Simplified Method for Estimating Thermal Stresses in Composite Bridges

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ABSTRACT

The North American codes of practice do not specify a temperature distribution throughout the depth of composite bridge superstructures; furthermore, no temperature differentials are given for accurately assessing the thermal stresses induced in such bridges. A realistic temperature distribution and temperature differentials are proposed in this paper. Simple formulas are deduced for use in the design office for estimating the thermal stresses in the concrete deck slab as well as in the steel girders in simple- and continuous-span composite bridges. It is shown that thermal stresses can be quite significant and should be included in the design of bridges.

Thermal stresses caused by temperature variation through the depth of a composite bridge can be relatively large compared with dead or live load stresses; such variation has been shown (1,2) to be nonuniform in composite concrete deck slab-on-steel beam bridges. These thermal stresses are known to cause considerable damage in structures (3-5); their presence will magnify the development of cracks in the concrete deck slab and thus in time cause corrosion of the steel reinforcing and the steel beams and deterioration of the concrete by allowing the seepage of salt-laden water. State-of-the-art surveys of thermal stresses in bridges were carried out by Reynolds and Emanuel (6) and more recently by Imbsen and Vandershaf (7). Surprisingly, none of the North American codes of practice provides guidance for the estimation of thermal stresses, or for the temperature distribution through the depth of the composite section; AASHTO (8) recently presented a range of temperature variation in bridges to account for the expansion movement but not for estimating thermal stresses.

The results reported in this paper are based on earlier work (9-11) that investigated the temperature distribution and temperature differentials through the depth of the composite section; simple formulas, suitable for the design office, are derived for estimating the thermal stresses in simple- and continuous-span bridges.

ANALYSIS

Following Zuk (9,12,13), the method of analysis is based on the following assumptions:

1. Plane sections remain plane after any temperature change;
2. The concrete deck slab is restrained in the transverse direction of the composite beam by the adjacent beam;
3. Perfect interaction exists between the concrete deck slab and the steel beam;
4. Both concrete and steel are elastic;
5. Temperature distribution is constant longitudinally and transversely along the beam but nonuniform throughout the depth;
6. Temperature of the reinforcing steel is the same as that of the surrounding concrete, and the concrete deck slab is symmetrically reinforced;
7. Fatigue stresses are negligible; and
8. The concrete deck slab and the steel beams are homogeneous and isotropic.

Temperature differentials through the depth of the composite section give rise to shearing forces (F) and couples (Q) at the interface between the concrete deck slab and the steel beams as shown in Figure 1(a). The resulting longitudinal strains at the bottom of the concrete deck slab and at the top of the steel beam can be shown to be (12)

$$\epsilon_{xc} = [2(1-\nu^2)/A_cE_c][2F-(3Q/2a)] + [(1+\nu)/2a]\,\alpha_c \int_{-a}^{+a} (T_{y_1} - T_o)dy_1 + [3(1+\nu)/2a^2]\,\alpha_c \int_{-a}^{+a} (T_{y_1} - T_o)\,y_1 dy_1 \tag{1}$$

$$\epsilon_{xs} = -(Qd_1/I_sE_s) - (F/E_s)[(d_1^2/I_s) + (1/A_s)] + (\alpha_s/A_s)\int_{-d_1}^{d_2} (T_y - T_o) b_y\,dy - (d_1\alpha_s/I_s)\int_{-d_1}^{d_2} (T_y - T_o) b_y\,y\,dy \tag{2}$$

where

ε_{sc}, ε_{xs} = total longitudinal strain at bottom of concrete deck slab and at top of steel beam, respectively;
F = interface shearing force, shown in Figure 1(a);
Q = interface couple, shown in Figure 1(a);
a = one-half concrete deck slab thickness;
w = slab width;
α_c, α_s = concrete and steel thermal expansion coefficients, respectively;
y_1 = distance from the centroid of the concrete deck slab to a fiber where the induced thermal stresses are determined, shown in Figure 1(b);
y = distance from the centroid of the steel section to a fiber where the thermal stresses are determined, shown in Figure 1(b);
b_y = width of steel section at distance y, shown in Figure 1(b);

