Rigid Bottom Considerations for Nondestructive Evaluation of Pavements

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ABSTRACT

Mechanistic analysis of dynamic deflection basins for evaluating in situ moduli of pavement-subgrade systems has become an important part of nondestructive pavement evaluation techniques. Discussed is the influence of a rock layer on the evaluation of in situ moduli by using the multilayered linear elastic theory. The value of Young's modulus of elasticity of the subgrade overlying a rock layer can be significantly underestimated if a semi-infinite subgrade is assumed in applying the linear elastic layer theory to analyze deflection basins. An algorithm has been developed to correct this type of error for two cases: (a) when the subgrade thickness is known and (b) when depth to the rock layer is unknown. For the Dynaflect and falling weight deflectometer systems, a rigid bottom can be considered for the second case by assigning a subgrade thickness as a function of the wave length of compression wave in the subgrade. The computer programs FPEDDL (for flexible pavements) and RPEDDL (for rigid pavements) incorporate procedures for evaluating in situ moduli with regard to rigid bottom considerations in pavement-subgrade systems.

Nondestructive testing (NDT) is an indispensable part of pavement condition monitoring procedures. Recent surveys (1-2) indicate that dynamic deflection measuring devices are used by a majority of agencies for nondestructive pavement evaluation. Among these, the Dynaflect is the single most popular and widely accepted NDT device, followed by the Road Rater and falling weight deflectometer (FWD). Several agencies are currently evaluating FWDs because of improvements in their operating characteristics and their ability to apply variable and heavy dynamic loads. These devices use seismic sensors to measure surface deflections when the pavement surface is excited by dynamic loads. The deflection basins formed by the dynamic deflection measurements from an array of seismic sen-
Sensors are used in conjunction with the thickness information and the multilayered linear elastic theory for calculating in situ Young's moduli of the pavement layers. This is accomplished through an iterative procedure of matching the measured deflections with theoretical deflections calculated by using an assumed set of Young's moduli. Uddin et al. presented a review of these iterative procedures (1). Several other papers are related to this topic [4-7]. The proposed AASHTO Guide (8) also recommends the use of in situ moduli, calculated from NDT deflection data, for overlay design of pavement structures.

In this paper the sources of errors in the moduli calculated from the iterative application of the multilayered linear elastic theory, particularly with regard to rigid bottom considerations.

Deflection Basin Matching Approach

Only the Dynaflect and FWD are discussed in this paper. The Dynaflect is a steady-state vibratory device that is instrumented to measure peak-to-peak dynamic deflections on the pavement surface. The Dynaflect applies a harmonic load of a 1,000-lb peak-to-peak amplitude through two steel wheels that are 20 in. apart. Peak-to-peak surface deflections are measured by five geophones secured on a lower-raise bar and spaced at 12 in. such that the first geophone is located midway between the loading wheels. The radial distances of the geophones from each loading wheel are 10.00, 15.62, 26.00, 37.36, and 49.03 in. Geophones are calibrated in the field at a driving frequency of 8 Hz before making deflection measurements.

An FWD applies an impulse load by dropping a known mass from a predetermined height. The Dynatest Model 8000 FWD system, used in this study, can generate a load of approximately 2,000 lb, as well as a very high peak force of more than 20,000 lb, by using different configurations of mass and height. The load is transmitted to the pavement surface through a loading plate 11.8 in. in diameter. A load cell is used to measure peak dynamic force. Use of a minimum of six and a maximum of seven geophones is assumed in this study. The first sensor is located in a hole at the center of the loading plate for measuring maximum peak deflection. The remaining sensors can be positioned along the lower-raise bar. The geophones are assumed to be 12 in. apart in this study, with radial distances 0.0, 12.0, 24.0, 36.0, 48.0, 60.0, and 72.0 in. from the center of the loading plate. Only peak deflections are recorded by the FWD.

