Use of Dynamic Analysis in Predicting Field Multilayer Pavement Moduli

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ABSTRACT

The response of a multilayer pavement system to dynamic loading excitations is discussed and compared with the static response. Field Road Rater deflection data previously obtained at the Pennsylvania Transportation Research Facility are used to backcalculate pavement layer stiffnesses using the elastodynamic technique. Sets of layer moduli and fundamental frequencies are developed for five sections of the test track. The amount of error associated with the use of the static analysis commonly used is also evaluated. The dynamic analysis incorporates the inertial effect (radiation damping and resonance) of the pavement structure that cannot be included within static analyses. Simply replacing the Young's modulus in a static analysis by the resilient modulus or the dynamic modulus does not change it to a dynamic analysis. Unless the inertia of the system is considered in the interpretation of the dynamic response of pavements, misleading results may develop.

Surface deflection measurements of pavements have gained wide acceptance in the past few decades. Unlike laboratory testing, the deflection measurement technique is fast and relatively accurate and can be used to evaluate the structural condition of a pavement system with a minimum of disturbance and cost. In this technique a load is applied to the pavement and the surface deflection is measured. With the exception of Benkelman beam and California continuous deflectometer, most of the deflection measurement devices are dynamic in nature. Among such devices are the Dynaflect, the Road Rater, various vibrators, and the falling weight deflectometer. The first three devices impart steady-state (harmonic) loading with either constant or variable frequencies, and the latter device imparts impulsive (transient) loading. In the case of harmonic loadings, peak-to-peak deflections are measured at several distances from the load from which the envelope of the surface movement is determined.

Mechanistic analyses of the data obtained from dynamic loading devices have hitherto been based on elastostatic and viscoelastostatic models (1-6) in which, obviously, the inertia of the pavement plays no part. Several computer programs are currently used in analyzing the dynamic response of pavement. These programs are based on static analyses such as Chevron, VESYS, BISTRO, BISAR, and so forth. Thus it is tacitly assumed that the dynamic response of pavement structures is similar (if not identical) to the static response.

The stress-strain relations of isotropic elastic materials are expressed in terms of moduli (Young's modulus, shear modulus, etc.). Stress-strain moduli such as the resilient modulus and the dynamic modulus are sometimes used to interpret the inelastic and time-dependent response of materials. The resilient modulus represents the stress-strain relationship after many load repetitions (i.e., current modulus of the material, which is normally different from the initial value). On the other hand, the dynamic modulus is a frequency-dependent parameter obtained from dynamic loading tests on a finite specimen.

The governing differential equations of elastodynamics include the inertial effect (radiation damping and resonance) of the pavement structure that cannot be incorporated within static analyses. Simply replacing Young's modulus in a static analysis by the resilient modulus or the dynamic modulus is insufficient to recover the elastodynamic equations.

Backcalculated material properties are sensitive to minor changes in surface deflections. Thus use of an erroneous static analysis in backcalculating the material properties from dynamic surface deflections may result in large error magnifications. Although nonlinear elastic models of pavement structures (3) likely offer some improvement over linear elastic models, more significant modeling errors may result from neglecting the inertial response of pavements.

The objective of this study is to use the elastodynamic analysis in backcalculating the stiffnesses of various layers of an actual pavement structure (Pennsylvania Transportation Research Facility). Field surface deflections, which were previously obtained using the Road Rater, are reanalyzed in the present study using dynamic analysis. The resonant frequencies of various pavement sections are determined. A comparison of static and dynamic analyses is presented.

CONCEPT OF DYNAMIC ANALYSIS

The governing equation for steady-state elastodynamics is the Helmholtz equation (7):

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} + \rho\omega^2 u_i = 0 \qquad (1)$$

where

- λ, μ = Lame's constants that are related to Young's modulus and Poisson's ratio,
- ρ = mass density,
- ω = circular frequency of excitation, and
- u_i = ith cartesian component of the displacement vector.

In Equation 1 cartesian indicial notation, in which the subscripts range from 1 to 3, is assumed; addition is implied over repeated subscripts; and a comma denotes differentiation with respect to the space variable (i.e., $u_{i,j} = \partial u_i/\partial x_j$). Also, the displacements are assumed to be time harmonic.

The usual assumptions of material linearity and isotropy are invoked. Soil and pavement layers are assumed to be unbounded laterally but are underlaid by a rigid bedrock or incompressible layer at a finite depth. Full interface bonding (no slip) conditions are assumed at the layer interfaces.

In addition to the usual elastic constants (Young's modulus and Poisson's ratio) and the mass density, a fourth constant (damping ratio) may be specified to characterize the material damping (internal energy dissipation) of each layer (8). In other words, the viscoelasticity of the pavement materials is considered through the use of the damping ratio. A typical value of 5 percent was assumed in this study (8). The damping of the materials can also be indirectly considered through the use of the complex modulus rather than the dynamic modulus without the viscous term.

