14. M. Ben-Akiva and S. Lerman; Cambridge Systematics, Inc. A Behavioral Analysis of Automobile Ownership and Modes of Travel. U.S. Department of Transportation, 1976.
15. J. Supernak. Toward a Better Understanding of Transportation-Related Choices. Proceedings of the 10th International Colloquium "Transportation and Stagnation: Challenges for Planning and Research," Delft, The Netherlands, 1983, pp. 269-288.

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Attribute Thresholds and Logit Mode-Choice Models

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ABSTRACT

The concept of thresholds has been mentioned in the transport choice literature from time to time. Few studies of mode choice have attempted to incorporate them into a modeling context, however. In this paper the concept of minimally perceived attribute differences is introduced into a logit choice model. For estimating the parameters of the model, maximum likelihood is employed and an experimental test is carried out on a sample of trip makers going to the Melbourne central business district. It was found that the average respondent required a 12-min (22 percent) difference in travel time or a 12-cent (32 percent) difference in travel cost before he would react to the variation in attribute ratings. The model is compared with a more traditional logit model with a linear additive measure of utility.

Transport planners have developed a variety of statistical techniques for analyzing mode choice (1-4). The common feature of all these models is that choice is seen as a function of the utility gained from each alternative. To calculate utility it was assumed that an alternative was characterized by a set of attributes that contribute to an index of total utility. A linear additive function was used to combine the attribute utilities into the index. In turn attribute utilities were assumed to be a continuous function of the satisfaction gained from each attribute. That is, every change in satisfaction, no matter how small, will influence the utility gained from an alternative and hence an individual's choice.

Evidence in the psychology (5,6), economics (7), and biology (8) literature suggests that people may be indifferent to changes in a stimulus unless it crosses a threshold of indifference. In the transport literature this suggestion has found support in several studies of the application of transport-choice models. Kovak and Demetsky (9) and Burns et al. (10) found that models that did not incorporate indifference thresholds tended to overestimate mode shift for small changes in attribute satisfaction. It was suggested that the inclusion of thresholds of indifference may overcome this problem because they would tend to dampen the effect of small changes in attribute satisfaction. In this paper the incorporation of such thresholds into logit choice models is investigated.

The paper is divided into six sections. The next section describes the incorporation of thresholds as used in a number of disciplines. The third section describes the incorporation of thresholds into logit choice models. The fourth section describes the data used in the study, and the fifth to seventh sections discuss the model estimation and compare model performance.

BACKGROUND

The existence of thresholds of acceptance has been discussed in many disciplines.

In psychology, sensory thresholds were suggested by Weber in 1830 (5). He introduced the concept of just noticeable differences and related their size to the magnitude of the stimulus. Fechner (6) extended Weber's law by relating the strength of the sensory process to the logarithm of the stimulus. Experimental studies that followed appeared to support Fechner's logarithm law and the existence of thresholds was accepted.

Similarly, economists analyze consumer choice of commodities by the application of indifference curves (5). In this approach it is considered that, in a choice between two commodities, the decision maker will choose one or the other or be indifferent. If the decision maker is indifferent, he will tend to randomize his decision. Indifference curves define all situations where the consumer is indifferent.

Biological experiments indicate that thresholds vary between subjects. The distribution of these thresholds was hypothesized to be normal. The resulting relationship between response and stimuli was therefore described by a probit model. Finney (8) analyzed a number of situations and concluded that the probit model predicted response relatively accurately. He also considered other relationships for the form of the threshold distribution; one of these was the logit model.

In transport planning, thresholds have been suggested in a number of contexts. Choice inertia, perception, and constraints (11) may to some extent exhibit threshold effects. Empirical studies of these thresholds have been directed along two lines.

