A Note on Accident Risk

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ABSTRACT

The use of accident rates as risk estimators, though widespread, presents a potential error. This may occur when the relationship between exposure and accidents is not linear (i.e., a decreasing derivative); then, an increase in exposure might be misinterpreted as leading to a decrease in accident risk. To obviate such error, a definition of risk as a triplet of exposure, accidents, and probability is presented. Accordingly, the risk level of a system can only be expressed in relation to a specific exposure level. The definition of exposure resulting from this definition of risk is simply any traffic situation from which the number of accidents can be estimated.

A common method of defining the safety level of a transport system is by means of risk and exposure. Risk estimates are used to describe the safety level of transportation systems in a manner that is invariable to their exposure level. This approach gains impetus in a "before and after" safety-improvement comparison or in a comparison of two structurally different systems (e.g., two different road sections or two intersections), where differences in exposure level are known to exist.

The most widely used means of describing transportation-system risk is the accident rate. According to Wolfe (1), ". . . comparison of accident rates can assist road safety researchers in developing safety countermeasures in ways that comparisons of absolute frequencies of accident cannot." Thus Frantzeskasis (2) compared highway risk in different countries on the basis of accident rates.

Accident rates are usually defined as the number of accidents (whether total number of accidents, certain types of accidents, or severity of accidents) divided by exposure measures. Exposure is generally defined as the number of opportunities for accidents--for example, total mileage or the number of pedestrians crossing, or as a certain function of traffic volumes at intersections.

A methodological problem, however, is inherent in the use of accident rates: the need to assume that the number of accidents increases by a constant amount with a certain increase in exposure. This assumption is equivalent to assuming the existence of a linear relationship between road accidents and exposure, a situation described in Figure 1. As can be observed, the linear relationship between exposure and accidents creates a constant risk (slope of the curve) for each exposure. Thus the risk in System 1 is always greater than the risk in System 2; for any given exposure level, there are always more accidents in System 1 than in System 2.

The situation changes when the derivatives of the curves decrease with an increase in exposure. This occurs, as shown in Figure 2, when increased exposure worsens the safety situation by decreasing units. Here the exposure levels in Systems 1 and 2 are F_1 and F_2, respectively; the risk (or the accident rate) in System 1 at point A is lower than that in System 2 at point B. Without prior knowledge of the type of curves, a wrong conclusion could be drawn, that is, that System 1 is less risky than System 2.

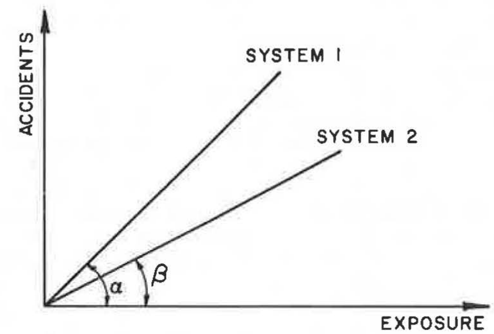

FIGURE 1 Hypothetical linear relationship between exposure and accidents.

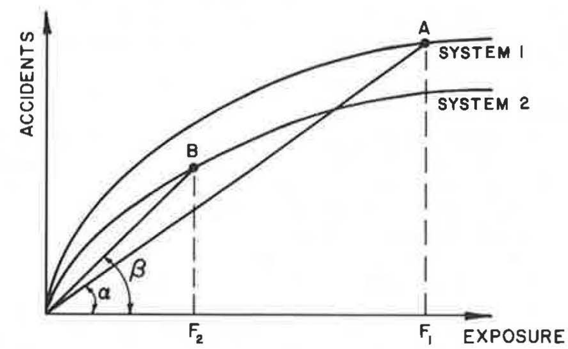

FIGURE 2 Hypothetical nonlinear relationship between exposure and accidents.

DEFINING THE PROBLEM

The popular definition of risk as a ratio between the number of road accidents and the amount of exposure [see, for example, Chipman (3); Cameron (4); Chapman (5); Wolfe (1); Hauer (6)] necessitating the existence of a linear relationship (with a zero intercept) between accidents and exposure appears log-

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ically appealing: it implies that risk is the probability of an accident's resulting from one exposure unit. Consequently, the number of accidents is seen as a binomial process whose expectation is the product of probability (risk) and the number of trials (amount of exposure).

