Development of a Statistically Based Specification for Unbound Aggregates

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ABSTRACT

The research conducted for developing a statistically based specification for the gradation of unbound aggregates is described. Statistical parameters that describe within-batch, between-batch, and sampling variances for three aggregates, Pennsylvania Department of Transportation (PennDOT) 2A (dense-graded subbase), PennDOT 1B (AASHTO B), and PennDOT 2 (AASHTO 67), were estimated from data that were collected according to a statistically designed sampling plan. This information was then used in a computer simulation program to generate a distribution of the estimated percentage of material within limits ($\hat{PWL}$). Operating characteristic curves and expected payment curves were developed based on the $\hat{PWL}$ and a discrete price adjustment schedule. The specification includes a statistically based acceptance plan and a system for assigning payment when multiple price adjustments are involved.

Aggregates are manufactured products and represent the bulk of the materials used in portland cement concrete, bituminous concrete, base courses, and subbases. The performance of highways and structures is affected by the quality and uniformity of the aggregate used in their construction. For this reason it is important that aggregates meet certain acceptance criteria and fall within specification limits.

Many state highway agencies, including the Pennsylvania Department of Transportation (PennDOT), currently use statistically based specifications for the acceptance of bituminous concrete mixtures and construction (1). In Pennsylvania, this type of specification is referred to as a restricted performance specification (RPS). The purpose of this paper is to consider the research that was done to extend restricted performance specifications to the gradation of unbound aggregates. As a result of the research, an acceptance plan was formulated for the gradation of three Pennsylvania aggregates: 2A, 1B, and 2. PennDOT 2A aggregate is a dense-graded aggregate that is commonly used as a subbase. PennDOT 1B and the aggregates are one-sized, similar to AASHTO gradations 8 and 67, respectively. However, for the purpose of this paper, the development of a statistically based acceptance plan will be discussed only with reference to the PennDOT 2A aggregate.

The distinguishing elements of a statistically oriented specification are

1. Performance-oriented acceptance criteria.
2. Use of statistical techniques for the purpose of
   • Ensuring unbiased, accurate information;
   • Effective and timely process control;
   • Objective evaluation of quality characteristics in terms of both central tendency and dispersion; and
   • Making acceptance decisions on a rational basis.
3. Clear delineation of responsibilities with respect to
   • Process control by the contractor or the agency, or both, in the case of maintenance force work; and
   • Acceptance sampling, testing, and inspection by the highway agency.
4. An equitable price adjustment schedule for materials and construction that are not fully in compliance with requirements.

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To develop such a specification, it was necessary to estimate the statistical parameters of the existing process capabilities of aggregate producers.

DEVELOPMENT OF STATISTICAL MODEL

Statistical parameters can be estimated by performing statistical analysis on historical data or by collecting data according to a statistically based sampling plan. Appropriate historical data were not available, and therefore a sampling plan was developed to establish the various components of variance necessary to develop the specifications.

Although specification limits are based on the overall variance of a material, it is necessary to analyze this variance and quantify its relative components. Of interest are the between-batch variance, the within-batch variance, and the variance due to testing error. For the purpose of this paper, a batch is defined as a mini-stockpile formed by dumping a randomly chosen loader bucket of aggregate on the ground. Other work done as part of this research has shown that the mini-stockpile is the preferred sampling location, and this procedure has subsequently been adopted by PennDOT for use in the interim maintenance specification.

The between-batch (or batch-to-batch) variance is an important component of variance because it may result in differences in the performance of different batches within a lot. The magnitude of this variance is a function of the efficiency of the method of handling, transporting, and storing aggregates and the resulting degree of segregation.

The within-batch variance is a measure of the homogeneity within a given batch. It is found by collecting and testing two test portions from suitably separated points within the same batch. This variance represents the nonuniformity of aggregate gradation within a batch.

Variance due to testing error occurs because of the lack of repeatability of the test procedure. This error is due to random variations associated with the testing technique. A large variance due to testing error would require a review of the testing procedure with the objective of reducing this variance. A statistical model was developed to define the hierarchical nature of the components of variance:

$$Y_{ijklm} = \mu + P_i + V_{j(i)} + B_{k(i,j)} + S_{l(i,j,k)} + E_{m(i,j,k,l)}$$

and

$$\text{Var}\ (Y_{ijklm}) = \sigma^2{}_p + \sigma^2{}_v + \sigma^2{}_b + \sigma^2{}_s + \sigma^2$$

