Developing Stochastic Flexible Pavement Distress and Serviceability Equations

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ABSTRACT

The development of a method of predicting pavement performance in terms of present serviceability index and four primary distress types (area and severity) and the application of the method to the design of flexible pavements are summarized. The method is based on an S-shaped performance curve, the curve fit parameters for which can be determined using the methodology developed by Garcia-Diaz and Riggins. These parameters have been found for 164 pavement test sections located throughout Texas. The pavement test sections were categorized as three main types: asphalt concrete pavement on unbound base course, asphaltic concrete pavement on bituminous base course, and asphaltic concrete overlay. Data describing the pavement structure, including the thickness and elastic modulus of each layer, the environment, and the traffic for these pavements, were used to develop regression models for the curve fit parameters. A sensitivity analysis was made of these models to determine the effects of climate on pavement performance in four widely separated highway districts in Texas. The regression models along with the proposed performance equations and the stochastic form of these equations have been incorporated in the Texas Flexible Pavement System (FPS) design computer program. The modified version of FPS provides a listing of the optimal pavement designs selected on the basis of least total cost including material and user costs, overlay costs, and salvage values.

Recent developments and actual applications of pavement performance equations that predict the loss of serviceability index and the deterioration of flexible pavements due to various types of distress are summarized. The performance model is an S-shaped curve that recognizes that the rate of deterioration of a pavement changes during its service life and that this deterioration process obeys boundary conditions at the beginning and end of its life. The methodology used to evaluate the curve fit parameters was developed by Garcia-Diaz and Riggins (1) and has been applied to field measurements of pavement performance obtained from the Texas flexible pavement data base, which is maintained at the Texas Transportation Institute. Pavement performance is evaluated in terms of the present serviceability index (PSI) and of the following types of distress (both area and severity):

- Rutting,
- Alligator cracking,
- Longitudinal cracking, and
- Transverse cracking.

The first two types of distress are regarded as load related and the latter two as non-load related. The primary load-related independent variable is the number of 18-kip equivalent single axle loads (ESALs) and the primary non-load-related independent variable is the number of months since construction or major rehabilitation.

The proposed performance models are readily adaptable to computer applications and have been incorporated in the Texas Flexible Pavement System (FPS) design program. Examples of calculated results are given.

In the first part of this paper is presented background information pertaining to performance equations and the characteristics of the S-shaped curve. The second section deals with the regression models that are used to determine the curve fit parameters. In the third section the methodology for predicting the number of 18-kip ESALs or the time required to reach a specified level of serviceability or distress is described. The final section deals with the application of the method to predicting the performance of Texas pavements, its incorporation into the FPS computer program, and its use in flexible pavement design in Texas.

FORMS OF THE PERFORMANCE EQUATION

The performance of flexible pavements is evaluated in terms of functional and structural performance. Functional performance is a measure of the riding quality of the pavement and can be quantified in terms of the present serviceability index (PSI). Structural performance is a measure of pavement deterioration as determined by the appearance of various forms of distress (e.g., rutting, cracking, patching). Distress types can be quantified in terms of the affected area or the degree of severity of the distress. The two types of performance are related in that the same variables (i.e., pavement structure, environment, and traffic) affect the overall performance and the equation used to predict performance can be of the same form for both functional and structural performance.

AASHO Functional Performance Curve

The form of the functional performance equation as developed from the AASHO Road Test is as follows:

$$g = (N/\rho)^{\beta} \quad (1)$$

where

- g = damage function ranging from 0 to 1;
- N = total number of applied loads (i.e., the accumulated 18-kip ESALs or total elapsed time);
- ρ = a curve fit parameter that represents the applied load when g reaches a value of 1 and thus gives the "scale" of the damage function (g); and
- β = a curve fit parameter that defines the degree of curvature of the damage function or the rate at which damage increases.

The damage function for functional performance is defined as the number g, given by the serviceability index ratio given in Equation 2.

$$g = [(P_o - P)/(P_o - P_t)] \qquad (2)$$

where

- P = present serviceability index (i.e., serviceability index at a specific value of the number of applied loads);
- P_o = initial serviceability index; a value of 4.2 to 4.5 is typical for a well-constructed pavement; and
- P_t = terminal serviceability index (i.e., the value of the serviceability index at which major repairs or rehabilitation must be done); a typical number is from 2.5 to 3.0.