This definition, however, creates a problem in that it negates situations in which certain systems function effectively (low risk) at a certain level of exposure and less effectively (high risk) at a different level of exposure. Traffic signals may be viewed as an example of a situation of this type: they are effective in reducing the number of accidents for high traffic volumes, but can cause an increase in the number of accidents for low traffic volumes.

Under certain simple assumptions (an increase in the number of accidents with an increase in density, and a linear relationship between speed and density), the relationship between accidents and an exposure estimator (traffic volumes) is not constant (see section on Variation of Risk in Accordance with Traffic Volumes). In other words, risk may vary with the amount of exposure, a phenomenon not permitting the use of risk as a constant scalar factor for characterizing a system.

To summarize, as a result of formal definitions, there appears to be a vicious circle in which, on the one hand, the accepted definition of risk necessitates a linear relationship between accidents and exposure; on the other hand, it is difficult (or even impossible) to find exposure estimators that fulfill this limitation. Therefore a lack of analytical or empirical tools exists whenever the need arises to evaluate the safety aspects of a specific facility. The question now is whether it is desirable to change the definition of risk to allow the use of an extensive set of exposure measures.

VARIATION OF RISK IN ACCORDANCE WITH TRAFFIC VOLUMES

In this section it is demonstrated how risk varies in accordance with the traffic-flow conditions under which the system exists during exposure measurements. According to the widespread approach, risk is defined as the ratio between the number of accidents (A) and the amount of exposure (E).

As a starting point, examine a road section in which the length is known (ℓ) for a time interval of t hours. The expected number of accidents in the section is assumed to be dependent on both the number of vehicles on the road section and their travel speed. The density (D) of these vehicles determines the relative proximity in space between them, and with travel speed, also their relative proximity in time. The use of density is appealing because it is possible to obtain the same level of traffic volumes for two different levels of density and speed. Roess et al. (7) also chose density and speed as recommended criteria in their proposal for revising procedures for level of services.

Earlier, Haight (8) proposed that the expected number of accidents in a road section be a quadratic function of density. From this simple model, two characteristics relating density to accidents can be defined:

1. The marginal increase in density to a road section increases the number of accidents $[(dA/dD) > 0]$; and
2. For a constant increase in density, the marginal increase in accidents increases $[(d^2A/dD^2) > 0]$ as D increases.

This second assumption does not necessarily exist in high densities. Following a decrease in travel speeds, it is possible that for a certain range of low speeds $d^2A/dD^2 < 0$.

The effect of traffic volumes on accidents may be obtained from the relationship

$$V = D \cdot S$$

where V is traffic volume (vehicles per time unit) and S is space mean speed (km/h).

Using Greenshields' (9) suggestion for the linear relationship between density and speed

$$S = a - bD \qquad (b > 0)$$

where a is mean free speed, $b = a/D_j$, and D_j is jam density, it follows that

$$V = D(a - bD) = dD - bD^2$$

The first derivative with respect to accidents will be

$$dV/dA = a(dD/dA) - 2\ bD(dD/dA) = dD/dA\ (a - 2\ bD)$$

using the relationship

$$dA/dV = (1/DV)/dA$$

gives the following

$$dA/dV = 1/(1/dA/dD)(a - 2bD)$$

Thus, the range of traffic volumes in which an increase in density follows an increase in volume $(D < a/2b)$, it follows that $dA/dV > 0$; for the range of traffic volumes in which volumes decrease with an increase in density,

$$dA/dV < 0$$

One conclusion from the foregoing is that the ratio A/V, which is widely used for risk, is not constant, being determined by traffic-flow conditions. In the range where $D > a/2b$, the risk (A/V) decreases with an increase in V; however, in the range where $D < a/2b$, the risk increases or decreases in accordance with the behavior of d^2A/dV^2.

