Revision of a Flawed Acceptance Standard

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ABSTRACT

A major revision of AASHTO Standard R9-84, Acceptance Sampling Plans for Highway Construction, has just been completed. The primary goals were to correct a major conceptual error and to reduce the level of complexity. In this paper the flaws in the original version are discussed, the basic changes that were made are described, and a significant addition to the new standard is presented. This addition is operating characteristic tables that enable the user to quickly and easily select acceptance plans that will provide the desired degree of quality assurance. Computer simulation is used to demonstrate that single-limit variables operating characteristic curves are sufficiently accurate for most double-limit applications. Two examples are included to illustrate the use of the revised standard.

In the early 1960s, the AASHO Road Test produced a wealth of statistical data that could be used to relate pavement quality to performance. Highway engineers began to recognize that various desirable quality characteristics could be described statistically, and, toward the end of that decade, several highway agencies had begun to develop acceptance procedures based on statistical concepts. Today, many highway agencies routinely use statistical acceptance procedures in one form or another.

The first statistical acceptance procedures were often far from optimal. Highway engineers were relatively unfamiliar with statistical terms and procedures, especially in regard to the construction of operating characteristic curves and the analysis of risks. Consequently, the early development of statistical specifications consisted largely of a trial-and-error process and several revisions were often required to obtain a workable specification.

More recently, there has been a significant improvement in the manner in which these specifications are developed. Highway engineers have acquired a better understanding of statistical methods (1-3) and the computer has emerged as a valuable aid (4,5) in performing much of the development and analysis work. The state of the art has now progressed to the extent that statistical specification writing must be regarded as a thoroughly scientific activity.

AASHTO Standard R9-84, Acceptance Sampling Plans for Highway Construction (6), was adopted in 1984 to document and standardize practices that had evolved over the previous two decades. It covers both attributes sampling for defects that are counted and variables sampling for characteristics that are measured on a continuous scale. Primary source documents for these two approaches are Military Standard 105 for attributes sampling (7) and Military Standard 414 for variables sampling (8), both published by the U.S. Department of Defense. The theory underlying attributes sampling is relatively simple and is covered in connection with the hypergeometric distribution in many texts on statistics and quality assurance (9-11). The theoretical basis for variables sampling is considerably more complex, involving both the beta and the noncentral t distributions, and is not as well known (11-13).

Unfortunately, the current version of Standard R9 is seriously flawed, both by what it includes and by what it omits. It is based on an early method that contains both technical and conceptual errors and it fails to cover the analysis of operating characteristic curves, one of the most important steps in the development of any acceptance procedure. A higher level of technical competence must be demanded of a work that is to serve as a procedural guide for the highway quality assurance profession.

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BASIC PROBLEMS AND CORRECTIVE MEASURES

The original developers of the methodology used in AASHTO Standard R9 undoubtedly had nothing but the best of intentions. At a time when statistical procedures were new and unfamiliar, and considerable resistance to the new methods was often encountered, it was understandably tempting to make various seemingly harmless modifications to make these procedures more palatable. Obviously, the arbitrary modification of any highly technical procedure by practitioners unfamiliar with the underlying theory is a dangerous business and, not surprisingly, the validity of some of these methods was seriously compromised. This is essentially what happened in the development of the procedures used in Standard R9. Specific shortcomings and the necessary corrective measures are as follows:

1. Both the attributes and variables plans described in Standard R9 are designed to control percent defective, the percentage of the lot falling outside a lower or upper specification limit, or outside both lower and upper specification limits, as illustrated in Figure 1. As presently written, however, Standard R9 is oriented partly toward percent defective and partly toward population means, which leads to considerable confusion. For example, it is stated in the current standard that, for a variables plan with the standard deviation unknown, only one risk (buyer's or seller's) can be controlled. Indeed, when quality is measured in terms of percent defective, both the buyer's risk and the seller's risk can be controlled by either variables or attributes plans. This basic contradiction has been corrected by basing the revised standard entirely on the percent defective parameter.

2. A major omission in the current standard is a convenient method of constructing the operating characteristic (OC) curves for the acceptance plans that are developed. OC curves give the probability

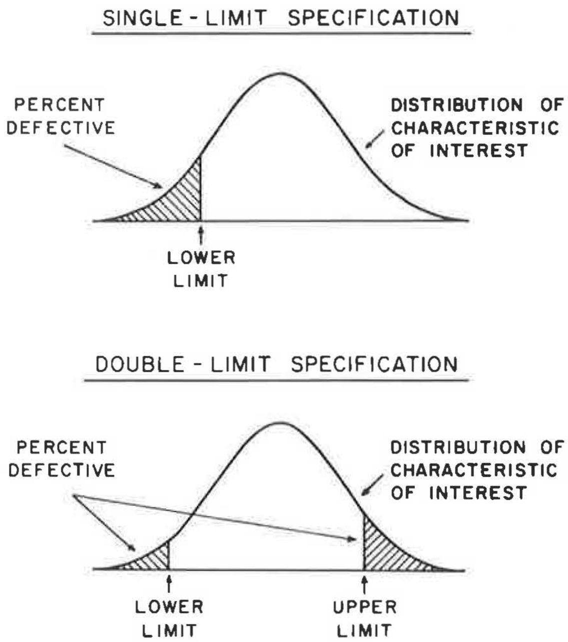

FIGURE 1 Illustration of the concept of percent defective.

of acceptance associated with various levels of submitted quality and provide a graphic representation of an acceptance plan's ability to discriminate between acceptable and unacceptable work. A typical example is shown in Figure 2. The importance of examining OC curves cannot be overemphasized. In this manner, the risks to both the specifying agency and the contractor can be determined in advance and modifications to the acceptance plan can be made, if necessary, before embarrassing and troublesome situations arise in the field. This shortcoming of the current standard has been corrected by the development of several new operating characteristic tables for both attributes and variables plans.

3. When constructing an OC curve for a variables plan, the problem cited in Item 1 becomes much more apparent. Because the variables approach was derived to control percent defective, there is a unique probability of acceptance associated with any particular level of lot percent defective, as can be seen in Figure 2. (This is precisely correct for single-limit plans and is approximately correct for double-limit plans.) However, if the acceptance procedure were oriented around population means, as it is in the current version of Standard R9, there would no longer be a unique OC curve because each level of population mean could correspond to a wide range of percent defective, depending on the value of the population standard deviation. Rewriting the standard entirely around the percent defective parameter has corrected this problem.

4. The table for the estimation of percent defective in the current version of the standard is not in the most logical or useful form and it omits several potentially useful sample sizes. The new table includes several additional sample sizes, it is accurate to a greater number of decimal places, and two revised formats are provided.

5. The current table for attributes sampling was taken from Military Standard 105 (7). It gives the recommended sample size and acceptance number (maximum allowable number of defective items in a sample) based on lot size and the user's definition of acceptable quality level (AQL). In its present form, it does not allow the user to know or control the risks that are involved and, as in the percent de-

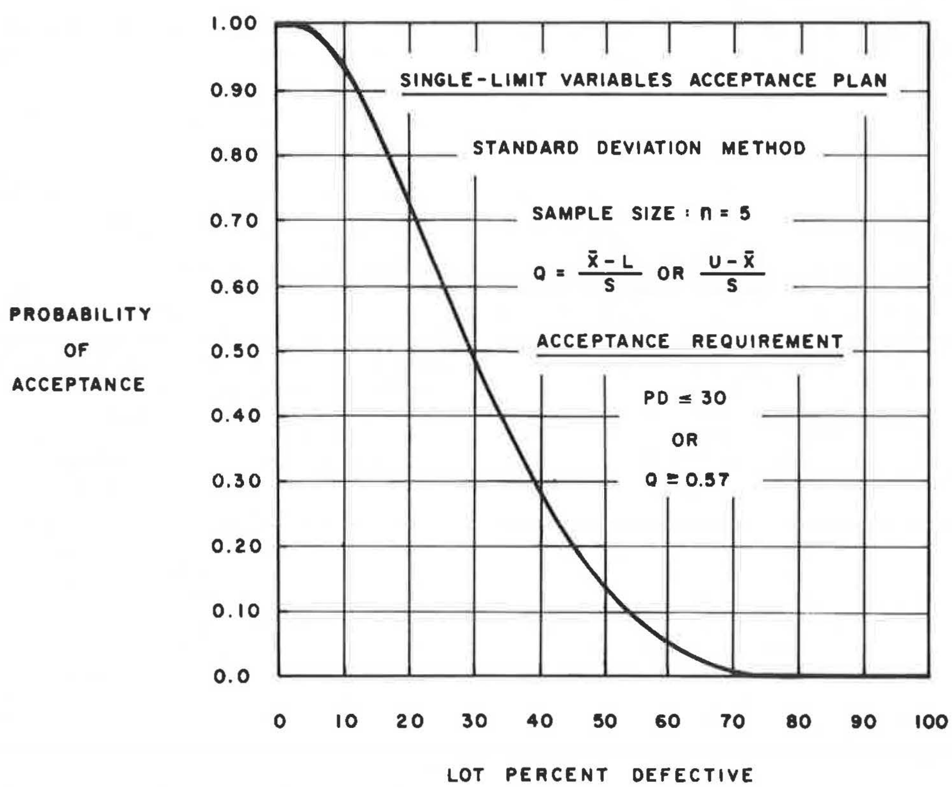

FIGURE 2 Typical operating characteristic curve for a variables acceptance plan.

fective estimation table, several useful sample sizes have been omitted. This table has been completely revised to be more suitable for highway construction applications.

6. The current version of the standard emphasizes the range method for variables acceptance plans. The standard deviation method is included, but the user is required to estimate the standard deviation from the range. For this reason, the current version fails to capitalize on the standard deviation's superior mathematical efficiency. To realize the cost savings associated with the smaller sample sizes required with the standard deviation method, this is made the primary procedure in the revised version of the standard. The range method has been retained and some new tables have been provided, but this procedure has now been relegated to an appendix.

7. If an acceptance specification developed by the method outlined in the current version of Standard R9 were to be challenged in court, it is possible that the weaknesses in the standard could be used to attack the validity of the specification. Although it is true that the acceptance plan could be perfectly satisfactory even though the methodology used to develop it was flawed, the highway agency might still be cast in an unfavorable light. This potential vulnerability can be avoided by using valid statistical procedures in a rigorous fashion. It is believed that the revised version of Standard R9 will encourage the proper use of these methods.

8. Finally, a major drawback of the present version of Standard R9 is its technical complexity. At best, it will fail to promote a wider use and acceptance of statistical quality assurance and, at worst, it could even be a deterrent. A primary goal in rewriting the standard was to make it considerably easier to understand and use.

DEVELOPMENT OF NEW TABLES

To correct the deficiencies of the current standard, it was first necessary to develop several new tables. These form the core around which the rest of the standard has been constructed and are discussed in the order in which they appear in the appendices of the revised standard.