where

Y_{ijklm} = percentage passing a given sieve on a single test;
μ = true population mean of the percentage passing a given sieve for all aggregates in Pennsylvania;
P_i = effect of the ith plant; P_i is assumed to be distributed normally with mean = 0 and variance = $\sigma^2{}_p$;
$V_{j(i)}$ = effect of the jth visit within the ith plant; $V_{j(i)}$ is assumed to be distributed normally with mean = 0 and variance = $\sigma^2{}_v$;
$B_{k(i,j)}$ = between batch with mean = 0 and variance = $\sigma^2{}_p$; $B_{k(i,j)}$ is assumed to be distributed normally with mean = 0 and variance = $\sigma^2{}_b$;
$S_{l(i,j,k)}$ = effect due to taking the lth sample within the kth batch within the jth visit within the ith plant; $S_{l(i,j,k)}$ is assumed to be distributed normally with mean = 0 and variance = $\sigma^2{}_s$; and
$E_{m(i,j,k,l)}$ = testing error on the lth sample within the kth batch within the jth visit within the ith plant; $E_{m(i,j,k,l)}$ is assumed to be distributed normally with mean = 0 and variance = σ^2.

This model is a random effects model based on the assumption that each source of variation (effect) is random; that is, that each plant, visit, batch, and sample is selected at random. Testing error is assumed to be a random error not an error due to an assignable cause such as a weighing error. In addition, it is assumed that the effects are independent of each other. The model has a nested or hierarchical structure. That is, the testing effect (error) is nested within the sampling effect, the sampling effect is nested within the batch effect, and so forth.

DEVELOPMENT OF SAMPLING PLAN

PennDOT has developed an interim statistically based specification for unbound aggregates used in maintenance work (2). During the 1983 construction season, aggregate producers from nine of the eleven engineering districts supplied material under this interim specification. For the development of the sampling plan used in this study, plants were selected to represent those nine engineering districts. These plants use either gravel or limestone as the source material for manufacturing aggregate. To meet the Environmental Protection Agency (EPA) requirements or to control the minus No. 200 sieve material, or both, some limestone plants employ a washing process to remove excess fines. However, due to the long-graded nature of the 2A aggregate, limestone plants do not use the washing process for manufacturing this material. In this paper 2A aggregate produced without washing is referred to as a "2A dry."

Sampling location is an important element of any acceptance plan. In its current acceptance plan, PennDOT collects samples from trucks. This is an unacceptable location because of safety problems and the difficulty of obtaining a representative sample. Mini-stockpiles are formed by dumping a randomly selected bucket load of aggregate on the ground while the trucks are being loaded for shipment. Details of this sampling procedure, which was used in this study, can be found elsewhere (3).

The design selected for the sampling plan was a compromise between budgetary limitations and statistical requirements. It was decided that

1. Four plants would be sampled for each combination of aggregate type and manufacturing process,
2. One visit would be made to each plant for the collection of samples, and
3. Samples would be collected to provide 16 degrees of freedom (df) for estimating the sampling (within-batch) variance and 12 df for estimating the between-batch variance.

Several combinations of sublot size (number of sublots) and number of samples per sublot would provide at least the necessary degrees of freedom. However, it was concluded that using four sublots and two samples per sublot provided an acceptable compromise among the statistical requirements of the project, the logistics of obtaining field samples, and the limitations of the laboratory. Figure 1 shows a graphic illustration of the sampling plan. Samples were properly identified and transported to the laboratory, where a sieve analysis was conducted in accordance with the appropriate testing methods.

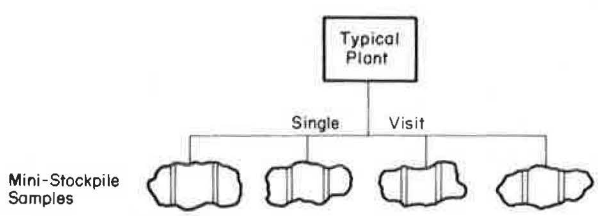

FIGURE 1 Illustration of components-of-variance sampling.

ANALYSIS OF VARIANCE

The gradation test results were used with the "nested" procedure of the Statistical Analysis System (SAS) to conduct an analysis of variance (ANOVA) (4). The results of this ANOVA are summarized in Tables 1-5. Because of the loss of two samples, the total number of degrees of freedom for the 2A-gravel combination was reduced from 31 to 29. In the case of 2A-dry, process samples collected for the sampling location study were also included in the ANOVA. This resulted in additional degrees of freedom. Because lot acceptance is affected by the between-batch ($\sigma^2{}_b$), sampling ($\sigma^2{}_s$), and testing ($\sigma^2{}_t$) variance, overall variance ($\sigma^2{}_o$) was determined by using the equation:

$$\sigma^2_o = \sigma^2_b + \sigma^2_s + \sigma^2_t$$

As indicated earlier, testing variance is a measure of the repeatability of the testing procedure. Because of budgetary considerations and time constraints, testing variance was estimated only for 2A-dry plants. It was assumed that this component of variance did not change with the process.

