

commute periods, should the one escalator at each platform be out of service. The Embarcadero and Montgomery Street stations will have problems because of patron resistance to climbing the long stairs at these stations. The Concord, El Cerrito Del Norte, Walnut Creek, Union City, and Hayward stations could use an additional escalator or stairwell. However, with their current configurations, the longer headway times at these five stations would allow patrons to exit the platform before the next train arrives.

The recommendations for additional escalators or stairwells are as follows:

<u>Station</u>	<u>Escalator or Stairwell Location</u>
Daly City	Two at platform 3
Pleasant Hill	One at each platform
Embarcadero	One at the west centroid
Montgomery	One at the west centroid
Concord	One for its single platform
El Cerrito Del Norte	One at the east platform
Walnut Creek	One at the east platform
Union City	One at the west platform
Hayward	One at the west platform

To facilitate a decision on constructing an escalator or stairwell at each station, cost estimates should be obtained and considered in light of the indicated severity of potential egress/ingress problems.

A Microcomputer-Based Fare Collection Dependability Model

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ABSTRACT

With the increasing sophistication of fare collection structures and consequently of fare collection equipment, equipment reliability and cost are becoming increasingly important issues. Techniques have been developed to analyze the interrelationships among reliability, cost, and the ability of a fare collection system to deliver dependable service to passengers. These techniques, based on mainframe computers and an investigation of the steady-state performance of the system, evaluate the performance of a given system, analyze its sensitivity to changes, determine specifications necessary for a given level of performance, and make trade-offs between system parameters. Microcomputers are becoming progressively more powerful, inexpensive, and readily available. So that the analysis techniques can be used more easily by transit personnel and analysts, a fare collection dependability model has been developed to run in a user-interactive microcomputer environment. The model determines the likelihood of equipment failures affecting system operation during a peak period. If equipment failures cause insufficient capacity to adequately process passenger demand, the fare collection system is defined as "in trouble." The likelihood of trouble is called the "trouble rate," whereas the likelihood of adequate capacity is called "peak period dependability." The technical approach for the performance and cost aspects of the model is discussed, both the probabilistic basis and the computational methodology to minimize execution time. The software to enable the user to interactively operate the model is described, and instructions are provided for its use. A sample fare collection dependability analysis session, consisting of four runs, is also provided.

The collection of transit system fares has been receiving increased attention as fares rise and federal operating subsidies decrease. Transit authorities are becoming more concerned about ways to maximize

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revenue and minimize costs while providing equitable fare and reliable, convenient service for passengers. Fare collection methods have a significant impact on total transit costs, amount of revenue generated, and passenger service (1,2). Fare collection costs range from 7 to 31 percent of passenger revenue at rail transit systems, and revenues generated from fares can vary from 40 to 90 percent of total transit

costs (3). Fare collection systems must therefore be selected only after careful examination of their cost, revenue, and service effects.

New fare collection concepts such as automated collection, barrier-free service, or credit-card use offer potential for reducing costs by minimizing the need for personnel to perform cumbersome, repetitive functions. However, the newer and more complex a piece of equipment, the more likely it is to have frequent failures, which can lead to significant passenger delay, lower throughput capacity, and general frustration (4-6). In improving this reliability, it should be known just how much of a reliability increase is required, as the extent of a reliability increase effort makes a significant difference in its cost and likelihood of success.

Because of the significant cost of fare collection equipment, the number and cost of equipment units to acquire for a given station also are quite important. Too many units can increase total system cost considerably, whereas too few will not be able to handle peak-period passenger demand, leading to significant passenger congestion and delay problems. One must be able to assess total system costs, and thus control them by determining and comparing the costs, as well as passenger performance, for various candidate system specifications.

To help carry out the aforementioned processes, the author developed and described an analysis technique and accompanying computer software (7). The analysis technique treats the fare collection system as a queue (or network of queues), the number of servers of which varies as equipment units fail and are then repaired. The computer software is interactive and runs on a mainframe computer in a time-sharing environment. The analysis technique obtains, for a specified fare collection system, average frequency distributions and mean values for passenger congestion and delay, as well as annualized costs. This allows a transit decision maker to evaluate a proposed system, evaluate the effects of possible changes in an existing or proposed system, or to determine the specifications necessary for a certain level of performance or cost.

One of the comments and feedback received on the software was that it should be able to run on microcomputers. Microcomputers have become inexpensive, readily available, and reasonably powerful (many of them reach or exceed the capabilities once represented by the IBM 360). With microcomputer software, more transit systems, individual transit staff members, consultants, analysts, and planners could make use of the fare collection system analysis technique.

An apparent drawback to implementing the software on a microcomputer was that the program took a significant amount of time to run even on a mainframe computer. It appeared that the time and space requirements on a microcomputer would be prohibitive. However, a situation that first arose as a problem became instead a key to formulating a microcomputer model.

The situation is that frequently transit systems do not carry out repairs of failed equipment during the peak period. High equipment reliability or spare equipment capacity is used instead to have available enough functional units for the peak-period passenger demand. Under this approach, measuring average steady-state performance over a long term is not very meaningful, because once the capacity of the system falls below the passenger demand as a result of equipment failures, the system cannot recover for the remainder of the peak period. A meaningful performance measure instead is the likelihood of the system running into trouble in this fashion during the peak period or, conversely, the likelihood of

the system not encountering significant passenger delay during the peak period.

As it turns out, this trouble rate, or, conversely, the peak-period dependability, can be calculated using a simple probability algorithm that requires a low mainframe computation time (less than 1 sec). Because of the low time requirement, the algorithm can be reasonably converted to a microcomputer environment.

Described in this report are the preceding algorithm, its implementation on a microcomputer, and its use for fare collection dependability analysis. Like the mainframe-based techniques (7), the microcomputer-based analysis technique is interactive and designed to help transit systems make more effective investment decisions in selecting fare collection methods, systems, and equipment to best fit their needs. It is also designed to help them minimize costs and provide equitable and convenient service to passengers.

TECHNICAL APPROACH

The interactive microcomputer fare collection dependability model evaluates the likelihood that during a specified length of time, such as a peak period, the number of equipment units available for use falls because of failures below the minimum number necessary to handle the passenger demand. If this happens, it is assumed that the fare collection service area will not be able to recover without significant passenger delay and is hence (by definition) in trouble. The likelihood that the area gets in trouble is thus the trouble rate, while the likelihood of the reverse, that the system does not get in trouble, is called the peak-period dependability. A trouble rate of 10 percent, which corresponds to a peak-period dependability of 90 percent, for example, means that on the average trouble will occur in 1 out of every 10 time periods. If the time period is a peak period, with two peak periods a day, 5 days a week, this means that trouble will occur during the peak period on an average of once a week.