In Appendix A of the new standard, the previous table for attributes sampling has been replaced with operating characteristic tables that give probability of acceptance for selected levels of population (lot) percent defective for many different combinations of sample size and acceptance number. Those plans that have relatively undesirable OC curves have been omitted, and not all plans in these tables will be suitable for all situations. The primary benefit of the new tables is that it is possible to tell at a glance how different plans will perform over a wide range of submitted quality.

The new attributes tables appear in four sections, one each for lot sizes of 20, 100, 500, and infinity. Two of these tables, for lot sizes of 100 and infinity, are shown in Figures 3 and 4. The tables are constructed so that it will never be necessary to interpolate between acceptance numbers or between sample sizes up to a sample size of n = 10. Some interpolation may be necessary for larger sample sizes or for specific lot sizes, although the OC curves are relatively insensitive to lot size. For plans with variable lot sizes, it will be necessary to plot bounding OC curves.

Appendix B of the new standard contains the corresponding operating characteristic tables for variables acceptance plans (standard deviation method), one of which is shown in Figure 5. The acceptance plans in these tables are specified by sample size and either the maximum allowable estimated percent defective (M) or the minimum allowable value (k) of the quality index (Q). The quality index is computed by Equation 1 or 2, as appropriate.

$$Q_L = (\bar{X} - L)/S \quad (1)$$

$$Q_U = (U - \bar{X})/S \quad (2)$$

where

Q = quality index,
$\bar{X}$ = sample mean,
S = sample standard deviation, and
L, U = lower and upper specification limits outside of which the material or work is defined as defective.

Because variables plans deal with continuous data, there are an infinite number of plans that might be used and it will occasionally be necessary to interpolate between the acceptance parameters shown in Figure 5. The operating characteristic tables for variables plans include a wide range of acceptance plans and, like the attributes tables, not all plans will be suitable for all situations.

Appendix C of the new standard provides a more complete table for the estimation of lot percent defective (standard deviation method). This table is the equivalent of Table B5 in Military Standard 414 on variables sampling (8) except that it includes several useful sample sizes that were omitted in both Military Standard 414 and AASHTO Standard R9. The new table consists of five sections, one of which is shown in Figure 6.

The percent defective estimation tables in Appendix C of the new standard cover a wide range of sample sizes, considerably more than would ever be used in a single acceptance procedure. For acceptance procedures that make use of only one or two sample sizes, it is possible to construct much more compact tables such as the one shown in Figure 7. With this format, there is a separate short table for each sample size.

Appendix D of the new standard contains two tables that have been developed for use with variables procedures based on the range as the measure of variability. The first, shown in Figure 8, gives the operating characteristics for a wide selection of range plans. The largest sample size included in this table is n = 15 because, above that sample size, range plans are considerably less efficient than standard deviation plans. The second, shown in Figure 9, gives the estimate of lot percent defective associated with the quality index (Q) computed by the range method in accordance with Equations 3 and 4. Because the range tends to be larger than the standard deviation, the Q-values tend to be smaller, and the table is more compact than its counterpart for the standard deviation method. Also, because it is believed that some precision is lost in adapting the standard deviation algorithms to construct the range table, the percent defective estimates in the body of the table have been printed to only a single decimal place.

$$Q_L = (\bar{X} - L)/R \quad (3)$$

$$Q_U = (U - \bar{X})/R \quad (4)$$

where

Q = quality index;

$\overline{X}$ = sample mean;
R = sample range, difference between largest and smallest values in the sample; and
L, U = lower and upper specification limits outside of which the material or work is defined as defective.

Still another useful format for operating characteristic tables is shown in Figure 10, although this particular version has not been included in the new standard. Whereas the more customary format lists lot percent defective in the heading of the table and probability of acceptance in the body of the table, this version does just the opposite. The advantage of this format is that it always provides an ample number of plotting points spaced conveniently throughout the length of each OC curve, a refinement that is especially useful when a wide range of sample sizes is included in a single table. This approach is appropriate primarily for variables plans, but it is also suitable for attributes plans when the lot size is divisible by 100.

ATTRIBUTES ACCEPTANCE PLANS LOT SIZE = 100

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE 5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.21	0.16	0.12	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.85	0.79	0.72	0.65	0.58	0.50	0.42	0.35	0.28	0.21
4	0	0.81	0.65	0.52	0.40	0.31	0.23	0.17	0.12	0.09	0.06	0.04	0.02	0.01	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.47	0.39	0.31	0.24	0.17	0.12	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.42	0.33	0.25	0.18	0.13	0.08	0.05	0.03
5	2	1.00	0.99	0.98	0.95	0.90	0.84	0.77	0.69	0.60	0.50	0.40	0.31	0.23	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.41	0.31	0.23	0.16	0.10	0.06	0.04	0.02	0.01
6	2	1.00	0.99	0.96	0.91	0.84	0.75	0.65	0.54	0.44	0.34	0.25	0.17	0.11	0.06
7	1	0.96	0.86	0.72	0.57	0.44	0.32	0.22	0.15	0.09	0.06	0.03	0.02	0.01	0.00
7	2	1.00	0.98	0.93	0.86	0.76	0.65	0.53	0.42	0.31	0.22	0.14	0.09	0.05	0.02
8	1	0.95	0.82	0.66	0.50	0.36	0.24	0.16	0.10	0.06	0.03	0.01	0.01	0.00	0.00
8	2	1.00	0.97	0.90	0.80	0.68	0.55	0.42	0.31	0.21	0.13	0.08	0.04	0.02	0.01
8	3	1.00	1.00	0.98	0.95	0.90	0.81	0.71	0.60	0.48	0.36	0.25	0.16	0.10	0.05
9	1	0.94	0.78	0.60	0.43	0.29	0.18	0.11	0.06	0.03	0.02	0.01	0.00	0.00	0.00
9	2	1.00	0.96	0.87	0.74	0.60	0.46	0.33	0.22	0.14	0.08	0.04	0.02	0.01	0.00
9	3	1.00	0.99	0.97	0.92	0.84	0.74	0.61	0.48	0.35	0.24	0.15	0.09	0.05	0.02
10	1	0.92	0.74	0.54	0.36	0.23	0.14	0.07	0.04	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.94	0.83	0.68	0.52	0.37	0.25	0.15	0.09	0.05	0.02	0.01	0.00	0.00
10	3	1.00	0.99	0.96	0.89	0.79	0.65	0.51	0.37	0.25	0.16	0.09	0.05	0.02	0.01
15	2	0.98	0.83	0.60	0.38	0.21	0.11	0.05	0.02	0.01	0.00	0.00	0.00	0.00	0.00
15	3	1.00	0.96	0.84	0.65	0.45	0.28	0.15	0.07	0.03	0.01	0.00	0.00	0.00	0.00
15	4	1.00	0.99	0.95	0.85	0.70	0.51	0.34	0.20	0.10	0.05	0.02	0.01	0.00	0.00
15	5	1.00	1.00	0.99	0.95	0.87	0.73	0.57	0.39	0.24	0.13	0.06	0.02	0.01	0.00
20	2	0.95	0.68	0.38	0.18	0.07	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.99	0.89	0.65	0.39	0.20	0.08	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.97	0.85	0.64	0.40	0.21	0.09	0.03	0.01	0.00	0.00	0.00	0.00	0.00
20	5	1.00	1.00	0.95	0.83	0.62	0.40	0.22	0.10	0.04	0.01	0.00	0.00	0.00	0.00
20	6	1.00	1.00	0.99	0.94	0.81	0.62	0.40	0.22	0.10	0.04	0.01	0.00	0.00	0.00
30	3	0.97	0.65	0.28	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	1.00	0.86	0.51	0.21	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.96	0.73	0.40	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.99	0.89	0.62	0.31	0.12	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	1.00	0.96	0.80	0.51	0.24	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.99	0.91	0.70	0.41	0.18	0.06	0.01	0.00	0.00	0.00	0.00	0.00
50	5	1.00	0.63	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	1.00	0.84	0.29	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.95	0.50	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.99	0.71	0.23	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	1.00	0.87	0.40	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	1.00	0.95	0.60	0.18	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.99	0.77	0.32	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	8	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION BUT IS DEPENDENT UPON LOT SIZE FOR ATTRIBUTES PLANS. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO CONSTRUCT BOUNDING OPERATING CHARACTERISTIC CURVES.

FIGURE 3 Operating characteristics of attributes acceptance plans with a lot size of n = 100.

OPERATING CHARACTERISTICS OF DOUBLE-LIMIT PLANS

Acceptance plans that have both lower and upper limits are referred to as double-limit plans. The operating characteristics for attributes plans shown in Figures 3 and 4 are correct for both single-limit and double-limit plans. For double-limit variables plans, there is no unique operating characteristic curve because probability of acceptance is influenced in part by the manner in which the percent defective is distributed between the two tails of the population. There exists, instead, a band of OC curves for each double-limit variables plan. It has been found (11,p.246), however, that this band is quite narrow and that the single-limit OC curves are sufficiently accurate for most double-limit applications. The table, generated by computer simulation, that is shown in Figure 11 provides a convincing demonstration of this fortunate property.

