

An Optimizing Model for Transit Fare Policy Design and Evaluation

MARK S. DASKIN, JOSEPH L. SCHOFER, and ALI E. HAGHANI

ABSTRACT

To support transit agencies in the design and evaluation of more equitable and efficient fare structures, an optimization-based model system has been developed and implemented on a microcomputer. This system seeks distance-based fares of the form: $\text{FIXED CHARGE} + (\text{MILEAGE CHARGE}) (\text{trip distance}) + (\text{TRANSFER CHARGE}) (\text{number of transfers})$. It maximizes estimated revenues subject to a minimum ridership constraint and constraints on the attributes of the fare structure, which provide the user with considerable control over the structure of the optimal fare, such that distance-based, zone, and flat fare schemes can be designed and tested. This model can be used to search for fare schemes meeting user-specified requirements, to perform sensitivity studies of fare characteristics, and to test user-supplied price structures. These applications are demonstrated through the use of a data set from part of a large urban transit system.

Among the challenges facing transit operators today is balancing service needs against financial resources. Virtually every U.S. transit system receives large public subsidies. These are justified in terms of the social goals to which transit contributes (e.g., mobility for low income people) and the externalities created by such services, including improved air quality, energy savings, and encouragement of efficient land use patterns.

Although there is support for continued transit subsidies, particularly from those who receive them, there is increasing concern about the magnitude of such subsidies. Federal policy makers have attempted to reduce federal contributions to operating subsidies. Local policy makers, facing many funding requests on limited revenue bases, have also become less inclined to support subsidy increases. Some have argued for increasing fares--and changing fare structures--to allocate a larger share of transit costs to the user. These suggestions are based on both efficiency and equity arguments. An efficient pricing system relates prices to the marginal cost of service; an equitable price structure relates prices to the user's ability to pay and to the amount of service consumed.

Because transit demand is generally inelastic with respect to price (1), fare increases have resulted in increased revenues, but not without significant losses in ridership. Thus, while transit properties have moved toward one of their goals through such actions, they have necessarily moved away from others, including increasing ridership and expanding service.

Farebox revenue generation has been constrained by the abandonment of differentiated pricing (e.g., zone or distance fares and time-of-day surcharges) in favor of flat fare schemes. This has resulted, in part, from a desire to simplify fare collection and reduce passenger confusion. It also reflects an interest in attracting non-central city market segments. Under flat fare schemes used in most U.S. cities, the price for traveling only a few blocks is the same as that for traveling distances in excess of 10 mi.

The era has passed in which transit operators,

facing assured subsidies, could turn their concerns toward maximizing service and ridership. The focus of transit policy today is more clearly on revenue maximization or, at least, subsidy minimization. Thus, it appears particularly appropriate to re-examine current pricing policies to ensure the financial viability of transit systems. Although revenue maximization appears to have become a primary objective, other objectives must not be ignored, including increasing ridership (or limiting ridership losses due to price changes) and developing a pricing policy that efficiently and equitably allocates costs to the users. In addition, the pricing policy should be simple enough to be understood by transit operators and passengers and cost-effective to implement and operate.

In this effort, transit operators should look beyond flat fares to consider more creative fare structures, including distance-based fares and time-of-day fares. Such schemes have been proposed and analyzed by researchers in recent years (2-8). Some operators have implemented alternative fare structures either under demonstration projects sponsored by UMTA (9-10), or independently, to achieve some of the objectives mentioned previously (11-13).

Design and analysis of alternative fare structures is not a simple task, particularly if innovative fare options are to be considered. The technical challenge is twofold. First, it is necessary to specify pricing options; second, the effects of these options on transit objectives must be explored. There is no systematic method for specifying pricing options. The approaches for testing different fare proposals range from "back of the envelope" calculations, based on an average price elasticity applied to the aggregate market, to line-by-line short-term travel forecasting methods. The former methods are most commonly used, whereas the latter methods tend to be cumbersome and costly and, thus, are reserved for specialized investigations.

Limitations of fare design and analysis techniques restrict both the range of fare options considered and the comprehensiveness of their evaluation. This is a particular problem for distance and zone fare options. Among the methodological requirements for

advanced fare policy design and analysis methods are the following:

- Methods should be responsive to the major issues associated with fare policy revisions (e.g., implications for revenues, ridership, and equity);
- Methods should be well-founded on appropriate theory;
- Methods should have transparent logic and face validity to enhance user comfort and confidence in their use;
- Methods should be compatible with available (or readily acquired) resources for transit planning, including personnel skills, computational facilities, and data; and
- Methods should be simple to apply and should support efficient design and analysis of alternatives.

In this paper, an optimization-based tool is described that meets these requirements and supports the design and analysis of alternative transit fare structures, including, but not limited to, distance and zone-based fares.

In the next section transit pricing issues, options, and methods are reviewed, followed by a qualitative formulation of the model and its solution technique. In the last section examples of applications of the model are given, and the paper ends with summary and conclusions.

TRANSIT PRICING ISSUES, OPTIONS, AND METHODS

Objectives for transit fare structures include revenue maximization, efficient allocation of demand and service resources, price differentiation to reflect costs and service quality, equity in pricing, and minimizing the cost (and/or assuring feasibility) of fare collection itself.

Several studies have concluded that under the common flat fare structure, long-distance and peak-period riders are cross subsidized by short-distance and off-peak riders (14). When efficiency of flat fare pricing is measured in terms of the farebox recovery ratio, or the ratio of revenue per passenger mile to the cost per passenger mile, the conclusion is the same: revenues from short trips pay a greater fraction of their costs than from long trips (3-5, 15). Thus, a flat fare system is considered to be the most inefficient pricing policy; distance or time-of-day based fare structures, or both, have been proposed to remedy the efficiency shortcomings of flat fares.