The damage function for structural performance is the same form of equation except that the damage (g) is expressed as a ratio of distressed area to a maximum acceptable level of distressed area, or a ratio of distress severity levels. Typical shapes of the functional and structural performance curves obtained from Equation 1 are shown in Figure 1.

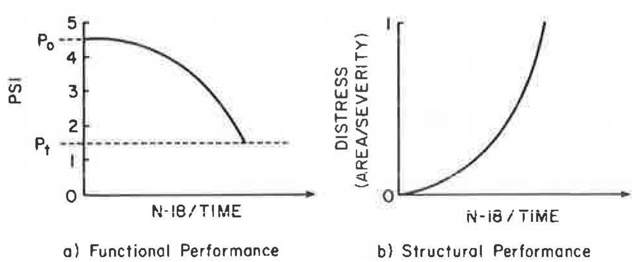

FIGURE 1 Illustration of typical AASHO Road Test functional and structural performance curves.

S-Shaped Performance Curve

There is a difficulty with using Equation 1 because it does not obey both of the following boundary conditions:

- The functional (structural) performance curve must have a maximum (minimum) value, at the traffic level or time equal to zero, and must be strictly decreasing (increasing) as the traffic level or time increases.
- The performance curve cannot predict negative values of serviceability index nor can it predict a distressed area greater than 100 percent of the total area for large values of traffic level or time.

Although the AASHO equation satisfies the first condition, it is found to be deficient in the second condition. To overcome this limitation, Texas Transportation Institute researchers (2,3) have adopted the S-shaped performance curve. The equation for pavement damage (g) is

$$g = e^{-(\rho/N)^{\beta}} \qquad (3)$$

In the case of serviceability index, the definition of damage is

$$g = [(P_o - P)/(P_o - P_f)] \qquad (4)$$

where P_f is the asymptotic value of serviceability index.

The functional and structural performance curves obtained from this equation are shown in Figure 2. It is observed from this figure that both of the boundary conditions are satisfied by Equation 3.

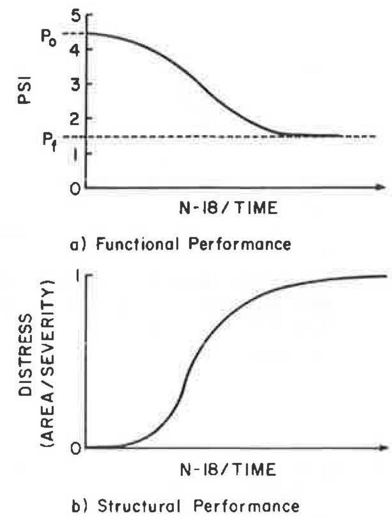

FIGURE 2 Illustration of the S-shaped functional and structural performance curves.

REGRESSION MODELS FOR DESIGN CONSTANTS

The methodology for determining the design parameters in Equation 3 (ρ, β, and P_f) was developed by Garcia-Diaz and Riggins (1). The analysis of 164 test sections has been conducted on pavements classified as follows:

- Hot mix asphaltic concrete on a bituminous base (51 sections),
- Hot mix asphaltic concrete on an unbound flexible base (36 sections), and
- Hot mix asphaltic concrete overlay placed on existing pavements (77 sections).

For each test section, the design parameters were evaluated for the following types of performance:

- Present serviceability index,
- Rutting (area and severity),
- Alligator cracking (area and severity),
- Longitudinal cracking (area and severity), and
- Transverse cracking (area and severity).

