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Controlling Longitudinal Cracking in Concrete Pavements

CHHOTE L. SARAF and B. FRANK McCULLOUGH

ABSTRACT

The objective of the study reported in this paper was to investigate the development of longitudinal cracks in wide concrete pavements (two or more lanes in one direction) and to develop a model to estimate the depth of saw cut needed to control these cracks within the groove. The model developed uses the concepts of variability in the material properties of the concrete (tensile strength), pavement thickness (as constructed in the field), and depth of saw-cut groove. It was observed that estimates of longitudinal cracking have a reasonable match with field observations. It was observed that the longitudinal cracking of concrete pavements (two or more lanes in one direction) was dependent on the type of aggregate used in the concrete mix. Two types of aggregates were investigated. Uniformity of concrete mix strength (tensile) represented by standard deviation (tensile strength) affected the development of longitudinal cracks. A lower value of standard deviation obtained for concrete mix using lime rock aggregate in the mix was responsible for confining more cracks within the saw cut compared with the mix using river gravel aggregate. A sensitivity analysis of the model indicated that substantial reduction in saw-cut depth can be achieved if the variability of concrete strength during construction can be reduced.

Wide concrete pavements (two or more lanes in one direction) will develop longitudinal cracks due to shrinkage of concrete soon after it is poured. The repair of these cracks is difficult and expensive, especially when they are spalled. The presence of these cracks in pavement is unsightly. Therefore longitudinal joints at reasonable spacing (12 ft or one lane wide) are provided to encourage development of controlled cracks along these joints.

Longitudinal joints are generally formed by cutting a groove in the green concrete with a power saw. Adequate depth of saw cut must be provided to ensure that the longitudinal cracks will be confined within the groove. This provides an aesthetically acceptable regular longitudinal joint in the pavement at a low maintenance cost.

The performance of any saw-cut joint depends on its depth. An inadequate depth of saw cut may result in the development of longitudinal cracks away from the groove. These cracks eventually will spall and require expensive repair and maintenance.

The objective of this study was to investigate the development of longitudinal cracks in concrete pavements and to develop a model to estimate the depth of saw cut needed to control these cracks within the groove.

DEVELOPMENT OF LONGITUDINAL CRACKS ALONG THE SAW-CUT GROOVE

Let us assume that a wide concrete pavement is constructed with a saw cut, as shown in Figure 1. Fur-

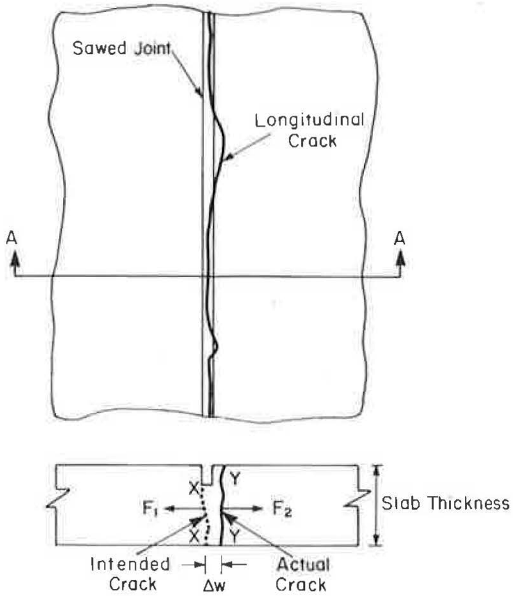

FIGURE 1 Concrete pavement with saw-cut joint and longitudinal crack.

ther, let us assume that, due to shrinkage of concrete, longitudinal cracks developed in the pavement as shown in Figure 1. This figure represents a typical longitudinal cracking pattern that was observed in a concrete pavement constructed in the Houston, Texas, area. The field observations of these cracks indicated that there were no cracks along the saw-cut joint where a longitudinal crack had occurred away from the saw cut (Figure 1).