TABLE 1 Components of Variance for 2A Aggregate, 3/4-in. Sieve

	Degrees of Freedom		Variance Component (%)	
Variance Source	Gravel	Dry	Gravel	Dry
Total	29	88	73.357	64.276
Plant	3	3	26.669	23.405
Visit	0	2		28.510
Between-batch	12	21	23.861	1.264
Within-batch	14	40	22.827	7.956
Error (testing)	0	22		3.141
Mean percentage passing (spec. 52-100)			77.27	82.90
Standard deviation (σ_o)			6.83	3.52
$(\sigma_b^2/\sigma_o^2) \times 100$			50	10

TABLE 2 Components of Variance for 2A aggregate, 3/8-in. Sieve

	Degrees of Freedom		Variance Component (%)	
Variance Source	Gravel	Dry	Gravel	Dry
Total	29	88	69.484	124.110
Plant	3	3	-1.830	50.354
Visit	0	2		42.092
Between-batch	12	21	36.041	8.428
Within-batch	14	40	33.443	19.234
Error (testing)	0	22		4.001
Mean percentage passing (spec. 36-70)			52.59	49.60
Standard deviation (σ_o)			8.34	5.63
$(\sigma_b^2/\sigma_o^2) \times 100$			52	27

TABLE 3 Components of Variance for 2A Aggregate, No. 4 Sieve

	Degrees of Freedom		Variance Component (%)	
Variance Source	Gravel	Dry	Gravel	Dry
Total	29	88	62.056	93.251
Plant	3	3	23.027	55.040
Visit	0	2		10.690
Between-batch	12	21	17.542	10.692
Within-batch	14	40	21.486	13.375
Error (testing)	0	22		3.454
Mean percentage passing (spec. 24-50)			37.66	30.85
Standard deviation, (σ_o)			6.25	5.25
$(\sigma_b^2/\sigma_o^2) \times 100$			45	39

TABLE 4 Components of Variance for 2A Aggregate, No. 16 Sieve

	Degrees of Freedom		Variance Component (%)	
Variance Source	Gravel	Dry	Gravel	Dry
Total	29	88	19.151	54.732
Plant	3	3	6.558	43.451
Visit	0	2		-0.174
Between-batch	12	21	5.974	6.195
Within-batch	14	40	6.619	3.582
Error (testing)	0	22		1.505
Mean percentage passing (spec. 10-30)			21.77	15.37
Standard deviation (σ_o)			3.55	3.36
$(\sigma_b^2/\sigma_o^2) \times 100$			47	55

TABLE 5 Components of Variance for 2A Aggregate, No. 200 Sieve

	Degrees of Freedom		Variance Component (%)	
Variance Source	Gravel	Dry	Gravel	Dry
Total	29	88	5.119	4.747
Plant	3	3	3.671	2.217
Visit	0	2		0.425
Between-batch	12	21	1.142	1.109
Within-batch	14	40	0.306	0.208
Error (testing)	0	22		0.788
Mean percentage passing (spec. 0-10)			6.90	7.04
Standard deviation (σ_o)			1.20	1.45
$(\sigma_b^2/\sigma_o^2) \times 100$			79	53

PRICE ADJUSTMENT SCHEDULE

So that the tentative acceptance plan would be similar to PennDOT's statistically based specification for bituminous concrete, it was decided that lot acceptance should be based on an estimate of the percentage of material that falls within specification limits. This estimate, commonly referred to as $\widehat{PWL}$, can be thought of as an index of the quality of a lot submitted by the producer for acceptance. A trial price adjustment schedule was developed (Table 6), which established a relationship between the $\widehat{PWL}$ (i.e., material quality) and payment. In developing this schedule, the authors kept in mind the relative criticality of the sieve sizes in the gradation of the aggregate. For example, because of the importance of the minus No. 200 sieve fraction, the tentative schedule for the No. 200 sieve is relatively more stringent than the payment schedules for the other sieves.

TABLE 6 Price Adjustment Schedule 1 for Tentative Acceptance Plan for 2A Aggregate (percentage of contract price to be paid)

	Sieve Size				
Estimated PWL	3/4 in.	3/8 in.	No. 4	No. 16	No. 200
91-100	100	100	100	100	100
86-90	95	95	95	95	90
81-85	90	90	85	90	80
76-80	80	80	75	80	70
71-75	70	70	65	70	60
65-70	60	60	-[a]	60	-[a]
<65	-[a]	-[a]		-[a]	

[a]The contractor shall remove and replace the lot to meet specification requirements, or the engineer and the contractor may agree in writing that, for practical purposes, the lot shall not be removed and will be paid for at 50 percent of the contract price.

OPERATING CHARACTERISTIC CURVES

A sample size of N = 5, PennDOT's gradation limits for 2A aggregate, and the estimated standard deviation computed (Tables 1-5) were then used in a computer simulation program to generate the distribution of the $\widehat{PWL}$ of a lot (5). For each sieve in the gradation, six lots with various true means (μ) were studied. The μ-values were selected so as to range between the specification mean and a specification limit. For the No. 200 sieve, the lot true means studied were 5.0, 6.0, 7.0, 8.0, 9.0, and 10.0. The computer program then generated values of $\widehat{PWL}$ by simulating the selection of 10,000 independent random samples of size N = 5 from each of the lots described.