The trouble rate and peak-period dependability are thus measures of overall system dependability, that is, the likelihood that a system will deliver its intended level of service to its users. They combine into one-term assurance measures, such as equipment reliability and maintainability, and level-of-service measures such as passenger arrival rate at the station, equipment passenger-processing rate, and number of equipment units. By doing so, they provide a top-level assessment of fare collection system performance to passengers at a station.

As mentioned previously, the model is especially suitable for situations in which no on-line repair is carried out during the time period under consideration or, nearly equivalently, where the on-line repair time (including the time for repair personnel to reach the site) is approximately equal to the time period itself. In either case, the fare collection service area will not be able to recover during the time period under consideration, once its capacity to process passengers becomes, because of failures, insufficient to handle passenger arrivals. Passenger congestion and delay will therefore continue to increase during the entire remainder of the time period, leading to the necessity to open emergency gates or other such remedial actions, passenger aggravation, staff aggravation, and, in short, trouble.

In situations in which fast on-line repair is indeed carried out, the model tends to overestimate the impact of failures because it is then possible to recover from an undercapacity situation. Nonethe-

less it provides a useful conservative estimate (an upper bound) of failure impact.

TECHNICAL APPROACH FOR PERFORMANCE MEASURES

The trouble-rate approach to the dependability analysis of a fare collection system is based on the field of probability theory, in particular the time to absorption of a Markov chain (8). The fare collection system can be described as being in one of a number of states, depending on how many equipment units are available for service relative to the minimum number necessary to adequately serve the passenger demand (see Figure 1). In state 0, not enough equipment units are available for service; thus the passenger demand is greater than system capacity and the system is in trouble. In state 1, enough units are available to handle passenger demand, but a loss of even one unit would cause trouble. In state 2, one unit could be lost from service without causing trouble; in state 3, two units could be lost, and so forth. In the maximal state, NMAX, all units are available for service. NMAX is given by $N - INT(ARR/SERV)$, where N is the number of units in the fare collection system, ARR the passenger demand (arrival rate), SERV the rate at which a unit processes passengers, and INT(X) the largest integer less than or equal to X. Note that $INT(ARR/SERV)$ is one fewer than the minimum number of units necessary for adequate service.

If the state of the fare collection system decreases to the trouble state 0 at any point during the peak period, the system gets into trouble and stays in state 0 for the rest of the period (state 0 is thus an absorbing state). The probability that this occurs is determined as a function of the number of passengers n who have arrived during the peak period. Let $P_0(n)$ denote this probability, with $P_i(n)$ being the probability that the system is in state i after n passengers have arrived, $i = 0, 1, \dots, NMAX$ (these probabilities are called the state probabilities). Let the equipment reliability be MCBF, the maintainability MTTR, and the passenger arrival rate

ARR. The probability of a transition from state i to the next lower state i-1 because of a failure is then the failure rate $F = 1/MCBF$, while the probability of a transition from state i (other than 0) to the next higher state i+1 because of a repair is the repair rate $R_i = (NMAX-i)/(MTTR*ARR)$. Note that $R_{NMAX} = 0$, so that the state of the system cannot go above the maximum state NMAX. The system has no failed units at the start of the peak period ($n = 0$), so that it is in state NMAX. Therefore, $P_{NMAX}(0) = 1$, with $P_i(0) = 0$ otherwise (for $i \neq NMAX$). The state probabilities of the system are then given by the following recursive equations:

$$P_0(n+1) = P_1(n)*F + P_0(n) \tag{1}$$

$$P_1(n+1) = P_2(n)*F + P_1(n)*[1-(F+R_1)] \tag{2}$$

$$P_i(n+1) = P_{i+1}(n)*F + P_{i-1}(n)*R_{i-1} + P_i(n)*[1-(F+R_i)] \quad 2 < i < NMAX \tag{3}$$

In words, Equation 3 states that the system will be in state i after n+1 passengers have arrived [i.e., $P_i(n+1)$] if it either was in state i+1 after n passengers have arrived [i.e., $P_{i+1}(n)$] and a failure then occurs (F), or it was in state i-1 after n passengers have arrived [$P_{i-1}(n)$] and a repair then occurs (R_{i-1}); or it was in state i after n passengers have arrived [$P_i(n)$] and neither a failure nor a repair then occur [$1-(F+R_i)$]. Similar word descriptions fit Equations 1 and 2. Notice that the multiplier of $P_0(n)$ in Equation 1 is 1. This puts into equation form the absorption property mentioned previously; that is, if the system is in the trouble state 0 after n passengers have arrived, it will remain in that state after n+1 passengers have arrived, and it therefore will remain so for the rest of the peak period.

Equations 1-3 can be expressed in condensed form as

$$P(n+1) = T*P(n) \tag{4}$$

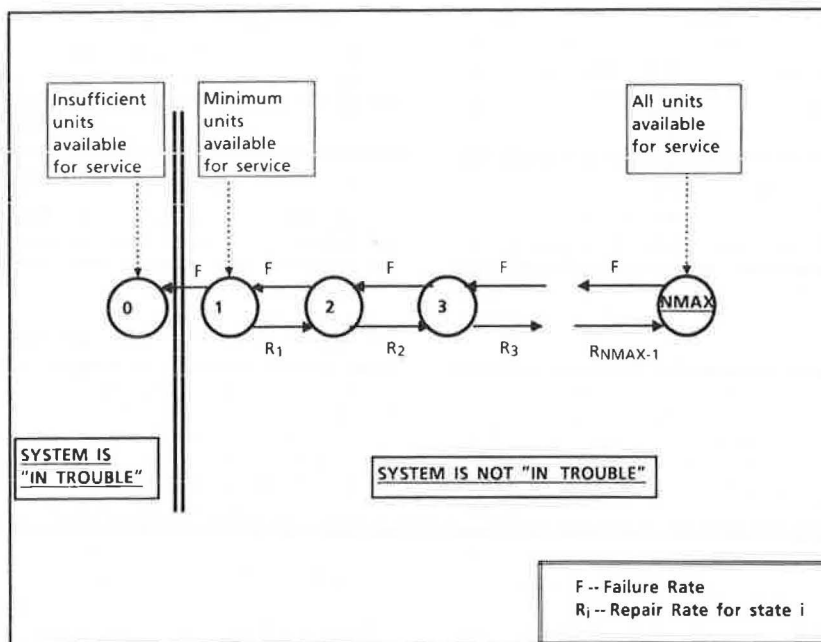


FIGURE 1 Reliability flow graph of fare collection system.

where $P(n)$ is the vector $[P_0(n), P_1(n), \dots, P_{NMAX}(n)]$ and T is the matrix (T_{ij}) , where

$$T_{00} = 1$$

$$T_{i,i-1} = F, i > 0$$

$$T_{i,i+1} = R_i, i > 0$$

$$T_{i,i} = 1 - (F + R_i), i > 0$$

T_{ij} represents the probability that if the fare collection system is in state i after the arrival of any particular passenger, it will be in state j after the arrival of the next passenger. Because these probabilities describe the transition of the system from one passenger to the next, T is called the transition matrix.