Let us consider a cross section of this pavement along A-A (Figure 1). If the resultant tensile forces along x-x and y-y are, respectively, F_1 and F_2, then, just before cracking occurs along y-y,

$$F_1 \approx F_2 \quad \text{if the distance } \Delta W \text{ is small} \tag{1}$$

The tensile forces F_1 and F_2 are resisted by the tensile strengths of the pavement sections along x-x and y-y. The tensile strength can be estimated by

$$T_1 = D_1 \cdot b \cdot t_1 \tag{2}$$

$$T_2 = D_2 \cdot b \cdot t_2 \tag{3}$$

where

T_1, T_2 = tensile strengths of pavement sections along x-x and y-y, respectively;
D_1, D_2 = depths of pavement sections along x-x and y-y, respectively;
b = assumed width of pavement sections; and
t_1, t_2 = unit tensile strength of concrete along sections x-x and y-y, respectively.

If the pavement section cracks along y-y, then $F_2 > T_2$ and $T_1 > F_1$ (because the section did not crack along x-x), but $F_1 \approx F_2$ (Equation 1). Therefore it is evident that if the section along y-y cracked, the tensile strength of concrete along x-x (T_1) should be greater than the tensile strength of concrete along y-y (T_2). Or $T_1 > T_2$ or $T_1/T_2 > 1.0$. Similarly, if cracking occurs along x-x, then $T_1/T_2 \leq 1.0$.

If the ratio T_1/T_2 is represented by R, then to develop a longitudinal crack along the saw cut (x-x), the following condition must exist:

$$R \leqslant 1.0 \tag{4}$$

Using this condition for cracking along x-x, it is now possible to estimate the probability of cracking along this section. This probability can be represented by

$$P\,[R \leqslant 1.0] \tag{5}$$

where P[R] represents the probability of the variable R.

An estimate of the probability represented by Equation 5 provides an assessment of the amount of cracking along x-x (saw cut).

STATISTICAL MODEL TO ESTIMATE THE PROBABILITY OF R FOR ($R \leq 1.0$)

The probability of the variable R, P[R] can be estimated if the distribution of R can be established. To determine the distribution of R, the following procedure is followed.

Because R is the ratio of T_1/T_2 as indicated earlier (Equation 4), the following expressions can be written:

$$R = T_1/T_2 \tag{6}$$

$$= (D_1 \cdot b \cdot t_1)/(D_2 \cdot b \cdot t_2) \quad \text{(see Equations 2 and 3)}$$

$$= (D_1 \cdot t_1)/(D_2 \cdot t_2) \tag{7}$$

Taking the natural log (ln) of both sides of this equation gives

$$\ln R = \ln D_1 + \ln t_1 - \ln D_2 - \ln t_2 \tag{8}$$

Assume that D_1, D_2, t_1, and t_2 are independent random variables such that $\ln D_1$, $\ln D_2$, $\ln t_1$, and $\ln t_2$ are normally distributed. Then the variable ln R, which is a linear combination of four normally distributed variables (Equation 8), is also normally distributed with mean and standard deviations as indicated:

$$\text{Mean of } \ln R = \overline{RL} = \overline{DL1} + \overline{tL1} - \overline{DL2} - \overline{tL2} \tag{9}$$

$$\text{S.D. of } \ln R = \sigma_{RL} = (\sigma^2_{DL1} + \sigma^2_{tL1} + \sigma^2_{DL2} + \sigma^2_{tL2})^{1/2} \tag{10}$$

(assuming independence of $\ln D_1$, $\ln D_2$, $\ln t_1$, and $\ln t_2$)

where

$\overline{DL1}, \overline{DL2}, \overline{tL1}, \overline{tL2}$ = mean values of $\ln D_1$, $\ln D_2$, $\ln t_1$, and $\ln t_2$, respectively, and
$\sigma^2_{DL1}, \sigma^2_{DL2}, \sigma^2_{tL1}, \sigma^2_{tL2}$ = variances of $\ln D_1$, $\ln D_2$, $\ln t_1$, and $\ln t_2$, respectively.

Equations 9 and 10 fully describe the distribution parameters of ln R. Therefore the probability of R for ($R \leq 1.0$) can now be redefined as

$$P\,[R \leqq 1.0] = P\,[\ln R \leqslant 0.0] \tag{11}$$

This probability can be estimated with the help of the standard parameter Z [Z is N(0,1)], where the parameter Z is defined as

$$Z = (\ln R - \overline{RL})/\sigma_{RL}$$

or

$$Z = (0 - \overline{RL})/\sigma_{RL} \quad \text{(if the probability for } \ln R \leqslant 0 \text{ is estimated)} \tag{12}$$

A standard table of normal distribution (1) can be used to estimate the desired probability.