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = INFINITE

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.22	0.17	0.13	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.84	0.78	0.72	0.65	0.57	0.50	0.43	0.35	0.28	0.22
4	0	0.81	0.66	0.52	0.41	0.32	0.24	0.18	0.13	0.09	0.06	0.04	0.03	0.02	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.48	0.39	0.31	0.24	0.18	0.13	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.43	0.34	0.26	0.19	0.13	0.09	0.05	0.03
5	2	1.00	0.99	0.97	0.94	0.90	0.84	0.76	0.68	0.59	0.50	0.41	0.32	0.24	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.42	0.32	0.23	0.16	0.11	0.07	0.04	0.02	0.01
6	2	1.00	0.98	0.95	0.90	0.83	0.74	0.65	0.54	0.44	0.34	0.26	0.18	0.12	0.07
7	1	0.96	0.85	0.72	0.58	0.44	0.33	0.23	0.16	0.10	0.06	0.04	0.02	0.01	0.00
7	2	1.00	0.97	0.93	0.85	0.76	0.65	0.53	0.42	0.32	0.23	0.15	0.10	0.06	0.03
8	1	0.94	0.81	0.66	0.50	0.37	0.26	0.17	0.11	0.06	0.04	0.02	0.01	0.00	0.00
8	2	0.99	0.96	0.89	0.80	0.68	0.55	0.43	0.32	0.22	0.14	0.09	0.05	0.03	0.01
8	3	1.00	0.99	0.98	0.94	0.89	0.81	0.71	0.59	0.48	0.36	0.26	0.17	0.11	0.06
9	1	0.93	0.77	0.60	0.44	0.30	0.20	0.12	0.07	0.04	0.02	0.01	0.00	0.00	0.00
9	2	0.99	0.95	0.86	0.74	0.60	0.46	0.34	0.23	0.15	0.09	0.05	0.03	0.01	0.00
9	3	1.00	0.99	0.97	0.91	0.83	0.73	0.61	0.48	0.36	0.25	0.17	0.10	0.05	0.03
10	1	0.91	0.74	0.54	0.38	0.24	0.15	0.09	0.05	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.93	0.82	0.68	0.53	0.38	0.26	0.17	0.10	0.05	0.03	0.01	0.00	0.00
10	3	1.00	0.99	0.95	0.88	0.78	0.65	0.51	0.38	0.27	0.17	0.10	0.05	0.03	0.01
15	2	0.96	0.82	0.60	0.40	0.24	0.13	0.06	0.03	0.01	0.00	0.00	0.00	0.00	0.00
15	3	0.99	0.94	0.82	0.65	0.46	0.30	0.17	0.09	0.04	0.02	0.01	0.00	0.00	0.00
15	4	1.00	0.99	0.94	0.84	0.69	0.52	0.35	0.22	0.12	0.06	0.03	0.01	0.00	0.00
15	5	1.00	1.00	0.98	0.94	0.85	0.72	0.56	0.40	0.26	0.15	0.08	0.03	0.01	0.00
20	2	0.92	0.68	0.40	0.21	0.09	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.98	0.87	0.65	0.41	0.23	0.11	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.96	0.83	0.63	0.41	0.24	0.12	0.05	0.02	0.01	0.00	0.00	0.00	0.00
20	5	1.00	0.99	0.93	0.80	0.62	0.42	0.25	0.13	0.06	0.02	0.01	0.00	0.00	0.00
20	6	1.00	1.00	0.98	0.91	0.79	0.61	0.42	0.25	0.13	0.06	0.02	0.01	0.00	0.00
30	3	0.94	0.65	0.32	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	0.98	0.82	0.52	0.26	0.10	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.93	0.71	0.43	0.20	0.08	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.97	0.85	0.61	0.35	0.16	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	0.99	0.93	0.76	0.51	0.28	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.97	0.87	0.67	0.43	0.22	0.09	0.03	0.01	0.00	0.00	0.00	0.00
50	5	0.96	0.62	0.22	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	0.99	0.77	0.36	0.10	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.88	0.52	0.19	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.94	0.67	0.31	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	0.98	0.79	0.44	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	0.99	0.88	0.58	0.26	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.94	0.71	0.38	0.14	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
100	8	0.94	0.32	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	0.97	0.45	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	0.99	0.58	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	0.70	0.16	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	0.80	0.25	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	0.88	0.35	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	0.93	0.46	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	0.96	0.57	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	0.98	0.67	0.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	0.99	0.76	0.27	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	0.84	0.36	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION BUT IS DEPENDENT UPON LOT SIZE FOR ATTRIBUTES PLANS. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO CONSTRUCT BOUNDING OPERATING CHARACTERISTIC CURVES.

FIGURE 4 Operating characteristics of attributes acceptance plans with an infinite lot size.

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** STANDARD DEVIATION METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
3	34	0.556	0.89	0.71	0.52	0.35	0.22	0.12	0.05
3	36	0.492	0.91	0.74	0.56	0.39	0.24	0.13	0.06
3	38	0.425	0.93	0.78	0.60	0.42	0.27	0.15	0.07
3	40	0.357	0.95	0.81	0.64	0.46	0.30	0.17	0.08
3	42	0.287	0.96	0.84	0.68	0.50	0.33	0.20	0.09
3	44	0.216	0.97	0.87	0.71	0.54	0.37	0.22	0.11
3	46	0.145	0.98	0.89	0.75	0.58	0.41	0.26	0.13
3	48	0.073	0.98	0.91	0.79	0.63	0.46	0.29	0.15
4	28	0.660	0.88	0.66	0.44	0.27	0.14	0.06	0.02
4	30	0.600	0.91	0.70	0.48	0.29	0.16	0.07	0.02
4	32	0.540	0.93	0.74	0.52	0.33	0.18	0.08	0.03
4	34	0.480	0.94	0.77	0.56	0.36	0.20	0.10	0.03
4	36	0.420	0.96	0.81	0.60	0.40	0.23	0.11	0.04
4	38	0.360	0.97	0.84	0.64	0.44	0.26	0.13	0.05
4	40	0.300	0.97	0.86	0.69	0.48	0.30	0.15	0.06
4	42	0.240	0.98	0.89	0.72	0.53	0.33	0.18	0.07
4	44	0.180	0.99	0.91	0.76	0.57	0.37	0.20	0.09
4	46	0.120	0.99	0.93	0.79	0.61	0.41	0.23	0.10
5	26	0.692	0.89	0.65	0.40	0.22	0.10	0.04	0.01
5	28	0.632	0.92	0.69	0.45	0.25	0.12	0.04	0.01
5	30	0.572	0.94	0.73	0.49	0.28	0.14	0.05	0.01
5	32	0.513	0.95	0.77	0.54	0.32	0.16	0.06	0.02
5	34	0.455	0.96	0.81	0.58	0.36	0.18	0.07	0.02
5	36	0.397	0.97	0.84	0.63	0.40	0.21	0.09	0.03
5	38	0.339	0.98	0.87	0.67	0.44	0.25	0.11	0.03
5	40	0.282	0.99	0.90	0.71	0.49	0.28	0.13	0.04
5	42	0.225	0.99	0.92	0.75	0.54	0.32	0.15	0.05
5	44	0.169	0.99	0.93	0.79	0.58	0.36	0.18	0.06
6	24	0.740	0.89	0.62	0.35	0.17	0.06	0.02	0.00
6	26	0.678	0.92	0.67	0.40	0.20	0.08	0.02	0.00
6	28	0.618	0.94	0.71	0.45	0.23	0.10	0.03	0.01
6	30	0.558	0.95	0.76	0.49	0.27	0.11	0.04	0.01
6	32	0.500	0.97	0.80	0.55	0.31	0.14	0.05	0.01
6	34	0.442	0.98	0.84	0.60	0.35	0.16	0.06	0.01
6	36	0.386	0.98	0.87	0.64	0.39	0.19	0.07	0.02
6	38	0.329	0.99	0.90	0.69	0.44	0.23	0.09	0.02
6	40	0.274	0.99	0.92	0.74	0.49	0.27	0.11	0.03
6	42	0.219	1.00	0.94	0.78	0.54	0.31	0.13	0.04
7	22	0.796	0.88	0.57	0.29	0.12	0.04	0.01	0.00
7	24	0.732	0.91	0.63	0.34	0.15	0.05	0.01	0.00
7	26	0.670	0.93	0.68	0.39	0.18	0.06	0.02	0.00
7	28	0.610	0.95	0.73	0.44	0.21	0.08	0.02	0.00
7	30	0.550	0.97	0.78	0.50	0.25	0.10	0.03	0.00
7	32	0.492	0.98	0.82	0.55	0.29	0.12	0.04	0.01
7	34	0.435	0.98	0.86	0.61	0.34	0.15	0.05	0.01
7	36	0.379	0.99	0.89	0.66	0.39	0.18	0.06	0.01
7	38	0.324	0.99	0.91	0.71	0.44	0.21	0.07	0.02
7	40	0.269	1.00	0.93	0.75	0.49	0.25	0.09	0.02
8	22	0.792	0.90	0.58	0.28	0.11	0.03	0.01	0.00
8	24	0.727	0.92	0.64	0.33	0.13	0.04	0.01	0.00
8	26	0.665	0.95	0.70	0.38	0.16	0.05	0.01	0.00
8	28	0.604	0.96	0.75	0.44	0.20	0.07	0.01	0.00
8	30	0.545	0.98	0.80	0.50	0.24	0.08	0.02	0.00
8	32	0.488	0.98	0.84	0.56	0.28	0.11	0.03	0.00
8	34	0.431	0.99	0.87	0.62	0.33	0.13	0.04	0.01
8	36	0.375	0.99	0.90	0.67	0.38	0.16	0.05	0.01
8	38	0.320	1.00	0.93	0.72	0.44	0.20	0.06	0.01
9	20	0.855	0.87	0.52	0.22	0.07	0.02	0.00	0.00
9	22	0.788	0.91	0.58	0.27	0.09	0.02	0.00	0.00
9	24	0.724	0.94	0.65	0.32	0.12	0.03	0.01	0.00
9	26	0.661	0.96	0.71	0.38	0.15	0.04	0.01	0.00
9	28	0.601	0.97	0.76	0.44	0.18	0.05	0.01	0.00
9	30	0.542	0.98	0.81	0.50	0.22	0.07	0.01	0.00
9	32	0.484	0.99	0.85	0.56	0.27	0.09	0.02	0.00
9	34	0.428	0.99	0.89	0.62	0.32	0.12	0.03	0.00
9	36	0.373	1.00	0.92	0.68	0.38	0.15	0.04	0.01
10	20	0.853	0.89	0.51	0.21	0.06	0.01	0.00	0.00
10	22	0.786	0.92	0.59	0.26	0.08	0.02	0.00	0.00
10	24	0.721	0.95	0.65	0.31	0.10	0.02	0.00	0.00
10	26	0.659	0.97	0.72	0.37	0.13	0.03	0.01	0.00
10	28	0.598	0.98	0.77	0.43	0.17	0.05	0.01	0.00
10	30	0.539	0.99	0.82	0.50	0.21	0.06	0.01	0.00
10	32	0.482	0.99	0.87	0.57	0.26	0.08	0.02	0.00
10	34	0.426	1.00	0.90	0.63	0.31	0.11	0.02	0.00

THE ACCEPTANCE PROBABILITIES IN THIS TABLE ARE ACCURATE FOR SINGLE-LIMIT PLANS AND ARE APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE-LIMIT PLANS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT PLANS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

FIGURE 5 Operating characteristics of variables acceptance plans (standard deviation method).