The first related to thresholds in the comparison of the utility gained from each alternative (12,13). Krishnan (12) contended that the difference in the utility gained from a number of alternatives must be large enough for the individual to recognize the difference; otherwise he will be indifferent. Krishnan introduced a threshold (δ) into the choice situation such that in a choice between A_1 with utility U_1 and A_2 with utility U_2

$$\text{Prob } (A_1 > A_2) = \text{prob } (U_1 > U_2 + \delta)$$

$$\text{Prob } (A_2 > A_1) = \text{prob } (U_2 > U_1 + \delta)$$

$$\text{Prob } (A_1 \sim A_2) = \text{prob } (|U_1 - U_2| \leq \delta)$$

This model was found to fit the travel data better than the traditional logit model [i.e., the model with the threshold (δ) equal to zero]. Kawakami and Hirobata (13) argued that the utility of an alternative must change by an amount greater than a threshold of inertia before people will change mode. Their study of mode choice on the Nagaya-Tokyo railway line, using before-and-after data, confirmed this hypothesis.

The second approach was developed in the area of noncompensatory lexicographic or elimination-by-aspects (EBA) models. These models hypothesize that the decision maker considers the attributes describing a set of alternatives in order of importance. An alternative is eliminated if its attribute satisfaction level falls below an acceptance threshold. The most common method for calculating the acceptable threshold (14,15) has been to use a criterion whereby attribute satisfaction levels are considered to be acceptable if they lie within a specific fractional tolerance of the best satisfaction level for the attribute over all alternatives for each individual. Thus

$$\text{Acceptable } S_{kjq} \geq (1 - T_k) \max_j (S_{kjq}) \tag{1}$$

where

S_{kjq} = satisfaction with the kth attribute of the jth mode for the qth individual,
T_k = tolerance for the kth attribute, and
$\max_j (S_{kjq})$ = maximum satisfaction for the kth attribute for the qth individual over all j modes.

This approach enables the concept of just noticeable differences to be incorporated into the model as well as the size of the stimulus. It is therefore in line with the psychological research of Weber.

This review illustrates that there is considerable evidence for the existence of thresholds. Particular emphasis in the literature appears to be directed at the decision maker's inability to discern small changes in stimulus levels.

INCORPORATION OF THRESHOLD TYPES

The literature review in the previous section alluded to the existence of two apparently different approaches to incorporating thresholds in mode-choice models. The first concentrates on thresholds in total utility and the second on attribute thresholds. Before these differences are discussed further, it is necessary to develop the modeling framework for the incorporation of the thresholds.

The most commonly used choice model in transport is the logit model. The most popular derivations of this model are the constant-utility approach (16) and the random-utility approach (17). With the latter approach, in the choice between two alternatives, it is assumed that the choice alternative will be the one that maximizes the decision maker's utility. That is, if x is chosen,

$$U_x > U_y \tag{2}$$

where U_x is the utility of alternative x.

If it is assumed that the total utility is an additive function of the utility gained from each attribute of the alternative, then x will be chosen if

$$\sum_{k=1}^{k} u_{kx} > \sum_{k=1}^{k} u_{ky} \tag{3}$$

where u_{kx} is the utility of attribute k for alternative x.

Further the attribute utility is assumed to be composed of two independent elements. These are the degree of importance associated with and the satisfaction gained from an attribute. Hence the utility function takes the form

$$u_{kx} = I_k \cdot S_{kxq} \tag{4}$$

where I_k is the importance of attribute k to the decision maker and S_{kxq} is the satisfaction gained from attribute k for mode x for individual q.

It can be shown that if there is an error function associated with the decision maker's perception of utility and that the error function is described by a Weibull distribution, then the multinomial logit model will describe the choice process (18). The form of this model is

$$p(x|K) = \exp(U_x) / \sum_{k=1}^{k} \exp(U_k) \tag{5}$$

For the binary case the logit model takes the form

$$p(x/xy) = \exp(U_x)/[\exp(U_x + \exp(U_y)] \tag{6}$$

As stated earlier, there are two methods for incorporating thresholds into this model. The first concentrates on attributes and the second on total utility. An approach that combines both methods into the binary logit model can be illustrated by reference to Figure 1. If the satisfaction levels are equal for the two alternatives, then there is no difference in the two alternatives. As the difference in the satisfaction levels increases, there is still no perceived difference until the difference

