Dynamic Analysis of Falling Weight Deflectometer Data

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ABSTRACT

The response of pavement systems to falling weight deflectometer (FWD) blows is evaluated by using a multidegree of freedom elastodynamic analysis. This analysis is based on a Fourier synthesis of a solution for periodic loading of elastic or viscoelastic horizontally layered strata. The method is verified for selected flexible pavement sections for which high-quality field and laboratory data are available in the literature. FWD deflection measurements at various geophone locations are compared by using dynamic as well as static analyses. The results indicate that inertial effects are important in the prediction of the pavement response. Conventional static analyses yield significantly different results and therefore yield erroneous (unconservative) predictions of pavement moduli backcalculated from deflection data. Elastodynamic analyses, based on fundamental material parameters (Young's modulus, mass density), appear to provide a useful vehicle for correlating pavement response between different loading modes (impulse, vibratory, etc.). Because resonance is a less important factor in the displacement response characteristics of pavements subjected to transient loading, deflection data obtained from transient loading devices are, in general, easier to interpret.

Rehabilitation of rapidly deteriorating pavements is receiving increasing attention from highway engineers. Because building new roads in the United States is a rare event at the current time, the main concern is directed toward upgrading the existing ones. Therefore, the need for rational and fast methods of assuring the integrity of pavement structures becomes more evident.

Nondestructive testing has proven to be an effective method for evaluating the load carrying capacity of pavements. Unlike seismic techniques, deflection measurements have gained widespread popularity among highway agencies. Deflection devices apply various types of loading: static (Benkelman beam), harmonic (Road Rater and Dynaflect), and impulsive [(falling weight deflectometer (FWD)]. Field studies have indicated that deflections of the FWD correlate closely with pavement deflections induced by moving wheel loads (1,2). However, interpretation of FWD deflection data remains somewhat problematic. A point of discrepancy arises because FWD measurements are interpreted as static deflections in many studies, thus ignoring the fact that the load of the FWD is dynamic.

Most previous analyses of the data obtained from dynamic loading devices have been based either on empirical approaches or elastostatic and viscoelastostatic models. Empirical approaches are not transferable, and in the static analyses the inertia of pavements is ignored. Unless the inertial effect of pavements is incorporated in the analysis, misleading results may be obtained.

One approach to dynamics is to represent the pavement system by a combination of masses, springs, and dashpots, that is, the single degree-of-freedom analysis (3-5). Although these single degree-of-freedom models can take into consideration inertial effects, one of their major shortcomings is the assumption that loads, deflections, stresses, and strains are applied in one direction, that is, the vertical direction. In reality, when a vertical load is applied, stresses and strains are developed in all directions throughout the pavement structure. Thus, by using a single degree-of-freedom representation, the deflections at points away from the load (at various geophone locations) cannot be predicted.

A three-dimensional elastodynamic solution was recently applied to nondestructive testing of pavements by Mamlouk and Davies (6-8) and by Roesset and Shao (9). It was demonstrated that the pavement surface profile differs in several respects from the stationary deflection basins assumed in static analyses. The pavement surface takes a wave form propagation outwards from the point of excitation in a manner similar to that produced by exciting a surface of water. These waves are highly affected by the mode of loading, loading frequency, or both. Static analyses of dynamic deflections may result in significant errors, particularly if a resonant condition is encountered or if the radiation damping is large. Also, it was reported that the material damping (the viscous effect) is insignificant compared with the radiation damping in the pavement structure (the dynamic effect). Field observations indicate that different dynamic devices result in different deflection readings--a behavior that cannot be explained by a static analysis (1,10,11).

The purpose of this study is to provide a rational interpretation of the FWD performance based on elastodynamic analysis. The dynamic response of selected flexible pavements is verified by using field data. In this study, static and dynamic analyses of pavement response are compared and the errors resulting from ignoring the inertia of pavements are evaluated.

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FWD OPERATING CHARACTERISTICS

The FWD delivers a transient force impulse to the pavement surface. The device uses a weight that is lifted to a given height on a guided system and then dropped. The falling weight strikes a plate that transmits the force to the pavement. By varying the mass of the falling weight, the drop height, or both, the impulse force can be varied. The FWD has a relatively small preload compared with the actual loading. The preload during the period the weights are dropping is usually in the range of 3 to 14 percent of the maximum load. The mass usually weighs about 200 lb and is dropped from a height varying from 0.13 to 1.3 ft. The weight drops onto a set of springs or a rubber buffer system to provide a load pulse in approximately a half-sine wave form. In most cases, the load is transmitted to the pavement through a 12-in.-diameter loading plate. The pulse duration is usually 30 to 40 msec.