A primary argument for either time-of-day or distance-based fare schemes is that they may improve the equity of fares. Both user charges and public subsidies should be allocated equitably. Although subsidies have been found to be progressive when compared with flat fare increases (16), certain trips tend to be more heavily subsidized than others; for example, long-distance trips receive greater subsidies than short trips; peak trips are more heavily subsidized than off-peak trips; and suburban trips receive greater subsidies than inner-city trips. All these features of current fare structures and subsidy policies tend to be regressive (16-17).

The importance of minimizing fare collection costs and delays is also clear. The fare structure should be easily understood by fare collectors and passengers. The shift to flat fares has responded to these concerns. Alternative fare structures may demand new technologies to support their implementation. To the extent that schemes involving other than flat fares are attractive for meeting primary operator objectives, incentives for innovation in fare collection

and passenger pricing information may be increased (18).

Efforts to evaluate the effects of transit fare changes (either structural changes or changes in the amount paid) have produced important specific results, including elasticity estimates for different services and rider groups, revenue impacts, and distributional consequences (1,9,10,19). The majority of published work assesses the impacts of various fare changes; principally, fare increases, pass programs, and unique concepts such as free fares. A few reports deal with methodologies for evaluating alternative proposed fare policies.

Wilbur Smith and Associates (7) studied existing and proposed pricing policies in the Detroit area, presenting and analyzing three zone pricing schemes that consisted of a flat fare system within the zones with different surcharge rates for crossing zone boundaries. Alternatives were ranked on the basis of financial (net revenue); social (patronage, equity, etc.); and operational (operating needs, enforcement, rider comprehension, etc.) criteria. Although the details of the policies differ, all resulted in increased ridership and decreased revenue due to the reduced average fare and the inelasticity of transit demand with respect to fare changes. No attempt to generate alternatives that reflected a different ordering of objectives was reported.

Cervero et al. (5) analyzed the effects of several pricing schemes on the Los Angeles, San Diego, and Oakland, California, transit systems. The policies included stage (zone) fare pricing and graduated pricing in which the distance-based fares were finely graduated either as a linear or logarithmic function of distance. Joint time-distance pricing policies were also tested. Policies were evaluated in terms of efficiency, equity, and ridership impacts. All of these policies increased the ratio of the revenue per mile to the cost per mile, which means that the more complex fare structures are more efficient in economic terms. It was concluded that more highly differentiated pricing schemes offer the most favorable balance between a modest patronage loss combined with significant revenue, efficiency, and equity gains.

Ballou and Mohan (20) developed a micro-simulation fare evaluation model aimed at evaluating not only systemwide ridership and revenue impacts but also equity impacts on different groups. The model is based on expanding the impacts projected for a sample of riders to systemwide impacts similar to that proposed by Cervero et al. (5). Seven combinations of distance-based and peak-period pricing policies were analyzed. The policies resulted in a range of ridership and revenue increases and decreases.

Both this and the Cervero models take the pricing schemes as a model input. No attempt is made to identify policies that attain specific ridership, revenue, efficiency, or equity objectives. Both models use a sample of transit riders and expand the results to the system's ridership.

Weiss and Hartgen (6,8) examined the financial, ridership, and equity implications of premium rush-hour fares on seven transit systems in New York State. They report that in all of the cities studied, no time-of-day based fare policy increases both revenue and ridership simultaneously. Certain combinations did improve equity while increasing either ridership or revenue with a less than 5 percent loss in the other. Again, fare structures were model inputs in these studies.

Taking fare policy as a model input results in two important shortcomings. First, analyses may fail to identify the policies most likely to attain specific objectives. Second, failing to identify the best alternative, the models cannot assess the op-

portunity costs associated with particular managerial or political constraints on the fare structures.

A search for the best pricing alternative may be conducted more efficiently and effectively by using an optimization model. Because of the requirements of such models, they will almost certainly be incapable of incorporating all the constraints that determine a viable pricing policy. The modeled policy is likely to be altered in response to these constraints before implementation. However, it is particularly desirable to identify the degree to which objectives can be achieved in the less constrained environment of an optimization model if the opportunity costs associated with imposition of the nonmodeled constraints are to be measured. Only when the opportunity cost of various constraints are known can it be decided whether the benefits of these constraints justify their cost.

In the next section an optimization model is described that determines the fare between any two points on a transit network through maximization of revenue subject to ridership and fare structure constraints. This model can deal with the different time-of-day pricing through the use of different ridership data and elasticity values. The model can produce a distance-based or a zone fare policy and estimates of the optimal transfer charges simultaneously.

MODEL DESCRIPTION

The model system is composed of seven programs designed to determine the optimal fare policy subject to user-supplied constraints, as described next, and to facilitate data input and model output analysis. The structure of the model system, together with the flow of information is shown in Figure 1. The system is designed for the IBM Personal Computer under DOS 1.1.

