Further Studies on Lateral Load Distribution Using a Finite Element Method

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ABSTRACT

A computer program, SALOD, has been written for the Florida Department of Transportation to evaluate the lateral load distribution characteristics of simple-span bridges in flexure. Bridges may be prestressed concrete girder, steel girder, T-beam, or flat slab. The program uses moment influence services generated by the STRUDL finite element system for representative simple-span bridges determined by a statewide survey. Up to three vehicles are placed in critical locations to determine the maximum distribution factors. The effect of span length, which is neglected in AASHTO, was found to be considerable. AASHTO results were found to be slightly unconservative for short spans and quite conservative for longer spans. Field testing, reported elsewhere, has been completed on eight bridges. Comparisons of results from finite element models and measurements of applied truck loading have been generally good. Comparisons of flexural distribution factors from SALOD and the Ontario Highway Bridge Design Code (OHBC) for prestressed girder bridges showed generally good agreement. However, OHBC indicates more sensitivity to girder spacing than does SALOD. AASHTO simple-span results compare quite well with SALOD for exterior girders. A limited study of shear distribution factors for girder-slab bridges showed that shear distribution factors do not vary significantly with span length and that AASHTO factors appeared adequate for design.

Lateral load distribution based on flexure in highway bridges has been the subject of previous research at the University of Florida (1). The AASHTO procedure for computing flexural distribution factors is generally used for bridge design by the Florida Department of Transportation (FDOT) and tends to be overly conservative for analyzing infrequent bridge overloads, which causes unnecessary rerouting of vehicles in some circumstances.

A computer program, Structural Analysis for Load Distribution (SALOD), was developed in prior research to compute accurate flexural distribution factors for a variety of girder-slab bridges under specific vehicular loading. The SALOD program uses a data base of influence surfaces that were generated using the finite element method of analysis with the STRUDL software package available on the FDOT computer system. The program has proven useful to the FDOT not only for large overload vehicles but also in evaluating bridges for legally permitted standard vehicles that may cause larger moments than AASHTO design vehicles because of close axle spacings.

A brief summary of the SALOD program, comparisons of the flexural distribution factors obtained using SALOD and the recommendations of the Ontario Highway Bridge Design Code (OHBC) for a wide range of prestressed concrete girders, and a brief study of shear distribution in prestressed concrete girders are presented. In addition, field studies were made to verify the finite element technique used in developing the SALOD program. These studies, reported elsewhere (2), demonstrated that the SALOD program could be used to obtain accurate predictions of flexural lateral load distribution.

SALOD COMPUTER PROGRAM

The SALOD program computes flexural distribution factors (or effective widths) for the following simple-span bridge systems: (a) prestressed concrete girders, (b) cast-in-place T-beams, (c) steel girders, and (d) flat slabs. A bridge can be loaded with as many as three standard (vehicle data stored in the program) or nonstandard vehicles. The program arranges the vehicles and locates them on the bridge in such a manner as to produce the maximum midspan girder moment. The distribution factor is computed as the ratio of this moment to half the simple beam moment due to one of the vehicles.

The midspan girder moment is obtained through the use of midspan moment influence surfaces. A permanently stored data base of influence surfaces has been generated for the four bridge types previously listed using the finite element method of analysis in conjunction with the McAuto STRUDL software package (3), which is available on the FDOT computer system. The selection of important bridge parameters, their range, and specific values within that range to be included in the data base were based on a statewide bridge parameter survey, practicality, and preliminary studies.

The SALOD program uses interpolation and a limited amount of extrapolation between the combinations of specific bridge parameter values represented in the data base to obtain an influence surface for the actual bridge data input for analysis. The SALOD program generates a mesh for the bridge being analyzed. This mesh is similar to the mesh used in the finite element model that was used to develop the set of influence surfaces for that particular bridge type.
The moment at midspan is computed by (a) distributing each wheel load to the finite element nodes, (b) multiplying each nodal load by the corresponding interpolated influence value at that node, and (c) summing the values obtained for all the wheel loads on the bridge. The maximum moment and critical locations of the vehicles are found by performing this operation with the vehicles systematically positioned at various longitudinal and lateral locations; vehicle spacing and clearance requirements are taken into consideration. For the girder bridges, this is done separately for each girder.

