. Random-sampling procedures are often tedious and time consuming but can be considerably simplified with computer assistance, either by using special forms generated by computer or by working directly at an interactive terminal. Examples of several applications are presented.

Of the various theoretical conditions on which statistical acceptance procedures are based, the assumption of random sampling is one of the most important. Only when all vestiges of personal bias are removed can the laws of statistical probability be relied on to function properly.

Random sampling is often defined as a manner of sampling that allows every member of the population (lot) to have an equal opportunity of appearing in the sample. This condition holds in the case of stratified random sampling for which the lot is divided into as many equal-sized sublots (strata) as there are samples to be drawn. A single random sample is then obtained from each sublot.

A more fundamental method of random sampling, sometimes called simple random sampling, allows every possible subset of the required sample size to have an equal chance of being selected. This is a less restrictive definition but it has some practical drawbacks that will be discussed shortly.

A variation of conventional stratified random sampling, discrete stratified random sampling, has also been found to be useful. With this type of sampling, discrete units (such as truckloads of material) are divided into subgroups and a random sample is chosen from each. Examples of this approach will also be given.

SIMPLE RANDOM SAMPLING

The least restrictive definition of random sampling is that of simple random sampling (1) for which all possible subsets of the required number of sample units are equally likely to be selected. However, a drawback of this type of sampling is that the sample locations occasionally tend to be clustered. For example, if a quarter mile of pavement were defined as a lot from which five thickness cores were to be obtained, it would be possible with simple random sampling for all five cores to be located in the first 100 ft of pavement. Although this sample would be statistically valid, neither the highway agency nor the contractor would believe that it adequately represented the lot. As a result, most agencies employ stratified random plans that force the sample locations to be spread more uniformly throughout the work.

STRATIFIED RANDOM SAMPLING

Stratified sampling plans for highway construction items are designed to avoid the clustering problem and tend to be quite similar. First, most plans divide the lot into equal-sized sublots on the basis of area, weight, or other appropriate measure. Then, within each sublot, provisions are made to select a single random sample. A typical example of this approach is shown in Figure 1. The uniform random numbers between 0 and 1 are obtained from standard tables or may be generated by computer.

In practice, some agencies carry this method one step further. In sampling bituminous concrete, for example, it may be more convenient to sample directly from the appropriate trucks than to wait until after the material has been placed. In this case, the random locations in Figure 1 are used to determine which trucks are to be sampled. This is normally done in advance on the basis of known total quantities and truck capacities.
This procedure has a minor flaw that can become more pronounced when the total lot size is small. If the random-sampling locations for two successive sublots both happen to fall close to the boundary separating the sublots, as in the case of samples 2 and 3 in Figure 1, they may both occur within the same truckload of material. When this happens, it is theoretically correct to take two samples from the same truck. However, because the material within a single truck is believed to be relatively homogeneous, this would provide little additional information about the quality of the lot. Consequently, most highway agencies require an alternate approach such as sampling the truck immediately preceding or following the truck that would have been double sampled. A provision such as this is a slight departure from truly random sampling but is considered by most practitioners to have little effect on the resultant quality estimates. However, it is possible to devise a truly random method for sampling directly from trucks as described in the next section.

DISCRETE STRATIFIED RANDOM SAMPLING

The stratified sampling plans just discussed divide the total quantity of the product into an appropriate number of equal-sized sublots and require that a single random sample be obtained from each. Not only is it desirable to develop an equivalent procedure for products that are measured in discrete units but, in many cases, such a procedure will prove to be useful for continuous products that are produced or delivered in discrete units such as batches or truckloads.

The objective is to develop a discrete stratified plan that performs like the continuous stratified plans (i.e., a plan that spreads the samples throughout the discrete population while allowing each member of the population an equal opportunity of being included in the sample). This is a simple task whenever the sublot size divides into the total lot an integral number of times but, unfortunately, this usually is not the case. For example, if 12 truckloads of concrete were scheduled for a particular structure and a sample size of six were desired, it would be a simple matter to randomly sample one of every two trucks arriving at the job site. However, if the scheduled number of trucks were 11, 13, or some other number not exactly divisible by six, the solution would be somewhat more involved.