In order to investigate the behavior of d^2A/dV^2, the second derivative of the inverse function should be evaluated

$$d^2V/dA^2 = d^2D/dA^2\ (a - 2bD) - 2b(dD/dA)^2$$

The second derivative of the function $A = f(v)$ is

$$\begin{aligned} d^2A/dV^2 &= [-(d^2V/dA^2)/(dV/dA)^2] \cdot dA/dV \\ &= \{-[(d^2D/dA^2)(a - 2bD) - 2b(dD/dA)^2] \\ &\quad \div [dD/dA(a - 2bD)]^3\} \end{aligned}$$

From the foregoing expression, it is possible to determine a series of conditions that determine the changes in A/V with an increase in V. For example, if traffic volumes increase with density $(D < a/2b)$ and if $d^2A/dD^2 > 0$ (i.e., a quadratic function between density and accident), it follows that $d^2A/dV^2 > 0$. In other words, an increase in V also increases A/V.

It should be emphasized that a convex function similar to that shown in Figure 2 can also be obtained when the volume is an increasing function of D. For example, when $D < a/2b$, $d^2D/dA^2 > 0$, and $d^2D/dA^2\ (a - abD) > 2b\ (dD/dA)^2$. Evidence for the existence of varying risk levels for different traffic volumes is described by Ceder and Livneh (10).

ALTERNATIVE DEFINITION OF RISK

Following ROWE (11)--"Risk is the potential for realization of unwanted negative consequences of an event"--the risk function of a road system must express the probability of a certain number of accidents for each possible traffic situation in that system. In other words, the risk function is aimed at estimating the number of expected accidents (or the probability of a certain number of accidents) at each exposure level in the system. Following Kaplan (12), at a given exposure level the risk (R_0) can be described by the triplet

$$R_0 = < E_0, A_0, P_0 >$$

where

- R_0 = risk at the E_0 exposure level;
- A_0 = expected number of accidents, or a vector describing the severity of accidents; and
- P_0 = probability of A_0 accidents (possibly a vector).

The definition of risk for any exposure level is the set of all triplets:

$$R = \{ < E, A, P > \quad E > 0 \}$$

The exposure level itself can be a vector of different exposure measures; for example, the number of pedestrians and traffic volumes.

The fundamental characteristic of this alternative definition of risk is its ability to express the expected number of accidents in a system or the probability of a certain number of accidents at any exposure level. Accordingly, the risk level of a system can be expressed only in relation to a specific exposure level.

The task of the researcher involved in risk analysis may be seen as the search for a black box in which input is exposure and in which output is accidents and probabilities. The image of the black box fits the situation in which the researcher seeks, not the physical law relating exposure to accidents, but a mathematical model relating the input variables to the output variables of a system.

Risk function can be described graphically with various cross-sections. Figure 3 shows such an example, describing the risk level of a number of systems at specific exposure levels. As can be seen, the probability of a certain number of accidents in System B is higher than in System A; thus at exposure level E_0, System B is more dangerous than System A. Figure 4 shows the probability for A_0 or more accidents in Systems A and B at each exposure level. Although System B is more dangerous than System A up to the exposure level of E_1, the reverse holds true for exposure levels greater than E_1.

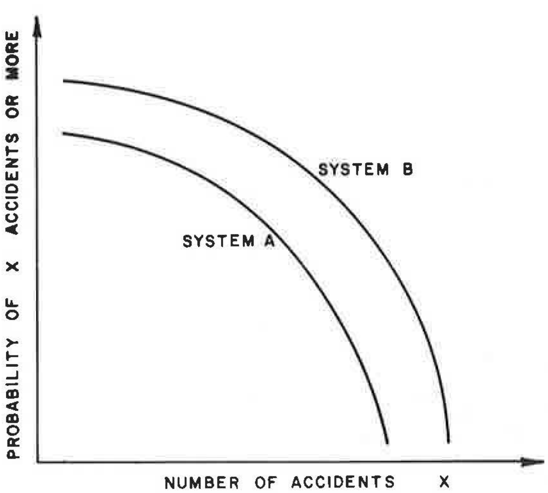

FIGURE 3 Risks at E_0 exposure level for two systems.