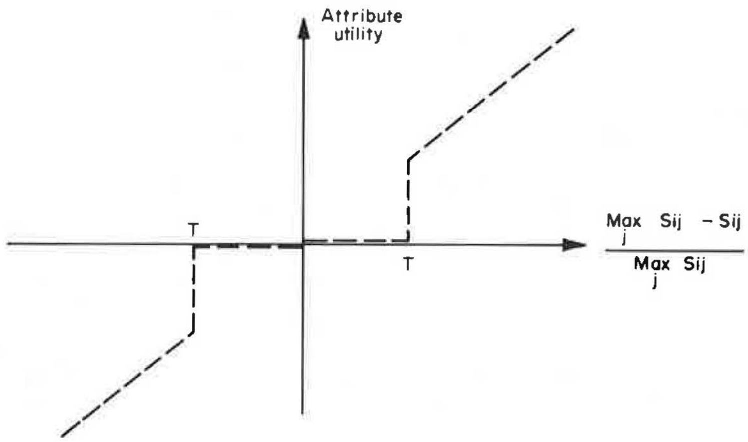

FIGURE 1 Threshold Type I.

crosses the acceptance tolerance. Once this occurs, the utility obtained from the attributes for each alternative is equal to the product of the importance and satisfaction ratings. This will be referred to as the Type I threshold in the ensuing discussion.

In mathematical terms this can be written as follows. If

$$|(S_{kxq} - S_{kyq})/S_{kxq}| < T \quad (7)$$

then

$$u_{kx} = u_{ky} = 0 \quad (8)$$

whereas if

$$|(S_{kxq} - S_{kyq})/S_{kxq}| \geq T \quad (9)$$

then

$$u_{kx} = I_k \cdot S_{kxq} \quad (10)$$

where $S_{kxq} > S_{kyq}$.

The utility for alternative x is then given by

$$U_x = \sum_{k=1}^{k} u_{kx} \quad (11)$$

A similar procedure can be used to obtain the utility associated with alternative y. The utility for alternatives x and y can then be substituted into Equation 6 and the choice probability calculated.

Another approach to thresholds is illustrated in Figure 2. Here the total utility gained from each attribute for each alternative is obtained once the threshold tolerance is crossed. This approach is consistent with studies by Recker and Golob (14) and Young and Brown (15). This approach will be referred to as the Type II threshold in the ensuing discussion. In mathematical terms this can be written using Equations 7-9. Then

$$u_{kx} = I_k \quad (12)$$

The utility for alternative x is then given by

$$U_x = \sum_{k=1}^{k} u_{kx} \quad (13)$$

A similar procedure can be used to obtain the utility associated with alternative y. The utility for alternatives x and y can then be substituted into Equation 6 and the choice probability calculated.

DESCRIPTION OF THE DATA

The data used in this study came from a survey of commuters going to the Melbourne central business district (CBD) in 1974 (19). A questionnaire survey was distributed to the employees of 35 CBD firms selected on a representative geographical and classification basis. A total of 3,737 correctly completed responses was received from a total of 7,400 issued questionnaires; of these 1,205 respondents reported a choice between automobile and train travel. It is these respondents who have been considered throughout the study.

The survey provided detailed information regard-

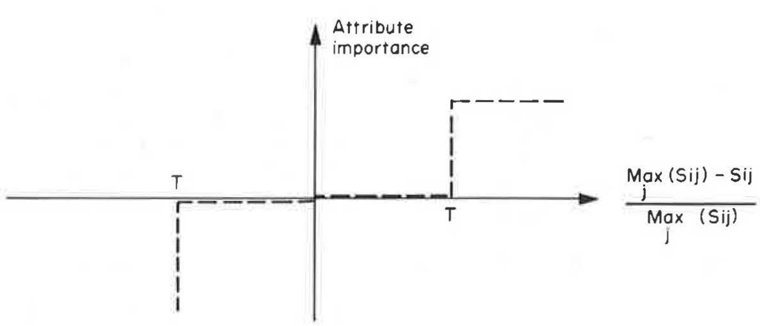

FIGURE 2 Threshold Type II.