The values of $\widehat{PWL}$ thus obtained were then employed with the payment schedule to develop the operating characteristic (OC) curves for the acceptance plan. OC curves were developed for each sieve and each aggregate-process combination. Appendix A gives a part of the computer output obtained for the No. 200 sieve of the 2A-dry process limestone. The OC curves shown in Figure 2 were based on the computer output and the payment schedule of Table 6. These curves provide a graphic illustration of the consequences of the acceptance plan.

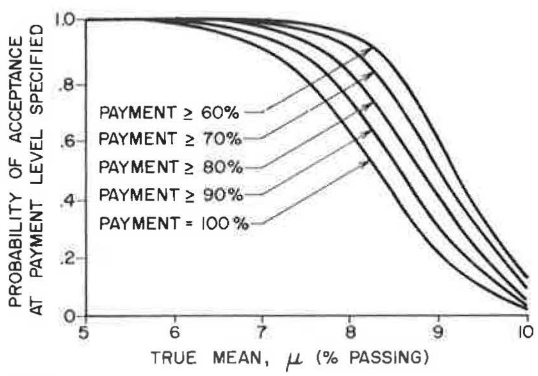

FIGURE 2 OC curves for No. 200 sieve of 2A-dry process aggregate.

EXPECTED PAYMENT CURVE

The expected payment curve indicates the average payment (over the long run) that a contractor will receive if he continues to supply material of a given conformity. Thus the curve illustrates the relationship between the conformity of the contractor's product and his expected payment.

The development of the expected payment curve for the No. 200 sieve is explained here with reference to the output (Appendix A) from the computer simulation program described earlier. According to the computer output, the probability that a lot with a true mean of μ = 8.00 will be assigned 100 percent payment is 1 - (3,740/10,000) = 0.626. The probability that this lot will be received at 90 percent payment is (3,740 - 2,662/10,000 = 0.1118. Similarly, the probability that this lot will be received at 80, 70, and 60 percent payment is 0.0874, 0.0758, and 0.0505, respectively. Now let us assume that when the $\widehat{PWL}$ is less than 71 percent the material will be accepted at 50 percent payment (in lieu of removal) in R(1 - L) percent cases, where

L = Offset of true mean from specification mean ÷ (Specification upper limit - Specification lower limit) [2]

and R = probability that the $\widehat{PWL}$ is rejectable. For the lot under consideration,

L = (8.00 - 5.00)/5.00 = 3/5 or 0.6

and the probability that the $\widehat{PWL}$ will be less than 71 percent (rejectable quality) is 0.0485. Thus the probability that 50 percent payment will be made is 0.0485 x (1 - 0.6) or 0.0194. Finally, the difference between the probability that $\widehat{PWL}$ is of rejectable quality and the probability that the lot will be accepted at 50 percent payment gives the probability of 0 percent payment. For the lot under discussion, the probability that no payment will be made is given by 0.0485 - 0.0194 = 0.0291. Expected payment is then determined by the relationship:

Expected payment = Σ [(Payment) x (Probability of receiving payment)]

Thus for a lot with μ = 8.00

Expected payment = [(100)(0.0626) + (90)(0.1118) + (80)(0.0874) + (70)(0.0758) + (60)(0.0505) + (50)(0.0194) + (0)(0.0291)] = 88.96%

This indicates that, under the tentative acceptance plan, a producer who supplies 2A aggregate such that its true mean on the No. 200 sieve is 8.00 will receive an average payment (over the long run) of 88.96 percent. The expected payment determined and the expected payments calculated for lots of other quality have been summarized in Table 7. This infor-

TABLE 7 Expected Payment Curve for No. 200 Sieve of 2A (dry process) Aggregate Based on Schedule 1, σ_o = 1.45, N = 5, and Acceptance Limits of 0 to 10 Percent

True (lot) Mean, μ	Probability of Receiving Indicated Payment (%) 100	90	80	70	60	50[a]	0	Expected[b] Payment (%)
5.00	0.9993	0.0005	0.0002					99.91
6.00	0.9913	0.0065	0.0016	0.0006				99.89
7.00	0.9118	0.0508	0.0229	0.0093	0.0034	0.0011	0.0007	98.49
8.00	0.6260	0.1118	0.0874	0.0758	0.0505	0.0194	0.0291	88.96
9.00	0.2122	0.0833	0.0920	0.1107	0.1100	0.0784	0.3134	54.35
10.00	0.0247	0.0133	0.0173	0.0292	0.0438	0.0000	0.8717	9.72

[a]Assumptions for 50% payment: percentage of cases in which 50% payment will be made in lieu of removal:

R (1 - L) x 100

where

L = offset of true mean from specification mean/(Specification mean - Specification lower limit) and
R = probability that PWL is rejectable.