In addition to performance data, there is also an extensive set of pavement structure, environmental, and traffic data available for each section. These data were studied to determine which variables could be considered independent and to determine the ef-

TABLE 1 Variables Used in the Regression Models

Environmental	Structural	Traffic
Thornthwaite index (TI)	Plasticity index (PI)	
Annual freeze/thaw cycles (F/T)	Equivalent thickness (H')[a]	N-18/month (N-18)[f]
Average temperature (T_{AVG})	Percentage asphalt binder (binder)[b]	
	Overlay Thickness (OVTH)[c]	
	Total asphalt thickness (ASPH)[d]	
	Surfacing thickness (HMAC)[e]	

[a] Equivalent thickness is the transformed pavement thickness.
[b] This term is for black base and hot mix asphalt concrete pavements.
[c] This term is for overlay pavements.
[d] This term is for black base pavements. It is the total asphalt thickness of black base plus surfacing course.
[e] This term is for hot mix asphaltic concrete pavements.
[f] The N-18/month value represents the observed value during the first performance period.

fect of each variable on the curve fit parameters ρ and β. The variables found to be most significant, which were used to develop regression models, are given in Table 1. In this table the equivalent thickness (H') represents the transformed thickness of the pavement, defined as

$$H' = \sum_{i=1}^{m} (E_i/E_s)^n \cdot t_i \tag{5}$$

where

- m = number of layers under consideration,
- E_i = elastic modulus for the ith layer,
- t_i = thickness of the ith layer,
- E_s = elastic modulus of the subgrade, and
- n = Odemark's constant that can be obtained from field data (4) but may be assumed to be 0.33.

To obtain an indication of surface deflections, the H' term was transformed to

$$HPR2 = [(E_s H')/10^5] \tag{6}$$

The H' term is also used to define an indication of the surface curvature, given by the expression

$$HPR3 = \{10^{10}/[E_s(H')^3]\} \tag{7}$$

The decade multipliers in Equations 6 and 7 serve to scale the variables so that their magnitudes will be of the same order as those of the remaining variables. The regression study consisted of determining the best linear models, which expressed the curve fit parameters in terms of the variables given in Table 1 as well as HPR2 and HPR3. The linear models for present serviceability are shown in Figure 3. The complete list of regression models for distress can be found elsewhere (5). All models have been incorporated in the revised Texas FPS program.

PREDICTION OF PAVEMENT PERFORMANCE

The prediction of pavement performance is concerned with determining the number of vehicles or time required to reach a minimum acceptable level of performance. In terms of functional performance, the prediction is made using the serviceability index damage function, Equation 2, with the S-shaped performance equation, Equation 3; that is,

$$g = [(P_o - P)/(P_o - P_f)] = e^{-(\rho/N)^\beta} \tag{8}$$

This expression represents the loss in serviceability between the initial (P_o) and final (P_f) states. However, in making the performance predictions, the primary concern is not with the attainment of the

HMAC Pavement on Bituminous Base

= -0.02182(F/T) - 0.00831(PI) + 0.04499(Binder) + 0.15019(HPR2)

= 0.01201(TI) + 0.03166(F/T) + 0.13775(T_{AVG}) + 0.00114(PI) - 0.31331(Binder) - 0.03234(HPR2)

P_f = -0.00637(F/T) - 0.01550(T_{AVG}) - 0.00658(PI) + 0.27714(Binder) + 0.05097(HPR2)

HMAC Pavement on Flexible Base

= -0.02000(TI) - 0.02481(F/T) - 0.03078(PI) + 0.60781(Binder) + 0.06424(HPR2)

= 0.04045(F/T) + 0.22931(T_{AVG}) - 0.53010(Binder)

P_f = -0.00665(F/T) - 0.07017(T_{AVG}) - 0.02472(PI) + 0.57235(Binder) + 0.00722(HPR2)

HMAC Overlays

= 0.26503(OVTH) + 0.07180(HPR2)

= 0.00413(TI) + 0.01036(F/T) + 0.04769(T_{AVG}) + 0.01707(N-18) - 0.09144(OVTH) - 0.01066(HPR2)

P_f = 0.33037(OVTH) + 0.07627(HPR2)

FIGURE 3 Linear regression models for the curve fit parameters (PSI).

final state but with the performance to a selected terminal state (P_t). Therefore, Equation 8 must be "scaled down" to represent the performance loss between the initial level (P_o) and the minimum acceptable or terminal level of performance (P_t). The terminal damage level (g_t) is given by

$$g_t = [(P_o - P)/(P_o - P_t)] = [(P_o - P_f)/(P_o - P_t)]\ e^{-(\rho/N)^\beta} \tag{9}$$

or

$$g_t = c \cdot e^{-(\rho/N)^\beta} \tag{10}$$

where c is $(P_o - P_f)/(P_o - P_t)$. Taking the natural log of Equation 10 and solving for N gives

$$N = \rho/[-\ln(g_t/c)]^{1/\beta} \tag{11}$$

This equation can be used to predict the number of vehicles or amount of time required to attain a minimum acceptable level of performance. For structural performance, Equation 11 can also be used by substituting the critical distress level (g_e) for the term (g_t/c).