The springs or the rubber buffers are usually loaded within their elastic range so that the force applied to the pavement can be found by equating the initial potential energy of the system to the stored strain energy of the springs when the mass is at rest. Therefore,

$$F = (2Mghk)^{1/2} \tag{1}$$

where

- F = applied force (lb),
- M = falling mass (lbm),
- g = acceleration of gravity (ft/sec²),
- h = drop height (ft), and
- k = spring constant (lb/ft).

The deflections are measured at the pavement surface at the center of the load and at several radial distances from the plate center. Velocity transducers (geophones) are usually used to measure deflections.

FWD DYNAMICS AND TRAFFIC SIMULATION

To investigate the dynamics of the FWD, two separate problems have to be addressed: the dynamics of the FWD and the dynamic response of the pavement system to the pulse loading. For the FWD, a simple discrete mass-spring model suffices to represent the device, as shown in Figure 1. The total displacement of the

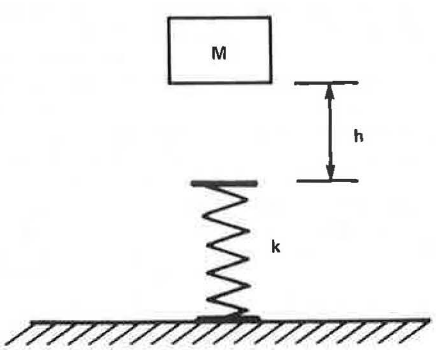

FIGURE 1 Discrete mass-spring model of FWD.

FWD system after time t can be given by the following equation:

$$z = s[(1 - \cos \omega t) + (2h/s)^{1/2} \sin \omega t] \tag{2}$$

where

- s = compression of the spring under the static loading (ft),
- ω = natural frequency of the oscillation of the system (Hz), and
- h = drop height (ft).

The compression-time relation described by Equation 2 is shown in Figure 2. The first term in Equation 2 represents the response of the spring if the free fall of the mass is zero. In this case, the compression of the spring reaches twice the static value, that is, 2s. The second term is dominant for large drop heights. For example, because FWD spring stiffness is approximately 6 kips/in. and the mass is approximately 200 lbm, the static compression of the spring is only 40 mils. Thus, even for drop heights as low as 1.5 in., the maximum spring compression is predicted to within 10 percent by the second term in Equation 2 alone (in this case, 0.35 in. instead of the correct 0.39 in.). Because for larger drop heights the error is even smaller, the compression-time relation for all practical purposes becomes

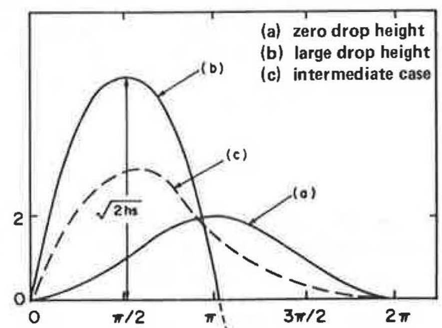

FIGURE 2 Three compression time relations.

$$z = (2h/s)^{1/2} \sin \omega t \tag{3}$$

Also, after the loading and unloading cycle is complete, the mass loses contact with the spring for a time period after which contact takes place again. A previous study indicated that, even for the most conservatively low drop heights, this elapsed time is approximately 180 msec, and therefore the initial impulse imparted by the FWD is effectively independent of the subsequent rebound (12).

As far as traffic simulation is concerned, previous studies concluded that a fixed-in-place nondestructive testing device of current design cannot simulate exactly the loading effect of a moving wheel (1,2). This conclusion is suggested by the disagreement between the considerably higher accelerations induced by the FWD (more than 4 g) and the low acceleration induced by moving wheels (approximately 0.1 g). Also, differences between the pulse duration of the FWD (25 to 60 msec) and that of moving trucks (about 200 msec) have been observed. Nevertheless, field results on deflections compare favorably. Moreover, better simulation can be achieved by simultaneously decreasing the drop height and increasing the mass of the drop weight. In this case, the full compression-time relation (Equation 2) may have to be adopted.