VARIABILITY-UNKNOWN PROCEDURE | STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	15	20	30	50	100
0.0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
0.01	49.72	49.67	49.64	49.63	49.63	49.62	49.62	49.62	49.61	49.61	49.60	49.60	49.60
0.02	49.45	49.33	49.29	49.27	49.25	49.24	49.24	49.23	49.22	49.21	49.21	49.21	49.20
0.03	49.17	49.00	48.93	48.90	48.88	48.86	48.85	48.85	48.83	48.82	48.81	48.81	48.81
0.04	48.90	48.67	48.58	48.53	48.50	48.49	48.47	48.46	48.44	48.43	48.42	48.41	48.41
0.05	48.62	48.33	48.22	48.16	48.13	48.11	48.09	48.06	48.05	48.04	48.02	48.02	48.01
0.06	48.35	48.00	47.86	47.80	47.75	47.73	47.71	47.70	47.66	47.64	47.63	47.62	47.61
0.07	48.07	47.67	47.51	47.43	47.38	47.35	47.33	47.31	47.27	47.25	47.24	47.22	47.22
0.08	47.79	47.33	47.15	47.06	47.01	46.97	46.95	46.93	46.88	46.86	46.84	46.83	46.82
0.09	47.52	47.00	46.80	46.70	46.63	46.59	46.57	46.54	46.49	46.47	46.45	46.43	46.42
0.10	47.24	46.67	46.44	46.33	46.26	46.22	46.18	46.16	46.10	46.08	46.05	46.04	46.03
0.11	46.96	46.33	46.09	45.96	45.89	45.84	45.80	45.78	45.71	45.69	45.66	45.64	45.63
0.12	46.69	46.00	45.73	45.60	45.51	45.46	45.42	45.40	45.33	45.29	45.27	45.25	45.24
0.13	46.41	45.67	45.38	45.23	45.14	45.08	45.04	45.01	44.94	44.90	44.88	44.86	44.84
0.14	46.13	45.33	45.02	44.86	44.77	44.71	44.66	44.63	44.55	44.51	44.48	44.46	44.45
0.15	45.85	45.00	44.67	44.50	44.40	44.33	44.29	44.25	44.16	44.13	44.09	44.07	44.05
0.16	45.58	44.67	44.31	44.13	44.03	43.96	43.91	43.87	43.78	43.74	43.70	43.68	43.66
0.17	45.30	44.33	43.96	43.77	43.65	43.58	43.53	43.49	43.39	43.35	43.31	43.29	43.27
0.18	45.02	44.00	43.60	43.40	43.28	43.21	43.15	43.11	43.01	42.96	42.92	42.89	42.88
0.19	44.74	43.67	43.25	43.04	42.91	42.83	42.77	42.73	42.62	42.57	42.53	42.50	42.48
0.20	44.46	43.33	42.90	42.68	42.54	42.46	42.40	42.35	42.24	42.19	42.15	42.11	42.09
0.21	44.18	43.00	42.54	42.31	42.17	42.08	42.02	41.97	41.85	41.80	41.76	41.73	41.70
0.22	43.90	42.67	42.19	41.95	41.80	41.71	41.64	41.60	41.47	41.42	41.37	41.34	41.31
0.23	43.62	42.33	41.84	41.59	41.44	41.34	41.27	41.22	41.09	41.03	40.98	40.95	40.93
0.24	43.34	42.00	41.48	41.22	41.07	40.97	40.89	40.84	40.71	40.65	40.60	40.56	40.54
0.25	43.05	41.67	41.13	40.86	40.70	40.59	40.52	40.47	40.33	40.27	40.22	40.18	40.15
0.26	42.77	41.33	40.78	40.50	40.33	40.22	40.15	40.09	39.95	39.89	39.83	39.79	39.77
0.27	42.49	41.00	40.43	40.14	39.97	39.85	39.77	39.72	39.57	39.50	39.45	39.41	39.38
0.28	42.20	40.67	40.08	39.78	39.60	39.48	39.40	39.34	39.19	39.12	39.07	39.03	39.00
0.29	41.92	40.33	39.72	39.42	39.23	39.11	39.03	38.97	38.81	38.75	38.69	38.65	38.62
0.30	41.63	40.00	39.37	39.06	38.87	38.75	38.66	38.60	38.44	38.37	38.31	38.26	38.24
0.31	41.35	39.67	39.02	38.70	38.50	38.38	38.29	38.23	38.06	37.99	37.93	37.89	37.86
0.32	41.06	39.33	38.67	38.34	38.14	38.01	37.92	37.86	37.69	37.61	37.55	37.51	37.48
0.33	40.77	39.00	38.32	37.98	37.78	37.65	37.55	37.49	37.31	37.24	37.18	37.13	37.10
0.34	40.49	38.67	37.97	37.62	37.42	37.28	37.19	37.12	36.94	36.87	36.80	36.75	36.72
0.35	40.20	38.33	37.62	37.27	37.05	36.92	36.82	36.75	36.57	36.49	36.43	36.38	36.35
0.36	39.91	38.00	37.28	36.91	36.69	36.55	36.46	36.38	36.20	36.12	36.05	36.01	35.97
0.37	39.62	37.67	36.93	36.55	36.33	36.19	36.09	36.02	35.83	35.75	35.68	35.63	35.60
0.38	39.33	37.33	36.58	36.20	35.98	35.83	35.73	35.65	35.46	35.38	35.31	35.26	35.23
0.39	39.03	37.00	36.23	35.84	35.62	35.47	35.37	35.29	35.10	35.01	34.94	34.89	34.86
0.40	38.74	36.67	35.88	35.49	35.26	35.11	35.00	34.93	34.73	34.65	34.58	34.52	34.49
0.41	38.45	36.33	35.54	35.14	34.90	34.75	34.64	34.57	34.37	34.28	34.21	34.16	34.12
0.42	38.15	36.00	35.19	34.79	34.55	34.39	34.29	34.21	34.00	33.92	33.85	33.79	33.76
0.43	37.85	35.67	34.85	34.43	34.19	34.04	33.93	33.85	33.64	33.56	33.48	33.43	33.39
0.44	37.56	35.33	34.50	34.08	33.84	33.68	33.57	33.49	33.28	33.20	33.12	33.07	33.03
0.45	37.26	35.00	34.16	33.73	33.49	33.33	33.21	33.13	32.92	32.84	32.76	32.71	32.67
0.46	36.96	34.67	33.81	33.38	33.13	32.97	32.86	32.78	32.57	32.48	32.40	32.35	32.31
0.47	36.66	34.33	33.47	33.04	32.78	32.62	32.51	32.42	32.21	32.12	32.04	31.99	31.95
0.48	36.35	34.00	33.12	32.69	32.43	32.27	32.15	32.07	31.85	31.77	31.69	31.63	31.60
0.49	36.05	33.67	32.78	32.34	32.08	31.92	31.80	31.72	31.50	31.41	31.33	31.28	31.24
0.50	35.75	33.33	32.44	32.00	31.74	31.57	31.45	31.37	31.15	31.06	30.98	30.93	30.89
0.51	35.44	33.00	32.10	31.65	31.39	31.22	31.10	31.02	30.80	30.71	30.63	30.57	30.54
0.52	35.13	32.67	31.76	31.31	31.04	30.87	30.76	30.67	30.45	30.36	30.28	30.23	30.19
0.53	34.82	32.33	31.42	30.96	30.70	30.53	30.41	30.32	30.10	30.01	29.93	29.88	29.84
0.54	34.51	32.00	31.08	30.62	30.36	30.18	30.07	29.98	29.76	29.67	29.59	29.53	29.49
0.55	34.20	31.67	30.74	30.28	30.01	29.84	29.72	29.64	29.41	29.32	29.24	29.19	29.15
0.56	33.88	31.33	30.40	29.94	29.67	29.50	29.38	29.29	29.07	28.98	28.90	28.85	28.81
0.57	33.57	31.00	30.06	29.60	29.33	29.16	29.04	28.95	28.73	28.64	28.56	28.51	28.47
0.58	33.25	30.67	29.73	29.26	28.99	28.82	28.70	28.61	28.39	28.30	28.22	28.17	28.13
0.59	32.93	30.33	29.39	28.93	28.66	28.48	28.36	28.28	28.05	27.96	27.89	27.83	27.79
0.60	32.61	30.00	29.05	28.59	28.32	28.15	28.03	27.94	27.72	27.63	27.55	27.50	27.46
0.61	32.28	29.67	28.72	28.25	27.98	27.81	27.69	27.60	27.38	27.30	27.22	27.16	27.13
0.62	31.96	29.33	28.39	27.92	27.65	27.48	27.36	27.27	27.05	26.96	26.89	26.83	26.80
0.63	31.63	29.00	28.05	27.59	27.32	27.15	27.03	26.94	26.72	26.63	26.56	26.50	26.47
0.64	31.30	28.67	27.72	27.26	26.99	26.82	26.70	26.61	26.39	26.31	26.23	26.18	26.14
0.65	30.97	28.33	27.39	26.92	26.66	26.49	26.37	26.28	26.07	25.98	25.90	25.85	25.82
0.66	30.63	28.00	27.06	26.60	26.33	26.16	26.04	25.96	25.74	25.66	25.58	25.53	25.49
0.67	30.30	27.67	26.73	26.27	26.00	25.83	25.72	25.63	25.42	25.33	25.26	25.21	25.17
0.68	29.96	27.33	26.40	25.94	25.68	25.51	25.39	25.31	25.10	25.01	24.94	24.89	24.86
0.69	29.61	27.00	26.07	25.61	25.35	25.19	25.07	24.99	24.78	24.69	24.62	24.57	24.54
0.70	29.27	26.67	25.74	25.29	25.03	24.86	24.75	24.67	24.46	24.38	24.31	24.26	24.23
0.71	28.92	26.33	25.41	24.96	24.71	24.54	24.43	24.35	24.15	24.06	23.99	23.95	23.91
0.72	28.57	26.00	25.09	24.64	24.39	24.23	24.11	24.03	23.83	23.75	23.68	23.64	23.60
0.73	28.22	25.67	24.76	24.32	24.07	23.91	23.80	23.72	23.52	23.44	23.37	23.33	23.30
0.74	27.86	25.33	24.44	24.00	23.75	23.59	23.49	23.41	23.21	23.13	23.07	23.02	22.99
0.75	27.50	25.00	24.11	23.68	23.44	23.28	23.17	23.10	22.90	22.83	22.76	22.72	22.69
0.76	27.13	24.67	23.79	23.37	23.12	22.97	22.86	22.79	22.60	22.52	22.46	22.42	22.39
0.77	26.76	24.33	23.47	23.05	22.81	22.66	22.56	22.48	22.30	22.22	22.16	22.12	22.09
0.78	26.39	24.00	23.15	22.74	22.50	22.35	22.25	22.18	21.99	21.92	21.86	21.82	21.79
0.79	26.02	23.67	22.83	22.42	22.19	22.04	21.94	21.87	21.70	21.62	21.57	21.53	21.50

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

FIGURE 6 First of five tables for estimation of percent defective (standard deviation method).