The development of a method to achieve the desired result was published recently (2). Subsequently, a number of modifications were made to improve the computational efficiency of the procedure. The most recent version is shown in Figures 2 and 3, which, for actual use, are printed back to back on single sheets of paper. This avoids the need for a separate table of random numbers and provides single-sheet documentation of the random selection.

### DETERMINATION OF RANDOM X COORDINATES

<table>
<thead>
<tr>
<th>SAMPLE NUMBER</th>
<th>RANDOM NUMBER</th>
<th>MULTIPLICATION TERM (SUBLOT LENGTH)</th>
<th>ADDITION TERM (CUMULATIVE LENGTH TO THIS SUBLOT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.603</td>
<td>x 1000</td>
<td>+ 0 = 603</td>
</tr>
<tr>
<td>2</td>
<td>0.992</td>
<td>x 1000</td>
<td>+ 1000 = 1992</td>
</tr>
<tr>
<td>3</td>
<td>0.086</td>
<td>x 1000</td>
<td>+ 2000 = 2086</td>
</tr>
<tr>
<td>4</td>
<td>0.214</td>
<td>x 1000</td>
<td>+ 3000 = 3214</td>
</tr>
<tr>
<td>5</td>
<td>0.551</td>
<td>x 1000</td>
<td>+ 4000 = 4551</td>
</tr>
</tbody>
</table>

### DETERMINATION OF RANDOM Y COORDINATES

<table>
<thead>
<tr>
<th>SAMPLE NUMBER</th>
<th>RANDOM NUMBER</th>
<th>MULTIPLICATION TERM (PAVEMENT WIDTH)</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.750</td>
<td>x 24</td>
<td>= 18</td>
</tr>
<tr>
<td>2</td>
<td>0.286</td>
<td>x 24</td>
<td>= 7</td>
</tr>
<tr>
<td>3</td>
<td>0.542</td>
<td>x 24</td>
<td>= 13</td>
</tr>
<tr>
<td>4</td>
<td>0.081</td>
<td>x 24</td>
<td>= 2</td>
</tr>
<tr>
<td>5</td>
<td>0.077</td>
<td>x 24</td>
<td>= 21</td>
</tr>
</tbody>
</table>

**FIGURE 1** Basic stratified random-sampling procedure applied to highway pavement.
process for each lot. The worksheet is essentially self-explanatory and can be completed in only a minute or two without the use of a hand calculator.

Selections from the random number tables are made manually and must be done in such a way that no obvious bias is introduced. A procedure frequently suggested for tables of this type is to gaze obliquely away from the page while touching the point of a pencil to the body of the table. The number touched by the tip of the pencil becomes the random selection. It is believed that this method is sufficiently random for most practical purposes although, if necessary, more sophisticated procedures can be devised.

The worksheet presented in Figures 2 and 3 is designed to select a stratified random sample of six items from a population of as many as 50 items and was developed for use with New Jersey's statistical specification for portland cement concrete. Figures 4 and 5 show another procedure designed to select a sample of five from a population of as many as 100 items. Other plans for different sample sizes or maximum population sizes can be patterned after these examples.

OTHER STRATIFIED RANDOM SAMPLING APPLICATIONS

Using this same general approach, it is a simple matter to construct worksheets for various other sampling applications. For example, if it were desired to base the sampling procedure in Figure 4 directly on tonnage rather than discrete truckloads, the procedure shown in Figure 6 could be used. With this procedure, the stratification and sampling locations are computed on the basis of tonnage and then converted to truck locations as the final step. However, this procedure does have one limitation. Whereas the procedure in Figure 4 can accommodate any lot size up to 100 trucks, the procedure in Figure 6 is suitable only for a lot size of 1,500 tons.