The trouble rate of the fare collection system is given by $P_0(n)$, that is, the probability that the fare collection system is in state 0 and so in trouble after n passengers have arrived. The peak-period dependability is given by $1 - P_0(n)$.

Computing the Performance Measures

The preceding approach for peak-period performance yields dependability as a function of the number of passengers processed. Because the peak-period length is based on elapsed time, the performance needs to be similarly expressed as a function of time. Let time be denoted by t , with t expressed in increments of 10 min up to a maximum peak period length of 120 min (the discussion can be easily generalized to increments of length dt up to a maximum peak-period length of $T_{max} = Md_t$). Let $Q_i(t)$ be the probability that the fare collection system is in state i at time t , so that $Q_0(t)$ is the time-based trouble rate (the probability that the system gets into trouble during a peak period of length t). Let $Q(t)$ be the vector $[Q_0(t), Q_1(t), \dots, Q_{NMAX}(t)]$. Then recursive equations for $Q(t)$ can be established in a manner similar to those for $P(n)$ (Equations 1-4) as

$$Q(t+10) = T^{PASS} * Q(t) \quad (5)$$

$$Q(0) = P(0) \quad (6)$$

where $PASS$ is the number of passengers arriving in 10 min. $PASS$ at first glance would appear to be $ARR/6$ (where ARR is the hourly passenger arrival rate). However, $ARR/6$ is not necessarily an integer. To determine the proper integer values of $PASS$ over the various 10-min intervals in Equation 5, the following algorithm was used:

1. Let $NUM = INT(ARR/6)$ and $ID = MOD(ARR,6)$, where $MOD(n,m)$ is the integer remainder when n is divided by m . Let $t = 0$, $ID1 = ID$, $INDEX = 1$, and $Q(0) = P(0)$.

2. If $INDEX = 1$, compute $Q(t+10) = T^{NUM} * Q(t)$. Otherwise, compute $Q(t+10) = T^{NUM+1} * Q(t)$.

3. Add 10 to t . If $t > 120$, then go to Step 5. Otherwise, continue to Step 4.

4. Add ID to $ID1$. If $ID1 > 6$, then subtract 6 from $ID1$ and set $INDEX = 2$. Otherwise, set $INDEX = 1$. Go to Step 2.

5. The trouble rate for time t (i.e., the probability that the fare collection system will run into trouble sometime during a peak period of time t) is given by $Q_0(t)$.

To compute the matrices T^{NUM} and T^{NUM+1} used in Step 2 is not a routine task. Because the hourly passenger arrival rate ARR can be in the thousands,

the value $NUM \approx ARR/6$ can be quite large. To calculate these matrices, the following acceleration algorithm (which requires no more than $2 \log_2 NUM + 1$ matrix multiplications, as against the NUM matrix multiplications required by the direct approach is used):

1. Let $k = 0$, $T_0 = T$, and $U = I$ (where I is the identity matrix, i.e., $A * I = I * A = A$ for any properly sized matrix A).

2. Convert the integer NUM into its binary representation $(b_k b_{k-1} \dots b_0)$, where $b_j = 0$ or 1. Note that $NUM = b_0 + 2b_1 + 4b_2 + \dots + 2^k b_k$.

3. If $b_k = 1$, multiply U by T_k .

4. If $k = K$, then go to Step 5. Otherwise, compute $T_{k+1} = T_k * T_k$ (note that $T_k = T^{2^k}$), add 1 to k and go to Step 3.

5. The matrix T^{NUM} is given by U , and T^{NUM+1} is given by $U * T$.

TECHNICAL APPROACH FOR COST MEASURES

In addition to assessing the performance of a fare collection system, the analysis software also examines system costs. The costs are computed on an annual basis and include equipment acquisition, spares provision, equipment operation, scheduled maintenance, and corrective maintenance and repairs. The costs are computed by the following formulas [a detailed description of the cost formulas is given by this author (7)].

1. Annualized capital cost

$$ACAP = ACQ * \{r / [1 - (1+r)^{-t}]\} * (n)$$

where

ACQ = acquisition cost,
 r = discount rate,
 t = useful life, and
 n = number of equipment units at the service area.

2. Annualized spares cost

$$SPRS = s * (ACAP)$$

where s is spares ratio.

3. Operating cost

$$OPER = n * (UOPR)$$

where $UOPR$ is annual operating cost per unit.

4. Cost of scheduled maintenance

$$SCHD = h * n * w$$

where h is annual hours of scheduled maintenance per unit, and w is hourly pay rate for repair personnel.

5. Cost of corrective maintenance and repair

$$CORR = (VOL) * (1/MCBF) * (MTR) * (w)$$

where

VOL = annual passenger volume at service area,
 $MCBF$ = reliability (mean cycles between failures),
 and
 MTR = maintainability (mean time to repair).

HOW TO USE THE MODEL FOR FARE COLLECTION SYSTEM ANALYSIS

The direct result of the microcomputer-based fare collection dependability model is information on the trouble rate (or, conversely, the peak-period dependability) and the annualized cost of a fare collection area at a particular station, given the system configuration and passenger demand. The information is given in terms of the trouble rate for peak periods of 10, 20, and 30 min, up to a maximum of 120 min. The costs are partitioned into acquisition cost, spares cost, operations cost, scheduled maintenance, and corrective maintenance.

The fare collection model can be used by transit authorities for a number of different purposes, such as

- Determination of required number of fare collection equipment units,
- Reliability and maintainability specifications,
- Impact of changes in passenger use level,
- Effect of maintenance policy changes, and
- Effect of changes in fare collection method.

In fulfilling these purposes, there are four basic kinds of analyses that can be conducted using the model: (a) evaluation, (b) sensitivity analysis, (c) specification determination, and (d) trade-off analysis. These are shown in Figure 2.

Evaluation examines a given fare collection system (sample question: "What is the system trouble rate for equipment with a reliability of 10,000 MCBF?") The required information about the system is collected and entered into the model as input data. The model produces estimates as to how well the system performs (or costs) under the given passenger demand, reliability, maintainability, and number of machine units.