VERIFICATION OF THE MODEL

Data from two projects were obtained to verify the crack prediction model described. The observed cracking along the saw-cut joint was estimated by measuring the total length of cracks in the saw-cut joint and expressing it as a percentage of total saw-cut joint length on the project. Figure 2 shows the locations of these projects. Brief descriptions of these projects follow.

Project 1

Project 1 is located near Houston, Texas, on TX-288 (inside the I-610 loop), as shown in Figure 2. About 2 mi of continuously reinforced concrete pavement (CRCP) were installed with a saw-cut joint at the center of a 24-ft-wide pavement. River gravel was used in the concrete mix for this project.

About 69 percent of the saw-cut joint developed longitudinal cracks. The remaining 31 percent of the cracks were observed to have developed away from the groove.

Project 2

This project is also located near Houston, Texas, on TX-288 (outside the I-610 loop), as shown in Figure 2. About 20 mi of CRCP were installed with a saw-cut joint at the 1/4 point of a 48-ft-wide pavement (a construction joint was at the center). Limestone aggregate was used in the concrete mix for this project.

About 99 percent of the saw-cut joint developed longitudinal cracks. The remaining 1 percent of the cracks was observed to have developed away from the groove.

Pavement cores 4 in. in diameter were obtained from both projects to measure the variations in thicknesses and tensile strengths along the saw-cut joint as well as away from but within about 2 ft of the joint. A summary of the test results is given in Table 1. Using these data, the probability of cracking along the saw-cut joint is calculated as follows:

Estimate of longitudinal cracks along saw-cut joint: Project 1

$$\overline{RL} = 1.969 + 6.369 - 2.236 - 6.269$$

$$= -0.167 \quad \text{(see Equation 9)}$$

$$\sigma_{RL} = (0.0195^2 + 0.237^2 + 0.0414^2 + 0.296^2)^{1/2}$$

$$= 0.382 \quad \text{(see Equation 10)}$$

$$Z = [0 - (-0.167)]/0.382$$

$$= 0.44 \quad \text{(see Equation 12)}$$

Using standard tables of normal distribution (1),

$$P[\ln R \leq 0] = 67 \text{ percent}$$

Therefore the estimated probability of cracking

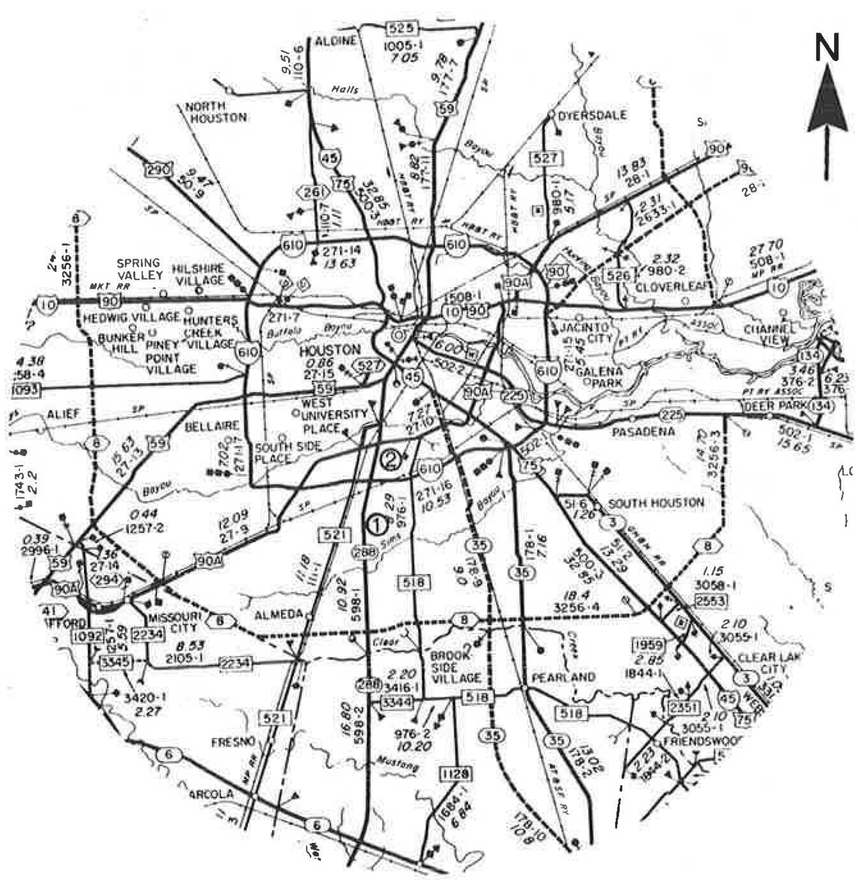

FIGURE 2 Locations of projects.