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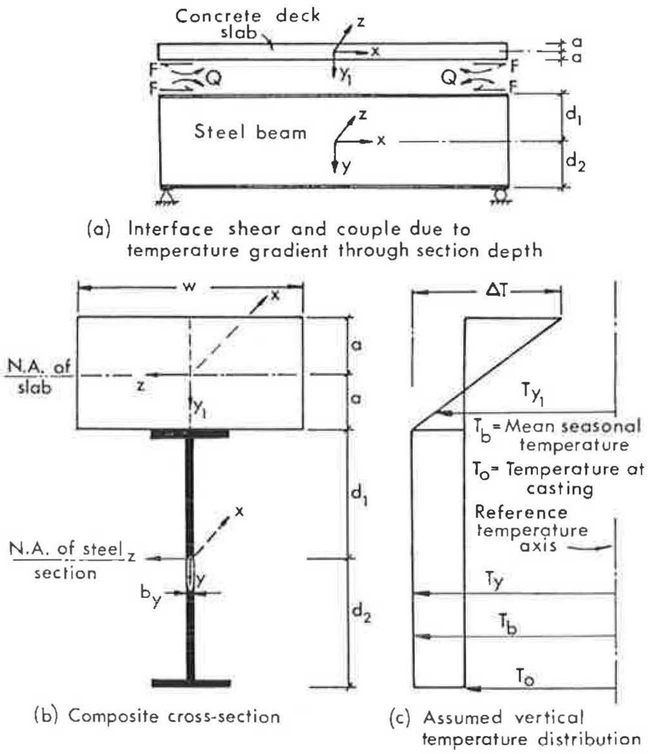

FIGURE 1 Interface shear and couple and assumed vertical temperature distribution.

E_c, E_s = concrete and steel modulus of elasticity, respectively;
A_s = area of the steel section;
I_s = moment of inertia of the steel section;
T_o = initial temperature at time of construction;
T_{y_1} = temperature of any fiber within the concrete deck slab, shown in Figure 1(c);
d_1, d_2 = distance from top and bottom fibers in the steel section, respectively, measured from the centroid of the steel section, shown in Figure 1(b); and
ν = Poisson's ratio of the concrete deck slab.

The shearing force (F) and the couple (Q) are determined from the compatibility conditions; namely, the strain and the radius of curvature for both the concrete deck slab and the steel beams must be the same at the interface. Thus equating Equations 1 and 2 for strain leads to

$$AF + BQ = [-(1+\nu)/2a]\,\alpha_c \left[\int_{-a}^{a} (T_{y_1} - T_o)\,dy_1 + (3/a)\int_{-a}^{a} (T_{y_1} - T_o)\,y_1\,dy_1\right] + \alpha_s\left[(1/A_s)\int_{-d_1}^{d_2} (T_y - T_o)\,b_y\,dy - (d_1/I_s)\int_{-d_1}^{d_2} (T_y - T_o)\,b_y\,y\,dy\right] \quad (3)$$

where A and B are coefficients defined in the Appendix. To deduce the radius of curvature for the concrete deck slab at the interface, difference in strains at the midplane of the slab and the bottom of the slab must be calculated. Such a difference in strains due to temperature change is given by

$$\Delta\epsilon_{xc1} = [3(1+\nu)/2a^2]\,\alpha_c \int_{-a}^{a} (T_{y_1} - T_o)\,y_1\,dy_1 \quad (4)$$

and the difference in strains due to F and Q is

$$\Delta\epsilon_{xc2} = [3(1-\nu^2)/2a^2 w E_c]\,(Fa - Q) \quad (5)$$

Hence the total difference in strain becomes

$$\Delta\epsilon_{xc} = \Delta\epsilon_{xc1} + \Delta\epsilon_{xc2} \quad (6)$$

From geometry the radius of curvature of the concrete deck slab is equal to the ratio $a/\Delta\epsilon_{xc}$; thus the radius of curvature (R_c) of the slab at the interface becomes

$$R_c = (a/\Delta\epsilon_{xc}) + a \quad (7)$$

The radius of curvature of the steel beam at the interface can be deduced from the moment on the steel beam given by

$$M_s = Fd_1 + Q + \alpha_s E_s \int_{-d_1}^{d_2} (T_y - T_o)\,b_y\,y\,dy \quad (8)$$