DYNAMIC ANALYSIS

In this analysis, the pavement system is idealized as layered viscoelastic continuum overlying the bedrock at a finite depth, as shown in Figure 3. Each material is characterized by Young's modulus, Pois-

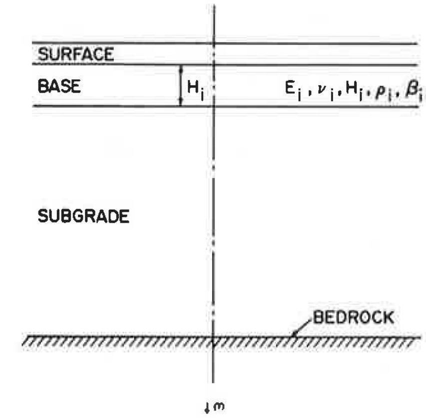

FIGURE 3 Flexible pavement characterization as a multilayer viscoelastic system.

son's ratio, material damping, and mass density. The assumptions of material linearity and isotropy as well as the no-slip conditions at the layer interfaces are invoked. The thicknesses of various layers as well as the depth to bedrock are assumed to be known.

The dynamic solution was obtained by using the multilayer computer program DYNAMIC. The solution is based on the Helmholtz equation of elastodynamics for harmonic loading (13). Because no direct solution to the elastodynamic equation is available for multilayered systems, a numerical solution must be used (7,14). This solution provides the in-phase and out-of-phase displacements at any location throughout the pavement structure. Unlike the single degree-of-freedom representations, the three-dimensional nature of the pavement system is considered in this solution. Further, the static solution can be easily derived from the dynamic solution simply by using a frequency of zero. This is valid because the Helmholtz equation of elastodynamics reduces to Navier's equation of elastostatics when the frequency is reduced to zero.

Because the FWD applies a transient load on the pavement surface, Fourier transformations can be used to represent the transient mode by a series of harmonic loadings with various frequencies and amplitudes. The FWD loading impulse is assumed to be periodic with a period T that is essentially divided into two time periods: (a) loading pulse width, T_p, and (b) rest period, T_R, as shown in Figure 4. The pulse width, T_p, is a function of the loading device as well as of the pavement system properties, varying from 25 to 60 msec for most FWD devices. Because a repeated loading function is used in the analysis, the second period, T_R, is chosen in such a way that the pavement fully recovers from deformation during this time so that the response to each impulse is isolated from earlier ones. After the evaluation of Fourier coefficients of the load impulse expansion, the phase lag, frequency, and amplitude of each harmonic load are obtained. Thus, the solution for the displacement response is sought in the so-called frequency domain (14). The responses of each of the harmonic loadings are summed in the time domain, which results in the full response of the impulse loading generated by Fourier expansion. The number of terms used in the Fourier expansion is a function of the pulse parameters and the degree of approximation sought.

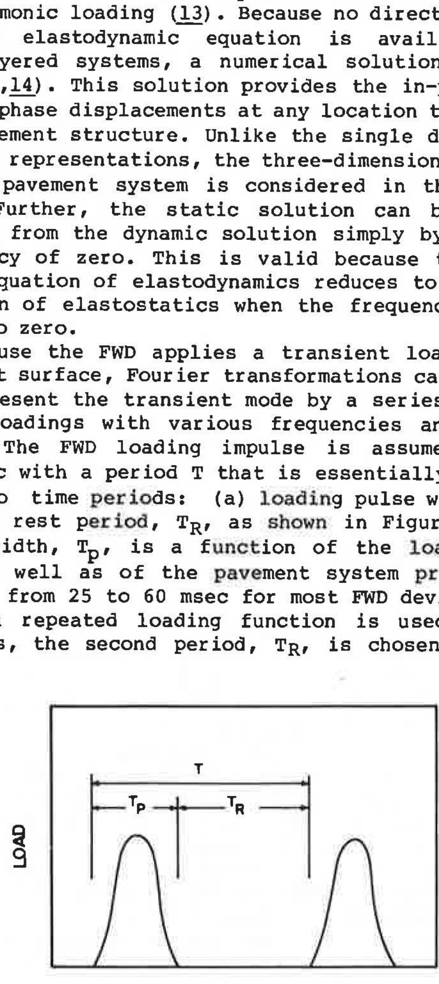

FIGURE 4 Assumed periodicity of FWD impulses.