Sensitivity analysis assesses the impact of a change in the fare collection system description (sample question: "What happens to the system trouble rate if the equipment reliability is 20,000 MCBF instead of 10,000?"). It is natural to want to know,

after a system has been evaluated, how the results would change if one or more of the input parameters were different from their current values (this would be particularly true if some of the input values were in doubt). To find out, several runs of the model are made with differing values for a given input parameter, and the changes that occur in system performance (or cost) are observed. This is called sensitivity analysis because it measures the sensitivity of system performance to changes in the input values.

Specification determination represents the opposite of evaluation and sensitivity analysis (sample question: "What equipment reliability must be specified in order to achieve a system trouble rate of no more than 3 percent?"). Instead of fixing an input value and determining the resulting performance, one fixes the desired performance and determines from that the value that a key input parameter, such as reliability, must be to meet that performance standard. In this manner, specifications may be determined so that the fare collection system will meet the desired standard.

Trade-off analysis examines the interaction between two input parameters under a fixed level of performance (sample question: "If there are four equipment units instead of three, by how much can the reliability decrease while still maintaining a trouble rate of no more than 3 percent?"). This allows one to determine how to trade off between two input parameters while achieving the same overall result. For example, if equipment reliability declines, by how much would maintainability have to improve to obtain the same overall performance? Trade-off analysis differs from sensitivity analysis in that trade-off analysis examines the interaction between two input parameters, whereas sensitivity analysis examines the interaction between an input and an output parameter.

SOFTWARE DESIGN AND USER'S MANUAL

An important part of any analysis software is its interaction with the user. The user should be able to

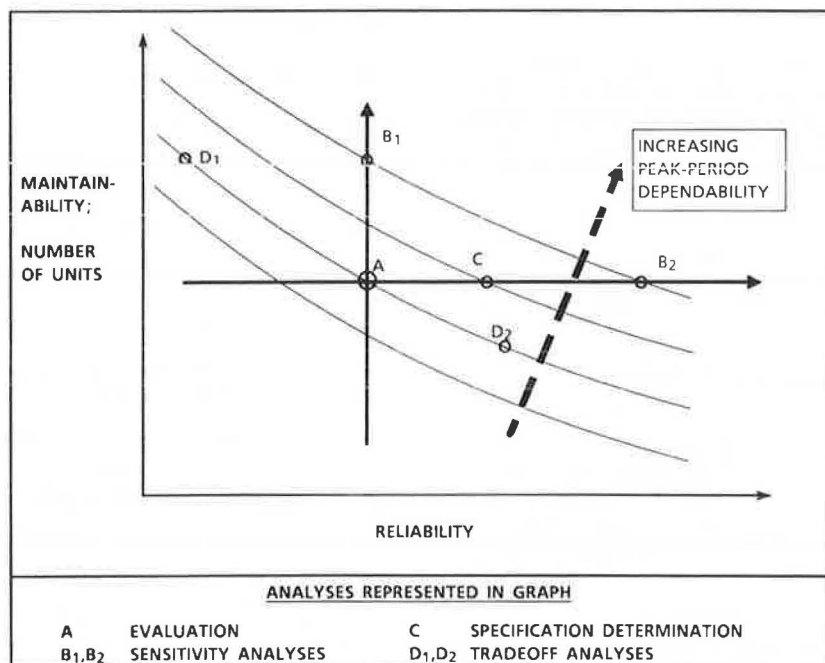


FIGURE 2 Types of dependability analyses.

easily access the program, enter data, make corrections, run analyses, make changes in the data, rerun analyses, and save input and output files. This is especially true with microcomputers, where the user converses directly with the computer instead of submitting runs and waiting for results. The software thus has various routines to communicate with the user and provides the following features:

- The user is guided through the program by prompts, most of which can be answered by "yes" or "no."
- Data can be entered either directly from the terminal or from a stored input file. If entered from the terminal, it can be stored for later use.
- The data to be used for an analysis run can be displayed to the user and, if desired, changes can be made in the data.
- Many runs can be made in the same session. Data for successive runs come either from changes in the previous data or from completely new data (from the terminal or from files).
- The output is compact enough for performance and cost results to fit on the same microcomputer screen.
- In addition to screen display of results, an output file is created for later use.
- Errors in data entry can be corrected, either during the entry of the data item or afterwards.

The full software package, including the analysis algorithm and its user-interface functions, contains six sections as shown in Figure 3:

- Login/Entry
- Terminal Input
- Disk-File Input
- Verification/Change
- Analysis
- Continue/Stop

It is designed to run on any IBM-compatible microcomputer using the MSDOS operating system. Below is a description of each section and its function in more detail, including user prompts and responses.

LOGIN/ENTRY SECTION

When FARE is typed, the operating system starts the program, eventually displaying the message:

Accept input from the terminal?
(Y or N):

This asks whether the data for the analysis will be typed in from the terminal or entered from a disk file. A "Y" answer transfers control to the Terminal Input section to accept the data, whereas an "N" response transfers control to the Disk-File Input section to read the file.

TERMINAL-INPUT SECTION

When data are being entered from the terminal, the program will display prompts for each data item required. The prompts consist of a question number, a description of the data item, and the data type [either integer (no fractional values allowed) or real (fractional values accepted)]. There are five performance data, and hence five prompts, as follows:

Prompt	Data	Measurement Units
1	Number of machine units	Number
2	Mean arrival rate	Passengers per hour

Prompt	Data	Measurement Units
3	Mean passenger service rate	Passengers per hour
4	Reliability	Mean cycles between failures
5	Mean repair time	Minutes

To indicate that no repairs are carried out during the peak period, "9999." is entered for the mean repair time.

After accepting the performance-related data, the program then prompts, "Do you wish to include costs in this run?" A "N" answer concludes the input. A "Y" response leads to the following prompts for cost data:

Prompt	Data	Measurement Units
6	Capital cost	Dollar per machine unit
7	Useful life	Years
8	Discount rate	Percent
9	Spares ratio	Percent
10	Operating cost	Dollar per machine unit per year
11	Scheduled maintenance	Hours per machine unit per year
12	Repair wage rate	Dollar per hour
13	Station passenger volume	Passengers per year