[b]Expected payment = Σ [(Payment) x (Probability of receiving payment)].

mation was then used to plot the expected payment curve shown in Figure 3. It can be seen from this curve that the contractor's expected payment is 9.72 percent when his process is centered at the specification upper limit, but that the expected payment rises sharply as he moves the process toward the mean of the acceptance limits. Because the lower limit on this sieve is zero, his expected payment will be 100 percent for any true mean less than 5.00.

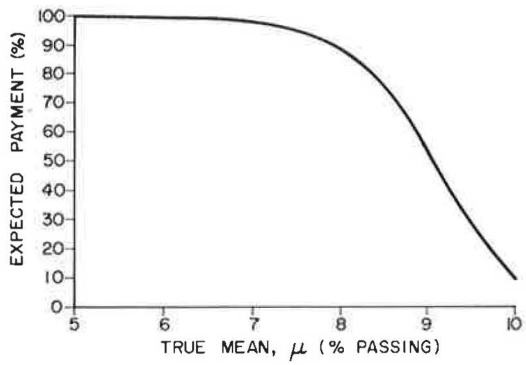

FIGURE 3 Expected pavement curve for No. 200 sieve of 2A-dry process aggregate, where $\sigma = 1.45$.

REVIEW OF ACCEPTANCE LIMITS

The objective of assigning numerical limits for a measurable characteristic such as aggregate gradation is to ensure uniformity or to ensure that some critical value that would affect performance is not exceeded, or both (6). The acceptance plan discussed previously was developed with the department's current specification limits given in Table 8 (7). Whether acceptance limits can be modified in accordance with the objectives for assigning numerical limits but without causing undue hardship to most aggregate producers and without incurring additional cost to the state will be assessed next.

TABLE 8 Gradation Limits for PennDOT's 2A Aggregate *(7)*

Sieve Size	Percentage Passing
3/4 in.	52-100
3/8 in.	36-70
No. 4	24-50
No. 16	10-30
No. 200	0-10

Consider the No. 200 sieve for the 2A aggregate with specification limits of 0 to 10 percent. This is a single-limit specification because one limit is zero. Therefore the limit of concern is 10. The data in Table 5 indicate that the dry process plants have an overall standard deviation of 1.45 percent and a mean percentage passing that is equal to 7.04 percent. The offset between the upper limit and the process mean is 10 - 7.04 = 2.96, which is equal to two standard deviations. Also, for the gravel producers, the between-plant component of variance is 79 percent of the total variance. This indicates that some of the plants sampled would be unable to produce aggregate within the limit. However, because of the critical nature of the material finer than the No. 200 sieve, it is not considered advisable to raise the upper limit of the specification.

A rationale similar to that used in reviewing acceptance limits for the No. 200 sieve was applied to the other sieves in the 2A gradation. Except for the 3/4-in. sieve, it was found that, with the producers' existing (1983) capabilities, the offset between the specification mean and a limit was less than three standard deviation units. In addition, for two of the four sieves (No. 4 and No. 16), the means for the two processes (gravel and dry) were located on opposite sides of the specification mean. If the acceptance limits are to be modified, fairness will require that the lower limit be lowered and that the upper limit be raised to accommodate both processes. This would, however, widen the specification band, which in turn could play havoc with the uniformity of the material and have an adverse effect on its performance. Consequently, acceptance limits were not changed for these sieves. For the same reason, the acceptance band for the 3/8-in. sieve also was not widened.

On the 3/8-in. sieve, the gravel process has the larger standard deviation (6.83). However, the existing limits on this sieve are such that the specification mean is more than three standard deviation units from a specification limit. Therefore these limits do not require any modification.

It should be mentioned here that an additional reason for not changing the acceptance limits for the 3/8-in. sieve and sieve Nos. 4, 16, and 200 is the belief that enforcement of a statistically oriented acceptance plan would provide the aggregate producers with the incentive to meet the specification limits.

MULTIPLE PRICE ADJUSTMENTS

The price adjustment schedule described earlier, which was incorporated into the tentative acceptance plan, was designed for individual sieves. However, it is possible for an aggregate gradation to be such that payment reductions must be applied to two or more sieves. A system had to be devised to determine the total payment in such cases. In general, three methods are possible:

1. Add price reductions,
2. Multiply payment percentages, and
3. Use smallest payment percentage.

Consider a 2A aggregate lot that has been tested for acceptance. Suppose the payment schedule indicates that the lot should be assigned 90, 90, and 70 percent payment for the 3/4-in., 3/8-in., and No. 4 sieves, respectively. If the first method is followed, the payment factor is 1.00 - (1 - 0.90) - (1 - 0.90) - (1 - 0.70) = 0.50 or 50 percent. The second method will result in 57 percent payment, and the third will accept the lot at 70 percent payment. Now consider a 2A aggregate lot for which the individual schedules for the same three sieves would allocate 80, 75, and 70 percent, respectively. In this case the lot would be accepted at 25, 47, and 70 percent payment by Methods 1, 2, and 3, respectively. It can be seen from this example that Methods 1 and 2 are excessively harsh. The third method, on the other hand, using the smallest payment percentage, encourages the producer to supply quality materials. Therefore this method was adopted as part of the tentative acceptance plan. The recommended tentative acceptance plan is given in Appendix B.