It should be noted that the material damping is virtually negligible because by far the major component of energy dissipation in continua results from radiation (geometric) damping; that is, the dispersion of energy from the source of excitation to the far field. The radiation damping is implicitly incorporated in the elastodynamic solution.

The solution of Equation 1 for a point load in a half-space can be expressed in the form:

$$u_i(x, \omega) = G_{ij}(x, \xi, \omega) P_j(\xi, \omega) \quad (2)$$

where

u_i = the ith displacement component,
G_{ij} = Green's function (a mathematical solution used to reduce the order of integration if the boundary values are known),
P_j = the jth load component,
x = coordinates of field point,
ξ = coordinates of load point, and
ω = circular frequency of excitation.

Analytical integration of the point load solution yields the disk load solution. However, no closed-form solutions are available for excitation of layered systems. Therefore solutions must be obtained by numerical means.

Kausel and Peek (9) have recently proposed a numerical technique that renders the elastodynamic problem of layered systems tractable. The solution is based on the assumption that the displacement field is linear in the direction of layering between adjacent interfaces. Thus sufficiently thin layers must be specified to ensure the validity of this representation. In practice, artificial sublayers may be introduced to satisfy this requirement. More details about the dynamic behavior of materials are found elsewhere (7-11).

FIELD MEASUREMENTS

Field measurements were obtained in March 1976 at the Pennsylvania Transportation Research Facility by Anani (5). The facility is a 1-mi, single-lane, oval-shaped, full-scale experimental highway. The construction, instrumentation, and operation of the research facility are discussed elsewhere (12).

In the present study, five test sections were considered: 1c, 1d, 2, 7, and 9, which had similar surface temperature, moisture content, and cumulative equivalent axle loads (∑EALs) at time of testing. Each pavement section consisted of four layers: surface, base, subbase, and subgrade, with material properties as given in Table 1. The layer thicknesses of the five test sections are given in Table 2. It should be noted that not enough information is available about the thickness of the compressible subgrade layer under the subbase and above the incompressible layer or the bedrock. A thickness of 20 ft was assumed.

TABLE 1 Material Properties of Test Sections of Pennsylvania Transportation Research Facility *(12)*

Layer	Material Type	Density (lb/ft^3)
Surface	Bituminous concrete	145
Base	Bituminous concrete	141
Subbase	Crushed limestone	141
Subgrade	Predominantly A-7 soil	124

TABLE 2 Layer Thicknesses of Test Sections of Pennsylvania Transportation Research Facility *(12)*

	Thickness (in.)		
Section	Surface	Base	Subbase
1c	1.5	6	8
1d	1.5	6	6
2	2.5	6	8
7	1.5	8	8
9	2.5	4	8

Surface deflection measurements (5) were obtained using the Road Rater device, Model 400. The Road Rater had two loading plates (4 in. x 7 in. each) and four deflection measurement sensors (geophones) 1 ft apart as shown in Figure 1. The Road Rater was operated to provide a simple harmonic loading with a frequency of 25 Hz and a peak-to-peak contact pressure of 13 psi under each plate in addition to a static pressure of approximately 27 psi. The peak-to-peak deflections were measured at each of the four geophone locations. Two sets of deflection readings obtained under approximately the same conditions from each test section were analyzed in the present study.

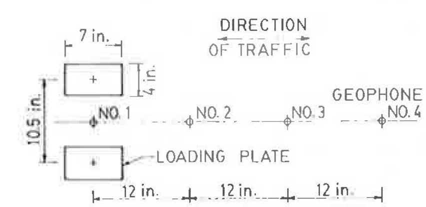


FIGURE 1 Schematic diagram of the Road Rater.

The surface temperature at the time of the measurements was 64°F and the moisture content ranged between 20.1 and 20.7 percent. The cumulative equivalent axle load experienced by the pavements before the measurements ranged between $1,296 \times 10^3$ and $1,336 \times 10^3$. The Road Rater deflection readings ($RR\delta_1$, $RR\delta_2$, $RR\delta_3$, and $RR\delta_4$) measured in an outward direction in addition to other associated data are given in Table 3.

BACKCALCULATION OF LAYER STIFFNESSES

The inverse problem of determining material properties from the response of the pavement structure to surface loading (from nondestructive deflection testing) is not easy to solve. No direct theoretical solution is available in the literature to determine the material properties of a multilayered system if the surface deflections and the layer thicknesses are known. Therefore it is necessary to employ iterative schemes based on the fact that surface deflections remote from the loaded area are primarily governed by the stiffness of the deeper layers. This has been indicated in several previous sensitivity analyses (5,6).