The curve fit parameters (ρ, β, and P_f) are determined from field data, and thus the variability of these parameters must be taken into account in predicting the level of reliability of a pavement. This can be achieved by finding the stochastic form of Equation 11. Taking the natural logarithm of this equation and expressing it in terms of the variance of ln(N) gives

$$\mathrm{Var}[\ln(N)] = \mathrm{Var}[\ln(\rho)] + \mathrm{Var}[(1/\beta) \cdot \ln(x)] \tag{12}$$

where x is $-\ln(g_t/c)$, which can also be expressed as

$$\mathrm{Var}[\ln(N)] = (\sigma_\rho/\bar{\rho})^2 + (1/\beta)^2 \cdot \mathrm{Var}[\ln(x)] \tag{13}$$

where $\hat{\rho}$ and $\hat{\beta}$ represent the mean values of ρ and β as determined from the regression equations.

The general form of the relation between the standard deviation (σ_ρ) and $\hat{\rho}$ was found to be hyperbolic for serviceability index loss, rutting, and alligator cracking. The relation is

$$\sigma_\rho = \hat{\rho}/(a + b\hat{\rho}) \quad (14)$$

A linear relation was found for longitudinal and transverse cracking. The relation is

$$\sigma_\rho = a + b\hat{\rho} \quad (15)$$

Values of the constants a and b for serviceability index loss and the different types of distress are given in Table 2. There was not a significant difference between the standard deviations of ρ for the different types of pavement or between area and severity of distress.

TABLE 2 Values of the Variance Parameters a and b

	a	b
Serviceability index loss	2.00	5.96
Rutting	3.90	4.91
Alligator cracking	6.05	5.04
Longitudinal cracking	0.00	0.0751
Transverse cracking	0.00	0.0762

The variance of ln(x) for the present serviceability index has been found to have a constant value of 0.125. This term is zero for the distress equations because the variable (g_t/c) is replaced by g_c or the critical distress level. Because this value is a constant, it can have no variance.

The design equation currently in use (6,7) is expressed in terms of the standard deviation of the common logarithm; therefore, for the sake of compatibility, it becomes necessary to express Equation 13 in the following form:

$$\mathrm{Var}[\ln(N)] = 2.302 \cdot \mathrm{Var}[\ln(N)] \quad (16)$$

The expression for the variance of log(N) is used to determine the number of 18-kip ESALs or the number of months for which a pavement must be designed in order to have a specified level of reliability. "Reliability" as it is used in the Texas FPS design program is defined as the probability that a pavement will not reach its terminal state within a specified number of load applications (N). The reliability (R) is a number between 0 and 100 and is given by Equation 17:

$$R/100 = \{1/[(2\pi)^{1/2}]\} \int_{-\bar{z}}^{\infty} e^{-z^2/z} dz \quad (17)$$

where z is the normal variable for a specified level of reliability in the performance of the pavement.

Typical values of $\bar{z}$ corresponding to difference levels of confidence or degrees of reliability are given in Table 3.

The "difference distribution" is given by the distribution of differences between the logarithm of the number of 18-kip ESALs that the pavement can withstand [log(N)] and the logarithm of the number of 18-kip ESALs that are actually applied to the pavement (log n). The mean difference (D) is approximately

$$\bar{D} = [\log(\bar{N})] - [\log(\bar{n})] \quad (18)$$

TABLE 3 Levels of Reliability Corresponding to Difference Levels of the Normal Variable

Reliability, R (%)	Normal Variable ($\bar{z}$)
50	0.00
80	0.84
90	1.28
95	1.65
99	2.33

and the variance of the difference distribution is

$$\mathrm{Var}(D) = \mathrm{Var}[\log(N)] + \mathrm{Var}[\log(n)] \quad (19)$$

A good estimate of the Var[log(n)] has been found by Darter and Hudson (8) to be between 0.0229 and 0.0333. Setting the difference between $\bar{D}$ and $\bar{z}\sigma_D$ equal to zero gives

$$\log N = \log n + \bar{z}\{\mathrm{Var}[\log(N)] + \mathrm{Var}[\log(n)]\}^{1/2} \quad (20)$$

It is Equation 20 that is used to estimate N, the number of 18-kip ESALs or the number of months for which a pavement should be designed in order to achieve a level of reliability that is specified by the normal variable ($\bar{z}$).