At the heart of the model system is FWFARE, which determines optimal fare structures by maximizing total revenue, the sum over all origin-destination (O-D) pairs of the fare charged for the O-D pair

multiplied by the number of riders between the origin and destination. The fare for each O-D pair is given by the following equation:

$$\begin{aligned} \text{Fare from Origin I to Destination J} &= \text{FIXED CHARGE} + (\text{MILEAGE CHARGE}) \\ &\quad \times (\text{Distance from I to J}) \\ &\quad + (\text{TRANSFER CHARGE}) \\ &\quad \times (\text{Number of Transfers from I to J}) \end{aligned} \quad (1)$$

The model determines the FIXED CHARGE, MILEAGE CHARGE, and TRANSFER CHARGE that maximize the total revenue subject to the user constraints. The number of riders between each O-D pair depends on the fare charged for trips between the origin and destination, as determined by the following equation:

$$\begin{aligned} \text{Ridership from Origin I to Destination J} &= \text{Base Case Ridership from I to J} \\ &\quad \times \{1 + [\text{ELASTICITY} \times (\text{NEW FARE} \\ &\quad - \text{BASE FARE})/\text{BASE FARE}]\} \end{aligned} \quad (2)$$

Equation 2 is a linear approximation to the demand curve at the base case ridership and fare. The NEW FARE is computed using Equation 1 once the model determines the FIXED CHARGE, MILEAGE CHARGE, and TRANSFER CHARGE. The use of this linear approximation results in a quadratic objective function and linear constraints that are easily solved as noted in the following paragraph. Use of a nonlinear demand model would result in nonlinear constraints and would greatly increase the difficulty involved in solving the optimization problem.

Transit demand is fare inelastic; that is, elasticities are negative and between -1.0 and 0.0 (1, 21, 22). Therefore, revenue may be increased by increasing the fare. However, fare increases will result in a decrease in ridership. Thus, the first constraint the user can place on the optimization model is a MINIMUM RIDERSHIP CONSTRAINT. This allows

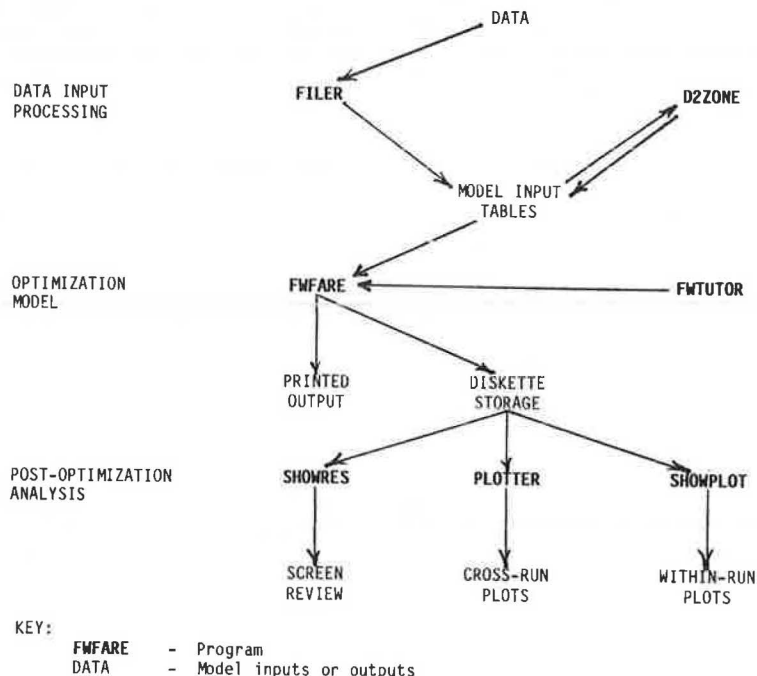


FIGURE 1 Model system structure.

the user to limit the ridership loss if fares are increased. Additional constraints can be placed on the:

1. MINIMUM and MAXIMUM FARE charged between any O-D pair,
2. MINIMUM and MAXIMUM MILEAGE CHARGE,
3. MINIMUM and MAXIMUM TRANSFER CHARGE,
4. MINIMUM and MAXIMUM FIXED CHARGE,
5. MINIMUM and MAXIMUM RATIO of the TRANSFER CHARGE to the FIXED CHARGE, and
6. MINIMUM and MAXIMUM DIFFERENCE between the FIXED CHARGE and the TRANSFER CHARGE.

Mathematically, an optimization model is obtained that maximizes a concave quadratic objective function subject to linear constraints on fixed, mileage, and transfer charges (23).

Model inputs include four tables--(a) the base case ridership (total or for a particular market segment) between each O-D pair, (b) the base case fare between each O-D pair, (c) the number of transfers required between each O-D pair, and (d) the distance between each O-D pair--and an estimate of the systemwide elasticity (for the study market segment). A systemwide elasticity is used to simplify the model inputs. Conceptually, the model may employ more realistic distance-based or origin-destination specific elasticities; however, the use of alternative elasticity measures would require minor recoding of the computer programs and would necessitate the analyst having additional information on the relationship between elasticity and trip length, for example. Model outputs include the optimal values of the FIXED CHARGE, MILEAGE CHARGE, and TRANSFER CHARGE as well as the REVENUE, RIDERSHIP, SMALLEST FARE, and LARGEST FARE.

Two programs facilitate the coding of the model inputs. FILER is a screen-oriented data input program. It allows the analyst to code the four input tables necessary to run the model. D2ZONE will transform these input tables to alternate forms. Thus, the analyst might code the actual number of transfers between each O-D pair and store the information on a diskette file. This matrix can be used if an additional charge is desired every time a transfer is made. If the analyst wants to test a fare policy with only a single charge for a transfer pass, independent of the number of transfers made, an alternate input matrix would be needed. D2ZONE can transform the information in the original file into the requisite input table that could then be stored as a new file. FWTUTOR is a brief tutorial program designed to assist analysts in using FWFARE.