Figures 7 and 8 show still another procedure de-
FIGURE 3 Random number tables printed on back of worksheet shown in Figure 2.
was not practical to construct the tables in the
numbers can then be obtained
stratified random manner. Because of the large range
of random numbers required for this application, it
was contained in a recent publication (3).

Small extent. For the table in Figure 7, a total of
significant degree of bias exists. The other two tables
are used.) However, because the tables contain
large quantities of each number that might be
chosen, little bias will occur even if sampling with
replacement is not strictly practiced. (This rela-
tively minor problem is completely avoided in the
previous examples because only a single selection is
made from each section of the various tables that
are used.)

The procedure shown in Figure 7 also permits the
user to randomly select the lanes from which the
cores will be taken. Alternatively, the lane for the
first sublot could be selected at random with the
subsequent lanes following in some predetermined
order.

The prominent role played by the computer in the
development of these procedures is quite evident.

<p>| LOT SIZE = .75 TRUCKS (MAXIMUM 100) |
| RANDOM STARTING VALUE LESS THAN OR EQUAL TO LOT SIZE | TRUCKS TO BE SAMPLED |
| ADD PREVIOUS SUBGROUP SIZE TO PREVIOUS ENTRY TO GET NEXT ENTRY | SUM OF PREVIOUS TWO COLUMNS -- SUBTRACT LOT SIZE FROM VALUES EXCEEDING LOT SIZE |</p>
<table>
<thead>
<tr>
<th>SUBGROUP SIZE</th>
<th>RANDOM NUMBERS LESS THAN OR EQUAL TO SUBGROUP SIZE</th>
</tr>
</thead>
<tbody>
<tr>
<td>15. 15. 15. 15. 15.</td>
<td>75</td>
</tr>
<tr>
<td>15. 15. 15. 15. 15.</td>
<td>11.</td>
</tr>
<tr>
<td>15. 15. 15. 15. 15.</td>
<td>14.</td>
</tr>
</tbody>
</table>

FIGURE 4 Typical worksheet outlining steps for selecting stratified random sample of five items from a discrete population of as many as 100 items.

signed to determine pavement coring locations in a
stratified random manner. Because of the large range
of random numbers required for this application, it
was not practical to construct the tables in the
same form as those for the previous procedures.
Consequently, true randomness has been compromised to a
small extent. For the table in Figure 7, a total of
2,280 single digits (228 each of the digits 0
through 9) have been randomly scrambled. A Fortran
subroutine developed to perform this operation is
contained in a recent publication (3). Three-digit
numbers can then be obtained horizontally, verti-
cally, or diagonally from any location in this
table. Although it would be a formidable task to
check if all possible three-digit random numbers are
equally represented by this table, the manner of
construction and use is sufficiently random that it
is believed to be extremely unlikely that any sig-
nificant degree of bias exists. The other two tables
in Figure 8 will exhibit no bias if sampling with
replacement is used (allowing identical selections to
occur; that is, same number and same location in
the table). However, because the tables contain
large quantities of each number that might be
chosen, little bias will occur even if sampling with
replacement is not strictly practiced. (This rela-
tively minor problem is completely avoided in the
previous examples because only a single selection is
made from each section of the various tables that
are used.)

The procedure shown in Figure 7 also permits the
user to randomly select the lanes from which the
cores will be taken. Alternatively, the lane for the
first sublot could be selected at random with the
subsequent lanes following in some predetermined
order.

The prominent role played by the computer in the
development of these procedures is quite evident.
Not only are the worksheets themselves printed by computer, the specially constructed random number tables are generated by computer programs written specifically for this purpose.

INVESTIGATION OF POTENTIAL BIAS

Because most sampling procedures in common use depart from perfect randomness to some extent, it was deemed worthwhile to empirically check for any bias that might be introduced. The results of such an investigation should provide useful guidance in the development of practical sampling plans for many applications.

For practical purposes, it was decided to limit the investigation to three basic types of sampling procedures:

1. Simple random sampling for which all possible subsets of the required sample size are equally likely to be chosen,

2. Conventional stratified random sampling for which the item to be sampled is divided into equally sized sublots and a single random sample is taken from each, and

3. Modified stratified random sampling that is subject to the additional restriction that no two sampling locations may be closer together than some specified minimum distance.