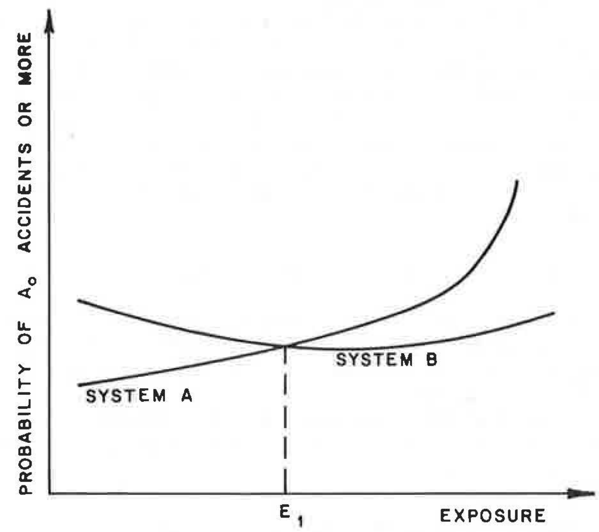

FIGURE 4 Probability of A_0 accidents or more as a function of exposure.

EXPOSURE ESTIMATORS

The present alternative definition of risk does not impose limitations on the choice of exposure estimators (such as the linear relationship to accidents); therefore, exposure may be defined as any traffic situation from which the number of accidents can be estimated. Nevertheless for a safety evaluation, the preference of one exposure estimator over others must be based on the following two criteria:

1. Data collection ability--the empirical ability to collect exposure data.
2. Validity--the analytical ability to estimate the vector of accidents and probabilities from exposure.

The criterion of empirical ability to collect exposure data gives priority to measures based on available data or easily collectable data. Such data as vehicle kilometers and total number of vehicles entering an intersection, will therefore receive preference over such exposure measurements as number of lane changes and number of stops. The second criterion determines the validity level of the exposure measures. The methodology for this determination involves standard statistical procedures in model-building, such as minimum least squares, correlation coefficients, and so forth. These two criteria assure that the justification for using a certain exposure measurement is primarily practical and empirical, and not methodological and theoretical.

The process of validity evaluation involves the building of a mathematical model that is used to calculate the vector of accidents and probabilities for each input level of exposure. For each system having a different exposure measure, a different model must be built; this process means a model for intersections, road sections, and so forth. Furthermore, the need for estimating a model sometimes arises for similar exposure measures, for example, straight road sections and horizontal curves. Separate models (or black boxes) also are often used for single-vehicle accidents and multivehicle accidents. In all the preceding cases, the degree of validity achieved is the criterion for building the separate black boxes. Successful examples for exposure models are described by Cleveland et al. (13), Cleveland and Kitamura (14), and Zegeer and Mays (15).

When it is empirically apparent or when it may be theoretically assumed that two systems have the same black box, a risk analysis can be carried out that is based solely on exposure data, without the need for accident data. In these situations, if an increasingly monotonous relationship exists between exposure and accidents, the number of accidents can be assumed to be greater when the amount of exposure is higher.

ADVANTAGES OF PRESENT APPROACH ILLUSTRATED

The example that follows emphasizes the advantages of the approach presented in this paper for evaluating risk as opposed to the conventional approach, which uses a constant scalar.

Assume that two alternatives were evaluated for a transport investment. To obtain risk estimators, the data given in Table 1 were collected, obtained from the doublets $<N_i,E_i>$. N_i is the mean number of accidents at Alternative 1 and E_i is traffic volumes or exposure at that site. A calculation of the mean number of accidents per one million exposure units shows that 520 accidents per one million units would apparently occur if Alternative 1 was used, whereas 598 accidents per one million exposure units would occur if Alternative 2 was used. The conclusion is that Alternative 2 is more dangerous and, therefore, inferior to Alternative 1.

A closer look at the existing relationship between exposure and accidents leads to the conclusion that it is possible to adapt a different model for each alternative.