The results of FWFARE, as well as the constraint values and other input information, may be stored on diskette files for future analysis. Three programs may be used to analyze these model outputs. SHOWRES displays the results stored in a results file on the screen for subsequent review and analysis. PLOTTER uses a dot matrix printer to plot any one of 22 variables (such as the total ridership, total revenue, smallest and largest fares, the range in fares, and all 13 constraint values) against any of the other values to allow the analyst to explore trade-offs between policy variables as identified by a series of model runs. Finally, SHOWPLOT allows the analyst to plot performance measures, including the fare, fare per mile, and the difference between the base case fare and the optimal fare, for a given model run.

Once an optimization problem has been solved, the most recently used parameter values become the default values for subsequent runs. This allows the user to perform sensitivity analyses rapidly by changing only one or two values for each subsequent

optimization. For example, if the user wants to analyze the trade-off between ridership and revenue, holding elasticity constant and all of the parameters of the fare policy fixed, he need only change minimum ridership and rerun the problem.

The Frank-Wolfe algorithm is used to solve this model (23). This involves maximizing a linear approximation to the objective function at any feasible value of decision variables. Having the solution to the linear program obtained by linearization of the objective function and the current feasible solution to the model, an improved solution is generated by averaging these two solutions through a one-dimensional search process. The algorithm proceeds by generating a sequence of solutions until they converge satisfactorily to the optimal solution. It is well known that when maximizing a concave function subject to linear constraints, this procedure will converge to the optimal solution.

Conceptually, FWFARE performs a form of constrained linear regression with objective function weights that differ somewhat from those used in ordinary least square regression. This is a limitation in the sense that the fare policies designed by the model are not likely to differ in significant structural ways from the current (input) fare policy unless explicitly constrained to do so. For example, in the model tests outlined using data from the Chicago Transit Authority (CTA), which employs a flat fare structure along with a transfer pass charge, the model always found a very small mileage charge unless constrained to do otherwise. On the other hand, this feature allows the user to specify an input fare table that approximates the desired policy. The use of the model system in policy analysis is outlined in the following section.

MODEL USE STRATEGIES AND EXAMPLES

The model system can be used to search for desirable fare policies and test candidate fare structures. Application strategies include (a) an experimental design approach of searching along critical policy dimensions to explore the sensitivity of fare structures and performance measures to key inputs, (b) a decision tree approach in which the user determines input values for subsequent model runs based on prior results, and (c) a policy emulation approach in which the model is constrained to replicate and test specific policies (and derivatives of them).

In the first mode, the analyst generally specifies all model runs to be conducted before the computer work is begun. For example, he might choose to explore the sensitivity of revenue and the structure of the fare policy to changes in (a) the minimum required ridership and (b) the elasticity of demand with respect to fare. To do so, he would specify a range of minimum allowable ridership values, for example, from 30 percent below the current ridership to 20 percent above this value. Similarly, a range of elasticity measures would be specified. The user would then run either all combinations of the minimum ridership and elasticity or selected combinations to cover the options of interest. The results would be stored and evaluated.

In the decision tree approach, the analyst uses the model to search for a fare policy that meets certain criteria, examining the results of each model run together with those of previous runs to select input values for subsequent runs. For example, the results of one test might produce an optimal transfer charge of \$0.108 per transfer. Because this is an impractical value, the user might constrain the transfer charge to be less than or equal to \$0.10 per transfer in the next model run. This would be a

more constrained problem and the revenue would decrease by a magnitude that measures the opportunity cost associated with being unable to charge the "optimal" transfer charge. Next, the user might explore the implications of alternate transfer charge policies by increasing the maximum transfer charge constraint to \$0.50 and setting the minimum transfer charge to \$0.15 to determine the effect of increasing the charge above its "optimal" value. The process would continue in this way until all options of immediate interest had been explored.

In the policy emulation mode of model use, the analyst attempts to replicate exogenously proposed fare structures in the model. The model system is used (a) to predict key outputs, including ridership and revenue; (b) to explore the sensitivity of these outputs to changes in uncertain input parameters such as the elasticity of demand with respect to fare; and (c) to identify other impacts of the proposed fare structure, including changes in the fare per mile paid by patrons.

To support development of the model system, the research team secured Chicago Transit Authority O-D travel data for parts of two rail transit lines and connecting bus services segmented by trip purpose, time of day, and fare class. Trips were coded into a 47-zone table. These data and the authors' analyses of them cannot be used to evaluate present and proposed CTA pricing policies for several reasons. First, only a portion of the CTA system has been used, and the representativeness of this portion was not tested. Second, CTA fares have changed since the O-D survey was taken, and this and other factors have led to potentially different ridership patterns (as well as levels) from those utilized. Third, a thorough assessment of CTA policy options would re-

quire a more extensive and detailed investigation than undertaken and reported here. Finally, transit operators respond to a variety of different goals and objectives; a valid analysis of CTA pricing would demand consideration of other important issues. Despite these limitations on the authors' ability to draw policy-related conclusions, the data set is useful in testing the model and in demonstrating the range of analyses that may be conducted by using the model system.

To illustrate the experimental design approach, runs were conducted to assess the sensitivity of revenue and the fare structure to changes in (a) the elasticity of demand with respect to fare, (b) the minimum allowable ridership, and (c) the time of day. Four elasticity values were used: -0.30, -0.25, -0.20, and -0.15. Three different ridership matrices were used that represented the entire day, the peak period only, and the off-peak periods. In addition to the elasticity and minimum allowable ridership, values were specified for all of the lower and upper bounds on the fare equation. The base case fare matrix corresponded to the fares currently charged; that is, \$1.00 and \$0.90 for trips with and without transfers, respectively.