ing the usual and next-best alternative mode available for the work trip. As well as actual time and cost data, perceptual data were solicited. The perceptual data related to the level of satisfaction experienced with the overall descriptors--comfort, convenience, and reliability. More specifically, satisfaction scores for the three factors relating to the overall trip, for both usual and alternative modes, were registered on a semantic scale of 1 to 7.

PARAMETER ESTIMATION AND MEASURES OF MODEL PERFORMANCE

The review of the literature indicated that decision makers may not be sensitive to small differences in attribute satisfaction. These differences were also thought to be related to the magnitude of the attribute satisfaction. The form of threshold most commonly used in transport research therefore takes the form presented in Equation 1. This expression incorporates both the maximum available attribute satisfaction ($\text{Max}_j\ S_{kjq}$) and the tolerable difference (T_k). It was therefore used in this study.

The aim of the model estimation procedures is to determine the most appropriate value of the tolerance (T_k). It was also necessary to determine the importance (I_k) placed on each attribute. Because the logit model was probabilistic, maximum likelihood was used to estimate these two parameters. The likelihood function took the form

$$L(\hat{I},\hat{T}) = \prod_q \prod_j P_q(j)^{g_{jq}} \quad (14)$$

where

$L(\hat{I},\hat{T})$ = likelihood at tolerance level $\hat{T}$ and importance level $\hat{I}$,

$P_q(j)$ = probability from the model that individual q chooses alternative j, and

g_{jq} = 1 if alternative j was selected by individual q, 0 otherwise.

Because both the threshold and logit models used maximum likelihood to estimate the parameters, it was possible to compare the overall fit of the model. Two tests were used. The first was the generalized likelihood ratio test (18), which was used to test the hypothesis that the probability that an individual would choose an alternative was independent of the value of the parameters in the choice model. If this hypothesis cannot be rejected, the tolerance and importance estimates used in the model may be assumed to have no effect on choice (i.e., the choice was a random one). The likelihood ratio test for the models took the form

$$-2\ln\lambda_T = -2[L^*(0,\infty) - L^*(\hat{I},\hat{T})] \quad (15)$$

where $L^*(0,\infty)$ is the log of the likelihood when the importance estimates are constrained to 0 and the tolerance estimates are constrained to a very high value, ∞, and $L^*(\hat{I},\hat{T})$ is the log of the likelihood for the best estimates of the importance and tolerance parameters. This $-2\ln\lambda_T$ value is distributed like a chi-squared distribution, with degrees of freedom equal to the number of parameters in the model.

The second test was the pseudo-r^2 value (18) where

$$\rho^2 = 1 - [L^*(\hat{I},\hat{T})./L^*(0,\infty)] \quad (16)$$

Although the statistic has the range $0 \leq \rho^2 \leq 1$, a value between 0.2 and 0.4 was considered to represent a good fit (18).

MODEL ESTIMATION

Model development consisted of a number of stages. First the model containing all the attributes was estimated. The attribute that was associated with the parameters that had the least influence on the model fit (lowest level of significance) was removed and the parameters for the new set of attributes were estimated. This refining procedure was continued until all remaining attributes had significant parameter estimates at the 5 percent level. The models presented in Tables 1 and 2 are the final product of this refining process.