APPLICATIONS FOR PAVEMENT PERFORMANCE

Sensitivity Analysis

A sensitivity analysis was made of the regression models described previously to determine the effects of various combinations of climate on the performance curves. Four highway districts in Texas were chosen to represent extremes in climate for the state. The districts and regions in the state in which they are located, along with the climate associated with each area, are as follows:

- District 1 (east)--wet, some annual freeze-thaw cycles;
- District 4 (north)--dry, many annual freeze-thaw cycles;
- District 17 (central)--wet, few annual freeze-thaw cycles; and
- District 21 (south)--dry, no annual freeze-thaw cycles.

The three types of pavement structure considered in this study are shown in Figure 4. The pavement, climate, structure, and traffic variables were used with the equations presented previously in Figure 3 to determine the curve fit parameters. The resulting performance curves, in terms of the present serviceability index, as a function of the number of 18-kip ESALs for each of the three pavement types are shown in Figures 5-7. Referring to these figures, the following conclusions can be drawn concerning the functional performance of pavements in Texas:

- Asphaltic concrete pavements on bituminous bases are susceptible to freeze-thaw and to a lesser extent to the presence of excess moisture,
- Hot mix asphaltic concrete pavements on flexible bases are susceptible to both freeze-thaw and excess moisture, and
- Overlaid pavements appear to be less dependent on climatic factors and depend to a greater extent on the existing pavement structure.

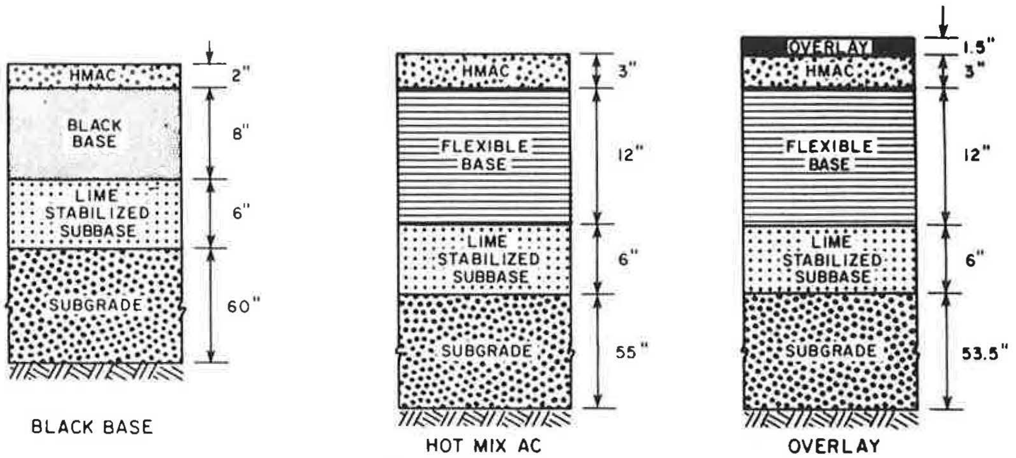

FIGURE 4 Pavement types used for sensitivity analysis.

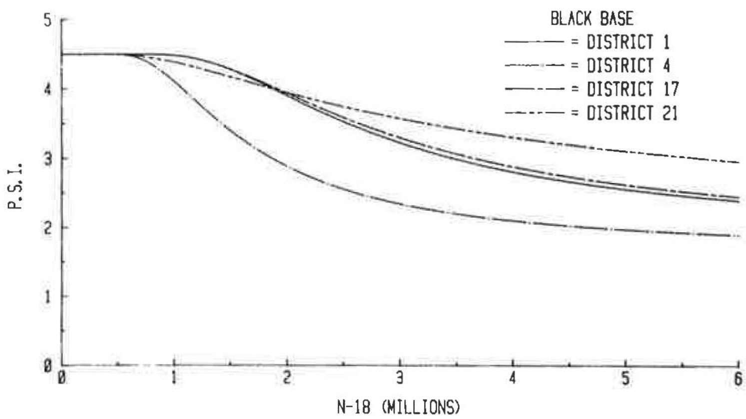

FIGURE 5 PSI sensitivity results for pavements on bituminous base.