Both the Dynaflect and FWD systems are mounted on lightweight trailers. The static weight of a Dynaflect trailer is approximately 1,800 lb and an FWD trailer weighs approximately 2,000 lb. Restated, deflections measured by these two devices are dynamic displacements of the pavement-subgrade system when the pavement surface is excited by the dynamic forces they generate.

Table 1: Tolerances for Activating Changes in Moduli (for use in basin-fitting subroutines)

<table>
<thead>
<tr>
<th>Pavement Type</th>
<th>Program</th>
<th>TOLR31</th>
<th>TOLR32</th>
<th>TOLR33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>RFPDDL (subroutine BASINR)</td>
<td>4.0</td>
<td>3.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Flexible</td>
<td>RFPDDL (subroutine BASINF)</td>
<td>4.0</td>
<td>2.0</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: TOLR31 is used for L (Young's modulus of surface layer); TOLR32 is used for intermediate layers; and TOLR33 is used for subgrade modulus.

The second type of tolerance is the maximum permissible discrepancy in measured and calculated deflections. The programs do not attempt any iteration if the maximum discrepancy, based on the first set of moduli, does not exceed 1.5 percent. Further iterations are stopped if the discrepancies increase in a given iteration. A second cycle of iterations is performed if the maximum discrepancy exceeds 10 percent. These tolerances make the basin-fitting procedures efficient.

The evaluation of in situ moduli by using the
deflection basin matching approach poses three major problems:

1. The possibility of nonuniqueness of the estimated in situ Young's moduli (in a multilayered pavement, several combinations of moduli can yield similar deflection basins);
2. Errors in the calculated in situ moduli due to nonlinear behavior of granular layers and subgrade; and
3. Errors due to the assumption of a semi-infinite subgrade when a rock layer exists at a shallow depth of subgrade.

The nonuniqueness of predicted moduli can lead to substantial errors, particularly in the moduli of pavement layers above the subgrade. The methodology incorporated in the PPDDEL and RPPDDL computer programs (3,9) ensures unique results. The methodology relies on generating seed moduli primarily as functions of measured deflections, radial distances of sensors from the load(s), and thicknesses of pavement layers. In addition, the seed moduli of upper layers of a pavement are also functions of the subgrade seed modulus. Applications of this methodology have been presented elsewhere (10,11). An example for verifying the methodology is shown in Figure 1; it shows a hypothetical concrete pavement with preassigned values of moduli (called true moduli), the seed moduli generated by the RPPDDL program, and the final predicted moduli derived by matching deflection basins. The pavement consists of a 10-in. portland cement concrete (PCC) layer, 6-in. stabilized base, and semi-infinite subgrade.

CONSIDERATION OF RIGID BOTTOM

The semi-infinite thickness of subgrade is an inherent assumption in the use of the elastic layered theory for calculating a deflection basin. The presence or assumption of a rock layer at some finite depth necessitates consideration of a rigid layer instead of a semi-infinite subgrade because it can significantly affect the deflection basin, as shown in Figure 2. Ignorance of this condition may result in significant errors in moduli derived from deflection basins, as demonstrated by McCallough and Taute (12). If the thickness of the subgrade overlaying a rock layer is known from design-construction records and other evidence then its value should be entered in the input of the basin-matching programs.

![Figure 1: Prediction of Young's moduli for a hypothetical rigid pavement.](image)

![Figure 2: Effect of the presence of a rigid layer at varying depths on theoretical Dynaflect deflection basins.](image)
Case of Unknown Thickness of Subgrade

This condition is undoubtedly more common in NDT data. The error involved in overpredicting deflection because of the assumption of a semi-infinite subgrade is obvious. Some researchers, such as Wiseman et al. (6), have considered using an arbitrary depth of subgrade equal to the rigid layer. Researchers at the U.S. Army Corps of Engineer Waterways Experiment Station (4) assume a rigid bottom at 20 ft for the evaluation of in situ moduli from their deflection basin matching programs. A subgrade thickness of 20 ft has been arbitrarily selected for the dynamic analysis of the Road Rater deflection basins (7). However, significant effects are reported for certain ranges of subgrade thicknesses by the researchers at the University of Texas at Austin based on their dynamic analyses of dynamic deflection tests (13). The assumption of an arbitrary thickness of subgrade may therefore significantly affect the moduli calculated from the static analyses of deflection basins.