TABLE 1 Comparison of Statistical Performance of Threshold and Logit Models: Part 1

	Threshold Type I		Threshold Type II		
Attribute	Tolerance	Importance	Tolerance	Importance	Logit
Time	−0.20	−0.40	−0.22	1.2	−0.041
Cost	−0.29	−0.28	−0.32	1.3	−0.030
Convenience	0.31	−0.40	0.33	1.6	−0.414
Constant					
Train		−1.32		1.54	−1.486
$-2\ln\lambda_T$		481		471	464
ρ^2		0.35		0.34	0.34

TABLE 2 Comparison of Statistical Performance of Threshold and Logit Models: Part 2

	Threshold Type I	Threshold Type II	Logit
Correct prediction			
Train	753	752	751
Market share	895	895	895
Percent	84	84	84
Automobile	168	167	166
Market share	310	310	310
Percent	54	54	54

Threshold Type I Model

The physical measures of travel time and travel cost and the perceptual measure of convenience remained in the threshold Type I model. Convenience was found to have the highest importance (-0.40) and largest tolerance (0.31), whereas travel time had the lowest tolerance (-0.20) and travel cost had the lowest importance (-0.28).

In terms of the overall fit, the model was encouraging. The $-2\ln\lambda_T$ value was significant at the 5 percent level ($-2\ln\lambda_T = 481 > 12.6 = \chi^2_{6,0.05}$) and the ρ^2-value was in the generally accepted range of 0.20 and 0.40. Further, the train mode was correctly predicted for 84 percent of the train users and the automobile mode was correctly predicted for 54 percent of the automobile users.

Threshold Type II Model

The threshold Type I and II models showed a number of similarities. Both models contained the same attribute set after refinement. There was also a marked similarity in the tolerance estimates. The major difference in the two models was the magnitude of the importance estimates. The threshold Type I model had importance ratings an order of magnitude lower than those of the Type II model.

The overall fit of the threshold Type II model was acceptable but was slightly poorer than that of the threshold Type I model.

Interpretation of Parameters

Three aspects of the estimated threshold model require further discussion: the interpretation of the estimated tolerance levels, the significance of the constant terms, and the relative magnitude of the estimated importance parameters. The threshold Type II model will be used to illustrate these points.

To facilitate this discussion, and to obtain a clearer picture of how the threshold model works, it is useful to consider the choice process for an average respondent in the sample, where such a respondent experiences the average satisfaction ratings of the sample. These average ratings (Table 3) are 54 min, 38 cents, and 4 units of convenience for the train user and 56 min, 64 cents, and 4 units of convenience for the automobile user.

TABLE 3 Model Allocation Process for Average Respondent

	Attribute				
	Travel Time (min)	Travel Cost (cents)	Convenience Units	Constant	Total
Satisfaction					
Train	54	38	4		
Automobile	56	64	4		
Tolerance	−0.22	−0.32	0.33		
Acceptable satisfaction	<66	<50	>3		
Estimated importances	1.2	1.3	1.6		
Allocation to sets					
Automobile	–	–	–	0.0	0
Train	–	1.3	0	1.5	2.8
Both	1.2	–	1.6	–	2.8

The first step in the threshold process is to determine the acceptance levels. These are obtained by using Equation 1. For example, consider the attribute travel time. The best satisfaction level for this attribute is the minimum travel time for each mode--54 min for the train mode. The tolerance level for this attribute is −0.22. Hence for the average respondent to react to any difference in the two modes there must be a 12-min [T_k x Max(S_{kjq}) = 0.22 x 54] difference in travel time. Given that the best travel time is 54 min, all travel times under 66 min are acceptable. That is, both the automobile and train modes are acceptable to the average respondent.

In the case of the travel-cost attribute, the average respondent will react to a difference in the two modes if there is a 12-cent [T_k x Max(S_{kjq}) = 0.320 x 38] variation in the cost of travel between the two modes. This is in fact the case; the train is more than 12 cents cheaper than the automobile for this trip.

Given the composition of each attribute set, its magnitude can be determined by summing the importances of each attribute as shown in Table 3. It is evident that time and convenience allocate their importance to the set that is satisfactory for both train and automobile. That is to say, these attributes have no influence on the final choice. The allocation of the cost importance level is to the train set.