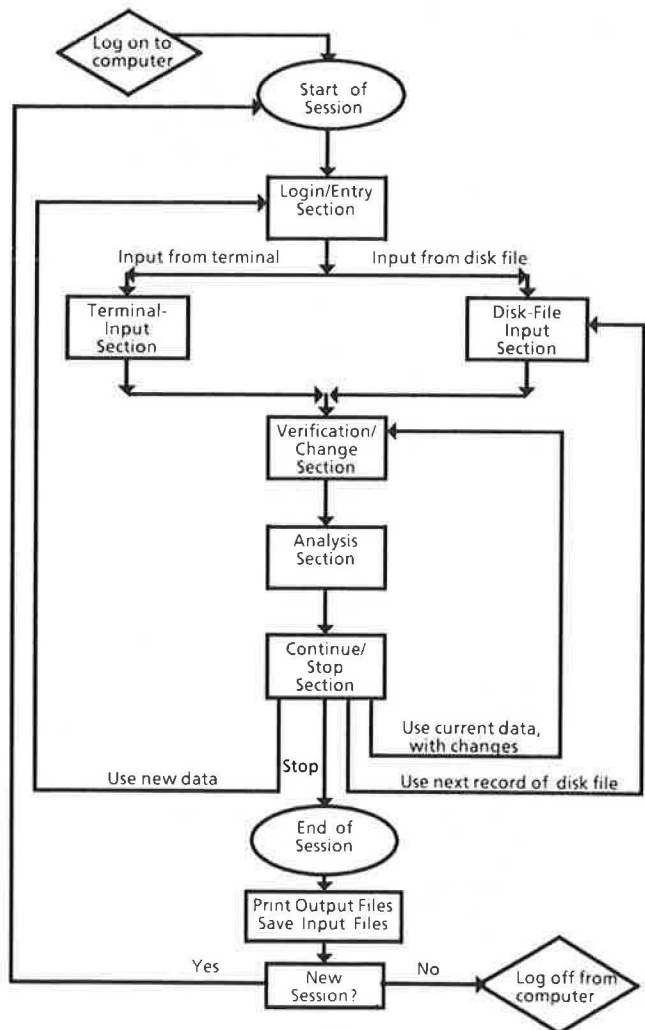


FIGURE 3 Software design—fare collection dependability model.

If costs are included in the analysis, but no repairs are carried out during the peak period, another prompt (Prompt 14) will ask for the mean time (in minutes) to carry out the repair during the off-peak period. This is necessary in order to calculate annual corrective maintenance costs. After accepting the cost data, the program proceeds to the Verification/Change section.

DISK-FILE INPUT SECTION

If the data are to be entered from an existing file, the program requests the name of the file by the message, "Enter input file name (up to 10 characters):" The input file consists of 6 lines if cost data are not included, and 14 or 15 lines if cost data are included. Each line, containing one value as shown below, occurs in the same order as that for the prompts in the Terminal Input section (thus the first line contains the number of equipment units, the second line contains the mean arrival rate, the third line contains the mean service or processing time, etc.). Line 5a contains either the value 0, if costs are not included in the analysis, or 1, if costs are included. Line 14 is necessary only if costs are included and no peak-period repair is carried out, in which case the line contains the off-peak mean repair time; for example:

Line Number	Data Value	Explanation
Performance data		
1	3	Three equipment units
2	2627.	2,627 arrivals per hour
3	1500.	Mean processing rate is 1,500/hr
4	10000.	Reliability is 10,000 MCBF
5	9999.	No peak-period repair
5a	1	Costs are included in the analysis
Cost data		
6	23000.	Capital cost is \$23,000 per unit
7	10	Useful life is 10 years
8	10.	10 percent discount rate
9	5.5	5.5 percent spares ratio
10	4100.	Operating cost
11	36.	Scheduled maintenance is 36 hr/year
12	15.66	Repair wage rate is \$15.66
13	2300000	Annual station volume is 2,300,000
14	12.	Mean off-peak time to repair is 12 min

VERIFICATION/CHANGE SECTION

This section displays the current data and any desired changes are made before running the model. The prompt message is, "Would you like to see the current input file?" A "Y" answer displays on the screen the data values to be used in the model run. The program then prompts, "Do you wish to make any changes?" A "N" answer transfers control to the Analysis section. A "Y" response, indicating a desire to make data changes, causes the program to request, "Enter the number of the data item you wish to change (Type '99' when you wish to make no further changes)". To select the data value to change, the corresponding prompt number as shown in the Terminal Input section is used (i.e., a "1" changes the number of equipment units, a "2" changes the passenger-arrival rate, a "3" changes the mean processing time, etc.). The program then requests the new data value and replaces the existing value with it. The program continues

requesting data items and making changes until the number "99" is given as a data item number, indicating no further changes.

The Verification/Change section is also used to change the inclusion of costs in the analysis. If costs are not included in the analysis, but they should be included, the answer "Y" is given to the make-changes prompt, then "6" is entered as the question number. The program then responds, "Costs are not included in the model. Do you wish to add cost data?" After a "Y" response, the program accepts the new cost data from the terminal in the same manner as it does in the terminal input section.

If costs are included in the analysis, but they should not be, the answer "Y" is given to the make-changes prompt, "6" is entered for the question number, and the acquisition cost is changed to zero. FARE will then ask, "Zero capital cost entered. Do you wish to include costs in this run?" After a "N" response, the program no longer includes costs in the analysis.

ANALYSIS SECTION

This section carries out the actual analysis of the fare collection system, using the data specified in the previous sections. The results, which are also stored in an external file, contain the following:

1. Reiteration of the input data.
2. The trouble rate (i.e., the likelihood of insufficient capacity during the peak period) for peak periods, in increments of 10 min, up to 2 hr in duration.
3. Annualized costs (if the cost option was selected): capital cost, spares cost, operating cost, scheduled maintenance cost, corrective repairs costs, and total cost.

CONTINUE/STOP SECTION

Whether or not further runs are desired is checked in this section. Further runs may be carried out in one of three ways:

1. Use the current input data (with possibly some changes).
2. Use the next record of the input file (if the current data were entered from a disk file).
3. Use completely new data.

The program asks, "Do you wish to make any further runs?" A "N" answer transfers control to the Stop procedure, described in the next paragraph. A "Y" answer leads to the prompt, "Do you wish to use the current data, either as is or with some changes? [answer 'Y' (current data or some changes) or 'N' (use a new data file)]". A "Y" response is used to make another run with exactly the same data as before or with a few changes (alternative 1). In this case, control passes to the Verification/Change section to accept whatever data changes need to be made. If an external input file is being used and the next record of this file is the data for the next run (alternative 2), a "N" response is used, followed by a "Y" response to the next prompt, "Are the new data the next file of (Name) DAT?" Control then passes to the Disk-File-Input section to read the next record of the input file. If completely new data are to be used (alternative 3), a "N" response to the first prompt is returned, and, if an external input file is being used, a "N" response to the second prompt, when it appears, is also returned. Control then passes to the entry procedure of the Log/Entry sec-

tion (displaying the prompt, "The new data are from the terminal or a new disk file. Accept input from the terminal?") so that the new data may be entered by either the terminal or a new external input file.