SALOD Finite Element Modeling

The following list gives the major assumptions and decisions made while developing the finite element models (Figure 1).

1. Linearly elastic behavior was assumed. This follows common practice (4) and results in a safe distribution of girder moments due to the ductile behavior of girder-slab bridges (5).

2. All girders, including the exterior girder, were assumed to have the same moment of inertia.

3. Plate bending elements were used for the finite element model of the bridge deck. Standard frame elements were used to model the girders and diaphragms.

4. The deck elements over the girders were artificially thickened to increase the transverse plate bending stiffness of the slab due to the girders for prestressed girder and T-beam bridges.

5. On the basis of the statewide survey, slab thickness was taken as 7.0 in. for prestressed girder and steel girder bridges and 7.5 in. for T-beam bridges. A study (1) showed that slab thickness has a minor effect on influence surface values.

6. Ten elements per half-span were used in the longitudinal (Y) direction for all the finite element models except that for flat slabs.

7. Two elements over the girders and four elements between adjacent girders were used in the lateral (X) direction for prestressed girder and T-beam bridges. Steel girder bridges had six equally spaced elements between girder centerlines.

8. For T-beam models, the ratio of girder spacing to girder width was held constant at five. Natesalier (6) showed that this gave generally good results for a wide range of actual T-beam dimensions.

9. Composite action between girders and deck slab was assumed for T-beams and prestressed girders; steel girders may be composite or noncomposite. Effective slab width was calculated on the basis of standard AASHTO recommendations, which are acceptable because minor variations in moment of inertia have little effect on influence surface values (1).

10. A torsional moment of inertia (J) of 20 in.$^4$ was used for steel girder bridges. Small J-values have little effect on load distribution (7). For all T-beam bridges, a torsional moment of inertia of 10,000 in.$^4$ was used. For prestressed girders, the torsional moment of inertia was obtained from a previous finite element solution (8).

11. Diaphragms with an 8- x 54-in. cross section were used only at the span ends for prestressed girder and steel girder bridges. Variation in the moment of inertia of end diaphragms had a negligible effect on influence values (3). End diaphragms were omitted from the T-beam model. Intermediate diaphragms were omitted from all models for simplicity because it has been shown that they often have a negligible effect on load distribution (9,10).

12. Half-span bridge models were used because of their structural symmetry about midspan (critical moment location).

13. The boundary conditions at midspan were set such that the midspan moments would be taken only by the girders and the slab moments at midspan would be neglected (prestressed girder and steel girder bridges). This simplification is acceptable because a study showed that the moment taken by the slab is negligible except for short spans (1). However, slab centerline moments were included for T-beam bridges, which generally have short spans.

General Assumptions and Procedures

The following list gives some general assumptions and procedures used in developing SALOD.