The size of the alternative specific constants is large when compared with the importances of the other attribute sets and hence it may be concluded that in this model unspecified attributes have a large effect on the final choice.

Furthermore, it can be seen from Table 3 that travel cost, travel time, and convenience have equal importance ratings.

COMPARISON OF THRESHOLD AND TRADITIONAL LOGIT MODELS

The empirical comparison between the threshold models and the logit model will be carried out on two levels. First, the statistical performance of the models and then the predictions resulting from the models will be compared.

Statistical Performance

The parameter estimates for the refined logit model were presented in Table 1. It can be seen that the refined logit model contained the same three attributes as the two threshold models. Further comparison of the parameter estimates is unlikely to be of value because of the difference in interpretation of the attribute satisfaction.

In terms of the overall fit there appeared to be little difference among the three models. All appeared to perform equally well. Hence on statistical grounds there appears to be little difference in these models.

Predictive Sensitivity

A full comparison of the predictive sensitivity of the three models would require the models to predict changes in the transport system. Those predictions could be compared with what actually takes place and the accuracy of the model determined. However, the data used in this study could not be used for such a test. It is worthwhile, nonetheless, to have the models predict what might occur if a system change were made. These predictions could then be used to determine whether the three models would in fact indicate different changes to the transport system for the same changes in attribute satisfaction.

The models were required to predict the magnitude of mode shift resulting from changes in attribute ratings between −95 and +100 percent in 5 percent increments for both time and cost. A similar prediction for changes in convenience ratings was not carried out because it was based on a semantic scale and would inevitably be of a discontinuous nature.

The changes in use of the train mode consequent on changes in the cost of travel by car and train are shown in Figure 3. It can be seen that the three models give a very similar prediction of changes in train use due to changes in train travel cost.

The changes in the use of the train mode consequent on changes in the travel time by car and train are given in Figure 4. Unlike Figure 3, there are a number of differences between the predictions provided by each model. In fact the only similarity in prediction is found when the three models predict changes in mode choice consequent on changes in automobile travel time between −40 and +100 percent. It is also of note that the traditional logit model tends to provide predictions that are greater than the threshold models.