APPLICATIONS AND RESULTS

To verify the validity of the model described in this paper, a theoretical analysis was performed on selected typical in-service flexible pavement sections. The theoretical results are compared with data reported from FWD measurements and laboratory testing on pavement sections Bement, Deland, Monticello, and Sherrard performed by Hoffman and Thompson (1). Each section consists of a surface layer and a base course above the subgrade, as shown in Figure 3. The material types and layer thicknesses of various pavement sections are given in Table 1. No information was available concerning the thickness of the subgrade above the bedrock. A 50- to 60-ft subgrade thickness was assumed based on field observations indicating that the bedrock is deep.

TABLE 1 Material Types and Layer Thicknesses of Pavement Sections

Section	Layer	Type	Thickness (in.)
Bement	Surface	Asphalt concrete	4
	Base	Soil cement	6
	Subgrade	A-7-6 (24)	720[a]
Deland	Surface	Surface treatment	0.5
	Base	Granular	8
	Subgrade	A-7-6 (21)	720[a]
Monticello	Surface	Asphalt concrete	3.5
	Base	Plant-mixed CAM	8
	Subgrade	A-6 (8)	720[a]
Sherrard	Surface	Asphalt concrete	4
	Base	Crushed stone	14
	Subgrade	A-4 (6)	720[a]

[a]Assumed values.

Table 2 gives the material properties of the pavement sections selected for analysis. The modulus values were reported by Hoffman and Thompson (1) as a result of an extensive laboratory program on the four pavement sections (Bement, Deland, Monticello, and Sherrard). The resilient properties of the subgrade soils were produced by subjecting the samples to repetitive loadings of varying magnitudes, according to the method proposed by Thompson and Robnett (15). For asphalt concrete layers and stabilized materials, the specimens were subjected to repeated diametral loads, and the diametral resilient modulus was computed. The modulus values used in this study were within ± 1 standard deviation from the laboratory mean values, and typical Poisson's ratios were assumed.

The experimental results indicated a difference in temperature between field and laboratory conditions, which was also taken into account. A typical

TABLE 2 Pavement Material Properties

Section	Layer	Moduli[a] (ksi)	Poisson's Ratio[b]	Density[b] (lb/ft^3)
Bement	Surface	170	0.35	145
	Base	1700	0.4	140
	Subgrade	7.5	0.45	115
Deland	Surface	30	0.35	145
	Base	9	0.4	140
	Subgrade	9	0.45	115
Monticello	Surface	450	0.35	145
	Base	650	0.4	140
	Subgrade	8	0.45	115
Sherrard	Surface	500	0.35	145
	Base	35	0.4	140
	Subgrade	10	0.45	115

[a] From laboratory testing.
[b] Assumed values.

TABLE 3 Frequency Content of FWD Load Impulse Obtained by Fourier Series Expansion

Frequency (Hz)	Relative Amplitude	Cumulative Amplitude (percent)
0	51	9
4.5	100	26.8
9.1	94	43.5
13.6	84	58.4
18.2	72	71.2
22.7	58	81.5
27.3	44	89.3
31.8	25	93.8
36.4	20	97.3
40.9	11	99.3
45.5	4	100.0

material damping ratio of 5 percent was used throughout the analysis (16). The FWD used to obtain the deflection measurements (1) had a 12-in.-diameter baseplate and produced an impulse load of 8 kips with a loading duration ranging from 30 to 40 msec. The deflections were measured with velocity transducers (geophones) located at 0, 1, 2, and 3 ft from the center of the loading plate. In the analysis, the loading impulse induced by the FWD was generated by a series of harmonic loads using the Fourier expansion, as discussed earlier. The half-sine impulse representation was found to be adequately modeled by using only the first 10 terms of the Fourier series.

By using a pulse duration of 40 msec and a rest period of 180 msec, it was found that the shape of the half-sine wave load impulse was accurately reproduced. Figure 5 shows the theoretical impulse synthesized from the Fourier expansion. The actual and theoretical impulses had the same duration and their amplitudes did not differ by more than 2 percent. The 10 frequencies used in the analysis and their relative amplitudes are given in Table 3 and shown in Figure 6. By examining Figure 6, it can be observed that the FWD pulse cannot be simply represented by a single load frequency because it is a complex combination of a number of harmonic loadings with different frequencies and magnitudes. It should be noted that the highest frequency is 45.5 Hz, which is sufficiently low to avoid the numerical difficulties associated with the high-frequency response of the discrete layer formulation (14).