An investigation such as this can most easily be accomplished with the aid of computer simulation (4). First, an array of randomly generated population values is created. As an added measure of realism, the program provides the capability of making successive population values correlated to any specified degree. The three sampling plans are then applied repeatedly a great many times and the sample values are compared to the true population parameters to assess how well each plan performs.

A typical run of this program is shown in Figure 9. The program was designed to check for bias in the
estimates of four commonly used statistical parameters: mean, standard deviation, variance, and percent defective. The average bias for 1,000 replications is given along with the one-tailed statistical significance level for each result. The significance levels for the bias of the estimates of the standard deviation and percent defective were computed from t-test values and, consequently, are only approximate.

It can be seen from the results in Figure 9 that all the biases are quite small and none achieved a significance level low enough to be attributable to anything other than chance. Numerous additional runs with a variety of different input values produced essentially the same results. Although far from an exhaustive study, this suggests that both conventional and modified stratified random sampling are equivalent to simple random sampling for most practical purposes.

Although it appears to have little effect, the requirement in modified stratified random sampling that all sampling locations be separated by some minimum distance should be used with caution. For the sample size of 5 used in the example in Figure 9, a minimum separation of 25 percent of the lot length would be totally unacceptable because it would exclude all sample combinations but one (samples taken at the ends and the quarter points of the lot). A rough rule of thumb might be to stay below 50/5 percent or 10 percent or 100 ft. An applica-

<table>
<thead>
<tr>
<th>STATION</th>
<th>0 + 00</th>
<th>TO</th>
<th>37 + 50</th>
<th>NUMBER OF LANES IN LOT = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOT LENGTH (FEET) =</td>
<td>3750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUBLOT LENGTH (TO NEAREST FOOT) =</td>
<td>750</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STARTING STATIONS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADD SUBLOT LENGTH</td>
<td>0.00</td>
<td>7.50</td>
<td>15.00</td>
<td>22.50</td>
<td>30.00</td>
</tr>
<tr>
<td>TO PREVIOUS</td>
<td>245</td>
<td>516</td>
<td>1266</td>
<td>2302</td>
<td>3613</td>
</tr>
<tr>
<td>OR EQUAL</td>
<td>2 + 45</td>
<td>12 + 66</td>
<td>16 + 62</td>
<td>23 + 02</td>
<td>36 + 13</td>
</tr>
<tr>
<td>TO GET NEXT STATION</td>
<td>2</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RANDOM NUMBERS LESS THAN</th>
<th>SUM OF</th>
<th>TRANSVERSE SAMPLING LOCATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO SUBLOT LENGTH</td>
<td>PREVIOUS TWO COLUMNS</td>
<td>LANE</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>7.50</td>
<td>15.00</td>
</tr>
<tr>
<td>2</td>
<td>245</td>
<td>516</td>
</tr>
<tr>
<td>3</td>
<td>2 + 45</td>
<td>12 + 66</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To further simplify the work of coring crews, a reorganization is presented in the next section.

INTERACTIVE SAMPLING PROGRAM

To further simplify the work of coring crews, a request was received to develop an interactive computer program to select random coring locations from areas of various shapes. The primary application is for straight or curved rectangular areas, such as main-line paving, but the program had to also have the capability of handling a variety of irregular shapes that might be encountered.

The input stage for the sampling of a rectangular area is shown in Figure 10. The initial entry enables the user to record project name, lot number, date, or any other identifying information. In response to the question, "is the area to be sampled rectangular?" the user has typed in "yes." For clarity, the starting and stopping stations have been entered in the form 45 + 67.89 but the program will also accept the form 4567.89. Finally, the total width and the number of stratified random samples to be taken are entered.