VARIABILITY-UNKNOWN PROCEDURE — SAMPLE SIZE 5 — STANDARD DEVIATION METHOD

Q	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	50.00	49.64	49.29	48.93	48.58	48.22	47.86	47.51	47.15	46.80
0.1	46.44	46.09	45.73	45.38	45.02	44.67	44.31	43.96	43.60	43.25
0.2	42.90	42.54	42.19	41.84	41.48	41.13	40.78	40.43	40.08	39.72
0.3	39.37	39.02	38.67	38.32	37.97	37.62	37.28	36.93	36.58	36.23
0.4	35.88	35.54	35.19	34.85	34.50	34.16	33.81	33.47	33.12	32.78
0.5	32.44	32.10	31.76	31.42	31.08	30.74	30.40	30.06	29.73	29.39
0.6	29.05	28.72	28.39	28.05	27.72	27.39	27.06	26.73	26.40	26.07
0.7	25.74	25.41	25.09	24.76	24.44	24.11	23.79	23.47	23.15	22.83
0.8	22.51	22.19	21.87	21.56	21.24	20.93	20.62	20.31	20.00	19.69
0.9	19.38	19.07	18.77	18.46	18.16	17.86	17.55	17.25	16.96	16.66
1.0	16.36	16.07	15.78	15.48	15.19	14.91	14.62	14.33	14.05	13.76
1.1	13.48	13.20	12.93	12.65	12.37	12.10	11.83	11.56	11.29	11.02
1.2	10.76	10.50	10.23	9.97	9.72	9.46	9.21	8.96	8.71	8.46
1.3	8.21	7.97	7.73	7.49	7.25	7.02	6.79	6.56	6.33	6.10
1.4	5.88	5.66	5.44	5.23	5.02	4.81	4.60	4.39	4.19	3.99
1.5	3.80	3.61	3.42	3.23	3.05	2.87	2.69	2.52	2.35	2.19
1.6	2.03	1.87	1.72	1.57	1.42	1.28	1.15	1.02	0.89	0.77
1.7	0.66	0.55	0.45	0.36	0.27	0.19	0.12	0.06	0.02	0.00

NUMBERS IN THE BODY OF THE TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF Q, THE QUALITY INDEX. FOR VALUES OF Q GREATER THAN OR EQUAL TO ZERO, THE ESTIMATE OF PERCENT DEFECTIVE IS READ DIRECTLY FROM THE TABLE. FOR VALUES OF Q LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

FIGURE 7 Alternate format for individual tables for estimation of percent defective.

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** RANGE METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
3	34	0.293	0.89	0.71	0.52	0.35	0.22	0.12	0.05
3	36	0.259	0.91	0.74	0.56	0.39	0.24	0.13	0.06
3	38	0.224	0.93	0.78	0.60	0.42	0.27	0.15	0.07
3	40	0.188	0.94	0.81	0.63	0.46	0.30	0.17	0.08
3	42	0.151	0.96	0.84	0.67	0.50	0.33	0.20	0.09
3	44	0.114	0.97	0.87	0.71	0.54	0.37	0.22	0.11
3	46	0.076	0.98	0.89	0.75	0.58	0.41	0.26	0.13
3	48	0.038	0.98	0.91	0.79	0.63	0.46	0.29	0.16
4	30	0.269	0.90	0.70	0.48	0.30	0.16	0.07	0.03
4	32	0.242	0.92	0.73	0.52	0.33	0.18	0.08	0.03
4	34	0.216	0.94	0.77	0.56	0.36	0.21	0.10	0.04
4	36	0.189	0.95	0.80	0.60	0.40	0.23	0.11	0.04
4	38	0.162	0.96	0.83	0.64	0.44	0.26	0.13	0.05
4	40	0.135	0.97	0.86	0.68	0.48	0.30	0.15	0.06
4	42	0.108	0.98	0.89	0.72	0.52	0.33	0.18	0.07
4	44	0.081	0.99	0.91	0.76	0.57	0.37	0.21	0.09
5	26	0.280	0.89	0.65	0.40	0.22	0.10	0.04	0.01
5	28	0.256	0.91	0.69	0.45	0.25	0.12	0.05	0.01
5	30	0.232	0.93	0.73	0.49	0.28	0.14	0.06	0.02
5	32	0.208	0.95	0.77	0.53	0.32	0.16	0.07	0.02
5	34	0.184	0.96	0.80	0.58	0.36	0.19	0.08	0.02
5	36	0.161	0.97	0.84	0.62	0.40	0.22	0.09	0.03
5	38	0.138	0.98	0.87	0.67	0.44	0.25	0.11	0.04
5	40	0.115	0.98	0.89	0.71	0.49	0.28	0.13	0.05
5	42	0.092	0.99	0.91	0.75	0.53	0.32	0.16	0.06
6	24	0.278	0.88	0.61	0.36	0.17	0.07	0.02	0.01
6	26	0.255	0.91	0.66	0.40	0.20	0.09	0.03	0.01
6	28	0.233	0.93	0.71	0.45	0.24	0.10	0.03	0.01
6	30	0.210	0.95	0.75	0.49	0.27	0.12	0.04	0.01
6	32	0.188	0.96	0.79	0.54	0.31	0.14	0.05	0.01
6	34	0.167	0.97	0.83	0.59	0.35	0.17	0.06	0.02
6	36	0.145	0.98	0.86	0.64	0.40	0.20	0.08	0.02
6	38	0.124	0.99	0.89	0.69	0.44	0.23	0.10	0.03
7	24	0.260	0.90	0.62	0.35	0.16	0.06	0.02	0.00
7	26	0.238	0.93	0.67	0.39	0.19	0.07	0.02	0.00
7	28	0.217	0.95	0.72	0.44	0.22	0.09	0.03	0.01
7	30	0.196	0.96	0.77	0.50	0.26	0.11	0.03	0.01
7	32	0.175	0.97	0.81	0.55	0.30	0.13	0.04	0.01
7	34	0.155	0.98	0.85	0.60	0.34	0.16	0.05	0.01
7	36	0.135	0.99	0.88	0.65	0.39	0.19	0.07	0.01
8	22	0.269	0.89	0.57	0.29	0.12	0.04	0.01	0.00
8	24	0.247	0.91	0.63	0.34	0.14	0.05	0.01	0.00
8	26	0.226	0.94	0.69	0.39	0.17	0.06	0.02	0.00
8	28	0.205	0.96	0.74	0.44	0.21	0.08	0.02	0.00
8	30	0.185	0.97	0.78	0.50	0.25	0.09	0.03	0.00
8	32	0.166	0.98	0.82	0.55	0.29	0.12	0.03	0.01
8	34	0.147	0.99	0.86	0.61	0.34	0.14	0.04	0.01

FIGURE 8 Operating characteristics of variables acceptance plans (range method).

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** **RANGE METHOD**

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
9	22	0.257	0.90	0.58	0.28	0.11	0.03	0.01	0.00
9	24	0.236	0.92	0.64	0.33	0.13	0.04	0.01	0.00
9	26	0.216	0.95	0.69	0.38	0.16	0.05	0.01	0.00
9	28	0.196	0.96	0.75	0.44	0.20	0.07	0.02	0.00
9	30	0.177	0.98	0.80	0.50	0.24	0.08	0.02	0.00
9	32	0.158	0.99	0.84	0.56	0.28	0.11	0.03	0.00
9	34	0.140	0.99	0.87	0.61	0.33	0.13	0.04	0.01
10	20	0.269	0.87	0.52	0.23	0.07	0.02	0.00	0.00
10	22	0.248	0.91	0.58	0.27	0.10	0.03	0.00	0.00
10	24	0.228	0.93	0.64	0.32	0.12	0.03	0.01	0.00
10	26	0.208	0.95	0.70	0.38	0.15	0.04	0.01	0.00
10	28	0.189	0.97	0.76	0.44	0.19	0.06	0.01	0.00
10	30	0.171	0.98	0.81	0.50	0.23	0.08	0.02	0.00
10	32	0.153	0.99	0.85	0.56	0.27	0.10	0.02	0.00
11	20	0.261	0.88	0.52	0.21	0.07	0.01	0.00	0.00
11	22	0.241	0.91	0.58	0.26	0.09	0.02	0.00	0.00
11	24	0.221	0.94	0.65	0.32	0.11	0.03	0.00	0.00
11	26	0.202	0.96	0.71	0.38	0.14	0.04	0.01	0.00
11	28	0.183	0.97	0.77	0.44	0.18	0.05	0.01	0.00
11	30	0.165	0.98	0.82	0.50	0.22	0.07	0.01	0.00
11	32	0.148	0.99	0.86	0.56	0.27	0.09	0.02	0.00
12	20	0.255	0.89	0.51	0.21	0.06	0.01	0.00	0.00
12	22	0.235	0.92	0.59	0.26	0.08	0.02	0.00	0.00
12	24	0.215	0.95	0.65	0.31	0.10	0.02	0.00	0.00
12	26	0.197	0.97	0.72	0.37	0.13	0.03	0.01	0.00
12	28	0.179	0.98	0.77	0.43	0.17	0.05	0.01	0.00
12	30	0.161	0.99	0.82	0.50	0.21	0.06	0.01	0.00
13	20	0.24[illegible]	0.89	0.51	0.20	0.05	0.01	0.00	0.00
13	22	0.[illegible]9	0.93	0.59	0.25	0.07	0.01	0.00	0.00
13	24	0.210	0.95	0.66	0.31	0.10	0.02	0.00	0.00
13	26	0.192	0.97	0.72	0.37	0.13	0.03	0.00	0.00
13	28	0.175	0.98	0.78	0.43	0.16	0.04	0.01	0.00
13	30	0.157	0.99	0.83	0.50	0.21	0.06	0.01	0.00
14	20	0.244	0.90	0.51	0.19	0.05	0.01	0.00	0.00
14	22	0.225	0.93	0.59	0.24	0.07	0.01	0.00	0.00
14	24	0.206	0.96	0.66	0.30	0.09	0.02	0.00	0.00
14	26	0.188	0.97	0.73	0.36	0.12	0.03	0.00	0.00
14	28	0.171	0.98	0.79	0.43	0.16	0.04	0.01	0.00
14	30	0.154	0.99	0.84	0.50	0.20	0.05	0.01	0.00
15	20	0.240	0.90	0.51	0.19	0.04	0.01	0.00	0.00
15	22	0.221	0.94	0.59	0.24	0.06	0.01	0.00	0.00
15	24	0.202	0.96	0.67	0.30	0.09	0.02	0.00	0.00
15	26	0.185	0.98	0.73	0.36	0.11	0.02	0.00	0.00
15	28	0.168	0.99	0.79	0.43	0.15	0.03	0.00	0.00

THE ACCEPTANCE PROBABILITIES IN THIS TABLE HAVE BEEN COMPUTED BY INTERPOLATION IN THE NONCENTRAL T DISTRIBUTION USING NONINTEGER DEGREES OF FREEDOM ASSOCIATED WITH RANGE ESTIMATES OF VARIABILITY. THESE PROBABILITY VALUES ARE QUITE ACCURATE FOR SINGLE-LIMIT PLANS AND APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE-LIMIT PLANS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT PLANS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

FIGURE 8 (continued)

UNDERLYING THEORETICAL PRINCIPLES

The operating characteristics for attributes acceptance plans are computed by means of the hypergeometric formula:

$$P = \sum_{x=0}^{x=c} C_{d,x} C_{N-d,n-x} / C_{N,n} \qquad (5)$$

where

- P = probability of acceptance;
- N = population (lot) size;
- n = sample size;
- d = number of defects in the population;
- c = acceptance number, maximum allowable number of defective items in the sample;
- $C_{m,n}$ = number of possible combinations of m items taken n at a time = $m!/[n!(m - n)!]$; and
- x = summation variable.