The primary output of these analyses is the trade-off between revenue and ridership for different elasticity values. Figure 2 shows this trade-off based on the ridership data for the entire day and for all trips. Two trends are illustrated in this figure. First, revenue increases as the ridership decreases below the base case value of 165,293 and decreases as ridership increases above this value. Second, the sensitivity of revenue to ridership increases as the demand becomes less elastic with respect to fare; that is, as demand becomes

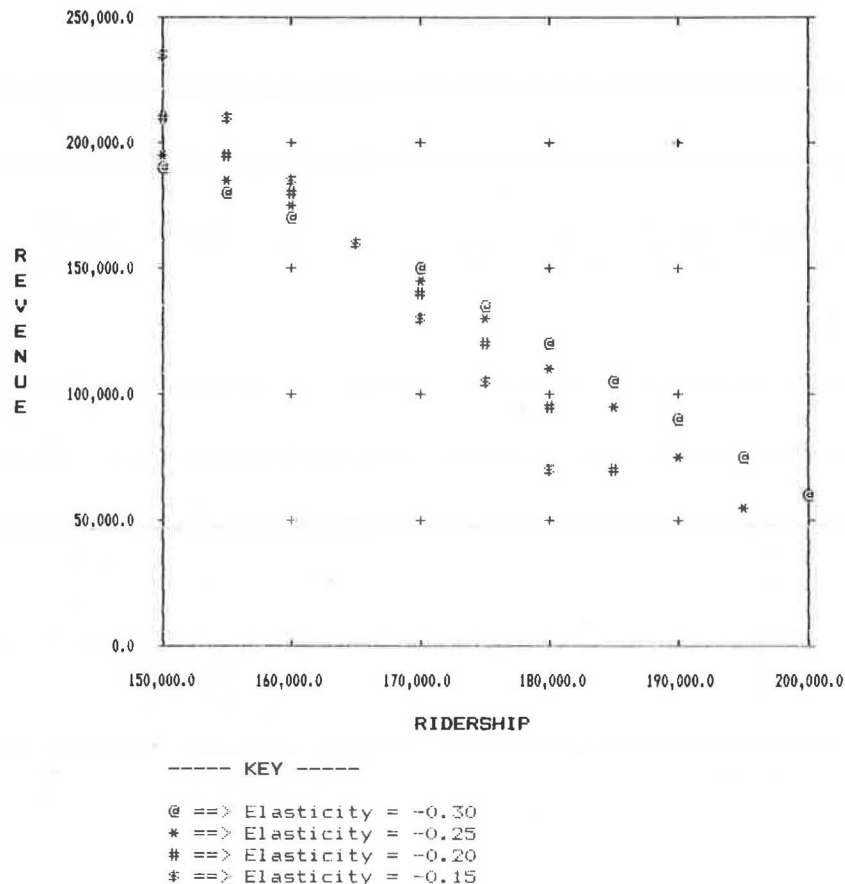


FIGURE 2 Ridership versus revenue by elasticity—all day.

more inelastic, a larger revenue change (reflecting a larger fare change) is necessary to produce a specific ridership change.

As the ridership changes in Figure 2, the coefficients in the fare equation change. In general, the transfer pass charge remains at its maximum allowable value (\$0.10) and a nominal mileage charge of less than \$0.01/mi is levied in all cases. To allow ridership to increase, the optimal fixed charge decreases until the smallest fare charged falls below the minimum allowable fare of \$0.25. At this point, the mileage charge is eliminated and the transfer charge must be decreased to allow further ridership increases. Similar experiments were conducted using the peak period and off-peak ridership data (23); the results showed identical trends.

The sensitivity of the fare structure and the total revenue across different market segments was also explored using appropriate elasticity ranges. The four market segments tested were: (a) all work trips, (b) peak work trips, (c) all nonwork trips, and (d) peak nonwork trips. For each of the work-trip segments, elasticities of -0.2, -0.15, and -0.10 were used. For nonwork trips, elasticities of -0.5, -0.4, and -0.3 were tested. In all cases, the minimum ridership was set at the base case ridership for that market segment.

In all cases, the fixed charge was between \$0.79 and \$0.90. The transfer pass charge always equaled the maximum allowable value of \$0.10. Because of the nominal mileage charges, the fare structure appeared to be nearly identical to the base case fare policy. Finally, the total revenue for any particular ridership matrix was nearly independent of the elasticity used. This is a result of using the base case ridership as the minimum allowable ridership. Figure 2 shows a similar result; the total revenue is nearly identical for all elasticity values when the ridership is fixed at the base case value.

None of the fare structures identified in the analyses outlined in this section exhibited large mileage charges. This is a direct result of (a) the absence of a distance component in the current CTA fare structure and (b) the similarity between the objective function used in FWFARE and that used in regression as discussed earlier. The model will try to find the fare structure that provides the closest fit to the existing fare structure. In this case, this will involve the use of transfer passes in preference to distance charges. From a policy perspective, the absence of a strong distance component in the fare structures identified by FWFARE reflects the fact that revenue maximization--the objective used by FWFARE--may not be consistent with the social and economic objectives that argue for distance-based fares. As a result, there is no cause for alarm by the absence of significant distance charges in the examples discussed previously.