The inherent weakness of the current state of the art is the use of static analyses (i.e., iterative application of multilayered linear elastic theory) to calculate in situ moduli from measured dynamic deflection basins. However, in the field of nondestructive pavement evaluation, the current state of knowledge for proper dynamic analyses of dynamic deflection basins is in the research and development stage. The layered theory is still the best tool available in the absence of any reliable and valid method of dynamic analysis.

EVALUATION OF MODULI FOR A KNOWN ROCK LAYER

If a rock layer is suspected at a pavement site, every effort should be made to extract information about the thickness of the subgrade. In this case, the pavement structure can be properly modeled for the iterative application of the elastic layered theory. However, the problem of nonuniqueness may still remain a source of significant error in the predicted moduli for the conventional basin matching approach in which iterations start from user-specified values of moduli.

The FPEDDL and RFEDDL programs ensure the uniqueness of predicted moduli by correcting the seed modulus of subgrade for the finite thickness of subgrade. For this purpose, a parametric study was made by using different rigid and flexible pavements to investigate the influence of variations in the depth of subgrade of Sensor 5 deflection. Based on layered theory computations, the ratio (RATS) of Dynaflect Sensor 5 deflection for several subgrade thicknesses (D, in.) to the Sensor 5 deflection for a semi-infinite subgrade was determined for various pavement structures. The ratio (RATS) approached zero if a rock layer was assumed at 1 ft or a shallower depth below the pavement. A power function was used to develop a regression equation based on the values of RATS and D. The programs compute RATS if the thickness of the subgrade layer is entered by the user. The equivalent Sensor 5 deflection Ws' for a semi-infinite subgrade case is then calculated by using the following relationship:

\[ W_s' = W_s / \text{RATS} \]  

(1)

where \( W_s \) is the measured Sensor 5 deflection.

The regression equations developed from numerous theoretical basins are then used to predict subgrade moduli. These equations are based only on Sensor 5 deflections. The \( R^2 \) values associated with these equations are above 0.95. In the case of the FWD, the Sensor 5 deflection is normalized by the program to 1,000 lb for substituting its value in Equation 1 and in the predictive equations. The subgrade seed modulus (corrected for the influence of subgrade thickness on measured deflections) is used to calculate seed moduli of upper layers of the pavement. The evaluation of in situ moduli is then performed by converging to measured deflections by using basin fitting routines.

Application

Several theoretical deflection basins generated for various hypothetical pavement-subgrade systems were analyzed to verify the predictions of moduli; this is a rational method of checking the accuracy of predictions before subjecting the methodology to field applications. Figure 3 shows this point; a deflection basin—predicted assuming 50 ft of subgrade—has been analyzed for a hypothetical pavement with preassigned moduli by using the FPEDDL program.

In this figure, HERRP represents maximum discrepancy in the theoretical and the best-fit deflections.

![Figure 3](image)

Figure 4 shows an example of predicted in situ moduli from an FWD deflection basin analyzed by the FPEDDL program. At this flexible pavement site, approximately 5 ft of subgrade soil exists over the bedrock, according to the available records. The false assumption of a semi-infinite subgrade would have resulted in a significant overprediction (more than 100 percent) of the subgrade modulus. An example of the moduli estimated from the Dynaflect and FWD deflection basins is another flexible pavement site where a 15-ft subgrade layer exists over the bedrock. The results are summarized in Tables 2 and 3. For this case, the FPEDDL program predicted a subgrade modulus of 16,700 psi from the analysis of the Dynaflect deflection basin. The program corrected this value to 16,300 psi for the nonlinear behavior.
of the subgrade soil. Obviously, differences exist in the two sets of moduli from the Dynaflect and FWD basins for surface and base layers; these differences could be attributed to the loading mode effect and device dependency as related to measured deflections. Figure 5 shows another example of a flexible pavement site where prior knowledge of the presence of a rock layer of a shallow depth was available.