Model for Alternative 1: $A = 3.10^{-3} * E^{0.8}$
Model for Alternative 2: $A = 0.1 * E^{0.4}$

where A is the expected number of accidents and E is the amount of exposure.

Figure 5 shows the risk curve for exposures of 5,000 and 10,000 vehicles. A Poisson model is used to calculate the probabilities. Alternative 1 can be seen to offer an advantage with a lower exposure level, whereas Alternative 2 is more attractive with higher exposure levels. The break point between the two alternatives is at traffic volumes of 6,457 vehicles per day, which, it should be remembered, completely disappears in traditional risk analysis.

TABLE 1 Accident Data and Traffic Volumes for Two Alternatives

	Alternative 1		Alternative 2	
Site	Mean No. of Accidents	Exposure (vehicles per day)	Mean No. of Accidents	Exposure (vehicles per day)
1	4.6	9,500	2.9	4,500
2	3.0	5,640	3.2	6,000
3	2.3	4,100	2.7	3,800
4	3.3	6,400	3.4	6,950
5	2.5	4,500	2.6	3,500
6	15.7	30,140	14.8	24,750

DISCUSSION OF APPROACH

In this paper a conceptual framework is presented with which, in the opinion of this author, risk analysis can be carried out more effectively.

The advantage of the present approach lies in its capacity to use a variety of mathematical models for describing risk in a system. Definition here does not limit or dictate linear or other assumptions during the empirical estimation of risk. Instead, the broad definition of risk that is presented allows for different traffic situations as exposure estimators in accordance with two criteria: one for

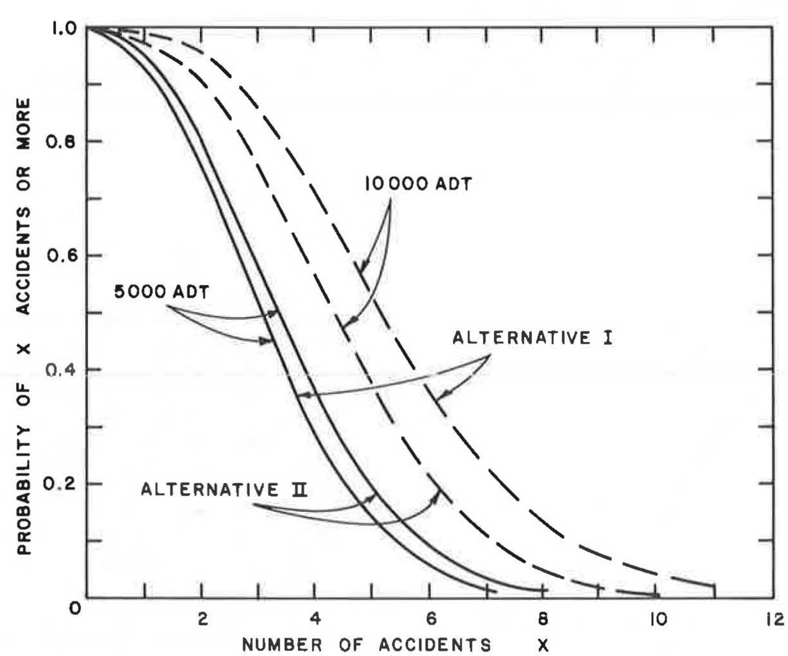

FIGURE 5 Risks at different exposure levels for the two alternatives.

facilitating data collection and another for validity. An empirical base supplies the justification for the preference of one exposure measurement over another. Risk, in accordance with the definition given in this paper, is the set of triplets that expresses the probabilities for the number of accidents at any given exposure.

Forfeiting the use of a constant scalar to describe risk is the main disadvantage of the present definition. This forfeiture, however, is not arbitrary, but results from the empirical fact that the relationship between exposure measures and accidents might not be linear. For those cases in which a certain exposure measure shows a linear relationship with the number of accidents, accident rates can then be used as a risk measure.

It should be emphasized that in situations in which a safety evaluation is required, the need for estimating risk function does not always arise. For example, in a comparison of two existing systems with similar exposure levels, the safety level can be evaluated directly by means of accidents thereby eliminating the need for the risk function.

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