The analysis indicated the existence of a short time lag between the application of the load impulse and the response of the pavement at all four geophone locations. Figure 7 shows the phase lag for the center of the baseplate at the Bement section, which is almost typical for the other three locations. The phase lag is small, only 4 to 6 msec at the peak deflection, and no discernible flattening occurs with respect to the initiating impulse. This phase lag is mainly a consequence of inertia and not of material damping. Also, the results indicate that small phase lags (not more than 1.5 msec) exist between peak deflections at successive geophones. This analysis proves also that little difference in the peak acceleration occurs between the falling mass itself and the pavement. This conclusion varies somewhat from the experimental results (1). This variation may be due to the existence of the cushioning material between the FWD and the pavement that was used to distribute the road more evenly.

In addition to the dynamic formulation discussed earlier, a static analysis was performed for comparison. The static solution was obtained by using the same DYNAMIC computer program at zero frequency. Samples of the DYNAMIC program results with zero frequency were verified by using the Chevron computer

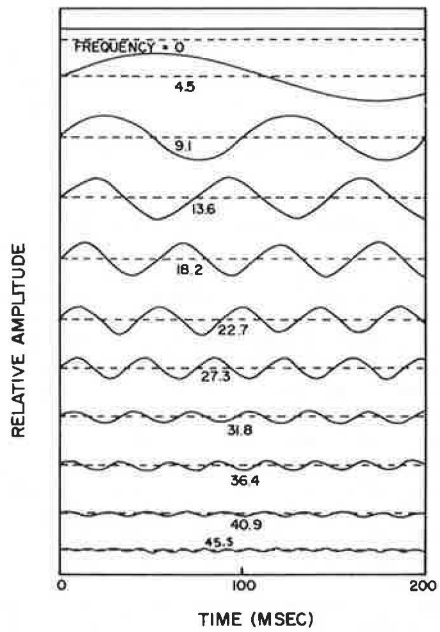

FIGURE 6 Equivalent theoretical harmonic components of FWD impulse.

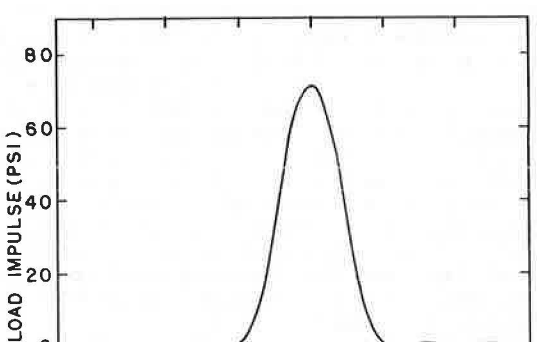

FIGURE 5 Theoretical representation of FWD impulse.

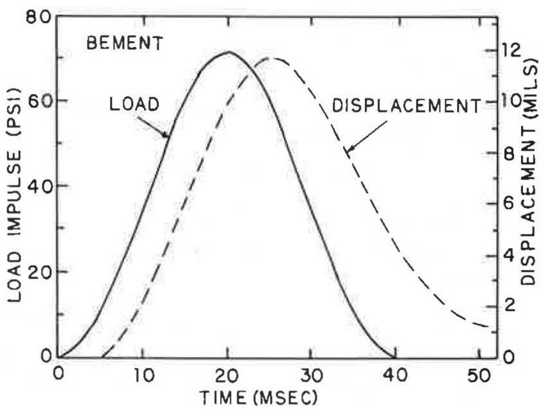

FIGURE 7 FWD load impulse and pavement displacement response at center of baseplate.

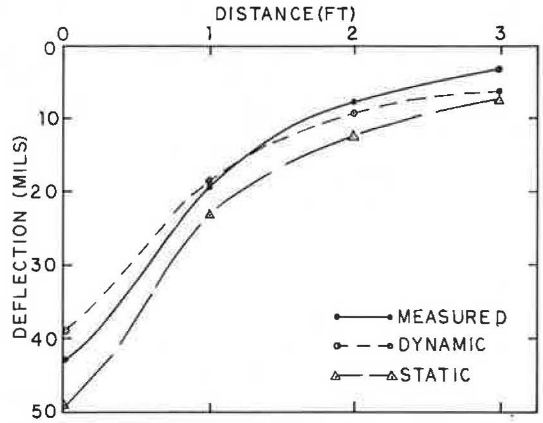

FIGURE 9 Measured, static, and dynamic deflections at various geophone locations for Deland section.

program under similar conditions with a rigid base. The two programs provided identical results.