SUMMARY AND CONCLUSIONS

The primary objective of the research project was to develop a statistically based specification for the gradation of unbound aggregates. The development of an acceptance plan for a dense-graded base course aggregate (PennDOT 2A) has been described. The acceptance plan was based on statistical parameters estimated with the help of a statistically designed sampling plan. The procedure can be adopted as a model for formulating a statistically based specification for any unbound aggregate used in highway construction or maintenance.

The acceptance plan developed here incorporates a trial price adjustment schedule based mainly on judgment. It should, therefore, be treated as a preliminary or tentative acceptance plan. It is important that a field simulation plan be designed, executed, and properly conducted to verify that the plan is implementable and fair both to the state and to industry. If the parties concerned, the state highway department and the aggregate producers, find that the acceptance plan is not reasonable, it may have to be modified in one or more of the following ways:

1. Loosen or tighten the acceptance limits,
2. Change the sample size (N),
3. Increase or decrease payment for a given PWL, and
4. Reduce or increase the number of payment levels in the schedule.

On the basis of the extensive field sampling and data analysis conducted as part of the research project, a number of conclusions and findings are relevant:

1. For a given sieve size, the statistical parameters (mean and standard deviation for percentage passing) varied significantly between dry process limestone and gravel aggregate.

2. Many of the plants sampled would not have any difficulty in meeting the specification limits. However, the magnitude of the between-plant component of variance indicated that there were several plants that were producing aggregate that would not meet this specification (2). It is expected that the adoption of the proposed acceptance plan will provide aggregate producers with an incentive for improved process control.

RECOMMENDATIONS

The acceptance plan described here should be considered tentative, especially because it is based on a trial price adjustment schedule. It is recommended that a continuous payment schedule be developed for the acceptance plan. The acceptance plan should then be evaluated and verified with an appropriately designed field simulation study. The simulation study should include sampling of both new construction and maintenance projects. A sample size of five was recommended in the tentative specification to allow for a comparative analysis of sample sizes ranging from three to five. Finally, the results obtained from the field simulation study should be used to modify the acceptance plan before it can be incorporated in a quality assurance program.