The examples just discussed indicate the usefulness of the methodology incorporated in the FPEDDI and RPEDDI computer programs. This study did not include any laboratory tests. Some researchers may consider comparisons with the moduli determined from laboratory tests to be more convincing. The following three points should be considered while making any inferences from a comparison of in situ moduli with laboratory moduli, particularly in the case of subgrade moduli:

1. Generally, a large scatter in resilient modulus relationships is obtained in the laboratory because of the influence of degree of saturation, water content, density, sampling disturbances, and variability associated with operators. In the case of subgrade soil, a possibility exists of even greater variability related to the depth from which the sample is extracted. On the other hand, the in situ subgrade modulus estimated from a given deflection basin is an average value over the entire depth of the subgrade.

2. The in situ stress and environmental conditions cannot be truly duplicated in laboratory.

3. The resilient modulus, based on laboratory tests, is calculated as a function of a stress parameter in the case of unbound materials and subgrade soils. It is interesting to note that the values of appropriate stress parameters are based on layered theory computations.

A RATIONAL APPROACH FOR CONSIDERING THE RIGID BOTTOM

In this section, a rational approach for assigning a finite thickness to the subgrade is presented. This approach eliminates the need for arbitrary selection of a depth to the rigid bottom. All dynamic deflection devices generate disturbance in the pavement-subgrade system. It is unlikely that the zone of influence of the dynamic test loads extends to infinite depth. In such cases, the FPEDDI and RPEDDI programs predict a finite thickness of subgrade. This predictive procedure is based on concepts taken from theory of stress wave propagation in elastic media. Basically, the predicted thickness of subgrade
is a function of frequency of loading (f) and compression wave velocity (Vp).

By looking at wave motion induced by the Dynaflect, long wavelengths result from low excitation frequency (8 Hz). For typical subgrade soils, half wavelengths will be more than 35 ft. It has been observed from the dynamic analysis of simulated Dynaflect tests that if the subgrade thickness is more than 35 ft, almost no significant difference in the pressure wave velocity (Vp).

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A simplified approach for calculating the predominant frequency excited by the FWD is the representation of the FWD load signal by an idealized load-time history: if the duration of the FWD load signal is 25 m/sec, the period can be approximated to be one-half of 25 m/sec by assuming a harmonic wave form. Therefore, frequency, being the inverse of period, can be taken as 20 Hz. This is supported by extensive field measurements using a Fourier Spectrum Analyzer made during a previous study at the University of Texas (14). It was found from the time history and power spectra for the FWD that the predominant frequency excited by the FWD was approximately 20 to 21 Hz.

The step-by-step procedure for assigning a finite thickness to the subgrade is described in the following list.

1. An initial estimate of the Young's modulus of the subgrade, ENAT, is made by using the predictive relationships based on the sensor 5 deflection. A value of Poisson's ratio (µ) for the subgrade is assigned.

2. The dynamic parameter (M) of the subgrade is calculated by using the following relationship:

   \[ M = \lambda + 2G \]  \hspace{1cm} (2)

   where \( \lambda \) and G are Lamé's constants,

   \[ G = ENAT/(1 + \mu) \], and

   \[ G = \mu \cdot ENAT/(1 + \mu)/(1 - 2\mu) \].

Equation 2 can be rewritten as follows:

   \[ M = (ENAT(1 - \mu))/((1 + \mu) \times (1 - 2\mu)) \]  \hspace{1cm} (3)

3. Mass density, \( \rho \), of the subgrade is calculated from the unit weight of the soil. (The unit weight of the soil is assigned based on the soil type.)