Comparisons between the experimental data of the FWD and the results of the numerical model used here are shown in Figures 8-11. Each figure shows field deflections as well as deflections obtained by using the static (zero-frequency) analysis and the dynamic model at various geophone locations. The results indicate that in all four pavement sections the dynamic analysis values were within ±15 percent of the field deflections and as low as 3 percent in some cases. The dynamic curves crossed the field results in all figures, indicating that in most cases the highest error occurs below the center of the baseplate. It is probable that part of the discrepancy at some locations is a consequence of the approximate estimation of the subgrade thickness. Meanwhile, the static analysis always yielded average deflection values approximately 25 to 30 percent higher than those obtained from the elastodynamic analysis. In most cases, these static deflections were 20 to 40 percent different from the field deflections.

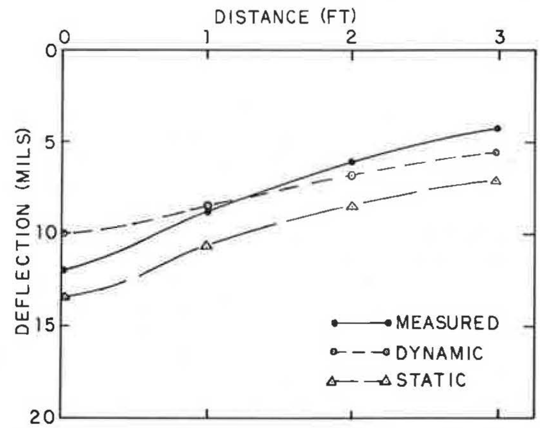

FIGURE 10 Measured, static, and dynamic deflections at various geophone locations for Monticello section.

SUMMARY

Published field data on pavement response to the FWD were analyzed by using an elastodynamic technique. Because of the inertia of the pavement system, the displacement impulse slightly lags the loading impulse, although its shape closely reflects the half-sine loading curve. Such analysis shows close agreement with the field data, with a maximum error of ±15 percent. On the other hand, the static analysis of the pavement response to the FWD always resulted in average surface deflections 20 to 40 percent larger than field measurements. This indicates that the static analysis of the FWD overestimates (by backcalculating from deflection data) the stiffnesses of the pavement layers. It is expected that larger differences between static and dynamic results would

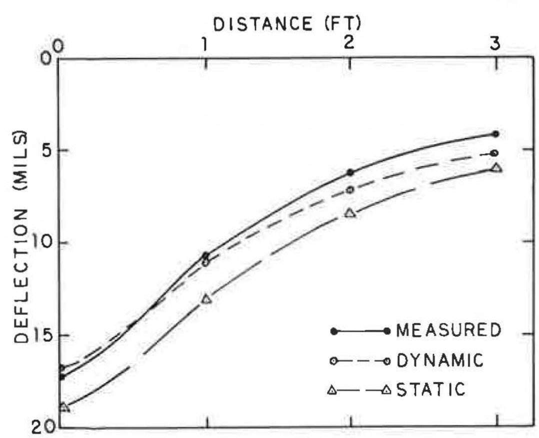

FIGURE 11 Measured, static, and dynamic deflections at various geophone locations for Sherrard section.

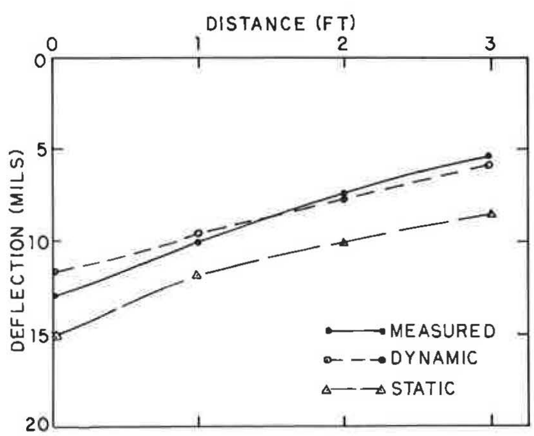

FIGURE 8 Measured, static, and dynamic deflections at various geophone locations for Bement section.

have been obtained if the subgrade was smaller than that used in this study.

The FWD appears to be suitable for the nondestructive testing of pavements because it simulates the shape and temporal nature of moving wheel loading reasonably closely. In addition, the hazards of resonance associated with periodic loading devices such as the Dynaflect and the Road Rater are less acute with the transient loading of the FWD.

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