1. The maximum moment was assumed to occur at midspan. This is not true for a series of consec-
trated loads; however, a study showed that the distribution factor is not significantly affected (1).
2. The modulus of elasticity was computed using the American Concrete Institute (4) recommendations for normal weight concrete.
3. Concrete 28-day compressive stress (f’c) of 3,400 psi was used for the deck slab as required by FDOT specifications.
4. Standard AASHTO prestressed concrete girder Types II, III, and IV were used with a 28-day f’c of 5,000 psi.
5. Only four-, five-, and six-girder bridges were considered. Studies (1,11) showed that a six-girder SALOD solution may be used to obtain generally conservative results for bridges with more than six girders.
6. The moment of inertia for steel girder bridges was calculated at the midspan cross section. A study showed that variation in moment of inertia along the span due to cover plate cutoff had little effect on influence surface values at the centerline of bridges (1).
7. Bridge skew was neglected. This will generally give conservative results for girder-slab bridges (12).
8. Standard FDOT vehicles stored by the SALOD program were SU 2, SU 3, SU 4, C 3, C 4, and C 5. Also, H 20 and HS 20 standard AASHTO vehicles were stored in the program. Nonstandard vehicles can be input by the user.
9. For vehicle clearance limitations used by the SALOD program, the vehicles’ wheels are assumed to be 9 in. wide. As applied loads, they are assumed to act as concentrated point loads.
10. Wheel loads are distributed to adjacent nodes assuming a series of simple stringers is acting.
11. A travel lane of 12 ft with a 10-ft load lane, which can be shifted to any position in the travel lane, is used to determine spacing limitations between vehicles for multiple vehicle loading (Figure 2). These spacing limits were developed using standard AASHTO vehicles; however, for nonstandard vehicles with different widths, the same spacing limitations are followed.

FORCE COMPUTER PROGRAM

The FORCE program was developed as a labor-saving aid in the analysis of bridges by the finite element procedure using the STRUDL software package. The program sets up a full-span finite element model for simply supported prestressed concrete girder, T-beam, steel girder, and flat slab bridges; computes the nodal loads for as many as three simultaneously acting vehicles placed at any location on the bridge; and generates a STRUDL program that can subsequently be executed. Many of the assumptions used in the finite element models that are generated by FORCE are similar to those previously described for the SALOD program. However, FORCE has more generality than SALOD and thus can be used for a wider range of bridges than permitted by SALOD. FORCE was used extensively in the shear studies and field studies (2). Details on the FORCE program are available elsewhere (11).

DETAILED COMPARISON OF SALOD AND OHBDC FLEXURAL DISTRIBUTION FACTORS

The OHBDC (13,14) method for computing flexural distribution factors takes span length, girder spacing, and stiffness properties into consideration. A comparison of SALOD and OHBDC distribution factors may help mutually reinforce their validity.

OHBDC Modification Factors and Critical Loading

In the development of the OHBDC graphs, modification factors were used to account for the probability of the presence of multiple vehicles on a bridge. The modification factors used were 1.0, 0.9, and 0.8 for one, two, and three vehicles, respectively. Also, the graphs were developed on the basis of the criti-
cal loading case. For example, a three-lane bridge could be loaded with one, two, or three vehicles simultaneously. The theoretical D-values were divided by the appropriate probability modification factor. To make a direct comparison with SALOD, the same modification factors were applied to the SALOD results.

Bridge and Loading Parameters Studied

The bridges used in this study were analyzed by SALOD and OHBDC and the results are presented in the form of a parametric study. There are two separate sets of graphs—one set for interior girders and one for exterior girders. Additional curves are included elsewhere (11).

The graphs showing the effect of span length are shown in Figure 3 for interior girders and in Figure 4 for exterior girders. The bridges used in this study had five Type III prestressed concrete girders spaced at 7.0 ft with span lengths of 30, 60, 90, and 120 ft. The overhang was selected as 3.0 ft to conform closely with OHBDC maximum overhang requirements. In the analyses, three design lanes with a width of 12.0 ft were used. This was slightly conservative because the bridge was only 34 ft wide. The bridge was loaded with one, two, and three standard H 20 vehicles. The SALOD solution for one H 20 vehicle was never critical for interior girders and the modified three-H 20 solution was never critical for exterior girders. Therefore, these two curves were omitted from the corresponding graphs.