Before ending the session, the current input file may be saved as an external disk file. This can save time in future runs if the current input data were entered from the terminal or if significant changes were made from an existing disk file. The program asks, "Do you wish to save the current input file?" A "N" answer ends the session. A "Y" answers leads to the prompt, "Enter the name of file in which to save input (up to 10 characters). (If such a file already exists, the data in it will be overwritten by the current input file.)" "Name of file," at which point the name of the file in which to save the input is entered.

As mentioned previously, the output from all the analysis runs in a given session is stored in an output file. This allows the results to be transmitted to a hardcopy printer and stored for later reference.

SAMPLE ANALYSIS RUNS

To demonstrate how the microcomputer fare collection dependability analysis model operates, a sample interactive computer session is presented. The base-case fare collection system (Run 1) uses performance data from Long Island Railroad plans (Heimann, unpublished analysis) and cost data from "A Reliability-Based Model to Analyze the Performance and Cost of a Transit Fare Collection System" (7). The session contains four analysis runs as follows:

1. The base case, with the data stored in a disk input file. The program displays the input data, then calculates the trouble rates and costs. The trouble rate for a 1-hr peak period turns out to be 2.9 percent, which means that slightly less than 3 out of every 100 peak periods will experience delay problems. At the usual frequency of 10 peak periods per week, this means that a delay problem occurs in this system somewhat less than once every 3 weeks. The cost of the fare collection system, on an annualized basis over the 10-year life of the equipment, is \$28,375 per year.

2. A new case, with data entered directly from the terminal. The reliability of the equipment is entered incorrectly at 10,000 MCBF, so the change feature is used to correct it to 1,000. No cost analysis is required. The trouble rate is 9 percent for a 1-hr peak period, which represents a delay in 1 peak period out of 11, or a frequency of slightly less than 1 peak period per week.

3. The reliability in Run 2 is increased to 3,000 MCBF. In addition, a cost analysis is conducted, using figures entered from the terminal. The trouble rate becomes 1.2 percent for a 1-hr peak period (1 peak period in trouble out of 80, or 1 peak period every 8 weeks). The annualized total cost is \$30,536.

4. Alternative fare collection equipment is available, performing similarly to the previously considered equipment in terms of passenger processing, reliability, and so forth. The major difference is that the alternative equipment costs more, \$30,000 instead of \$23,000, but has lower operating costs, \$3,000 annually instead of \$4,100. This run compares the respective total annualized costs. Because only costs need be recomputed, the performance software is not used in this run. The annualized total cost turns out to be \$30,842 per year, about the same as the previous total cost.

The text of the interactive session follows (the responses by the user are underlined). Note that the session has been retyped.

FARE

B>DEL OUTPUT.DAT

B>FARET

ACCEPT INPUT FROM TERMINAL? RUN 1
('Y' OR 'N') : N

ENTER INPUT FILE NAME (UP TO 10 CHARACTERS) :
INPUT.DAT

THE INPUT FILE IS: INPUT.DAT

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS	3
2. MEAN ARRIVAL RATE IS	2627.0 /HOUR
3. MEAN SERVICE RATE IS	1500.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURES IS	9999.
5. MEAN TIME TO REPAIR IS	(NO ON-LINE REPAIRS)
6. CAPITAL COST PER UNIT IS	\$ 23000.00
7. USEFUL LIFE IS	10 YEARS
8. DISCOUNT RATE IS	10.00%
9. SPARES RATIO IS	5.50%
10. ANNUAL UNIT OPERATING COST IS	\$ 4100.0
11. ANNUAL HOURS OF SCHEDULED MAINTENANCE IS	36.00
12. REPAIR WAGE RATE IS	\$ 15.66 /HOUR
13. ANNUAL STATION PASSENGER VOLUME IS	2700000
14. MEAN TIME FOR OFF-LINE REPAIR IS	12.00 MIN.

DO YOU WISH TO MAKE ANY CHANGES ?
('Y' OR 'N') : N

ANALYSIS IN PROCESS

FARE COLLECTION PASSENGER-FLOW ANALYSIS
(TROUBLE-RATE MODEL)

NUMBER OF UNITS = 3
ARRIVAL RATE = 2627. /HR.
SERVICE RATE = 1500. /HR.
MCBF = 9999.
MTTR = (NO ON-LINE REPAIR)

CAPITAL COST = \$ 23000.00
USEFUL LIFE = 10 YEARS
DISCOUNT RATE = 10.0%
SPARES RATIO = 5.5%
OPERATING COST = \$ 4100.00
ANNUAL SCHEDULED MAINTENANCE HOURS = 36.0
REPAIR WAGE RATE = \$ 15.66 /HR
ANNUAL PASSENGER VOLUME = 2700000
OFF-LINE REPAIR TIME = 12.0 MIN.

TROUBLE RATE (PROBABILITY OF INSUFFICIENT CAPACITY)
DURING PEAK PERIOD (FOR PEAK PERIODS UP TO
2 HOURS IN DURATION)

LENGTH OF PERIOD	TROUBLE RATE
0 HR. 10 MIN.	.0%
0 HR. 20 MIN.	.4%
0 HR. 30 MIN.	.8%
0 HR. 40 MIN.	1.4%
0 HR. 50 MIN.	2.1%
1 HR. 0 MIN.	2.9%

LENGTH OF PERIOD	TROUBLE RATE
1 HR. 10 MIN.	3.8%
1 HR. 20 MIN.	4.9%
1 HR. 30 MIN.	6.0%
1 HR. 40 MIN.	7.2%
1 HR. 50 MIN.	8.5%
2 HR. 0 MIN.	9.8%

--ANNUAL COSTS OF FARE COLLECTION SERVICE AREA--

ANNUALIZED CAPITAL COST \$ 11229.43
 ANNUALIZED SPARES COST \$ 617.62
 OPERATING COST \$ 12300.00
 SCHEDULED MAINTENANCE COST \$ 1691.28
 CORRECTIVE REPAIR COST \$ 2537.17

 TOTAL COST \$ 28375.50

DO YOU WISH TO MAKE ANY FURTHER RUNS? **RUN 2**
 (ANSWER 'Y' OR 'N') Y

DO YOU WISH TO USE THE CURRENT DATA, EITHER AS IS OR WITH SOME CHANGES?
 (ANSWER 'Y' (CURRENT DATA OR SOME CHANGES) OR 'N' (USE A NEW DATA FILE)) : N