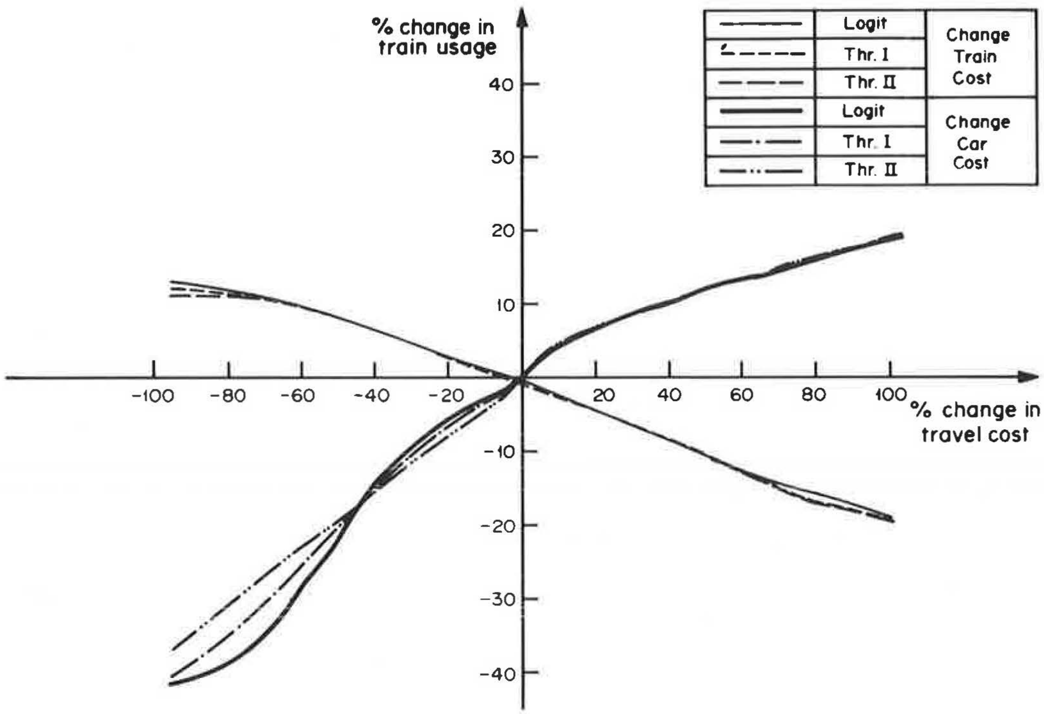

FIGURE 3 Response sensitivity to changes in travel cost.

The differences among the predictions of the three models imply that each model would give a different valuation of travel time. The logit model provides a value of travel time of 16 percent of the wage rate. The importance parameters from the threshold model cannot be interpreted in this way, but inspection of the sensitivity curves indicates that the value of travel time implicit in the threshold models is lower than that given by the logit model.

CONCLUSION

The inclusion of attribute thresholds into logit mode-choice models has been investigated. The sta-

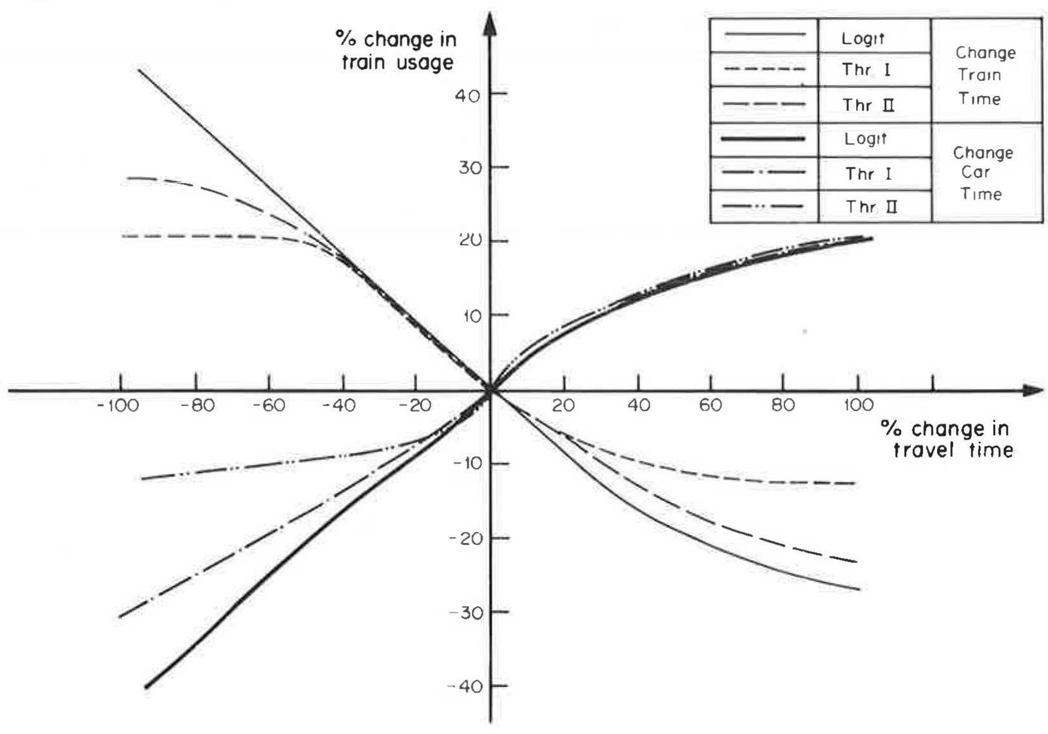

FIGURE 4 Response sensitivity to changes in travel time.

tistical performance and the predictions of these models were compared with a traditional logit model that did not contain thresholds. It was found that on the basis of statistical fit there was little difference in the performance of the models. However, each model responded differently to changes in the attribute satisfaction level and would thus predict different outcomes for certain system changes. The traditional logit model was found to be more sensitive to attribute level changes than the models that incorporated thresholds.