4. The wavelength of the P-wave, \( L_P \), is then calculated by using the following relationship:

   \[ L_P = (M/\rho)^{0.5}/f \]  \hspace{1cm} (4)

   where \( f \) is the frequency of the driving force (8 Hz for the Dynaflect and for the FWD; the predominant frequency can be taken as 20 Hz, as discussed previously).

5. The thickness of the subgrade is assumed to be one-half of \( L_P \).

In the FPEDD1 and RPEDD1 programs, the procedure just described for assigning a finite thickness to the subgrade is activated through an option in the input data. An example of using this option is shown in Figure 6.

**SUMMARY AND CONCLUSIONS**

Various sources of errors associated with the in situ moduli of pavement layers based on the application of elastic layered theory and basin fitting approach have been discussed in this paper, particularly with respect to the rigid bottom considerations. It is shown that the use of seed moduli generated by self-iterative procedures is desirable to ensure uniqueness of the predicted in situ moduli and eliminate user-dependency aspects of these procedures. This methodology has been successfully used in the FPEDD1 and RPEDD1 computer programs. These programs also incorporate equivalent linear analyses for correcting the Dynafl eet moduli with regard to the nonlinear behavior of granular layer and cohesive subgrade. The principal findings are as follows.

1. Ignorance of rigid bottom considerations may lead to substantial errors in the predicted moduli of a pavement-subgrade system. The subgrade modulus may be significantly overpredicted if a semi-infinite subgrade is falsely assumed, when actually bedrock exists at a shallow depth.

2. A procedure has been described for taking into account the influence of a rock layer if the subgrade thickness is known. This procedure relies on correcting the seed modulus of subgrade for the known depth to the rock layer.

3. A rational approach has been outlined to assign a finite thickness to the subgrade if no rock exists at shallow depths. This depth is taken as a function of the frequency of loadings and the velocity of compression wave in the subgrade. This option eliminates the need to assign an arbitrary thickness to the subgrade if the consideration of rigid bottom is required in the deflection basin analysis.

Various procedures discussed in this paper have been applied to the Dynaflect and FWD. However, the concepts can be equally applied to other dynamic load NDT devices, for example, the Road Rater.

**ACKNOWLEDGMENTS**

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**Table: Predicted Moduli**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Semi-Infinite</th>
<th>Rigid Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subgrade</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Semi-Infinite</td>
<td>Rigid Bottom</td>
</tr>
<tr>
<td></td>
<td>5,200,000</td>
<td>4,870,000</td>
</tr>
<tr>
<td></td>
<td>468,000</td>
<td>371,000</td>
</tr>
<tr>
<td></td>
<td>169,000</td>
<td>50,000</td>
</tr>
<tr>
<td></td>
<td>36,100</td>
<td>25,200</td>
</tr>
</tbody>
</table>

*Predicted Subgrade thickness = 15.1 Feet

**Figure 6** Analysis of an FWD deflection basin by the PREDD1 program using the rigid bottom option.
Texas State Department of Highways and Public Transportation and the FHWA, U.S. Department of Transportation, at which time the first author was a graduate student at the University of Texas at Austin. Appreciation is extended to K.H. Stokoe II, Professor of Civil Engineering at the University of Texas at Austin, for his interest during the course of this research. Thanks are also due to John Nixon of ARB, Inc., for providing some field deflection basin data for the evaluation of in situ moduli.

Discussion

Anastasios M. Ioannides*

Since the publication of Westergaard's pioneer works in the 1920s, the behavior of slabs-on-grade has been investigated on the basis of plate theory. On the other hand, Uddin, Meyer, and Hudson, as well as a few other investigators (15-17), have proposed using layered elastic analysis for both flexible and rigid pavements. Such a unified procedure is philosophically and practically attractive, and would allow the characterization of the pavement as a multilayered system. This is more realistic than the current use of the subgrade modulus, k, in a two-layer (or at most three-layer) system assumed when plate theory is employed.