In terms of the hypergeometric distribution, the lot percent defective would be expressed as 100d/N. This distribution was used to develop the table shown in Figure 3.

As the population size increases, the hypergeometric distribution approaches the binomial distribution as a limit. For very large or infinite lot sizes, the operating characteristics for attributes acceptance plans are computed as follows:

$$P = \sum_{x=0}^{x=c} C_{n,x} p^x (1 - p)^{n-x} \qquad (6)$$

where

- P = probability of acceptance;
- n = sample size;
- p = fraction defective of the population;
- c = acceptance number, maximum allowable number of defective items in the sample;
- $C_{m,n}$ = number of possible combinations of m items taken n at a time = $m!/[n!(m - n)!]$; and
- x = summation variable.

In terms of the binomial distribution, the lot

VARIABILITY-UNKNOWN PROCEDURE | RANGE METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	11	12	13	14	15
0.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
0.01	49.5	49.3	49.1	49.0	48.9	48.9	48.8	48.8	48.7	48.7	48.7	48.7	48.6
0.02	49.0	48.5	48.2	48.1	47.9	47.8	47.7	47.6	47.5	47.4	47.4	47.3	47.3
0.03	48.4	47.8	47.4	47.1	46.9	46.7	46.5	46.4	46.2	46.1	46.1	46.0	45.9
0.04	47.9	47.0	46.5	46.1	45.8	45.6	45.3	45.2	45.0	44.9	44.7	44.6	44.5
0.05	47.4	46.3	45.6	45.1	44.8	44.5	44.2	44.0	43.8	43.6	43.4	43.3	43.2
0.06	46.9	45.6	44.8	44.2	43.7	43.4	43.0	42.8	42.5	42.3	42.1	42.0	41.8
0.07	46.3	44.8	43.9	43.2	42.7	42.3	41.9	41.6	41.3	41.0	40.8	40.6	40.5
0.08	45.8	44.1	43.0	42.2	41.6	41.2	40.7	40.4	40.1	39.8	39.6	39.3	39.1
0.09	45.3	43.3	42.1	41.3	40.6	40.1	39.6	39.2	38.8	38.5	38.3	38.0	37.8
0.10	44.7	42.6	41.3	40.3	39.6	39.0	38.5	38.0	37.6	37.3	37.0	36.7	36.5
0.11	44.2	41.8	40.4	39.4	38.5	37.9	37.3	36.9	36.4	36.1	35.8	35.4	35.2
0.12	43.7	41.1	39.5	38.4	37.5	36.8	36.2	35.7	35.2	34.9	34.5	34.2	33.9
0.13	43.1	40.4	38.7	37.5	36.5	35.8	35.1	34.6	34.1	33.6	33.3	32.9	32.6
0.14	42.6	39.6	37.8	36.5	35.5	34.7	34.0	33.4	32.9	32.5	32.1	31.7	31.4
0.15	42.1	38.9	36.9	35.6	34.5	33.7	32.9	32.3	31.7	31.3	30.9	30.5	30.1
0.16	41.5	38.1	36.1	34.6	33.5	32.6	31.8	31.2	30.6	30.1	29.7	29.3	28.9
0.17	41.0	37.4	35.2	33.7	32.5	31.6	30.8	30.1	29.5	29.0	28.5	28.1	27.7
0.18	40.4	36.6	34.4	32.8	31.5	30.6	29.7	29.0	28.4	27.8	27.4	26.9	26.5
0.19	39.9	35.9	33.5	31.9	30.6	29.5	28.6	27.9	27.3	26.7	26.2	25.8	25.4
0.20	39.3	35.2	32.7	30.9	29.6	28.5	27.6	26.9	26.2	25.6	25.1	24.7	24.2
0.21	38.8	34.4	31.8	30.0	28.6	27.5	26.6	25.8	25.1	24.6	24.1	23.6	23.1
0.22	38.2	33.7	31.0	29.1	27.7	26.6	25.6	24.8	24.1	23.5	23.0	22.5	22.1
0.23	37.7	32.9	30.2	28.2	26.8	25.6	24.6	23.8	23.1	22.5	21.9	21.4	21.0
0.24	37.1	32.2	29.3	27.3	25.8	24.6	23.6	22.8	22.1	21.5	20.9	20.4	20.0
0.25	36.5	31.4	28.5	26.5	24.9	23.7	22.7	21.8	21.1	20.5	19.9	19.4	19.0
0.26	35.9	30.7	27.7	25.6	24.0	22.8	21.7	20.9	20.1	19.5	18.9	18.4	18.0
0.27	35.4	29.9	26.9	24.7	23.1	21.9	20.8	19.9	19.2	18.5	18.0	17.5	17.0
0.28	34.8	29.2	26.0	23.9	22.2	21.0	19.9	19.0	18.3	17.6	17.1	16.5	16.1
0.29	34.2	28.4	25.2	23.0	21.4	20.1	19.0	18.1	17.4	16.7	16.2	15.6	15.2
0.30	33.6	27.7	24.4	22.2	20.5	19.2	18.1	17.2	16.5	15.8	15.3	14.8	14.3
0.31	33.0	27.0	23.6	21.4	19.6	18.4	17.3	16.4	15.6	15.0	14.5	13.9	13.5
0.32	32.3	26.2	22.8	20.5	18.8	17.5	16.4	15.6	14.8	14.2	13.6	13.1	12.7
0.33	31.7	25.5	22.0	19.7	18.0	16.7	15.6	14.7	14.0	13.4	12.8	12.3	11.9
0.34	31.1	24.7	21.2	18.9	17.2	15.9	14.8	14.0	13.2	12.6	12.1	11.6	11.1
0.35	30.4	24.0	20.5	18.1	16.4	15.1	14.0	13.2	12.5	11.8	11.3	10.8	10.4
0.36	29.8	23.2	19.7	17.4	15.6	14.4	13.3	12.4	11.7	11.1	10.6	10.1	9.7
0.37	29.1	22.5	18.9	16.6	14.9	13.6	12.5	11.7	11.0	10.4	9.9	9.4	9.1
0.38	28.5	21.7	18.1	15.8	14.1	12.9	11.8	11.0	10.3	9.7	9.3	8.8	8.4
0.39	27.8	21.0	17.4	15.1	13.4	12.2	11.1	10.3	9.7	9.1	8.6	8.2	7.8
0.40	27.1	20.2	16.6	14.4	12.7	11.5	10.5	9.7	9.0	8.5	8.0	7.6	7.2
0.41	26.4	19.4	15.9	13.6	12.0	10.8	9.8	9.1	8.4	7.9	7.4	7.0	6.7
0.42	25.6	18.7	15.2	12.9	11.3	10.2	9.2	8.5	7.8	7.3	6.9	6.5	6.2
0.43	24.9	17.9	14.4	12.2	10.7	9.5	8.6	7.9	7.3	6.8	6.4	6.0	5.7
0.44	24.1	17.2	13.7	11.6	10.0	8.9	8.0	7.3	6.7	6.3	5.9	5.5	5.2
0.45	23.3	16.4	13.0	10.9	9.4	8.3	7.4	6.8	6.2	5.8	5.4	5.0	4.8
0.46	22.5	15.7	12.3	10.2	8.8	7.8	6.9	6.3	5.7	5.3	4.9	4.6	4.3
0.47	21.7	14.9	11.6	9.6	8.2	7.2	6.4	5.8	5.3	4.7	4.5	4.2	4.0
0.48	20.8	14.1	11.0	9.0	7.6	6.7	5.9	5.3	4.8	4.4	4.1	3.8	3.6
0.49	19.9	13.4	10.3	8.4	7.1	6.2	5.4	4.9	4.4	4.0	3.7	3.5	3.2
0.50	19.0	12.6	9.6	7.8	6.5	5.7	5.0	4.4	4.0	3.7	3.4	3.1	2.9
0.51	18.0	11.9	9.0	7.2	6.0	5.2	4.5	4.0	3.7	3.3	3.1	2.8	2.6
0.52	17.0	11.1	8.3	6.7	5.5	4.8	4.1	3.7	3.3	3.0	2.8	2.5	2.4
0.53	15.9	10.3	7.7	6.2	5.1	4.3	3.8	3.3	3.0	2.7	2.5	2.3	2.1
0.54	14.7	9.6	7.1	5.6	4.6	3.9	3.4	3.0	2.7	2.4	2.2	2.0	1.9
0.55	13.5	8.8	6.5	5.1	4.2	3.6	3.0	2.7	2.4	2.2	2.0	1.8	1.6
0.56	12.1	8.0	5.9	4.7	3.8	3.2	2.7	2.4	2.1	1.9	1.7	1.6	1.5
0.57	10.5	7.2	5.4	4.2	3.4	2.9	2.4	2.1	1.9	1.7	1.5	1.4	1.3
0.58	8.6	6.5	4.8	3.8	3.0	2.6	2.2	1.9	1.7	1.5	1.3	1.2	1.1
0.59	6.2	5.7	4.3	3.3	2.7	2.3	1.9	1.6	1.5	1.3	1.2	1.0	1.0
0.60	1.4	4.9	3.7	2.9	2.4	2.0	1.7	1.4	1.3	1.1	1.0	0.9	0.8
0.61	0.0	4.1	3.2	2.6	2.1	1.7	1.4	1.2	1.1	1.0	0.9	0.8	0.7
0.62	0.0	3.3	2.8	2.2	1.8	1.5	1.2	1.1	0.9	0.8	0.7	0.7	0.6
0.63	0.0	2.5	2.3	1.9	1.5	1.3	1.1	0.9	0.8	0.7	0.6	0.6	0.5
0.64	0.0	1.7	1.9	1.6	1.3	1.1	0.9	0.8	0.7	0.6	0.5	0.5	0.4
0.65	0.0	0.9	1.5	1.3	1.1	0.9	0.7	0.6	0.6	0.5	0.4	0.4	0.4
0.66	0.0	0.0	1.1	1.0	0.9	0.7	0.6	0.5	0.5	0.4	0.4	0.3	0.3
0.67	0.0	0.0	0.8	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3	0.3	0.2
0.68	0.0	0.0	0.4	0.6	0.5	0.5	0.4	0.3	0.3	0.3	0.2	0.2	0.2
0.69	0.0	0.0	0.2	0.4	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2
0.70	0.0	0.0	0.0	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.1	0.1
0.71	0.0	0.0	0.0	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1
0.72	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.73	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.74	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0
0.75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.76	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.77	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

FIGURE 9 Table for estimation of percent defective (range method).