From the flexural formula, $(1/R) = (M/EI)$, the radius of curvature of the steel beam at the interface becomes

$$R_s = (E_s I_s/M_s) - d_1 \quad (9)$$

The last terms in Equations 7 and 9 are relatively quite small and therefore may be ignored without significant error. With this simplification, equating R_c to R_s yields an equation of the form:

$$KF + RQ = E_c E_s w\left[3(1+\nu)\alpha_c I_s \int_{-a}^{a} (T_{y_1} - T_o)\,y_1\,dy_1 - 2a^3\alpha_s \int_{-d_1}^{d_2} (T_y - T_o)\,b_y\,y\,dy\right] \quad (10)$$

where the coefficients K and R are as defined in the Appendix. F and Q can be explicitly determined from Equations 3 and 10 for any given temperature distribution through the depth of the cross section.

TEMPERATURE DIFFERENTIALS

From an extensive review of the pertinent literature, it was found that the most realistic and simple temperature distribution through the composite section is the one-dimensional distribution shown in Figure 1(c); the upper portion of the distribution is linear through the depth of the concrete deck slab, and the lower portion is uniform through the depth of the steel beams. On the basis of previous results (7), the temperature differential (ΔT) between the top of the concrete deck slab and the bottom of the steel beam can be assumed to be as given in Table 1. The maximum temperature differentials pertain to the case in which the concrete deck slab is exposed to the sun's radiation during the summer and winter, which results in a deck slab that is warmer than the steel beams; the minimum temperature differentials refer to the case in which the concrete deck slab is suddenly drenched with cold rain or snow and thus cools at a faster rate than the steel beams.

TABLE 1 Recommended Values for Temperature Differential, ΔT

ΔT	Summer (°F)	Winter (°F)
Maximum	40	20
Minimum	-7.5	-7.5

It can be shown (14) that for the linear uniform temperature distribution shown in Figure 1(c) Q and F can be expressed as

$$Q = \Delta T\, k_1 - (T_b - T_o)k_2 \quad (11)$$

$$F = (T_b - T_o)k_3 - Qk_4 \quad (12)$$

where

- ΔT = temperature differential between the top and the bottom surfaces of the concrete deck slab;
- T_b = maximum (or minimum) seasonal temperature obtained from a map of isotherms;
- T_o = temperature at casting of concrete; and
- k_1, k_2 ... = constants with values presented in Tables 2-7 for different steel beam sections, recommended by the U.S. Department of Transportation (15); three concrete deck slab thicknesses; and different concrete compressive strengths.

Explicit expressions for k_1, k_2 ... are given in the Appendix. The moment (M_s) in the steel beam, acting as an individual element, is

$$M_s = Fd_1 + Q \quad (13)$$

When F and Q are known, the thermal stresses in the concrete deck slab and in the steel beams can be calculated from (12)

$$\sigma_{xc} = (F/2aw) + [3(Fa - Q)y_1/2a^3w] - [\alpha_c E_c/(1-\nu)]\,(T_{y_1} - T_o) + [\alpha_c E_c/2a(1-\nu)] \int_{-a}^{+a} (T_{y_1} - T_o)dy_1 + [3\alpha_c E_c/2a^3(1-\nu)]\, y_1 \int_{-a}^{+a} (T_{y_1} - T_o)y_1 dy_1 \quad (14)$$