The output stage for this run is shown in Figure 11. The first item to be printed is the identifying information. The computer then prints the random generator seed number used to initiate the sampling sequence. (This would be needed if it were desired to conduct a complete check of the operations performed by the computer.) The stationing for the section to be sampled is printed next followed by the necessary intermediate results: random numbers so that the procedure can readily be verified, if desired. The station and offset for each sampling location are given to the nearest foot in the last two columns of the table.

In addition to the obvious longitudinal stratification, it can be observed from the offsets in the last column that the procedure also performs a
EXECUTION BEGINS...

ENTER MINIMUM REQUIRED SEPARATION FOR MODIFIED STRATIFIED RANDOM SAMPLING (PERCENT OF TOTAL LOT LENGTH)
?
5

ENTER CORRELATION COEFFICIENT FOR SUCCESSIVE POPULATION VALUES
?
0.5

ENTER POPULATION MEAN, STANDARD DEVIATION, AND PERCENT DEFECTIVE
?
10 1 10

ENTER SAMPLE SIZE, NUMBER OF REPLICATIONS, AND RANDOM GENERATOR SEED NUMBER
?
5 1000 7654321

ACTUAL POPULATION CHARACTERISTICS

MEAN = 10.02
STANDARD DEVIATION = 1.01
VARIANCE = 1.02
PERCENT DEFECTIVE = 9.87
CORRELATION OF SUCCESSIVE VALUES = 0.51

RESULTS OBTAINED WITH SIMPLE RANDOM SAMPLING

MEAN = 10.03
STD. DEV. = 1.02
BIAS = 0.01
SIGNIF. LEVEL = 0.281
VARIANCE = 1.04
BIAS = 0.02
SIGNIF. LEVEL = 0.176
PCT. DEF. = 9.91
BIAS = 0.04
SIGNIF. LEVEL = 0.449

RESULTS OBTAINED WITH CONVENTIONAL STRATIFIED RANDOM SAMPLING

MEAN = 10.01
STD. DEV. = 1.01
BIAS = -0.01
SIGNIF. LEVEL = 0.349
VARIANCE = 1.02
BIAS = -0.00
SIGNIF. LEVEL = 0.482
PCT. DEF. = 10.17
BIAS = 0.30
SIGNIF. LEVEL = 0.196

RESULTS OBTAINED WITH MODIFIED STRATIFIED RANDOM SAMPLING

MEAN = 10.02
STD. DEV. = 1.01
BIAS = 0.00
SIGNIF. LEVEL = 0.438
VARIANCE = 1.03
BIAS = 0.01
SIGNIF. LEVEL = 0.314
PCT. DEF. = 9.95
BIAS = 0.08
SIGNIF. LEVEL = 0.407

FIGURE 9 Typical run of computer simulation program to test for bias.

EXECUTION BEGINS...

ENTER PROJECT IDENTIFICATION (MAXIMUM 50 CHARACTERS)
?
route 123 -- lot 45 -- 3/14/84

IS THE AREA TO BE SAMPLED RECTANGULAR
?
yes

ENTER STARTING STATION
?
45 + 67.89

ENTER STOPPING STATION
?
65 + 43.21

ENTER TOTAL WIDTH OF AREA TO BE SAMPLED
?
24

ENTER NUMBER OF STRATIFIED RANDOM SAMPLES TO BE TAKEN (MAXIMUM 30)
?
5

FIGURE 10 Input stage for random sampling from a rectangular area.
transverse stratification. The total width is divided into quarters, the quarters are alternated in a random manner, and random transverse locations are selected within each quarter. Unlike the procedure shown in Figure 7, the transverse stratification is based directly on width rather than on the number of traffic lanes. This approach is more generally applicable for the various lane and shoulder configurations that might be encountered.

Although it is invisible to the user, the procedure contains another useful refinement that can readily be incorporated when the computations are performed by computer. Conventional stratified random sampling will occasionally produce sampling locations that are quite close together, such as points 2 and 3 in Figure 1. This is undesirable from a practical standpoint because a second measurement made at nearly the same location usually provides little additional information about the population being sampled. This difficulty can be overcome by the use of modified stratified random sampling, discussed in the previous section, which requires that all sampling locations be separated by a specified minimum distance. This is accomplished by completely discarding any unsuitable combination of locations and repeating the entire selection procedure. The program has been designed to do this and will default to the conventional procedure only if the computation time becomes excessive.