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APPENDIX A
COMPUTER OUTPUT FROM SIMULATION PROGRAM

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THIS PROGRAM IN FILE PWL18W200COR MARCH 31,1985
  N  NM  NIT    MEANS                                       LIMITS
  5    610000  5.00  6.00  7.00  8.00  9.00 10.00  0.00 10.00
```

DIST OF CONTENT FOR MEAN= 8.00 OFFSET OF POP. MEAN= -3.00

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	10	10	11	11	12	14	14	16	17	18
51	18	20	21	21	21	21	21	21	21	21
52	23	23	23	23	23	23	23	24	24	24
53	24	24	25	25	26	26	26	26	26	26
54	26	26	26	26	26	27	28	28	31	31
55	31	33	33	34	34	34	35	36	39	39
56	39	39	39	39	39	39	40	40	40	41
57	41	43	44	46	47	50	50	51	52	53
58	54	56	57	58	59	60	61	61	63	63
59	63	64	67	68	68	69	70	74	76	79
60	80	82	83	86	89	90	90	91	92	94
61	96	99	101	102	103	106	106	106	108	112
62	115	116	116	117	119	120	121	121	123	125
63	128	133	138	139	143	147	150	152	159	160
64	163	169	174	175	180	187	190	194	196	196
65	201	203	205	208	213	215	217	221	222	225
66	226	229	233	235	239	242	248	252	256	263
67	268	271	273	274	284	288	291	299	304	309
68	313	321	328	333	340	344	353	355	360	366
69	375	380	387	393	401	405	412	420	422	427
70	431	438	448	453	459	463	467	472	481	485
71	494	503	511	521	527	537	544	557	566	575
72	585	599	607	615	622	631	638	650	663	674
73	684	695	709	720	730	740	755	763	774	783
74	788	799	804	807	815	830	842	852	861	868
75	881	894	911	919	930	938	951	969	979	990
76	1003	1022	1036	1050	1062	1081	1092	1109	1122	1132
77	1144	1165	1179	1196	1213	1228	1244	1264	1279	1301
78	1317	1325	1342	1358	1369	1387	1399	1410	1433	1451
79	1469	1476	1487	1503	1519	1532	1549	1567	1575	1591
80	1607	1619	1631	1648	1667	1681	1699	1710	1729	1748
81	1771	1789	1808	1821	1839	1863	1881	1894	1917	1938
82	1956	1965	1983	1994	2006	2022	2043	2059	2070	2086
83	2104	.2120	2141	2162	2177	2189	2209	2223	2243	2258
84	2272	2290	2309	2327	2353	2371	2399	2415	2433	2452
85	2484	2500	2512	2528	2541	2559	2569	2579	2599	2622
86	2632	2662	2684	2706	2725	2748	2768	2786	2810	2838

APPENDIX B
RECOMMENDED ACCEPTANCE PLAN

Acceptance Sampling

Sampling Location

Aggregate will be sampled from mini-stockpiles at the source of supply (quarry) or the processing plant as it is loaded on trucks for shipment.

Lot Size

Each 1,000 tons of material shipped from a plant will be treated as a lot for acceptance purposes. However, if the purchase order quantity is less than 1,000 tons, the quantity on the purchase order will constitute the lot size.

Sample Size

Each lot will be divided into five equal sublots, and replicate samples will be obtained from each sublot.

Sampling Procedure

A stratified random sampling procedure will be used to collect a pair (replicate) of sample increments from each sublot (2). The 10 sample increments so collected will be separated into two sample sets designated sample set 1 and sample set 2. Each sample set will include one increment from each sublot.

Referee Sample

Sample set 2 will constitute the referee sample. This sample will be tested for gradation analysis, and the results will be employed for acceptance purposes in the event that gradation results from sample set 1 are questioned.

Evaluating Material Acceptability

Testing Procedure

Sample set 1 will be tested for gradation in accordance with the appropriate Pennsylvania test methods.

The test results thus obtained for each lot will be used to compute the sample mean (x_1) and sample standard deviations (s_1) for each sieve. The subscript 1 indicates that the statistics are associated with sample set 1. These results will be used in the acceptance procedure unless the statistics for one or more sieves are questioned. In that event the contractor or the department may request that gradation results for the entire sample set 1 be disregarded and that acceptance be based on the mean (x_2) and the standard deviation (s_2) computed from the gradation analysis of the referee sample. If the request for testing the referee sample is made by the contractor, he should pay for the additional testing of the lot at a previously determined rate. However, the department has the option to waive the charge for the additional testing. The contractor will have the option of monitoring all acceptance sampling and testing.

Acceptance Procedure

Acceptance for aggregate gradation will be based on the estimated percentage of the material that is within the specification limits (PWL). The specification limits for the sieves used to control the gradation of the aggregate are given in Table 8. The standard deviation method will be used for estimating the PWL. For each sieve the PWL will be estimated with the help of two quality indices, Q_U and Q_L:

$$Q_U = (U - x_i)/s_i$$