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** STANDARD DEVIATION METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	LOT PERCENT DEFECTIVE VALUES PRODUCING THE LISTED ACCEPTANCE PROBABILITIES								
			0.99	0.95	0.90	0.80	0.50	0.20	0.10	0.05	0.01
3	30	0.679	1	4	7	12	28	48	59	68	81
3	35	0.524	3	7	10	16	32	53	63	71	84
3	40	0.357	4	10	14	21	38	58	67	75	86
3	45	0.181	6	13	18	26	44	63	72	79	88
4	25	0.750	2	5	8	12	25	42	52	60	74
4	30	0.600	3	7	10	15	29	47	56	64	76
4	35	0.450	5	10	14	19	34	51	60	68	79
4	40	0.300	7	13	17	24	39	56	65	71	82
4	45	0.150	9	17	22	29	44	61	69	76	85
5	25	0.723	3	6	9	13	25	40	49	56	69
5	30	0.572	5	9	12	17	30	45	53	60	72
5	35	0.426	7	12	16	21	34	50	58	65	76
5	40	0.282	9	15	20	26	40	55	63	69	79
5	45	0.141	12	19	24	30	45	60	67	73	82
6	20	0.869	2	5	7	11	21	34	42	49	61
6	25	0.709	4	7	10	14	25	39	47	53	65
6	30	0.558	6	10	14	18	30	44	51	58	69
6	35	0.414	8	14	17	23	35	49	56	62	73
6	40	0.274	11	17	21	27	40	54	61	66	76
7	20	0.862	3	6	8	11	21	33	40	46	58
7	25	0.701	5	8	11	15	25	38	45	51	62
7	30	0.550	7	11	15	19	30	43	50	56	66
7	35	0.407	9	15	18	23	35	48	55	60	70
7	40	0.269	12	19	23	28	40	53	59	65	74
8	20	0.858	3	6	9	12	20	32	38	44	55
8	25	0.696	5	9	12	16	25	37	44	49	60
8	30	0.545	8	12	15	20	30	42	48	54	64
8	35	0.403	11	16	19	24	35	47	53	59	68
9	20	0.855	4	7	9	12	20	31	37	43	53
9	25	0.692	6	10	12	16	25	36	42	48	58
9	30	0.542	9	13	16	20	30	41	47	53	62
9	35	0.400	12	17	20	25	35	46	52	57	66
10	20	0.853	4	7	10	13	20	30	36	41	51
10	25	0.690	7	10	13	17	25	35	41	46	56
10	30	0.539	9	14	17	21	30	41	46	51	60
10	35	0.398	12	18	21	25	35	46	51	56	65
15	15	1.037	4	6	8	10	15	23	27	31	39
15	20	0.848	6	9	11	14	20	28	33	37	45
15	25	0.683	9	13	15	18	25	33	38	42	50
15	30	0.533	12	16	19	22	30	39	43	47	55
20	15	1.036	5	7	8	10	15	22	25	29	35
20	20	0.846	7	10	12	15	20	27	31	34	41
20	25	0.680	10	14	16	19	25	32	36	40	46
30	15	1.036	6	8	9	11	15	20	23	26	31
30	20	0.844	9	12	13	15	20	26	29	31	37
30	25	0.678	13	16	18	20	25	31	34	37	42
50	10	1.277	4	6	6	8	10	13	15	17	20
50	15	1.036	7	9	10	12	15	19	21	23	27
50	20	0.843	11	13	15	16	20	24	27	29	32
100	10	1.279	5	7	7	8	10	12	14	15	17
100	15	1.036	9	11	12	13	15	18	19	20	23
100	20	0.842	13	15	16	17	20	23	25	26	29

THE ACCEPTANCE PROBABILITIES IN THE HEADING OF THIS TABLE ARE ACCURATE FOR SINGLE-LIMIT PLANS AND ARE APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE LIMIT APPLICATIONS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT APPLICATIONS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

FIGURE 10 Alternate format for operating characteristic table for variables plans.

percent defective would be expressed as 100p. This distribution was used to develop the table shown in Figure 4.

The estimates of lot percent defective for the standard deviation method contained in the table shown in Figure 6 are obtained by numerically integrating the beta distribution function (13):

$$p = \int_{x=0}^{x=\mathrm{Max}\{0,\ 1/2 - Qn^{1/2}/[2(n-1)]\}} \beta(a,b,x)\,dx \qquad (7)$$

where

p = fraction defective of the population for single-limit applications (for double-limit applications, two separate integration steps must be performed and the results added to obtain the total fraction defective);

$\beta(a,b,x)$ = beta distribution function;

a,b = parameters of the beta distribution = $n/2 - 1$;

VARIABILITY-UNKNOWN PROCEDURE STANDARD DEVIATION METHOD

SAMPLE SIZE	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE	PERCENT DEFECTIVE			PROBABILITY OF ACCEPTANCE	
		LOWER TAIL	UPPER TAIL	TOTAL	SINGLE LIMIT (COMPUTED)	DOUBLE LIMIT (SIMULATED)
3	42	0	10	10	0.96	0.96
3	42	5	5	10	0.96	0.96
3	38	20	0	20	0.78	0.78
3	38	10	10	20	0.78	0.77
3	34	10	50	60	0.12	0.11
3	34	25	35	60	0.12	0.12
5	36	10	0	10	0.97	0.97
5	36	5	5	10	0.97	0.98
5	32	0	30	30	0.54	0.54
5	32	15	15	30	0.54	0.52
5	26	60	0	60	0.04	0.04
5	26	30	30	60	0.04	0.03
10	22	0	10	10	0.92	0.93
10	22	5	5	10	0.92	0.92
10	24	20	0	20	0.65	0.65
10	24	10	10	20	0.65	0.65
10	28	10	30	40	0.17	0.15
10	28	20	20	40	0.17	0.15

EACH SIMULATION RESULT WAS OBTAINED BY INDEPENDENTLY GENERATING 5000 RANDOM SAMPLES OF THE APPROPRIATE SIZE FROM A CONTINUOUS NORMAL POPULATION.

FIGURE 11 Demonstration that single-limit operating characteristic curves are sufficiently accurate for most double-limit variables acceptance plans.

n = sample size;
Q = quality index, $(\bar{X} - L)/S$ or $(U - \bar{X})/S$ for single-limit applications, both required for double-limit applications;
$\bar{X}$ = sample mean;
S = sample standard deviation;
L,U = lower and upper specification limits; and
x = integration variable.

The area under the beta distribution obtained in this manner is the fraction defective that must be multiplied by 100 to yield the estimate of percent defective. Although this integration can be done manually using tables of the beta function (14), it is far more practical to use computer assistance with subroutines developed specifically for this purpose.

The operating characteristics for variables plans based on the standard deviation are obtained by numerically integrating the noncentral t distribution function (12):

$$P = 1 - \int_{x=-\infty}^{x=kn^{1/2}} t(\nu,\delta,x)\,dx \qquad (8)$$

where

P = probability of acceptance,
$t(\nu,\delta,x)$ = noncentral t distribution function,
ν = degrees of freedom = n - 1,
n = sample size,
δ = noncentrality parameter = $K_p n^{1/2}$
K_p = normal z-score associated with each level of population percent defective for which the computation is made,
k = acceptance constant, and
x = integration variable.

If the acceptance procedure is stated in terms of the maximum allowable estimated percent defective (M) rather than the minimum allowable value (k) of the quality index (Q), this must first be converted to a k-value using tables such as those shown in Figures 6, 7, or 9. The integration step indicated in Equation 8 may be performed manually using tables of the noncentral t distribution (12) although, like the integration of the beta distribution in Equation 7, it is much more practical to use computer assistance. The table shown in Figure 5 was generated in this manner.

When these same operations are to be performed for acceptance plans based on the range (R), minor modifications must be made to account for the reduced degrees of freedom associated with range estimates of variability. The following values are obtained from Duncan (11).

Sample Size	Conversion Factor (d_2*)	Degrees of Freedom (range method)
3	1.91	2.0
4	2.24	2.9
5	2.48	3.8
6	2.67	4.7
7	2.83	5.5
8	2.96	6.3
9	3.08	7.0
10	3.18	7.7
11	3.27	8.4
12	3.35	9.0
13	3.42	9.6
14	3.49	10.2
15	3.55	10.8

To obtain estimates of lot percent defective using the range method, the upper integration limit in Equation 7 must be changed (13) to

$$x = \text{Max}\,\{0,\ 1/2 - d_2^*\,Q\,[(\nu + 1)^{1/2}]/2\nu\} \qquad (9)$$

where

x = integration variable;
Q = quality index computed by the range method,

$(\overline{X} - L)/R$ or $(U - \overline{X})/R$ for single-limit applications, both required for double-limit applications;

$d_2{}^*$ = factor that, when divided into the range computed from the sample, converts it into an estimate of the standard deviation; and

ν = degrees of freedom (the appropriate noninteger values associated with the range method must be used).

To develop operating characteristic curves for variables acceptance plans based on the range, Equation 8 may be used except that is is necessary to account for the appropriate noninteger degrees of freedom associated with range estimates of variability (personal conversation with G.J. Resnikoff, California State University, Hayward, 1985). In this case, it is necessary to compute two probability values for integral degrees of freedom in order to obtain the desired value by interpolation.

POTENTIAL PROBLEM WITH VARIABLES PLANS

Although such occurrences are rare, it is possible when using variables acceptance plans that a lot may be judged rejectable even though none of the individual test results falls outside the specification limits. Provided no fundamental assumptions (normal population, random sampling, etc.) have been violated, this is a theoretically correct result. The proper inference is that, based on the mean and standard deviation (or range) estimated from the sample, the population percent defective is unacceptably large.

This same result may also be caused by one or more outliers, test results that deviate unusually far from the norm because of some assignable cause such as equipment malfunction or operator error. Because such a result may be challenged by a contractor who is unfamiliar with its theoretical basis, and may indeed be an indication of a breakdown in the sampling and testing process, it is advisable to investigate and reevaluate any lot rejected in this manner.

PAVEMENT THICKNESS EXAMPLE

A highway agency wishes to develop an acceptance procedure for pavement thickness that is as uncomplicated as possible and involves no statistical calculations or special tables. The pavement will be considered satisfactory if at least 90 percent of it has a thickness greater than the design value. Therefore the acceptable quality level (AQL) may be considered to be 10 percent defective and it is desired that this level of quality have a relatively high probability of acceptance. At the other extreme, if 40 percent of more of the pavement is less than the design thickness, it has been decided that this will be defined as the rejectable quality level (RQL) and a correspondingly low probability of acceptance is desired.

For purposes of this example, suppose that a seller's risk of $\alpha = 0.05$ and a buyer's risk of $\beta = 0.10$ are desired. The corresponding probabilities of acceptance are $P = 0.95$ at the AQL and $P = 0.10$ at the RQL.

The requirement for simplicity dictates an attributes plan. When attributes acceptance procedures are applied to continuous data (thickness in this case), the lot size is considered to be infinite. By scanning the rows and columns of the table shown in Figure 4, it is observed that a plan with a sample size of $n = 15$ and an acceptance number of $c = 3$ produces very nearly the desired risk levels. (Because the sample size and acceptance number are discrete values, it is not possible to match the risks exactly.) The following values are obtained:

Lot Percent Defective	Probability of Acceptance
10 (AQL)	0.94
20	0.65
30	0.30
40 (RQL)	0.09
50	0.02

It can be seen from these values that the basic objectives have been well satisfied. A good quality pavement that has 10 percent or less defective will have a probability of acceptance of at least $P = 0.94$. If the pavement is 40 percent or more defective, the probability of acceptance will be $P = 0.09$ or less.