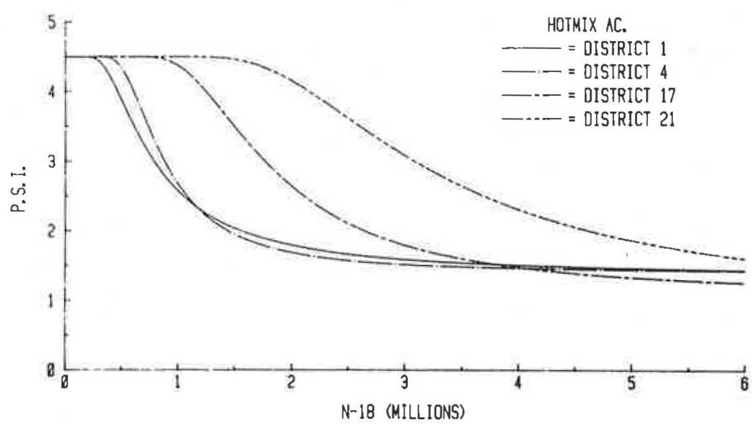

FIGURE 6 PSI sensitivity results for hot mix AC pavements.

Applications to Pavement Design

The regression equations that determine the shape of the performance curve and the probabilistic equations that determine the number of load applications for which the pavement should be designed (Equations 13-15 and 20) have been incorporated in the Texas FPS. Figure 8 shows a flow chart of subroutine "time," which calculates the time required to attain a specified minimum level of performance.

The calculation of pavement life for the present serviceability index, rutting, and alligator cracking cases involves an iterative procedure that calculates time as a function of the accumulated number of 18-kip ESALs. This procedure is required because these types of performance have been found to be load related (5). The iteration procedure consists of comparing the performance loss due to traffic (and expansive clay for the present serviceability index case) over a specified period of time to the minimum acceptable performance level. If the minimum performance level is not attained, the performance time is increased by a time increment and the comparison is repeated. This process continues until

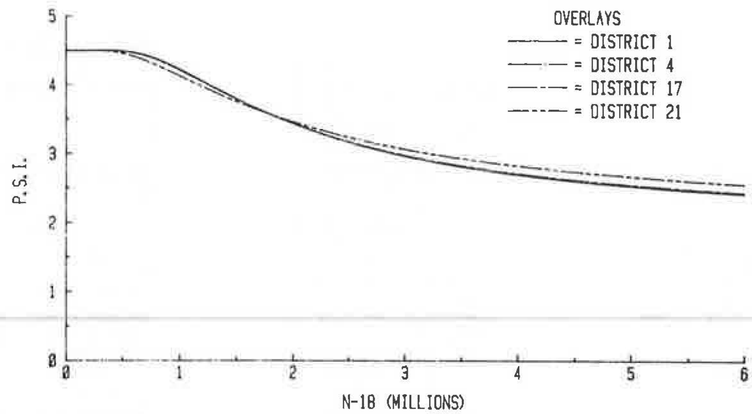

FIGURE 7 PSI sensitivity results for overlay pavements.

the minimum performance level is attained, thus giving the time to failure.

For the longitudinal and transverse cracking cases, this procedure is not required because the distress types have been found to be time related (5). Therefore the time required to reach a given level of distress can be calculated directly. After the first overlay has been placed, the program uses the overlay model for all subsequent performance periods. The output of the FPS program provides a listing of the optimal pavement designs, as well as subsequent overlay strategies, if required.