TABLE 3 Road Rater Deflection Measurements and Associated Data *(5)*

Section	Deflection Measurements ($\times 10^{-6}$in.) $RR\,\delta_1$	$RR\,\delta_2$	$RR\,\delta_3$	$RR\,\delta_4$	Surface Temperature (°F)	Moisture Content (%)	ΣEAL (x 1,000)
1c	645	457	285	167	64	20.1	1,296
	555	427	265	168	64	20.7	1,336
1d	801	543	311	167	64	20.1	1,296
	661	474	266	162	64	20.7	1,336
2	423	340	253	166	64	20.1	1,296
	446	355	247	182	64	20.7	1,336
7	410	333	247	177	64	20.1	1,296
	405	343	250	187	64	20.7	1,336
9	817	558	266	140	64	20.1	1,296
	795	517	245	138	64	20.7	1,336

Using the iteration technique, the number of unknown parameters must be less than or equal to the number of the measured surface deflections. Because there are only four geophones in the Road Rater used, the maximum number of unknown material properties that may be determined is four. If Poisson's ratios and material damping factors are assumed, the material stiffnesses of the four layers may be calculated. The procedure, however, can be easily adapted for pavement systems with more than four layers if more sensors are used in each Road Rater run.

An iterative process was used in this study to backcalculate the in situ layer moduli for the pavements at the Pennsylvania Research Transportation Facility. The iterative procedure used here is similar to the procedure followed by Kilareski et al. (6), except that the dynamic analysis is used. A computer program, DYNAMIC, was developed for this purpose (see flow chart in Figure 2). The program starts with input of the Road Rater data ($RR\delta_1$, $RR\delta_2$, $RR\delta_3$, and $RR\delta_4$). Initial layer moduli (E_1, E_2, E_3, and E_4) are assumed to represent the moduli of surface, base, subbase, and subgrade materials, respectively. Poisson's ratios of 0.35, 0.4, 0.4, and 0.45 are assigned to the four layer materials, respectively. Surface deflections consistent with the assumed material properties are computed using the procedure developed by Kausel and Peek (9). The loading platens of the Road Rater device are idealized by twin flexible circular plates (28 in.^{2} each) spaced 10.5 in. center to center.

The calculated deflections (δ_1, δ_2, δ_3, and δ_4) are compared with the Road Rater deflection measurements ($RR\delta_1$, $RR\delta_2$, $RR\delta_3$, and $RR\delta_4$). The differences between the calculated and the measured deflections are assumed to be entirely due to incorrectly assumed E-values. The correction starts with the outermost reading and the lowest layer (subgrade), with the assumption that the difference between δ_4 and $RR\delta_4$ is primarily due to an erroneous assumption for the subgrade modulus (E_4). A new value of E_4 is calculated for the next iteration as follows:

$$E_{i(new)} = E_{i(old)} \times [(RR\delta_i + \delta_i)/2]/RR\delta_i \qquad (3)$$

where

$RR\delta_i$ = measured deflection,
δ_i = calculated deflection, and
$i = 4$.

This method of correction adjusts only one-half of the discrepancy to assure a gradual convergence. This correction will reduce the value of E_4 if the calculated deflection is too small, which implies that the assumed value of E_4 is too large.

The correction of E_4 will influence all other calculated deflections, so the next interation produces a new set of δ-values. Using the newly computed values of δ, the subbase modulus (E_3) is adjusted using $RR\delta_3$ and δ_3 using Equation 3 with i = 3. A new set of deflections is then computed using the new E_3 value and previous E_1, E_2, and E_4 values. The value of E_2 is then adjusted followed by E_1 using similar procedures. Thus, after four calculations have been made, a new set of moduli has been generated. This interative process is followed until the differences between the calculated and the measured deflections for all geophones are within a predetermined tolerance (δ_{tol}) of 4 percent, which was found to provide reasonably accurate results. The percentage differences in deflection measurements [$\%\delta_{i(diff)}$] are calculated as follows:

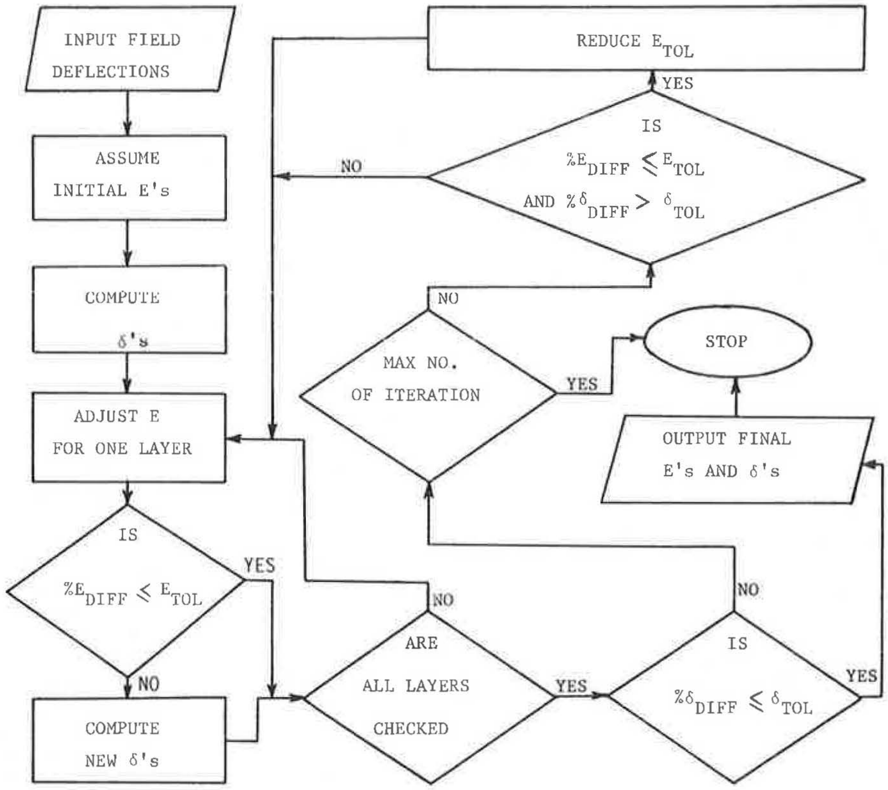
$$\%\delta_{i(diff)} = \mathrm{ABS}\,[(RR\delta_i - \delta_i)/RR\delta_i] \times 100\% \qquad (4)$$

where i takes values of 1, 2, 3, or 4, representing the deflection points under consideration. To reduce unnecessary computer calculations, some limitations were imposed as shown in Figure 2.

RESULTS AND ANALYSIS

Because the Road Rater deflection measurements were obtained at an operating frequency of 25 Hz in the field, that frequency was incorporated in the backcalculation process. Using the aforementioned procedure, a set of layer moduli is obtained for each section of the test road analyzed. In addition to the dynamic analysis, a static analysis (with a frequency of zero) is used to backcalculate the layer moduli of the same sections using the same field deflection measurements obtained at 25 Hz. The main reason for the static analysis was to estimate the amount of error made when using the static analysis in analyzing the dynamic response of pavements. The average moduli of various pavement sections obtained using static and dynamic analyses, as well as the amount of error, are given in Table 4.

As the data in Table 4 indicate, the static analysis resulted in higher moduli for the upper two layers than those obtained from the dynamic analysis with average errors of 9 and 11 percent for the two layers, respectively. On the other hand, the static analysis resulted in a smaller subgrade modulus with an average error of 12 percent. The ratios of modulus values using dynamic analysis to modulus values using static analysis for individual road sections and various layer materials are shown in Figure 3. A careful look at this figure would show that the mod-



Notes: E_{DIFF} = absolute difference between new and old E values

δ_{DIFF} = absolute difference between measured and calculated δ values.

E_{TOL} and δ_{TOL} = tolerances in E and δ values (%)

FIGURE 2 Flow diagram for the DYNAMIC computer program.

TABLE 4 Average Moduli (ksi) of Various Layers Using Static and Dynamic Analyses

	Analysis		
Layer	Static	Dynamic	Percentage Error
Surface	707	647	9
Base	845	762	11
Subbase	15	15	0
Subgrade	29	33	-12

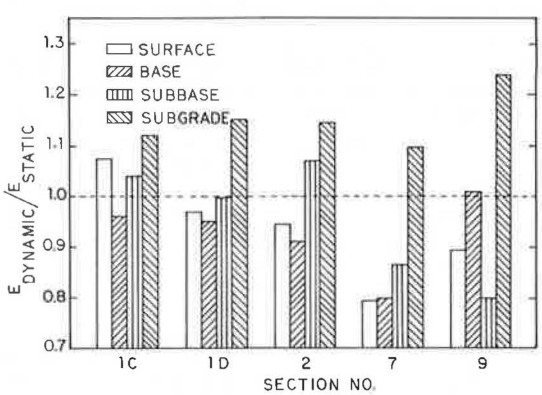


FIGURE 3 Ratios of modulus values obtained using dynamic and static analyses for various road sections.

uli of the top three layers were overestimated in some road sections and underestimated in other sections using static analysis, with no consistent trend. The modulus for the subgrade material, however, was always underestimated when static analysis was used. The largest amount of modulus overestimation due to the use of static analysis was obtained for the surface of Section 7 with an average error of 21 percent, and the largest amount of modulus underestimation was obtained for the subgrade of Section 9 with an average error of 24 percent. No general conclusion can be derived to evaluate the amount of error when static analysis is used because this error is a function of several factors including layer thicknesses, material stiffnesses, and the Road Rater operating frequency. Although the amount of error due to the use of static analysis appears to be relatively small in this study, a more serious situation might occur if the operating frequency of the deflection measurement device were close to the natural vibration frequency of the pavement system as discussed in subsequent paragraphs.