The sets of graphs showing the effect of girder spacing are shown in Figure 5 (a and b) for interior girders and in Figure 6 (a and b) for exterior girders. The bridges used in this study had a constant distance of 28 ft between centerlines of exterior girders with an overhang of 3.0 ft on each side. Bridges with span lengths of 30, 60, 90, and 120 ft were used. However, results are shown herein for only the 30- and 120-ft spans. Each bridge had four, five, or six Type III prestressed concrete girders with corresponding 9.33-, 7.0-, or 5.6-ft spacings, respectively. All other conditions were the same as previously described for the span length study.

Discussion of Results

Figure 3 shows that the distribution factor varies significantly with span length for both SALOD and OHBDC solutions. Both show about the same percentage change in the distribution factors with changing...
The critical SALOD curve is generally conservative, compared with that of OHBDC, by about 8 percent. This can be attributed mainly to two averaging processes used by OHBDC when idealizing the bridge as an orthotropic plate. First, this method uses a smeared, or average, stiffness across the width of the bridge, whereas SALOD accounts for the exact position of increased stiffness due to the girders. Second, OHBDC graphs are based on moments that are averaged over a certain transverse width of plate to eliminate highly localized intensities of longitudinal moments resulting under concentrated loads as predicted by orthotropic plate analysis (15).

Figure 5 (a and b) compares the distribution factor variation with girder spacing at different span lengths. The critical SALOD distribution factors are always slightly conservative compared with those from OHBDC. The difference changes slightly depending on span length and girder spacing; however, both methods show essentially the same variation in the distribution factor with girder spacing.

Figure 4 shows the variation of SALOD and OHBDC distribution factors with span length for exterior girders. Both methods exhibit the same trends with span length variation. The critical SALOD values are about 10 percent higher than those of OHBDC. However, Figure 6 (a and b) illustrates that the OHBDC distribution factors show a much more pronounced effect with changing girder spacing than SALOD. The method used by OHBDC for averaging the peak moments may be less accurate for exterior girders than for interior ones because of eccentric loading. Also, the exterior girder distribution factor is quite sensitive to vehicle positioning relative to the position of the exterior girder.

In Figure 7a for interior girders, it can be seen that AASHTO, which neglects the span length effect, matches OHBDC for the 30-ft span length and agrees with the SALOD results for the 60-ft span length. SALOD and OHBDC both become less conservative with increasing span length and differ by about 17 percent. This large difference is the result of OHBDC's having more liberal modification factors than AASHTO and other reasons discussed previously. Also, it should be noted that the basic design vehicles specified by the Ontario code (13) are heavier than those recommended by AASHTO (18). Both SALOD and OHBDC show a definite span length effect—about a 22 percent change in the distribution factor between the 30- and 120-ft span lengths—whereas AASHTO shows no span effect.
Figure 7b for exterior girders shows that SALOD agrees well with the AASHTO simple-beam results. The OHBDC distribution factors are again lower than those computed by SALOD. The effect of span-length on the distribution factors for exterior girders is quite small.

Shear Distribution Study for Prestressed Concrete Girder Bridges

Variations of shear and moment distribution factors along the span lengths for exterior girders and critical interior girders were determined using STRUDL finite element models generated by the FORCE program. These variations were plotted along with the results obtained using the SALOD program and OHBDC. Both shear and moment distribution factor variations were plotted because AASHTO recommendations for computing the shear capacity of prestressed concrete girders include an equation for the combined effect of shear and moment. The STRUDL and SALOD distribution factors were modified using OHBDC probability factors for a direct comparison of methods.

Bridge and Loading Parameters

The bridges used in the following studies are the same as those used in the flexural distribution factor study presented in this paper, except the 120-ft span bridges are not included in the shear study.

Vehicle loading for all bridges consisted of one H 20 and then two-H 20 standard AASHTO vehicles. All vehicles were facing in the forward (positive Y) direction. The critical lateral positioning of the vehicles to create the maximum girder shear was obtained by using the vehicle and bridge clearance limitations used by the SALOD program (Figure 2).