Sampling from irregularly shaped areas posed a somewhat more challenging problem. A method had to be found to uniquely define a wide variety of shapes and the computer program had to be capable of recognizing the shapes and dealing with them properly. As with the procedure for rectangular areas, it was still necessary that each unit of area have an equal chance of being sampled. Finally, some form of stratification was required to avoid the possibility that the samples might be clustered together in a relatively small area.

It was decided to limit the shapes to those having straight sides (or whose sides can be closely approximated by straight lines). By so doing, it is possible to uniquely define the shape by listing the coordinates (station and offset) of its vertices in either clockwise or counterclockwise order. Another limitation is that the procedure is not designed to handle shapes that contain holes (areas within the figure that are not part of the item to be sampled). Holes can be accommodated either (a) by rerunning the program if any of the sample locations happen to fall within a hole or (b) by breaking the area into two or more parts that do not contain holes.

The procedure for sampling an irregularly shaped area is conceptually quite simple. First, using the coordinates of the vertices, the maximum width and height of the figure are determined. On the basis of these overall dimensions, a suitable grid size is chosen and a grid system is superimposed on the figure. Next, the grid system is scanned and all points falling within the figure are counted and their coordinates are stored in memory. Finally, the discrete stratified random-sampling procedure described earlier is applied to the population of internal grid points to determine the coordinates of the sampling locations. Like the procedure for rectangular areas, the separation between all sampling locations is checked and, if any are too close together, the complete sampling sequence is repeated.

The procedure for determining whether any particular grid point lies within the area to be sampled is tedious but relatively fast when performed by computer. From each individual grid point, the vertices of the figure are scanned in clockwise or counterclockwise order. The algebraic sum of the central angles swept out by this process is computed using the law of cosines. If the point lies within the figure, the total angle will be 360 degrees; otherwise it will be zero.

The input stage for this application is shown in Figure 12. This time, in response to the question, "is the area to be sampled rectangular?", the user has entered "no." This causes a different series of input instructions to be printed out asking for the number of vertices, their coordinates, and the sample size. Here again, the coordinates have been entered in the form 35 + 00, 41 but the program will accept several variations of this.

Figure 13 shows the output stage for this application. As before, the first items to be printed out are the lot identification and the random generator seed number. Next are the coordinates of the vertices and other pertinent information so that, if desired, the various computations can be checked. Like the previous example for a rectangular area, the station and offset for each sampling location are given to the nearest foot in the last two columns of the table. A plot of the random sampling locations selected for this area is shown in Figure 14.

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>SUBLOT BOUNDARIES</th>
<th>STATION NUMBERS</th>
<th>RANDOM NUMBERS</th>
<th>SAMPLING LOCATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45 + 67.89</td>
<td>STATION OFFSET</td>
<td>0.10560</td>
<td>STATION OFFSET</td>
</tr>
<tr>
<td>2</td>
<td>49 + 62.95</td>
<td>0.34415</td>
<td>54 + 94</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>53 + 58.01</td>
<td>0.5120</td>
<td>52 + 10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>57 + 53.06</td>
<td>0.4942</td>
<td>60 + 64</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>61 + 48.14</td>
<td>0.8578</td>
<td>63 + 41</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 11 Output stage for random sampling from a rectangular area.
IMPLEMENTATION

At the time of this writing, the worksheet shown in Figure 2 and the interactive program have both received extensive use. Field personnel have experienced little difficulty in learning the new procedures and report a considerable savings in both time and effort. The worksheet has enabled field inspectors to quickly and easily determine which trucks arriving at the job site are to be sampled. With the aid of the interactive program, a series of core-sampling calculations that used to require a full day can now be done effortlessly in less than an hour.
REFERENCES


Publication of this paper sponsored by Committee on Quality Assurance and Acceptance Procedures.