TABLE 1 Summary of Data Analysis

Project No.	Item	Original Measurements n	Mean	S.D.	Transformed Results Using Log_e Mean	S.D.
1	D_1	8	7.17	0.140	1.969	0.0195
	D_2	17	9.37	0.375	2.236	0.0414
	t_1	19	598	134	6.369	0.237
	t_2	34	549	147	6.269	0.296
2	D_1	6	6.39	0.195	1.854	0.0303
	D_2	13	9.19	0.200	2.218	0.0219
	t_1	12	495	90	6.188	0.188
	t_2	18	497	92	6.191	0.194

along a saw-cut joint is 67 percent. The observed cracking was 69 percent as described under Project 2. This indicates a reasonable match with the estimated value.

Project 2

$$\overline{RL} = 1.854 - 2.28 + 6.188 - 6.191$$
$$= -0.367$$
$$\sigma_{RL} = (0.0303^2 + 0.0219^2 + 0.188^2 + 0.194^2)^{1/2}$$
$$= 0.2727$$
$$Z = [0 - (-0.367)]/0.2727$$
$$= 1.346$$

Using standard tables of normal distribution, P $[\ln R \leq 0]$ = 91 percent.

Therefore the estimated probability of cracking along the saw-cut joint is 91 percent. The observed cracking was 99 percent (see Project 2 description). This indicates a reasonable match with the estimated values.

IMPLEMENTATION OF THE MODEL

The model can be used to estimate the amount of longitudinal cracking that is expected to develop along the saw-cut groove, as illustrated earlier. For this purpose, the following information is obtained:

1. Mean and standard deviation of pavement thickness along the saw-cut joint and away from it and
2. Mean and standard deviation of tensile strength along the saw-cut groove and away from it.

Equations 9, 10, and 12 can be used to estimate the longitudinal cracking along the saw-cut groove, as illustrated earlier for Projects 1 and 2.

The model can also be used to determine the depth of saw-cut that would induce a specified amount of longitudinal cracking along the groove. To illustrate this, two curves were developed, as shown in Figure 3. The data obtained for two projects (described earlier) were used in these curves. These curves show the effect of depth ratio on longitudinal cracking along the saw-cut groove.

Suppose, for example, that it is desired to have at least 90 percent of the cracks confined within the saw-cut groove; then the saw-cut depths can be determined in each of the following cases:

1. Concrete mix with river gravel and
2. Concrete mix with limestone aggregate.

Using Figure 3, if 90 percent longitudinal cracks is selected, then D_1/D_2 = 56 percent (river gravel mix) and D_1/D_2 = 71 percent (limestone mix) because saw-cut depth = $D_1 - D_2 = D_2(1 - D_1/D_2)$. Therefore desired saw-cut depths are

1. River gravel mix = 0.44 D_2 and
2. Limestone aggregate mix = 0.39 D_2.

If it is assumed that D_2 = 10 in., the saw-cut depths are 4.4 and 3.9 in., respectively, for the given mixes.

DISCUSSION OF RESULTS

The results of this study are summarized in Equation 12, which makes possible the estimation of the prob-