In the longitudinal direction, seven loading positions were used for Load Position 1, and the vehicles were positioned with their rear axle at the span end (Y = 0). For Load Positions 2 through 7, the vehicles were positioned with their rear axle at Y/L equal to 0.05, 0.1, 0.2, 0.3, 0.4, and 0.5, respectively (where Y is the distance from the span end and L is the span length).

STRUDL Distribution Factors

Finite element solutions were obtained with the vehicles at each of these load positions. The distribution factors were computed by dividing the output shears and moments by one-half of the corresponding simple-beam shears and moments at the same locations.

OHBDC Shear Distribution Factors

Shear distribution factors used for design by the OHBDC (13,14) are based on bridge type, number of lanes, and a correction factor that is used when the girder spacing is less than 2.0 m (6.56 ft). Also, OHBDC does not distinguish between exterior and interior girders for shear.

Variation of Distribution Factors Along Span

Complete results of this shear study are presented elsewhere (11) in graphs similar to those shown in Figures 8 and 9. The figures show the variation of shear and moment distribution factors along the span computed using STRUDL. Also the results from SALOD and OHBDC are included. The SALOD distribution factors are based on and shown only for flexure.

As can be seen in these graphs, for interior girders, the two-H 20 solution is always critical for both shear and flexure. The shear distribution factors (STRUDL) vary significantly along the span, especially close to the span end. The sharp decrease in the distribution factor near the end is due to the end diaphragm. The end diaphragm was assumed to be in contact with the slab and thus was connected at all the nodes between the exterior girders. If the slab is not in contact with the diaphragm, the model will overestimate the diaphragm's effect at the end of the span.

The maximum shear distribution factor appears to occur at Y/L of about 0.05 and the distribution factor begins to decrease at positions farther from the span end. The short-span bridges show a continued decrease in the shear distribution factor to span centerline. For longer spans (not shown here), the curves decrease to a minimum at about quarter-span and then rise slightly at positions close to the span centerline.

The flexural distribution factors also vary along the span. However, this variation is much less than for shear distribution, except for positions of Y less than about 0.2 L where moments are small. The SALOD distribution factors agree well with the STRUDL...
flexural distribution factors at midspan as expected. Thus SALOD flexural distribution factors are sufficiently close to the STRUDL factors for flexure at Y/L greater than about 0.2.

Because the AASHTO specifications for the design of prestressed concrete girders include an equation that considers the ratio of girder shear to moment, it is interesting to study the ratio of STRUDL shear distribution factors to STRUDL flexural distribution factors. This ratio was computed at Y/L in the region of 0.2 to 0.3 for all the bridges (11) and was found to vary widely from bridge to bridge, ranging from about 0.8 to about 1.2 for interior girders. This ratio increased with increasing span length and decreased with increasing girder spacing. However, the ratio for short spans showed little change with girder spacing.

The STRUDL and OHBDC shear distribution factors were found to match best at Y/L between 0.2 and 0.3, except at the wider girder spacings. However, the OHBDC procedure was apparently developed for maximum shear near the span end (3).

Shear Distribution Factor Parameter Study

The shear distribution factor parameter study shown in Figures 10 (a and b) for interior girders and 11 (a and b) for exterior girders contains results for AASHTO, OHBDC, and STRUDL. Additional graphs are included elsewhere (11). The STRUDL distribution factors were determined using the critical loading condition without modifying the results for probability of loading for a more direct comparison with AASHTO.

The STRUDL analysis for the shear study (11) developed shear distribution factors at several positions along the span. However, shear in prestressed girders may be more critical at quarter-span (19). Thus the shear distribution factors plotted in Figures 10 and 11 were determined from the largest value in the region of Y/L between 0.2 and 0.3 for the critical loading.

AASHTO Shear Distribution Factors

For loads at the support, AASHTO recommends computing the shear distribution factors assuming simple-beam action between girders in the transverse direction. For loads away from the support, AASHTO recommends using the flexural criteria for computing shear distribution factors. This requires using the formula S/5.5 for interior girders and the simple-beam approach for exterior girders.