In the second step of the analysis the average layer moduli estimated from the two Road Rater measurements for each road section using the dynamic analysis were considered. Using these moduli, the dynamic responses of various road sections were evaluated at different Road Rater operating frequencies ranging from zero to 50 Hz. The ratios of dynamic surface deflection to static surface deflection for various road sections are shown in Figures 4-8, respectively. In each figure, four curves are shown

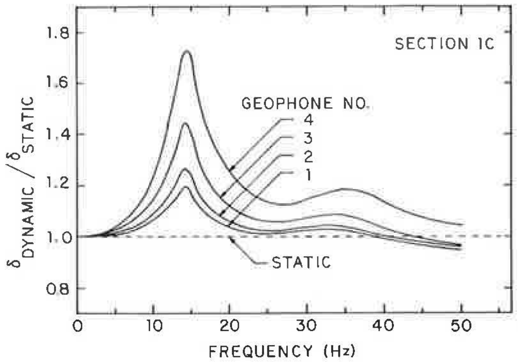


FIGURE 4 Dynamic-to-static deflection ratio for Section 1c at various frequencies and geophone locations.

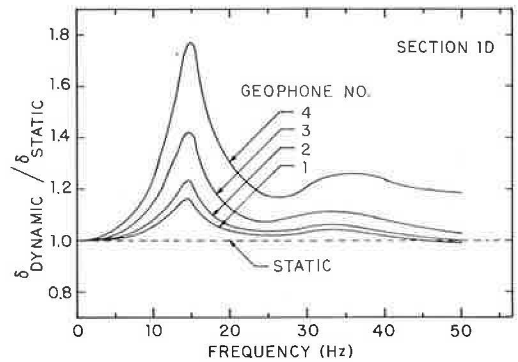


FIGURE 5 Dynamic-to-static deflection ratio for Section 1d at various frequencies and geophone locations.

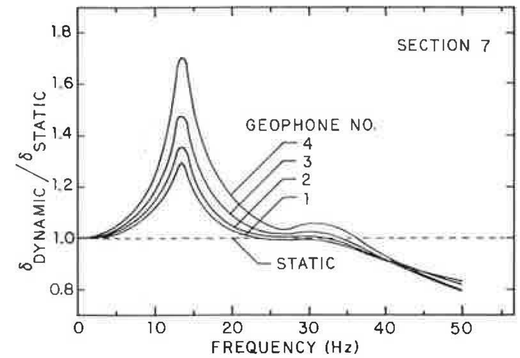


FIGURE 7 Dynamic-to-static deflection ratio for Section 7 at various frequencies and geophone locations.

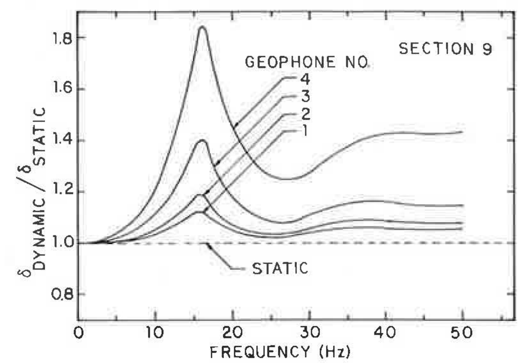


FIGURE 8 Dynamic-to-static deflection ratio for Section 9 at various frequencies and geophone locations.

for various geophones of the Road Rater. From these figures it can be concluded that the dynamic responses of the pavement system are more apparent when the distance from the load is increased. This is indicated by the gradual increase in the dynamic-to-static deflection ratio from geophone 1 through geophone 4 in all road sections. Therefore the amount of error made using static analysis in evaluating the dynamic response of pavement gets large when the deflection is measured at a large distance from the center of the load.

A more serious finding is that the natural vibration frequencies (fundamental frequencies) fall

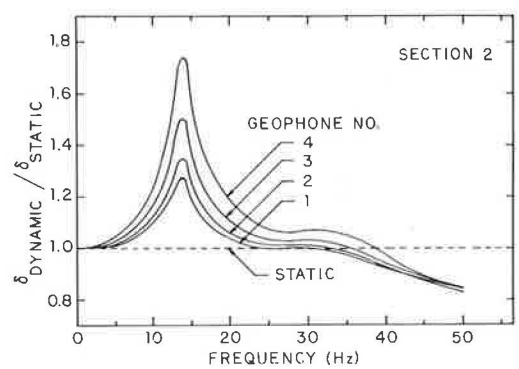


FIGURE 6 Dynamic-to-static deflection ratio for Section 2 at various frequencies and geophone locations.

within the common range of the operating frequency of the Road Rater. This is indicated by the large ratio of dynamic-to-static deflections (large magnification factors). The resonant response of the pavement system occurs when the frequency of the applied load is equal to a natural vibration frequency of the pavement system. Note that for any pavement system there is a series of natural vibration frequencies, namely first fundamental frequency, second fundamental frequency, and so forth. The first fundamental frequency can be defined as the lowest frequency at which the magnification factor reaches a local maximum. The subsequent fundamental frequencies can be obtained at frequencies equal to the first fundamental frequency multiplied by certain factors that are functions of shear moduli, densities, Poisson's ratios, and thicknesses of various layers (8). The first fundamental frequency is usually the most important one because the magnification factor is high at that frequency. In this study, the first fundamental frequencies of various road sections ranged between 13 and 16 Hz, and the second fundamental frequencies ranged between 30 and 42 Hz as given in Table 5. The fundamental frequencies of each road section are slightly changed at different geophone locations, especially the second fundamental frequency as indicated by the ranges given in Table 5.