ARE THE NEW DATA THE NEXT RECORD OF: INPUT. DAT?
 (ANSWER 'Y' OR 'N') N

THE NEW DATA ARE FROM THE TERMINAL OR A NEW DISK FILE.
 ACCEPT INPUT FROM TERMINAL?
 ('Y' OR 'N') : Y

1. ENTER NUMBER OF MACHINES (ONE INTEGER VALUE) : 3
2. ENTER MEAN ARRIVAL RATE PER HOUR- (ONE REAL VALUE) : 500
3. ENTER MEAN SERVICE RATE PER HOUR- (ONE REAL VALUE) : 300
4. ENTER MEAN CYCLES BETWEEN FAILURES- (ONE REAL VALUE) : 10000
5. ENTER MEAN TIME TO REPAIR IN MINUTES- (IF NO ON-LINE REPAIR IS DONE, ENTER '9999.') (ONE REAL VALUE) : 9999

DO YOU WISH TO INCLUDE COSTS IN THIS RUN?
 ('Y' OR 'N') : N

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
 ('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS 3
2. MEAN ARRIVAL RATE IS 500.0 /HOUR
3. MEAN SERVICE RATE IS 300.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURE IS 10000.
5. MEAN TIME TO REPAIR IS (NO ON-LINE REPAIRS)

DO YOU WISH TO MAKE ANY CHANGES ?
 ('Y' OR 'N') : Y

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
 (TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER CHANGES).

(ONE INTEGER VALUE) : 4

4. ENTER MEAN CYCLES BETWEEN FAILURES- (ONE REAL VALUE) : 1000

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
 (TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER CHANGES).

(ONE INTEGER VALUE) : 99

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
 ('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS 3
2. MEAN ARRIVAL RATE IS 500.0 /HOUR
3. MEAN SERVICE RATE IS 300.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURES IS 1000.
5. MEAN TIME TO REPAIR IS (NO ON-LINE REPAIRS)

DO YOU WISH TO MAKE ANY CHANGES ?
 ('Y' OR 'N') : N

ANALYSIS IN PROCESS

FARE COLLECTION PASSENGER-FLOW ANALYSIS (TROUBLE-RATE MODEL)

NUMBER OF UNITS = 3
 ARRIVAL RATE = 500. /HR.
 SERVICE RATE = 300. /HR.
 MCBF = 1000.
 MTTR = (NO ON-LINE REPAIR)

TROUBLE RATE (PROBABILITY OF INSUFFICIENT CAPACITY) DURING PEAK PERIOD
 (FOR PEAK PERIODS UP TO 2 HOURS IN DURATION)

LENGTH OF PERIOD	TROUBLE RATE
0 HR. 10 MIN.	.3%
0 HR. 20 MIN.	1.2%
0 HR. 30 MIN.	2.6%
0 HR. 40 MIN.	4.4%
0 HR. 50 MIN.	6.6%
1 HR. 0 MIN.	9.0%

LENGTH OF PERIOD	TROUBLE RATE
1 HR. 10 MIN.	11.6%
1 HR. 20 MIN.	14.4%
1 HR. 30 MIN.	17.3%
1 HR. 40 MIN.	20.3%
1 HR. 50 MIN.	23.3%
2 HR. 0 MIN.	26.4%

DO YOU WISH TO MAKE ANY FURTHER RUNS?
 (ANSWER 'Y' OR 'N') Y

RUN 3

DO YOU WISH TO USE THE CURRENT DATA, EITHER AS IS OR WITH SOME CHANGES?
 (ANSWER 'Y' (CURRENT DATA OR SOME CHANGES) OR 'N' (USE A NEW DATA FILE)) : Y

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
 ('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS 3
2. MEAN ARRIVAL RATE IS 500.3 /HOUR
3. MEAN SERVICE RATE IS 300.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURES IS 1000.
5. MEAN TIME TO REPAIR IS (NO ON-LINE REPAIRS)

DO YOU WISH TO MAKE ANY CHANGES ?
 ('Y' OR 'N') : Y

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
 (TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER CHANGES).

(ONE INTEGER VALUE) : 4

4. ENTER MEAN CYCLES BETWEEN FAILURES- (ONE REAL VALUE) : 3000

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
(TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER
CHANGES).

(ONE INTEGER VALUE) : 6

COSTS ARE PRESENTLY NOT INCLUDED IN THIS MODEL.
DO YOU WISH TO ADD COST DATA?
('Y' OR 'N') : Y

6. ENTER THE CAPITAL COST PER UNIT
(ONE REAL VALUE) : 23000

7. ENTER THE USEFUL LIFE OF THE UNIT, IN YEARS
(ONE INTEGER VALUE) : 10

8. ENTER THE DISCOUNT RATE, IN PERCENTAGE TERMS
(ONE REAL VALUE) : 10

9. ENTER THE SPARES RATIO, IN PERCENTAGE TERMS
(ONE REAL VALUE) : 5.5

10. ENTER THE ANNUAL OPERATING COST PER UNIT
(ONE REAL VALUE) : 4100

11. ENTER THE ANNUAL SCHEDULED MAINTENANCE HOURS
PER UNIT
(ONE REAL VALUE) : 36

12. ENTER THE REPAIR WAGE RATE, PER HOUR
(ONE REAL VALUE) : 15.66

13. ENTER ANNUAL PASSENGER VOLUME AT STATION
(ONE INTEGER VALUE) : 1500000

14. ENTER TIME TO DO OFF-PEAK REPAIR, IN MINUTES
(FOR COST PURPOSES: IF YOU DO NOT WISH TO
INCLUDE THIS REPAIR IN THE COST FIGURES,
ENTER '0')
(ONE REAL VALUE) : 12

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS 3
2. MEAN ARRIVAL RATE IS 500.0 /HOUR
3. MEAN SERVICE RATE IS 300.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURES IS 3000.
5. MEAN TIME TO REPAIR IS (NO ON-LINE REPAIRS)

6. CAPITAL COST PER UNIT IS \$ 23000.00
7. USEFUL LIFE IS 10 YEARS
8. DISCOUNT RATE IS 10.00%
9. SPARES RATIO IS 5.50%
10. ANNUAL UNIT OPERATING COST IS \$ 4100.00
11. ANNUAL HOURS OF SCHEDULED MAINTENANCE IS 36.00
12. REPAIR WAGE RATE IS \$ 15.66 /HOUR
13. ANNUAL STATION PASSENGER VOLUME IS 1500000
14. MEAN TIME FOR OFF-LIFE REPAIR IS 12.00 MIN.

DO YOU WISH TO MAKE ANY CHANGES ?
('Y' OR 'N') : N

SHOULD THE NEW RUN RECOMPUTE COSTS ONLY?
(ANSWER 'Y' (COSTS ONLY) OR 'N' (PERFORMANCE
AND COSTS)) : N

ANALYSIS IN PROCESS

FARE COLLECTION PASSENGER-FLOW ANALYSIS
(TROUBLE-RATE MODEL)

NUMBER OF UNITS = 3
ARRIVAL RATE = 500. /HR.