To illustrate the decision tree approach, the model was used to identify a distance-based fare structure. The data matrices used in this analysis were: the network distance matrix; the base case fare charges of \$0.90 and \$1.00 for nontransfer and transfer trips, respectively; the matrix of the number of transfers between each O-D pair (as opposed to the transfer pass matrix used previously); and the all-day, all-purpose ridership matrix. Throughout this analysis an elasticity of -0.25 was used. A minimum allowable ridership equal to the base case total ridership of 165,293 was used initially.

To obtain a distance-based fare policy, the mileage charge was constrained to equal \$0.10/mi by setting the lower and upper bounds on the mileage charge to this value. Because the maximum distance in the base case data was 22.5 mi, the smallest feasible maximum fare would be \$2.25 with a \$0.10/mi mileage

charge. In this example, a range in fares was allowed from \$0.25 to \$4.50 per trip. In addition, transfer charges up to \$0.10 per transfer were permitted, and a maximum fixed charge of \$0.25 was tried initially. Finally, the constraints on the ratio of, and difference between, the transfer and fixed charges were set so that they would not be binding. The results are shown in Figure 3. The upper bounds on all three components of the fare equation are binding. The minimum ridership constraint is not binding, because the fare equation results in an estimated ridership of 1,701 passengers in excess of the minimum allowable (base case) value. Revenue declines approximately 7.3 percent when compared with the base case value. Notice that the range in fares has changed dramatically from \$0.10 (\$0.90 to \$1.00) to \$2.40 (\$0.30 to \$2.70).

To allow the ridership to decrease to its base case level, the analyst might next change the constraints on the fare equation coefficients as follows:

1. Reduce the minimum allowable mileage charge to \$0.075,
2. Increase the maximum allowable transfer charge to \$0.15, and
3. Increase the maximum allowable fixed charge to \$0.50.

These changes allow the optimization program greater freedom in identifying a desirable fare policy. In addition, fares are likely to increase sufficiently to reduce the total ridership to the base case value. Indeed, this is exactly what happens: only the maximum fixed charge, the minimum ridership, and the minimum allowable mileage charge constraints are binding. In the unconstrained cases, the optimal fare policy consists of only a nominal mileage charge and rather large fixed charges. Thus, the model attempts to reduce the mileage charge as much as possible and to increase the fixed charge to the greatest possible extent.

To illustrate the use of the model for analyzing specific policies, again, the CTA sample data are used. One recent proposal for a revised fare structure on the CTA called for the elimination of transfer charges; riders would need to pay a new fare each time they board another service. The proposed fare was \$0.50 for bus trips and \$0.75 for rail trips. The model system may be used to analyze this policy as well as related policies.

In the data set coded for this work, all riders use at least one rail line. Thus, under the proposed fare structure, all riders will pay a fixed charge of (at least) \$0.75, so an approximation of the proposed fare structure was begun by constraining the fixed charge to \$0.75. In addition, some riders also use feeder buses to go to and from the rail line. The additional bus fares paid by these passengers were emulated by constraining the transfer charge to \$0.50. The number of transfers table (as opposed to the zero/one transfer pass table) was used. The proposed policy does not call for distance-based charges, and so the mileage charge is constrained to \$0.00. Finally, because it is uncertain if ridership will increase or decrease under this policy, the minimum allowable ridership must be reduced to ensure a feasible solution. The ridership that results from the proposed policy will be a model output. All other model inputs, including the elasticity of demand with respect to fare, were kept at their default values.

Figure 4 shows the results of this analysis. The model suggests that for the sampled O-D pairs the ridership will decrease about 4.3 percent and the revenue will increase 7.6 percent under the proposed

Maximize REVENUE

Subject to:

			RIDERSHIP	>=	165,293.000	nb
nb	0.250	<=	FARE	<=	4.500	nb
b	0.100	<=	MILEAGE CHARGE	<=	0.100	b
nb	0.000	<=	TRANSFER CHARGE	<=	0.100	b
nb	0.000	<=	FIXED CHARGE	<=	0.250	b
nb	0.000	<=	TRANSFER / FIXED	<=	1.000	nb
nb	0.000	<=	FIXED - TRANSFER	<=	5.000	nb

b ==> BINDING CONSTRAINT nb ==> NON-BINDING CONSTRAINT

VARIABLE	NEW VALUE	BASE VALUE	DIFFERENCE	PERCENT
Ridership	166,994	165,293	1,701	1.029
Revenue	\$147,244	\$158,813	-\$11,570	-7.285
Low Fare	\$0.300	\$0.900	-\$0.600	-66.667
High Fare	\$2.700	\$1.000	\$1.700	170.000

Fare = 0.250 + 0.100 * DISTANCE + 0.100 * TRANSFERS

775 O-D Fares Increased... 477 O-D Fares Decreased... 0 Stayed the same

DISTANCE Matrix: DISTANCE.CTA TRANSFER Matrix: TRANSFER.CTA
 FARE Matrix: BASEFARE.CTA RIDERSHIP Matrix: ALLTIME.CTA

Elasticity = -0.250

FIGURE 3 Decision tree approach—initial distanced-based fare.