All three models indicated that the main attributes influencing choice were travel time, cost and convenience. The threshold models indicated that the average traveler would not react to differences in travel time of less than 12 min and travel costs of less than 12 cents from the best available alternatives satisfaction rating.

Unfortunately, this study provided no clear indication of the need to include thresholds. Rather, it was found that if thresholds are included, the model will perform well statistically and provide a different prediction to the model that does not include thresholds. The unfortunate conclusion is therefore a call for further research to reconcile this dilemma.

REFERENCES

1. D.A. Quarmby. Choice of Travel Mode for the Journey to Work: Some Findings. Journal of Transport Economic Policy, Vol. 1, 1967, pp. 273-314.
2. C.A. Lave. A Behavioural Approach to Mode Split Forecasting. Transportation Research, Vol. 3, 1969, pp. 463-480.
3. P.R. Stopher. A Probability of Travel Mode Choice for the Work Journey. In Highway Research Record 283, HRB, National Research Council, Washington, D.C., 1969, pp. 57-65.
4. C.F. Daganzo. Calibration and Prediction with Random Utility Models: Some Recent Advances and Unresolved Questions. Presented at Fourth International Conference on Behavioural Travel Modelling, Eibsee, West Germany, 1980.
5. J.P. Guilford. Psychometric Methods. McGraw-Hill, New York, 1954.
6. S.S. Stevens. The Surprising Simplicity of Sensory Metrics. American Psychologist, Vol. 17, 1962, pp. 26-29.
7. E.E. Slutsky. On the Budget of the Consumer. In Readings in Price Theory, Irwin, New York, 1952.
8. D.J. Finney. Probit Analysis. Cambridge University Press, Cambridge, England, 1964.
9. C.F. Kovak and M.H. Demetsky. Behavioural Modelling of Express Bus-Fringe Parking Decisions. In Transportation Research Record 534, TRB, National Research Council, Washington, D.C., 1975, pp. 10-23.
10. L.D. Burns, T.F. Golob, and G.C. Nicolaidis. A Theory of Urban Households' Automobile Ownership Decisions. In Transportation Research Record 569, TRB, National Research Council, Washington, D.C., 1975, pp. 56-72.
11. A.J. Richardson. Advanced Demand Modelling for Advanced Transport Systems. Journal of Advanced Transportation, Vol. 14, No. 2, 1980, pp. 113-131.
12. K.S. Krishnan. Incorporating Thresholds of Indifference in Probabilistic Choice Models. Management Science 23, 1977, pp. 1224-1233.
13. S. Kawakami and Y. Hirobata. Predicting Changes of Modal Choice after Improvement of Transport Services. Presented at World Conference on Transport, Hamburg, West Germany, 1983.
14. W.W. Recker and T.F. Golob. A Non-Compensatory Model of Transportation Behaviour Based on Sequential Consideration of Attributes. Transportation Research, Vol. 13B, 1979, pp. 269-280.
15. W. Young and H.P. Brown. A Revealed Importance Elimination-By-Aspects Model of Mode Choice. Proc., PTRC Summer Annual Meeting, Brighton, England, PTRC Education and Research Services, Ltd., London, 1983.
16. R.D. Luce. Individual Choice Behaviour: A Theoretical Analysis. John Wiley and Sons Inc., New York, 1959.
17. J. Marschak. Binary Choice Constraints and Random Utility Indicators. In Mathematical Methods in Social Sciences (K.J. Arrow et al., eds.), Stanford University Press, Stanford, Calif., 1959.
18. D.A. Hensher and L.W. Johnson. Applied Discrete Choice Modelling. Croom Helm, London, 1981.
19. H.P. Brown. Attitudinal Measures in Models of Mode Choice. Presented at Third Annual Meeting, Australian Transport Research Forum, 1977.

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