However, using the layered elastic theory inevitably restricts the scope of all analyses to the case of interior loading. Current layered elastic theory computer codes cannot provide any information on pavement response under edge and corner loads or on the efficiency of load transfer systems at the pavement joints. This latter aspect of behavior is evidently considered an overriding consideration by the Federal Aviation Administration, U.S. Department of Transportation. A 1978 Advisory Circular entitled Airport Pavement Design and Evaluation changed the design criterion for portland cement concrete (PCC) airport pavements from the maximum stress under an interior load to that of the maximum stress when the load is placed near an edge (18). This approach to design is more realistic because it can be shown that the maximum stress in PCC pavements occurs under edge loading. Furthermore, it is almost impossible to place an aircraft on most pavement slabs in a manner that produces anything approaching a true interior loading condition (19). It has also been shown that nearly all distress in jointed concrete pavements is related to joints (20).

The possibility of analyzing the top layer of a pavement system as a plate, while retaining Burmister's multilayer theory for the remainder, has been investigated by Pickett and Al (21). However, these authors point out that such a substitution "without modification would result in appreciable error in cases of practical importance, since it does not take into account the effects of shear in the pavement on deflection and does not properly take into account horizontal shear at the interface between subgrade and pavement." On the other hand, Parker et al. contend that "the representation of the top layer as a thin elastic plate or as an elastic layer is really not that different when the top layer is a PCC slab" (16).

To investigate this issue further, a comparison is presented here of the layered elastic and plate theories for the case of interior loading on a semi-infinite pavement system. This system consists of a top layer or PCC slab resting on a Boussinesq half-space, characterized by $E_s$ and $\mu_s$. From the point of view of the layered elastic theory, this half-space is the special case of a multilayered system consisting only of one layer. The problem with plate theory is the standard one: a plate on an elastic solid foundation. The analyses were performed using computer programs ELPL5 (22) and WESTER (23), respectively. The equations presented by Losberg are used in WESTER for the responses according to plate theory (24).

The parameters selected for the various system components ranged between wide limits in order to ensure that most practical conditions are included in the factorial of runs conducted. The plots of nondimensional maximum response in Figures 7-9 indicate that the governing consideration when comparing the layered elastic and plate theories is the

![FIGURE 7](image_url)  
Comparison of layered elastic and plate theories: maximum deflection.
value of the ratio \(a/k_o\) of the radius of the applied load to the radius of relative stiffness of the pavement system.

Good agreement between the two theories is obtained when \(a/k_o\) is between 0.2 and 1.0; considerable discrepancies occur outside this range. Values of \(a/k_o\) greater than 1.0 are principally of academic interest only, but in practice several cases may arise in which \(a/k_o\) takes values below 0.2. In such cases, widely divergent results may be expected from the two theories.

Note that the comparison presented here refers only to the interior loading condition. Similar comparisons for edge and corner loading are not feasible because of the inherent limitations of the layered elastic theory. Furthermore, the comparison is with the plate theory used in conjunction with an elastic solid foundation. It is generally admitted that the dense liquid and elastic solid models can only be correlated in a limited number of special cases. As a result, a comparison between the layered elastic theory and the plate theory used in conjunction with a dense liquid foundation would have limited significance and would not be of a general nature. Such a comparison was attempted by Barker [17].

The fairly good agreement between the two theories within the prescribed range of \(a/k_o\) values has only been established for the case of the semi-infinite half-space. A similar comparison for the multilayered foundation case is feasible, at least theoretically, by using WESLAYER [25] or RISC [26] finite element programs for the plate theory investigation. Unfortunately, the application of the finite element method in conjunction with a multilayered subgrade has not yet been developed to the point where it is generally practical to use because such programs make substantial demands on computer resources.
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