Maximize REVENUE

Subject to:

			RIDERSHIP	>=	100,000.000	nb
nb	0.250	<=	FARE	<=	3.500	nb
b	0.000	<=	MILEAGE CHARGE	<=	0.000	b
b	0.500	<=	TRANSFER CHARGE	<=	0.500	b
b	0.750	<=	FIXED CHARGE	<=	0.750	b
nb	0.000	<=	TRANSFER / FIXED	<=	1.000	nb
nb	0.000	<=	FIXED - TRANSFER	<=	5.000	nb

b ==> BINDING CONSTRAINT nb ==> NON-BINDING CONSTRAINT

VARIABLE	NEW VALUE	BASE VALUE	DIFFERENCE	PERCENT
Ridership	150,206	165,293	-7,087	-4.288
Revenue	\$170,870	\$158,813	\$12,057	7.592
Low Fare	\$0.750	\$0.900	-\$0.150	-16.667
High Fare	\$2.250	\$1.000	\$1.250	125.000

Fare = 0.750 + 0.000 * DISTANCE + 0.500 * TRANSFERS

908 O-D Fares Increased... 344 O-D Fares Decreased... 0 Stayed the same

DISTANCE Matrix: DISTANCE.CTA TRANSFER Matrix: TRANSFER.CTA
 FARE Matrix: BASEFARE.CTA RIDERSHIP Matrix: ALLTIME.CTA
 RESULTS Matrix: RESMSD.CTA

Elasticity = -0.300

FIGURE 4 Policy emulation analysis—initial results.

fare structure. However, these results must be interpreted with caution, not only because of the data limitations outlined previously, but also because the model as currently structured does not accurately reflect the proposed fare structure. Fares range from \$0.75 to \$2.25 in the model (Figure 4), but passengers using two rail lines as well as bus access and egress routes would pay \$2.50 under the proposed scheme--two \$0.75 rail fares (free transfers are not now permitted between these rail lines) and two \$0.50 bus fares. The model charges such passengers one rail fare of \$0.75 and three transfer (bus) fares of \$0.50. All passengers using both rail lines are modeled as paying \$0.25 less than the proposed fare structure might call for them to pay. Thus, the model is likely to underestimate both the revenue increase and the ridership decrease that would result from such a proposal.

To replicate the proposed fare structure more accurately, the transfer table was used to indicate the number of rail trips needed by an O-D pair in addition to the one rail needed by all riders in the sample. The distance table was used to provide the number of bus trips needed by passengers between each O-D pair. The fixed charge and the transfer charge--now used to capture the second rail trip made by some passengers--were both constrained to equal \$0.75. The distance charge, which now reflects bus use, was constrained to \$0.50. With these inputs, the model estimates a 7 percent decline in ridership and an 11.5 percent increase in revenue.

Finally, the model can be used to explore variations on the proposed policy using the last two constraints in the model formulation. For example, suppose we wish to identify the optimal rail fare, if the bus fare is held fixed at \$0.50 and ridership is to be retained at the current level. By using the constraint on the ratio of the transfer charge to the fixed charge, the two fees were constrained to equal each other, while the model was asked to determine the optimal value of the charge. The model suggests that the rail fare must be reduced to less than \$0.60 to maintain the current ridership with an elasticity of -0.3. At this point, the proposed fare structure results in slightly more than 2 percent reduction in revenue.

SUMMARY AND EVALUATION

Alternative price structures, including distance-based and zone fares, as well as time-of-day pricing, offer ways to enhance revenue generation while maintaining greater control of distributional consequences. Such fare structures may also permit increases in efficiency by linking user charges more closely to operator costs. The challenge is to find feasible ways to design, explore, and evaluate alternative fare structures. A microcomputer model system has been described that can support such fare policy studies.

The system is composed of seven programs that support the determination of an optimal fare policy subject to user-supplied constraints on fare characteristics and ridership. The core model maximizes total revenues over all O-D pairs. Fare is comprised of a fixed charge, a mileage charge, and a transfer charge, all internally determined by the model, and all subject to some degree of user control through the constraint specifications. Because it is structured around an optimization formulation, the model system provides strong support for the search for promising fare policies; in response to user-supplied requirements, it designs the best fare policy and provides a variety of evaluation measures. The model

system also permits the evaluation of specific, user-defined fare policies.

Because it has been developed for a microcomputer, the model system allows fast and easy user interaction in the search for desirable fare policies. This feature encourages users to test a variety of options in an efficient manner. Outputs from each run guide successive runs, so that a comprehensive and systematic search for promising fares may be carried out.

Even if users do not want to explore fares that are structurally different from current fares, this system supports rapid testing of proposed fares using an analysis process at least as sophisticated as that commonly used by transit properties. The speed of response, and the comprehensiveness of the evaluation measures, suggest that this model system is superior to traditional hand computation or mainframe computer methods. The model system makes it easy to explore and evaluate distance-based and zone fare policies. In addition, with a time-of-day data base, it supports the assessment of time-of-day pricing options if O-D data are available for time-based market segments.

The optimization process at the core of the model system can help the user determine the opportunity costs associated with unmodeled constraints. An understanding of these costs may lead to both better fare analyses and better fare decisions.

The requirement for a recent O-D ridership data base may appear to be a limitation of this model system. However, a reasonable analysis of fare policies cannot be conducted without such a data base, no matter what the approach. Of course, with an aggregate measure of system ridership, simple elasticity methods can be used to estimate revenue and ridership impacts of changes in flat fare schemes. Yet such approaches cannot provide information on distributional implications of fares, nor do they permit evaluation of alternatives to flat fare pricing.

The system utilizes a simple treatment of the travel demand function, approximated as a linear relationship. This, of course, is the same type of assumption that is now made in aggregate, elasticity-based fare policy analysis. It does not reflect the possibility that changes in fares may shift the spatial orientation of trips, nor does it evaluate the impacts on other modes of trips driven off transit. The former is likely to be a long-term effect, better treated through the use of a traditional travel forecasting process. The same is true of mode shifts, although the magnitude of transit ridership is such that this may be a minor issue.