The completed acceptance procedure will require that $n = 15$ cores be taken at random locations within a specified lot size. Because attributes acceptance theory makes no assumptions about the distributional form of the population, there is considerable latitude to define the lot size in any manner that the highway agency believes is appropriate. Provided that no more than $c = 3$ cores are less than the design thickness, the lot will be judged acceptable.

GRADATION EXAMPLE

An acceptance procedure is to be prepared for a crushed stone base course. The percentage by weight of material passing the No. 200 sieve is known to be a significant performance characteristic. Experience has shown that bases that have 7.0 percent or less of minus No. 200 material have performed well but bases that have more than 10.0 percent of minus No. 200 material have poor stability and drainage and tend to be frost susceptible. For this example, it is assumed that an analysis of historical data has shown the test results on minus No. 200 material to be approximately normally distributed with a typical standard deviation of about $\sigma = 1.0$ percent.

The information provided in this example is sufficient to develop a workable acceptance plan but it is not in the most useful form. For the types of acceptance plans covered in Standard R9, definitions of acceptable and unacceptable quality must be stated in terms of the percentage of material falling outside some specification limit (or pair of limits). Instead, the information is presented in terms of two average levels of minus No. 200 material that experience has shown have produced satisfactory and unsatisfactory results, respectively. As a reasonable approximation, these average values can be associated with the typical standard deviation of $\sigma = 1.0$ percent by means of normal distribution theory to provide guidance in establishing both the AQL and the RQL in terms of percent defective. The acceptance plan will then perform as desired as long as the standard deviation is reasonably close to the typical value and, if conservatively designed, it should provide ample protection even when the standard deviation is larger than usual.

Because there is no reason to impose a lower limit on minus No. 200 material, this will be a single-limit specification. A logical choice for this limit is 7.0 percent, the level of minus No. 200 material that is known to be clearly satisfactory. It is believed that the base will perform well as long as 90 percent or more of the material has a minus No. 200 value of 7.0 percent or less. Therefore the AQL is defined as 10 percent defective above the limit of 7.0 percent. This is a relatively conservative definition because, even if the standard deviation were considerably larger than the typical value, there is little chance that any of the material in the normal distribution representing AQL quality would reach the known critical value of 10.0 percent minus No. 200 material. The AQL is illustrated in the upper diagram in Figure 12.

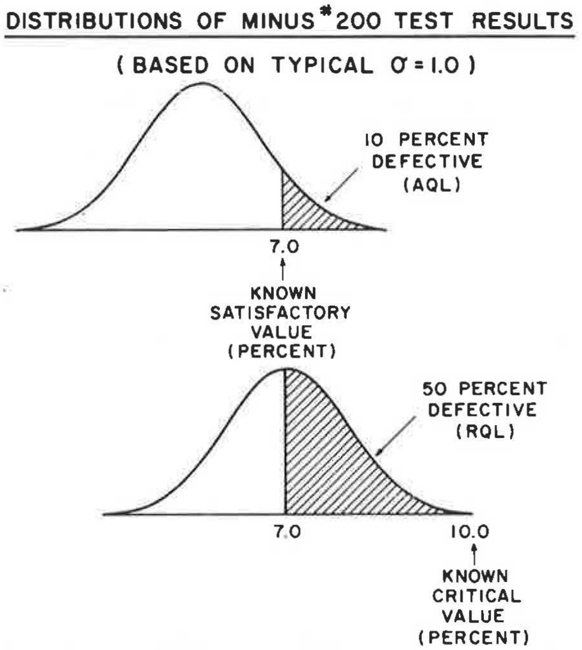

FIGURE 12 Illustration of definitions of AQL and RQL for gradation example.

To determine the level of percent defective to be defined as the RQL, it is noted that if this same distribution had 50 percent of its material above the limit of 7.0 percent, its upper tail would extend just to the critical value of 10.0 percent minus No. 200 material. On those few occasions in which the standard deviation was substantially larger than the typical value of $\sigma = 1.0$ percent, a relatively small portion of the distribution would extend above the critical value of 10.0 percent. As the amount of material exceeding 7.0 percent minus No. 200 material increases above 50 percent, however, progressively more will exceed the critical value of 10.0 percent and performance problems might be expected to develop. This provides a rational basis for defining the RQL as 50 percent defective above the limit of 7.0 percent minus No. 200 material, as illustrated in the lower diagram in Figure 12.

For this example, it will be assumed that the highway agency wishes to control both the seller's risk and the buyer's risk at $\alpha = \beta = 0.05$. The required acceptance probabilities at the AQL and RQL are P = 0.95 and P = 0.05, respectively. It is seen from the table shown in Figure 5 that a variables plan with a sample size of n = 8 and a maximum allowable estimated percent defective of M = 26 meets these requirements.

Lot Percent Defective	Probability of Acceptance
10 (AQL)	0.95
20	0.70
30	0.38
40	0.16
50 (RQL)	0.05
60	0.01

A suitable lot size must be chosen and the method of testing specified. Because variables acceptance theory assumes sampling from a normal population, care must be taken not to combine distinctly different populations into a single lot. The acceptance procedure will require that the mean ($\bar{X}$) and standard deviation (S) be calculated from n = 8 random samples and used to compute the Q-statistic in Equation 10. The corresponding percent defective estimate is obtained from tables such as the one shown in Figure 6 or the type shown in Figure 7. For the lot to be judged acceptable, the estimated percent defective must be no larger than M = 26. (Alternatively, it could be required that the Q-statistic be equal to or greater than k = 0.665.)

$$Q = (7.0 - \bar{X})/S \tag{10}$$

PAY ADJUSTMENT CLAUSES

Because it is seldom possible to define a single level of quality that differentiates between satisfactory and unsatisfactory work, it has become customary to define two distinctly different quality levels--the AQL and the RQL--when developing statistical acceptance procedures. The AQL represents a clearly acceptable level of quality that the highway agency expects the contractor to deliver. The RQL represents a much lower level of quality that, when detected, requires some sort of remedial action.

In actual practice, highway agencies are often faced with the dilemma of having to deal with marginal quality, items of work that fall between the AQL and the RQL. Many agencies have found the use of adjusted pay schedules, which award payment in proportion to the quality received, to be a practical and effective solution. The percent defective parameter, on which the revised version of Standard R9 is based, is particularly well suited for this purpose. For the reader interested in pursuing this refinement, the development of pay adjustment clauses is extensively covered in the recent literature (1-3, 15-21).

SUMMARY AND CONCLUSIONS

A major revision of AASHTO Standard R9, Acceptance Sampling Plans for Highway Construction, was described. The primary goals were to correct several technical flaws and to reduce the level of complexity of the standard. The new version is oriented around the concept of percent defective as the quality measure and advocates the standard deviation method rather than the less efficient range method for variables acceptance plans. Several new tables were developed, including operating characteristic tables for a wide range of both attributes and variables acceptance plans, and it was demonstrated by computer simulation that single-limit variables operating characteristic curves are sufficiently accurate for most double-limit applications. Finally, two examples were presented to illustrate the use of the revised standard.

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REFERENCES

1. Statistically Oriented End-Result Specifications. NCHRP Synthesis of Highway Practice 38. TRB, National Research Council, Washington, D.C., 1976.
2. Quality Assurance. NCHRP Synthesis of Highway Practice 65. TRB, National Research Council, Washington, D.C., 1979.
3. R.M. Weed. Statistical Specification Development. FHWA, U.S. Department of Transportation, 1982.
4. R.M. Weed. An Introduction to Computer Simulation. Demonstration Projects Division, FHWA, U.S. Department of Transportation, 1976.
5. R.T. Barros, R.M. Weed, and J.H. Willenbrock. Software Package for the Design and Analysis of Acceptance Procedures Based on Percent Defective. In Transportation Research Record 924, TRB, National Research Council, Washington, D.C., 1983, pp. 85-93.
6. AASHTO Interim Specifications--Materials. AASHTO, Washington, D.C., 1984.
7. Sampling Procedures and Tables for Inspection by Attributes. Military Standard 105. U.S. Department of Defense, 1963.
8. Sampling Procedures and Tables for Inspection by Variables for Percent Defective. Military Standard 414. U.S. Department of Defense, 1957.
9. I. Miller and J.E. Freund. Probability and Statistics for Engineers. Prentice Hall, Inc., Englewood Cliffs, N.J., 1965.
10. W.J. Dixon and F.J. Massey, Jr. Introduction to Statistical Analysis. McGraw Hill Book Co., New York, 1969.
11. A.J. Duncan. Quality Control and Industrial Statistics. Richard D. Irwin, Inc., Homewood, Ill., 1965.
12. G.J. Resnikoff and G.J. Lieberman. Tables of the Noncentral t Distribution. Stanford University Press, Stanford, Calif., 1957.
13. J.H. Willenbrock and P.A. Kopac. Development of a Highway Construction Acceptance Plan. In Transportation Research Record 691, TRB, National Research Council, Washington, D.C., 1978, pp. 16-22.
14. K. Pearson. Tables of the Incomplete Beta Function. Biometrika Office, University College, London, England, 1934.
15. J.H. Willenbrock and P.A. Kopac. Development of Price-Adjustment Systems for Statistically Based Highway Construction Specifications. In Transportation Research Record 652, TRB, National Research Council, Washington, D.C., 1977, pp. 52-58.
16. R.M. Weed. Equitable Graduated Pay Schedules: An Economic Approach. In Transportation Research Record 691, TRB, National Research Council, Washington, D.C., 1978, pp. 27-29.
17. R.M. Weed. Unbiased Graduated Pay Schedules. In Transportation Research Record 745, TRB, National Research Council, Washington, D.C., 1980, pp. 23-28.
18. R.M. Moore, J.P. Mahoney, R.G. Hicks, and J.E. Wilson. Overview of Pay Adjustment Factors for Asphalt Concrete Mixtures. In Transportation Research Record 821, TRB, National Research Council, Washington, D.C., 1981, pp. 49-56.
19. R.M. Weed. Method to Establish Pay Schedules for Rigid Pavement. In Transportation Research Record 885, TRB, National Research Council, Washington, D.C., 1982, pp. 18-24.
20. P. Puangchit, R.G. Hicks, J.E. Wilson, and C.A. Bell. Development of Rational Pay Adjustment Factors for Asphalt Concrete. In Transportation Research Record 911, TRB, National Research Council, Washington, D.C., 1983, pp. 70-79.
21. R.M. Weed. Adjusted Pay Schedules: New Concepts and Provisions. In Transportation Research Record 986, TRB, National Research Council, Washington, D.C., 1984, pp. 32-38.

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