TABLE B-1 Table for Estimating Percentage of Lot Within Limits (PWL) (standard deviation method)

Percent Within Limits	Negative Values of Q_U or Q_L					Percent Within Limits	Positive Values of Q_U or Q_L				
	n=3	n=4	n=5	n=6	n=7		n=3	n=4	n=5	n=6	n=7
50	.0000	.0000	.0000	.0000	.0000	99	1.1510	1.4700	1.6719	1.8016	1.8893
45	.1806	.1500	.1406	.1364	.1338	98	1.1476	1.4400	1.6018	1.6990	1.7615
						97	1.1439	1.4100	1.5428	1.6190	1.6662
40	.3568	.3000	.2823	.2740	.2689	96	1.1402	1.3800	1.4898	1.5500	1.5868
39	.3912	.3300	.3106	.3018	.2966	95	1.1367	1.3500	1.4408	1.4892	1.5184
38	.4252	.3600	.3392	.3295	.3238						
37	.4587	.3900	.3678	.3577	.3515	94	1.1330	1.3200	1.3946	1.4332	1.4562
36	.4917	.4200	.3968	.3859	.3791	93	1.1263	1.2900	1.3510	1.3813	1.3990
						92	1.1170	1.2600	1.3091	1.3328	1.3465
35	.5242	.4500	.4254	.4140	.4073	91	1.1087	1.2300	1.2683	1.2866	1.2966
34	.5564	.4800	.4544	.4426	.4354	90	1.0977	1.2000	1.2293	1.2421	1.2494
33	.5878	.5100	.4837	.4712	.4639						
32	.6187	.5400	.5131	.5002	.4925	89	1.0864	1.1700	1.1911	1.2001	1.2045
31	.6490	.5700	.5424	.5292	.5211	88	1.0732	1.1400	1.1538	1.1592	1.1615
						87	1.0596	1.1100	1.1174	1.1196	1.1202
30	.6788	.6000	.5717	.5586	.5506	86	1.0446	1.0800	1.0819	1.0813	1.0798
29	.7076	.6300	.6018	.5880	.5846	85	1.0286	1.0500	1.0469	1.0437	1.0413
28	.7360	.6600	.6315	.6178	.6095						
27	.7635	.6900	.6619	.6480	.6395	84	1.0118	1.0200	1.0125	1.0073	1.0032
26	.7905	.7200	.6919	.6782	.6703	83	.9940	.9900	.9782	.9718	.9673
						82	.9748	.9600	.9453	.9367	.9315
25	.8164	.7500	.7227	.7093	.7011	81	.9555	.9300	.9123	.9028	.8966
24	.8416	.7800	.7535	.7403	.7320	80	.9342	.9000	.8798	.8693	.8626
23	.8661	.8100	.7846	.7717	.7642						
	.8896	.8400	.8161	.8040	.7964	79	.9122	.8700	.8479	.8363	.8290
21	.9122	.8700	.8479	.8363	.8290	78	.8896	.8400	.8161	.8040	.7964
						77	.8661	.8100	.7846	.7717	.7642
						76	.8416	.7800	.7535	.7403	.7320
						75	.8164	.7500	.7227	.7093	.7011
20	.9342	.9000	.8798	.8693	.8626	74	.7905	.7200	.6919	.6782	.6703
19	.9555	.9300	.9123	.9028	.8966	73	.7635	.6900	.6619	.6480	.6395
18	.9748	.9600	.9453	.9367	.9315	72	.7360	.6600	.6315	.6178	.6095
17	.9940	.9900	.9782	.9718	.9673	71	.7076	.6300	.6018	.5880	.5846
16	1.0118	1.0200	1.0125	1.0073	1.0032	70	.6788	.6000	.5717	.5586	.5506
15	1.0286	1.0500	1.0469	1.0437	1.0413	69	.6490	.5700	.5424	.5292	.5211
14	1.0446	1.0800	1.0819	1.0813	1.0798	68	.6187	.5400	.5131	.5002	.4925
13	1.0597	1.1100	1.1174	1.1196	1.1202	67	.5878	.5100	.4837	.4712	.4639
12	1.0732	1.1400	1.1538	1.1592	1.1615	66	.5564	.4800	.4544	.4426	.4354
11	1.0864	1.1700	1.1911	1.2001	1.2045	65	.5242	.4500	.4254	.4140	.4073
10	1.0977	1.2000	1.2293	1.2421	1.2494	64	.4917	.4200	.3968	.3859	.3791
9	1.1087	1.2300	1.2683	1.2866	1.2966	63	.4587	.3900	.3678	.3577	.3515
8	1.1170	1.2600	1.3091	1.3328	1.3465	62	.4252	.3600	.3392	.3295	.3238
7	1.1263	1.2900	1.3510	1.3813	1.3990	61	.3912	.3300	.3106	.3018	.2966
6	1.1330	1.3200	1.3946	1.4332	1.4562	60	.3568	.3000	.2823	.2740	.2689
5	1.1367	1.3500	1.4408	1.4892	1.5184	55	.1806	.1500	.1406	.1364	.1338
4	1.1402	1.3800	1.4898	1.5500	1.5868	50	.0000	.0000	.0000	.0000	.0000
3	1.1439	1.4100	1.5428	1.6190	1.6662						
2	1.1476	1.4400	1.6018	1.6990	1.7615						
1	1.1510	1.4700	1.6719	1.8016	1.8893						

and

$Q_L = (x_i - L)/s_i$

where

x_i = mean of the measurements on the lot;
U = specification upper limit;
L = specification lower limit;
s_i = standard deviation of the measurements on the lot; and
i = 1 or 2, depending on whether gradation results for sample set 1 or 2 were used for determining acceptance.

The value of Q_U thus obtained will be used with Table B-1 to determine the estimated percentage of the material below the upper limit (PWL_U) for the sieve. Similarly, the value of Q_L, used in conjunction with Table B-1, will provide the estimated percentage of the material above the lower limit (PWL_L). In the case of a given sieve, the PWL estimate for the lot will then be calculated as

$PWL = (PWL_U + PWL_L) - 100$

Price Adjustment

The price adjustment for a given sieve size based on the estimated PWL will be determined by reference to the appropriate price adjustment schedule (Table 8).

Multiple Price Adjustments

If the estimated PWL values for a particular lot of material indicate price adjustments for more than one sieve, the total pay factor for the lot will be determined by the smallest individual pay factor (in decimal form). For example, if the estimated PWL values for a lot of aggregate indicated pay factors of 90, 90, and 80 percent for the 3/8-in., No. 4, and No. 8 sieve, respectively, the total pay factor for the lot would be 0.80 (or 80 percent of the unit bid price).

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