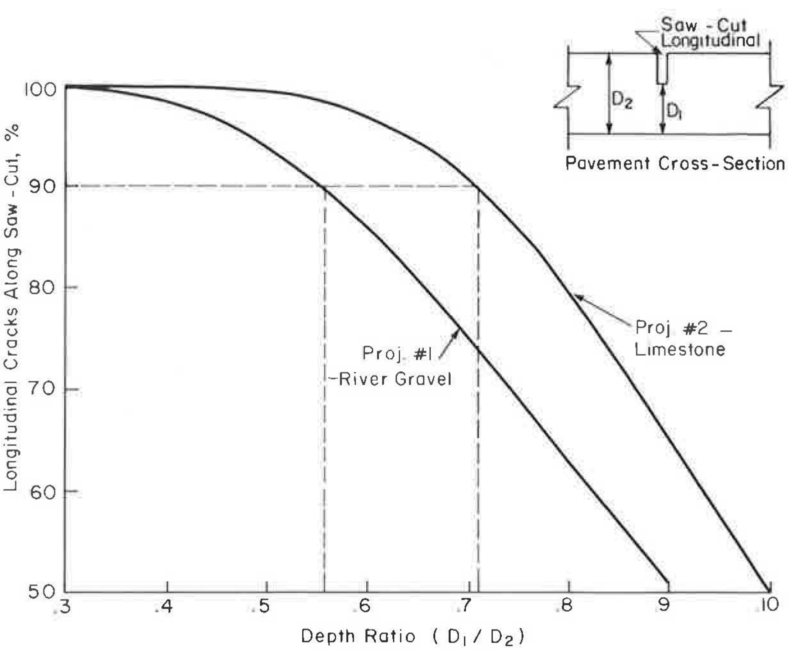

FIGURE 3 Effect of depth ratio on longitudinal cracking along saw cut.

ability of cracking along the saw-cut groove. As the value of Z increases, the probability of cracking increases (see standard table of normal distribution). Because Z is a ratio of $\overline{RL}$ and σ_{RL}, Equations 9 and 10 can be used to rewrite Equation 12 as follows:

$$Z = (\overline{DL2} + \overline{tL2} - \overline{DL1} - \overline{tL1}) / [(\sigma^2_{DL1} + \sigma^2_{tL1} + \sigma^2_{DL2} + \sigma^2_{tL2})^{1/2}] \qquad (13)$$

A study of Equation 13 indicates that the numerator will be largest when $\overline{DL1}$ and $\overline{tL1}$ are smallest. However, from a practical point of view, the tensile strength (tL1) alone cannot be reduced along the groove without changing tL2. Therefore the only parameter that can be controlled is $\overline{DL1}$. This means that the depth ratio (D_1/D_2) should be reduced or the saw-cut depth should be increased to increase the percentage of cracks along the groove. This is shown in Figure 3.

Alternatively, the denominator can be reduced to increase the value of Z. Because it is a combination of the variances of all four parameters (ln D_1, ln D_2, ln t_1, ln t_2), to reduce this quantity it will be necessary to obtain uniformity of thicknesses and concrete strengths (tensile) along the groove and away from the groove. If it is possible to achieve this, considerable reduction in saw-cut depth can be obtained, as shown in Figures 4 and 5. These figures were drawn for each project using the data given in Table 1 and assuming that the standard deviations are 1/2 and 1/4 of the values listed in the table.

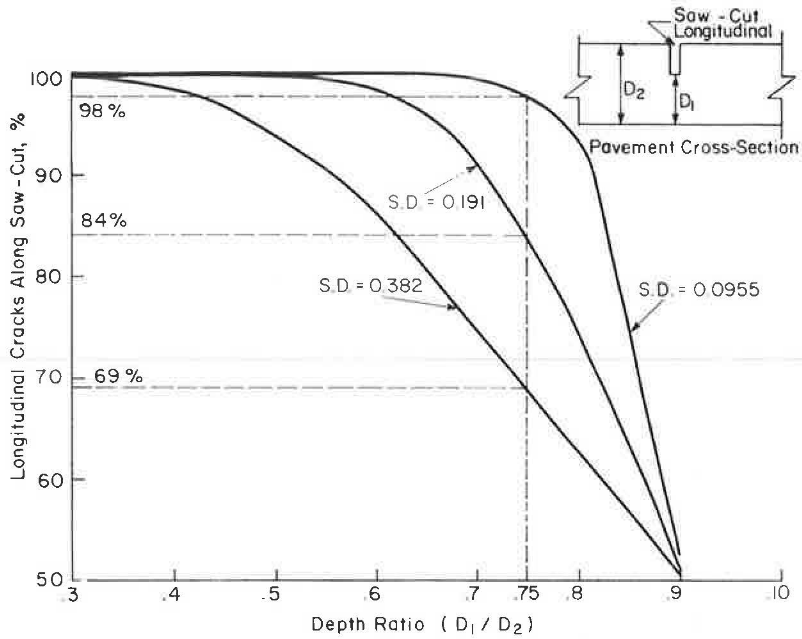

FIGURE 4 Effect of depth ratio on longitudinal cracking in river gravel mix (Project 1).