If the Road Rater is operated at or close to any fundamental frequency of the pavement system, especially the first one, a resonant response will occur that might be detected by the unsteady geophone readings resulting from the large vibration amplitudes of the pavement surface. This resonant response

TABLE 5 Natural Frequencies (fundamental frequencies) of the Test Road Sections (Hz)

	Fundamental Frequency	
Section	First	Second
1c	14	34-35
1d	14	34-35
2	14	31-32
7	13	30-31
9	16	36-42

was reported by some researchers [e.g., Sharpe et al. (1)]. It should be noted that the actual field resonant frequencies of Pennsylvania Transportation Research Facility sections might be slightly different from those obtained in this study because of the assumption of a subgrade thickness of 20 ft above a rigid layer. Other factors that might affect the resonant response include temperature, moisture content, random variation in material properties, and existence of cracks in asphalt concrete layers as well as various experimental errors.

A large amount of error may occur when the deflection measurement device has only one operating frequency. The Dynaflect is such a device with a typical frequency of 8 Hz. If the natural frequency of the pavement system is equal or close to the operating frequency of the deflection measurement device, a resonant condition will occur. Unless dynamic analysis is used, misleading results may develop.

Typical surface deflections were examined to further compare static and dynamic pavement responses. Figure 9 shows surface deflection under static and dynamic loads for Section 9. For the static case (using the dynamic solution with zero frequency), a stress of 6.5 psi was used on each Road Rater plate; a stress of ±6.5 psi was used in the dynamic case as well as in the field study. It is noted that the difference between static and dynamic responses is not large for that road section.

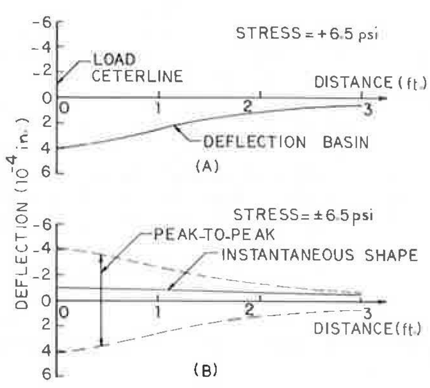


FIGURE 9 Surface deflections for Section 9 under (A) static load condition and (B) steady-state vibrating load condition at 25 Hz.

This happened because, accidentally, the magnification factor at 25 Hz is close to unity as shown in Figure 8. Larger amounts of error might occur due to the use of static analysis in other cases if the magnification factor were largely different from unity. This situation might occur at some combinations of layer thicknesses, material stiffnesses, and loading frequency.

Furthermore, if a static load is applied to the pavement system, the pavement response will be in phase with the load. However, if a vibrating (harmonic) load is superimposed on an initial static load, the instantaneous pavement response will be generally out of phase with the load because of both geometric and material dampings. In this case, the pavement surface at a point may be moving upward when the vibrating load is being increased. Indeed, the pavement surface takes a wave form propagating away from the load. When a Road Rater with two loading plates is used, the surface waves will be similar to waves produced on a smooth surface of water when two stones are simultaneously thrown into it. With the Road Rater geophones, only the peak-to-peak surface deflections are recorded and no information is obtained regarding the instantaneous pavement response or the out-of-phase condition. The dynamic response of the pavement can be represented by a complex number in which the real part represents the in-phase response and the imaginary part represents the 90-degree out-of-phase response. Resonance occurs when the response of the pavement system is 90 degrees out of phase with the applied load and, consequently, the applied load is exactly balanced by the damping force (10).

The phase angles at various geophone locations for Section 9 at 25 Hz are given in Table 6. This indicates that the instantaneous deflection lags the instantaneous load, and this lag is different from one location to another. The instantaneous shape of the pavement surface when the load is zero and increasing is shown in Figure 9. The instantaneous shape fluctuates between the peak-to-peak limits. It should be noted that the wave length of the instantaneous surface shape for Section 9 is about 11 ft, which is longer than the 3-ft distance considered in the present study. Thus no wave shape appears in Figure 9.