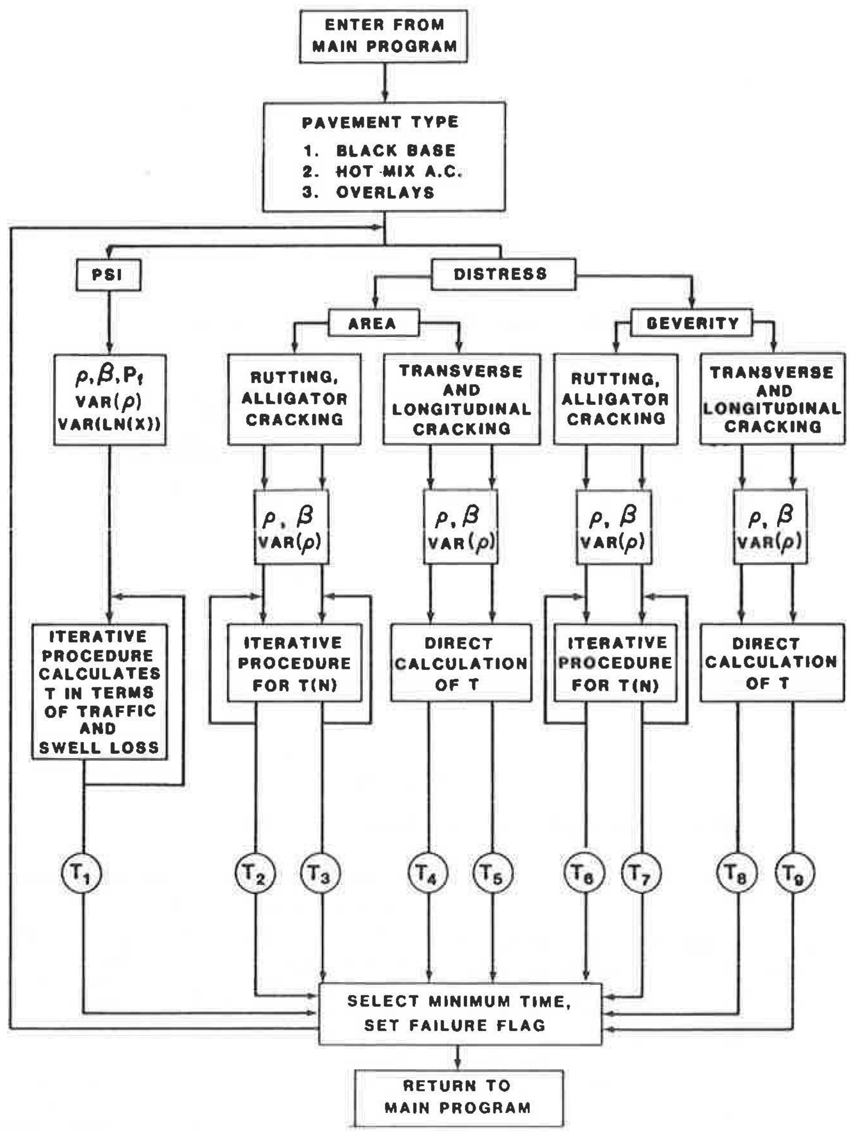

FIGURE 8 Flow chart for calculating the time an overlay is required.

CONCLUSIONS

The development of functional (PSI) and structural (distress) performance equations for Texas and the incorporation of these equations in the Texas FPS computerized pavement design system make it possible to develop optimal design strategies for flexible pavements in Texas. The equations are based on observations of performance trends on 164 pavement sections within the state. The equations make use of the elastic modulus of each layer as determined nondestructively by a deflection survey (4) or as determined in the laboratory, which is a major change in the Texas flexible pavement structural subsystem. The probabilistic form of these equations has been incorporated in the new FPS, making it possible for the designer to select the level of reliability that is appropriate for the level of traffic to be served. Because the program now checks several criteria including PSI, rutting, alligator cracking, longitudinal cracking, and transverse cracking to determine whether the useful service life has terminated, rules for selecting a desired level of reliability do not appear to depend solely on the level of traffic.

Some form of distress usually appears to be the principal cause of rehabilitation and that makes the desired level of reliability depend not only on the traffic level but on the climate in which the pavement is to be built. Studies are continuing to develop new rules for selecting a desirable level of reliability, but, in the meantime, even this result is encouraging for it indicates that the new FPS program, revised as described in this paper, is more realistic than the former version, more sensitive to the factors that cause pavement deterioration, and thus more useful in the design of flexible pavements.

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