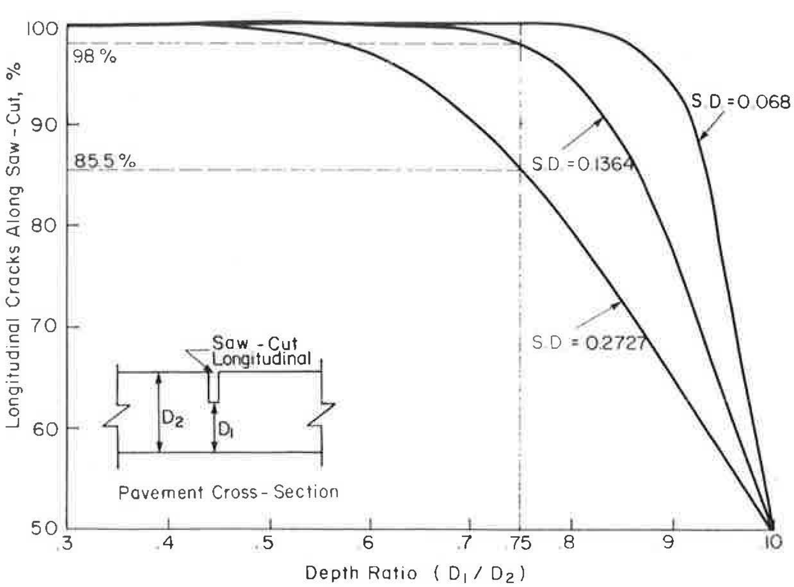

FIGURE 5 Effect of depth ratio on longitudinal cracking in limestone mix (Project 2).

It is clear from both of these figures that a substantial increase in longitudinal cracking can be obtained by reducing the standard deviation (see the examples in Figures 4 and 5 shown by a dotted line for a depth ratio = 0.75). This indicates that reasonable control of the quality of this construction material (concrete), pavement thickness, and saw-cut depth, all combined, can contribute to a reasonable saw-cut depth.

CONCLUSIONS

The results of this study can be summarized as follows:

1. The development of longitudinal cracks in a saw-cut groove can be explained by a model using the concepts of variability in concrete strength and thickness of pavement sections.
2. The model developed for this study is sensitive to the construction quality of pavement. An improvement in construction quality can result in reduction of saw-cut depths. The reliability of longitudinal cracks (being confined to saw-cut groove) is also improved. This can save construction costs as well as future maintenance and repair costs.
3. Figures 3-5 show that it is possible to induce any desired amount of longitudinal cracking along the saw-cut groove if an appropriate saw-cut depth is provided.
4. The aggregates used in concrete affect the development of longitudinal cracks along a saw-cut groove. This finding is based on a study of two aggregates (river gravel and limestone).

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Westergaard Solutions Reconsidered

A. M. IOANNIDES, M. R. THOMPSON, and E. J. BARENBERG

ABSTRACT

The pioneering analytical work of Harold Malcom Westergaard (1888-1950) has been at the heart of slab-on-grade pavement design since the 1920s. Every code of practice published since then makes reference to the "Westergaard solutions." These solutions are only available for three particular loading conditions (interior, edge, and corner) and assume a slab of infinite or semi-infinite dimensions. Since their first appearance, beginning in the early 1920s, Westergaard equations have often been misquoted or misapplied in subsequent publications. To remedy this situation, a reexamination of these solutions using the finite element method is described in this paper. A number of interesting results are presented: (a) Several equations ascribed to Westergaard in the literature are erroneous, usually as a result of a series of typographical errors or misapplications, or both. The correct form of these equations and their limitations have now been conclusively established. (b) Westergaard's original equation for edge stress is incorrect. The long-ignored equation given in his 1948 paper should be used instead. (c) Improved expressions for maximum corner loading responses have been developed. (d) Slab size requirements for the development of Westergaard responses have also been established.