TABLE 6 Phase Angles at Various Geophone Locations for Test Section 9 at 25 Hz

Geophone	Phase Angle (degrees)
1	-14.4
2	-19.3
3	-33.3
4	-57.2

SUMMARY AND CONCLUSIONS

In this study, the concept of dynamic response of the pavement is discussed and compared with the static response. Field Road Rater data that were obtained at the Pennsylvania Transportation Research Facility during a previous study (5) are used in the present study to backcalculate the pavement layer moduli using elastodynamic analysis. Static analysis was also used and the associated error was evaluated. The resonant frequencies of the different sections of the test track are estimated. The following conclusions are obtained:

1. The dynamic response of a multilayer pavement system is materially different from its static response. Dynamic analysis incorporates the inertial effect (radiation damping and resonance) of the pavement structure, which cannot be incorporated within static analyses. Simply replacing Young's

modulus in a static analysis by the resilient modulus or the dynamic modulus is insufficient to recover the elastodynamic equations.

2. If the operating frequency of the deflection measurement device (e.g., 25 Hz for the Road Rater and 8 Hz for the Dynaflect) coincides with one of the fundamental frequencies of the pavement system, a resonant condition will occur and a large magnification of the deflection measurements will result. Unless dynamic analysis is used in the interpretation of the dynamic response of the pavement system, misleading results may develop. Although the amount of error resulting from using the static analysis in this study did not exceed 24 percent, larger amounts of error may result in other cases with different operating frequencies of the deflection measurement device, different layer thickness, or different material properties (which might occur at different temperatures or moisture contents for the same materials). No simple relation between static and dynamic pavement responses exists.

3. No "direct" mechanistic solution is currently available to backcalculate material properties from surface deflection data obtained by either static or dynamic loading. Iterative processes are usually used for this purpose.

4. The number of deflection measurements for one run of the deflection measurement device should at least equal the number of unknown material properties. Because the material properties obtained are sensitive to any error in the deflection measurements, a larger number of deflection data points may provide more accurate results.

5. Further research is needed to study the transient loading of the pavement systems obtained from the path of vehicles or from the use of the falling weight deflectometer.

ACKNOWLEDGMENTS

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Discussion

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Formulation of a dynamic response analysis for interpretation of dynamic deflection basins is a significant contribution to the state of the art. In addition to the author's paper, two more papers on this subject were presented during the 64th Annual Meeting of the Transportation Research Board (1,2). The author's elastodynamic analysis has been used only on Road Rater Model 400 deflection basins. The basic constitutive law in this analysis is still linear elasticity. Pavement materials and subgrade in the real world do not exhibit linear elastic behavior. Therefore caution should be exercised in applying the findings from the Road Rater study to other vibratory devices. The dynamic response of a pavement system is device dependent in addition to its known dependency on frequency and loading mode effects. Several aspects of the study need further elaboration:

1. The author's static analysis is based on the formulation of his dynamic analysis at zero frequency. How good is the static analysis? It can be judged only if the author provides a comparison with the well-established layered elastic theory. A comparison with responses predicted by ELSYM5 or BISAR programs should be included. These programs have been validated in several studies by comparison with measured responses (3).

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2. The paper lacks any documentation of the validity of the proposed dynamic analysis. An appropriate way to validate the author's procedure is to check the backcalculated moduli with some independent measurements of in situ dynamic moduli such as the SASW method (4).

3. A companion paper by the author (2, Figure 6) shows static deflection to be larger than dynamic deflection for a similar Road Rater study. This is contrary to the findings in this paper. Moreover, material damping can be quite significant for subgrade soils (5), which the author has totally ignored.

4. The selection of subgrade thickness appears to be arbitrary. A subgrade of 20 ft in thickness is assumed in this paper and 12.5 ft was assumed elsewhere (2). Both dynamic and static responses are significantly influenced if a rigid bottom is assumed at a shallow depth. The resonance condition discussed by the author will probably be insignificant if a deeper subgrade is assumed.

5. Several aspects of the backcalculation procedures need further clarification: (a) The procedure is user dependent because "guess" moduli are required as input. How is the uniqueness of derived moduli ensured? (b) What typical values of tolerance in E were used? (c) What is the validity of the procedure for a pavement with known properties?

6. The peak-to-peak force generated by this model of Road Rater is smaller than the 1,000-lb force of a standard Dynaflect. Therefore shear strains in granular subbase-base and subgrade for Road Rater loading will be of low amplitude and the backcalculated moduli of these materials are maximum dynamic moduli. Determination of effective moduli corresponding to standard design load conditions will require a procedure such as an equivalent linear analysis (6). This method is based on a strain sensitivity approach used in earthquake engineering.

7. It has been emphasized in this paper and elsewhere (2) that any NDT device operating at a frequency close to the fundamental frequency of pavement will result in large magnification of surface deflections. How were the natural frequencies of the test pavements established? It is observed from the data in these two papers and related publications that the first natural frequency of pavements is generally above 10 Hz. The discussions in this paper and elsewhere (2) imply that the Dynaflect is inferior because it operates at a fixed frequency of 8 Hz. A device operating at a lower frequency (e.g., Dynaflect) should not be susceptible to resonance condition.