The fare policy design model system presented in this paper represents an important step toward developing efficient, operational strategies for fare policy design and evaluation. The result, ultimately, should be a more powerful capacity on the part of transit managers to identify, evaluate, and implement creative and responsive pricing schemes.

ACKNOWLEDGMENT

This paper was funded by the Urban Mass Transportation Administration, U.S. Department of Transportation. The authors wish to acknowledge Dennis Ryan and Mary Kay Fitzgerald, Chicago Transit Authority Operations Planning Department, who provided the data used in the study.

REFERENCES

1. P.D. Mayworm, A.M. Lago, and J.M. McEnroe. Patronage Impacts of Changes in Transit Fares

- and Services. Report UMTA-MD-06-0054-81-1. UMTA, U.S. Department of Transportation, 1980.
2. D.P. Ballou and L. Mohan. A Decision Model for Evaluating Transit Pricing Policies. *Transportation Research A*, Vol. 15A, No. 2, 1981, pp. 125-138.
 3. R. Cervero. Efficiency and Equity Impacts of Current Transit Fare Policies. *In* *Transportation Research Record* 799, TRB, National Research Council, Washington, D.C., 1981, pp. 7-15.
 4. R. Cervero. Flat Versus Differentiated Transit Pricing: What's a Fair Fare? *Transportation*, Vol. 10, 1981b, pp. 211-232.
 5. R. Cervero, M. Wachs, R. Derlin, and R.J. Gephart. Efficiency and Equity Implications of Alternative Transit Fare Policies. Final Report. UMTA, U.S. Department of Transportation, 1980.
 6. D.T. Hartgen and D.L. Weiss. Differential Time-of-Day Transit Fare Policies: Revenue, Ridership and Equity. *In* *Transportation Research Record* 625, TRB, National Research Council, Washington, D.C., 1977, pp. 43-48.
 7. Wilbur Smith and Associates. Regional Fare Study Final Report, UMTA, U.S. Department of Transportation, 1979.
 8. D.L. Weiss and D.J. Hartgen. Revenue, Ridership, and Equity of Differential Time-of-Day Fares. Prelim. Research Report 99. Planning Research Unit, New York State Department of Transportation, Albany, 1979.
 9. T.J. Atherton and E.S. Eder. CBD Fare-Free Transit Service in Albany, New York. Report UMTA-NY-06-0064-81-1. UMTA, U.S. Department of Transportation, 1981.
 10. D.L. Connor. Off-Peak Fare-Free Transit: Mercer County, New Jersey. Report UMTA-MA-06-0049-80-3. UMTA, U.S. Department of Transportation, 1982.
 11. G.D. Fox. Tri-Met's Self Service Fare Collection Program. *In* *Transportation Research Record* 857, TRB, National Research Council, Washington, D.C. 1982, pp. 32-38.
 12. R.G.P. Tebb. Differential Peak/Off-Peak Bus Fares in Cumbria: Short Term Effects. *Transportation and Road Research Laboratory, TRRL Supplementary Report* 368, Crowthorne, Berkshire, England, 1978a.
 13. R.G.P. Tebb. Differential Peak/Off-Peak Bus Fares in Cumbria: Short Term Passenger Responses. *Transportation and Road Research Laboratory, TRRL Supplementary Report* 391, Crowthorne, Berkshire, England, 1978b.
 14. W.R. Ugolik and C.B. Leutze. Who Pays the Highest and the Lowest Per Kilometer Transit Fares. *In* *Transportation Research Record* 719, TRB, National Research Council, Washington, D.C., 1979, pp. 32-34.
 15. R. Cervero and M. Wachs. An Answer to the Transit Crisis: The Case for Distance-Based Fares. *Journal of Contemporary Studies*, Vol. V, No. 2, 1982, pp. 59-70.
 16. J. Pucher. Who Benefits from Transit Subsidies? Recent Evidence From Six Metropolitan Areas. *Transportation Research A*, Vol. 17A, No. 1, 1983, pp. 39-50.
 17. S.M. Rock and D.A. Zavatiero. Flat Fares, Transit Trip Distance and Income Redistribution. *Proc. 20th Annual Transportation Research Forum*, 1979, pp. 291-296.
 18. The Electronic Revolution and Farebox Management. *Metropolitan*, March-April 1983, pp. 20-34.
 19. J. Attanucci, D. Vozzolo, and I. Burns. Evaluation of the July 1980, SCRTD (Los Angeles) Fare Increase. Report UMTA-MA-06-0016-82-8. UMTA, U.S. Department of Transportation, 1982.
 20. D.P. Ballou and L. Mohan. Evaluation of Ridership, Revenue and Equity Implications of Distance-Based Fares for Transit Systems. Report UMTA-NY-11-0016-80-1. UMTA, U.S. Department of Transportation, 1979.
 21. Barton Aschman Associates, Inc. Traveller Response to Transportation System Changes. UMTA, U.S. Department of Transportation, 1981.
 22. M.A. Kemp. Some Evidence of Transit Demand Elasticities. *Transportation*, Vol. 2, 1973, pp. 27-38.
 23. M.S. Daskin, J.L. Schofer, and A.E. Haghani. An Optimization-Based Model for Designing and Evaluating Transit Fare Policies. Office of Technical Assistance, UMTA, U.S. Department of Transportation, 1984.

The authors are solely responsible for the contents of this paper.

Publication of this paper sponsored by Committee on Public Transportation Marketing and Fare Policy.