SERVICE RATE = 300. /HR.
MCBF = 3000.
MTTR = (NO ON-LINE REPAIR)

CAPITAL COST = \$ 23000.00
USEFUL LIFE = 10 YEARS
DISCOUNT RATE = 10.0%
SPARES RATIO = 5.5%
OPERATING COST = \$ 4100.00
ANNUAL SCHEDULED MAINTENANCE HOURS = 36.0
REPAIR WAGE RATE = \$ 15.66 /HR.
ANNUAL PASSENGER VOLUME = 1500000
OFF-LINE REPAIR TIME = 12.0 MIN.

TROUBLE RATE (PROBABILITY OF INSUFFICIENT CAPACITY)
DURING PEAK PERIOD
(FOR PEAK PERIODS UP TO 2 HOURS IN DURATION)

LENGTH OF PERIOD	TROUBLE RATE
0 HR. 10 MIN.	.0%
0 HR. 20 MIN.	.1%
0 HR. 30 MIN.	.3%
0 HR. 40 MIN.	.6%
0 HR. 50 MIN.	.9%
1 HR. 0 MIN.	1.2%

LENGTH OF PERIOD	TROUBLE RATE
1 HR. 10 MIN.	1.7%
1 HR. 20 MIN.	2.1%
1 HR. 30 MIN.	2.6%
1 HR. 40 MIN.	3.2%
1 HR. 50 MIN.	3.8%
2 HR. 0 MIN.	4.5%

--ANNUAL COSTS OF FARE COLLECTION SERVICE AREA--

ANNUALIZED CAPITAL COST	\$ 11229.43
ANNUALIZED SPARES COST	\$ 617.62
OPERATING COST	\$ 12300.00
SCHEDULED MAINTENANCE COST	\$ 1691.28
CORRECTIVE REPAIR COST	\$ 4698.00
TOTAL COST	\$ 30536.33

DO YOU WISH TO MAKE ANY FURTHER RUNS? RUN 4
(ANSWER 'Y' OR 'N') : Y

DO YOU WISH TO USE THE CURRENT DATA, EITHER AS IS OR
WITH SOME CHANGES?
(ANSWER 'Y' (CURRENT DATA OR SOME CHANGES) OR 'N'
(USE A NEW DATA FILE)) : Y

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS 3
2. MEAN ARRIVAL RATE IS 500.0 /HOUR
3. MEAN SERVICE RATE IS 300.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURES IS 3000.
5. MEAN TIME TO REPAIR IS (NO ON-LINE REPAIRS)

6. CAPITAL COST PER UNIT IS \$ 23000.00
7. USEFUL LIFE IS 10 YEARS
8. DISCOUNT RATE IS 10.00%
9. SPARES RATIO IS 5.50%
10. ANNUAL UNIT OPERATING COST IS \$ 4100.00
11. ANNUAL HOURS OF SCHEDULED MAINTENANCE IS 36.00
12. REPAIR WAGE RATE IS \$ 15.66 /HOUR
13. ANNUAL STATION PASSENGER VOLUME IS 1500000
14. MEAN TIME FOR OFF-LINE REPAIR IS 12.00 MIN.

DO YOU WISH TO MAKE ANY CHANGES ?
('Y' OR 'N') : Y

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
(TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER
CHANGES).

(ONE INTEGER VALUE) : 6

6. ENTER THE CAPITAL COST PER UNIT
(ONE REAL VALUE) : 30000

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
(TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER
CHANGES).

(ONE INTEGER VALUE) : 10

10. ENTER THE ANNUAL OPERATING COST PER UNIT
(ONE REAL VALUE:) 3000

ENTER THE NUMBER OF THE DATA ITEM YOU WISH TO CHANGE
(TYPE '99' WHEN YOU WISH TO MAKE NO FURTHER
CHANGES).

(ONE INTEGER VALUE) : 99

WOULD YOU LIKE TO SEE THE CURRENT INPUT FILE?
('Y' OR 'N') : Y

1. NUMBER OF MACHINES IS 3
2. MEAN ARRIVAL RATE IS 500.0 /HOUR
3. MEAN SERVICE RATE IS 300.0 /HOUR
4. MEAN CYCLES BETWEEN FAILURES IS 3000.
5. MEAN TIME TO REPAIR IS (NO ON-LINE REPAIRS)
6. CAPITAL COST PER UNIT IS \$ 30000.00
7. USEFUL LIFE IS 10 YEARS
8. DISCOUNT RATE IS 10.00%
9. SPARES RATIO IS 5.50%
10. ANNUAL UNIT OPERATING COST IS \$ 3000.00
11. ANNUAL HOURS OF SCHEDULED MAINTENANCE IS 36.00
12. REPAIR WAGE RATE IS \$ 15.66 /HOUR
13. ANNUAL STATION PASSENGER VOLUME IS 1500000
14. MEAN TIME FOR OFF-LINE REPAIR IS 12.00 MIN.

DO YOU WISH TO MAKE ANY CHANGES ?
('Y' OR 'N') : N

SHOULD THE NEW RUN RECOMPUTE COSTS ONLY?
(ANSWER 'Y' (COSTS ONLY) OR 'N' (PERFORMANCE AND
COSTS)) : Y

ANALYSIS IN PROCESS

--ANNUAL COSTS OF FARE COLLECTION SERVICE AREA--

ANNUALIZED CAPITAL COST	\$ 14647.09
ANNUALIZED SPARES COST	\$ 805.59
OPERATING COST	\$ 9000.00

SCHEDULED MAINTENANCE COST	\$ 1691.28
CORRECTIVE REPAIR COST	\$ 4698.00

TOTAL COST	\$ 30841.96
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DO YOU WISH TO MAKE ANY FURTHER RUNS?
(ANSWER 'Y' OR 'N') N

DO YOU WISH TO SAVE THE CURRENT INPUT FILE?
(ANSWER 'Y' OR 'N') Y

ENTER NAME OF FILE IN WHICH TO SAVE INPUT (UP TO 10
CHARACTERS).

(IF SUCH A FILE ALREADY EXISTS, THE DATA IN IT
WILL BE REPLACED BY THE CURRENT INPUT FILE.)

NAME OF FILE: DATA.DAT

END OF SESSION.

THE RESULTS OF THIS SESSION ARE CONTAINED IN A FILE
CALLED 'OUTPUT.DAT'. IF YOU WISH TO SAVE THIS FILE,
BE SURE TO RENAME IT BEFORE THE NEXT SESSION.

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