8. It turns out that the low excitation frequency is a merit and provides a rational justification for using static analysis to interpret the Dynaflect deflection basins. Uddin (7) has shown that, for all practical purposes, a static analysis of Dynaflect deflection basins using layered theory is a reasonable approach because the peak-to-peak harmonic force of a Dynaflect can be considered as an equal pseudostatic force. This is further confirmed by the results of an earlier study of the Texas Transportation Institute (8). The TTI study showed that the Dynaflect deflections measured at the surface are independent of the frequencies in the range of 6 to 10 Hz.

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Author's Closure

The author would like to thank the discusser for his instructive comments on the paper. It should be noted that in this study, the term "deflection basin" was not used because, when a harmonic load is applied to pavements, the pavement surface takes on a wave form. Currently, only the peak-to-peak deflections are measured and no information is obtained regarding the instantaneous basin deflections. Replies to the discusser's comments follow in order:

1. Although not reported in the paper, pavement Section 9 of the Pennsylvania Transportation Research Facility was analyzed using the computer program with zero frequency and using the Chevron computer program (with a very stiff semi-infinite layer underlying the subgrade). The deflections of the two programs did not differ by more than 5 percent. This is to be expected because the Helmholtz equation reduces to Navier's equation when the frequency is reduced to zero. The latter equation is, of course, the governing differential equation of elastostatics.

2. The validity of the dynamic analysis has been established by Kausel and Peek (1). Verification of the applicability of the analysis to pavement systems by the methods proposed by the discusser would be welcome. However, it should be pointed out that whereas dynamic analysis can easily reproduce real loading conditions, analyses and testing of a seismic nature will necessarily involve extrapolation to field conditions (as noted by the discusser, comment 6).

3. The dynamic deflection can be either smaller or larger than the corresponding static deflection depending on several factors such as the operating frequency of the loading device, material properties, layer thicknesses, number of layers, and depth to bedrock. Note that the author has not "totally ig-

nored" material damping. A material damping ratio of 5 percent was used (see discussion between Equations 1 and 2). Further, radiation damping is believed to be more significant than material damping during dynamic vertical translation.

4. That the pavement response is significantly influenced by the depth to bedrock has been discussed elsewhere (2,3).

5. In the backcalculation procedure, the initial modulus values affect the number of iterations required to reach a certain accuracy. In both static and dynamic analyses, there is no guarantee that the solutions are unique although the range of admissible solutions may be relatively narrow. A 3 percent tolerance in E was used in this study. Within the constraints of the assumed model (linearity), the solutions are thought to have greater validity than do those of models that ignore the dynamic effect.

6. The author agrees with the discusser that test loading conditions should correspond as closely as possible with field conditions and that some appropriate method should be used to extrapolate test data in cases in which field conditions are not replicated. However, this latter procedure is difficult to implement because many factors influence the response of a pavement system (as distinct from a single soil sample tested in the laboratory) including, for example, cyclic loading, pavement layer thicknesses and stiffnesses, and subgrade material properties. Thus the best procedure appears to be to test pavement systems under design loads and analyze these data directly. The author's analysis is not restricted to low loading levels.

7. The natural frequencies of pavements are functions of material properties: layer thicknesses, number of layers, and depth to bedrock. The first natural frequency of typical pavement sections can be below 10 Hz as demonstrated by Hoffman and Thompson (4). For example, field tests (4) showed that the first natural frequency of "Sherrard" section with 4 in. of asphalt concrete surface and 14 in. of crushed stone base and an AASHTO A-4(6) subgrade is between 8 and 10 Hz, whereas "Viola" section (Stations 13 and 18) with a 9-in. bituminous aggregate mixture surface and a 6-in. AASHTO A-6(9) subgrade has a first natural frequency of 8 Hz. These examples show clearly that the Dynaflect may result in resonating the pavement system, and unless dynamic analysis is used, misleading results may develop. The natural frequencies of the pavement sections in the current study were determined by running the computer program for various frequencies. By definition, the frequencies that resulted in relatively large deflections are the resonant frequencies.

8. As discussed before, resonance may occur at low excitation frequencies under certain conditions. That these conditions were not encountered by Uddin (5) and Cogill (6) does not mean that they do not occur in practice [see Hoffman and Thompson (4)].

In conclusion, true dynamic analysis has a most useful role to play in pavement evaluation. It is the only means whereby the resonant condition can be predicted. Further, it shows clearly the range of validity of purely static analyses and thus serves to warn the user of such analyses of the conditions under which error may result. Of course, it is not proposed that the present analysis can cope with all the complexities of pavement response, but, in regard to one important factor (pavement inertia under dynamic loading), it is, the author hopes, a step forward.

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