In the figures, graph (a) shows the variation in the shear distribution factors with changing span length for the bridges with girder spacing of 7.0 ft. As seen in this figure, there is no significant variation in the shear distribution factors with changing span. STRUDL varies only 3 percent, and AASHTO and OHBDC do not consider span length.

Figure 10b shows the variation in the shear distribution factor with changing girder spacing for interior girders with a 60-ft span. The STRUDL curves are generally slightly less sloped than are those of the other two methods. That is, STRUDL shear distribution factors show less sensitivity to changing girder spacing. OHBDC is generally unconservative compared with STRUDL. This is primarily due to the probability factors that are implicit in the OHBDC.
solution. However, at wider girder spacings, OHBDC tends to become conservative. The AASHTO curves are consistently close to the STRUDL curves (within 14 percent) and are usually on the conservative side. At shorter girder spacings, AASHTO becomes unconservative by about 3 to 9 percent depending on the span length. AASHTO appears to be adequate for design.

Figure 11 compares the shear distribution factors for STRUDL, AASHTO, and OHBDC for exterior girders. Figure 11a shows the variation with span length. This figure shows that all methods have no significant variation with span length.

Figure 11b shows the variation in the shear distribution factor with changing girder spacing for exterior girders. STRUDL shows a 14 percent variation with girder spacing at the 30-ft span length. OHBDC varies 38 percent with girder spacing and differs from STRUDL by as much as 31 percent on the conservative side and 11 percent on the unconservative side.

The exterior girder curves for AASHTO are conservative compared with STRUDL for practically all bridges studied. This conservatism is generally less than 10 percent, except for the 30-ft span (not shown here).

**FIELD STUDIES**

The finite element method is today a well-accepted method of analysis. However, any method of analysis or modeling technique requires some degree of approximation when applied to a real structure. Thus it was prudent to verify the modeling assumptions made in the finite element analysis used to generate the data base for SALOD. A total of eight spans were tested, two of each of the following types:

1. Prestressed concrete girder bridges,
2. Steel girder bridges,
3. T-beam bridges, and
4. Flat slab bridges.

All of the bridges were simple span and tested under static load conditions. Strain and deflection data were taken near midspan using a data acquisition system. Complete details on the testing program and evaluation of results are available elsewhere.

**CONCLUSIONS AND RECOMMENDATIONS**

Parameter studies were done for prestressed concrete girder bridges covering a wide range of span lengths and girder spacings. Comparisons of OHBDC, AASHTO, and SALOD flexural distribution factors for interior girders show that, unlike AASHTO, both SALOD and OHBDC exhibit significant variation with span length. Both OHBDC and SALOD show the same percentage change in the distribution factors with changing girder spacing. However, OHBDC is generally about 8 percent unconservative compared with SALOD because of model-
ing differences. AASHTO is conservative compared with SALOD except at short span lengths.

For exterior girders, the GRBDC flexural distribution factors tend to be more sensitive to changing girder spacing than is SALOD or AASHTO. The simple-beam criterion used by AASHTO is a good representation of the flexural distribution characteristics of exterior girders.

In the design of prestressed concrete girders, considering the combined effect of shear and moment in the quarter-span region, distribution factors should be computed using the following guidelines:

1. Use the AASHTO criteria for computing shear distribution factors.
2. Use the SALOD program for computing flexural distribution factors. This procedure should give sufficiently accurate results for most prestressed concrete girder bridges. However, it should be noted that AASHTO shear distribution factors may be conservative at short girder spacings.

The SALOD program used influence surfaces developed using the finite element method. A series of tests (2) was conducted to validate the modeling techniques used in developing the influence surfaces for the SALOD program. The test program was believed to generally confirm the applicability of the finite element modeling techniques used in SALOD as a useful tool for predicting the moments in bridges for purposes of analysis and design.

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