$$\sigma_{xs} = -\alpha_s E_s(T_y - T_o) + (\alpha_s E_s/A_s)\int_{-d_1}^{d_2} (T_y - T_o)b_y dy + (\alpha_s E_s/I_s)\int_{-d_1}^{d_2} (T_y - T_o)b_y dy + (Q/I_s)\,y + F[(d_1/I_s)y - (1/A_s)] \quad (15)$$

$$\sigma_{zc} = \nu\sigma_{xc} - \alpha_c E_c(T_{y_1} - T_o) \quad (16)$$

where

- σ_{xc}, σ_{xs} = thermal stresses induced through the depth of the concrete deck slab and the steel beams in the longitudinal direction (x), respectively, and
- σ_{zc} = thermal stress induced through the depth of the concrete deck slab in the transverse direction (z).

For the assumed linear uniform temperature distribution in Figure 1(c), Equations 14 and 15 reduce to

$$\sigma_{xc} = (F/2aw) - (Q/I_c)\,y_1 + (Fa/I_c)\,y_1 \quad (17)$$

$$\sigma_{xs} = (-F/2aw) + (Q/I_s)\,y + (Fd_1/I_s)\,y \quad (18)$$

where I_c is the moment of inertia of the concrete deck slab about its own centroid.

TABLE 2 Thermal Coefficients k_1 to k_4 for f'_c = 3 ksi

Steel Section	k_1 for 2a =			k_2 for 2a =			k_3 for 2a =			k_4 for 2a =		
	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.
36x300	-13.46	-17.68	-22.40	-0.19	-0.22	-0.23	0.03	0.04	0.05	-0.13	-0.10	-0.08
36x280	-13.11	-17.22	-21.83	-0.18	-0.20	-0.22	0.03	0.04	0.05	-0.13	-0.10	-0.07
36x260	-12.72	-16.71	-21.19	-0.17	-0.19	-0.20	0.03	0.04	0.04	-0.12	-0.09	-0.07
36x245	-12.43	-16.33	-20.72	-0.16	-0.18	-0.19	0.03	0.04	0.04	-0.12	-0.09	-0.07
36x230	-12.12	-15.93	-20.24	-0.15	-0.17	-0.17	0.03	0.04	0.04	-0.12	-0.09	-0.06
36x210	-11.48	-15.10	-19.20	-0.13	-0.14	-0.15	0.03	0.03	0.04	-0.11	-0.08	-0.06
36x194	-11.17	-14.73	-18.78	-0.12	-0.13	-0.13	0.03	0.03	0.04	-0.10	-0.07	-0.05
36x182	-10.92	-14.41	-18.40	-0.12	-0.12	-0.12	0.03	0.03	0.04	-0.10	-0.07	-0.05
36x170	-10.65	-14.08	-17.99	-0.11	-0.11	-0.11	0.03	0.03	0.03	-0.10	-0.07	-0.05
36x160	-10.41	-13.77	-17.61	-0.10	-0.11	-0.10	0.03	0.03	0.03	-0.09	-0.06	-0.04
36x150	-10.20	-13.50	-17.29	-0.10	-0.10	-0.09	0.03	0.03	0.03	-0.09	-0.06	-0.04
36x135	-9.71	-12.90	-16.55	-0.08	-0.08	-0.08	0.02	0.03	0.03	-0.08	-0.05	-0.03
33x221	-11.83	-15.51	-19.66	-0.14	-0.16	-0.16	0.03	0.04	0.04	-0.11	-0.08	-0.06
33x201	-11.39	-14.96	-18.99	-0.13	-0.14	-0.14	0.03	0.03	0.04	-0.11	-0.08	-0.06
33x141	-9.83	-12.99	-16.60	-0.09	-0.09	-0.08	0.02	0.03	0.03	-0.08	-0.06	-0.04
33x130	-9.52	-12.61	-16.13	-0.08	-0.08	-0.07	0.02	0.03	0.03	-0.08	-0.05	-0.03
33x118	-9.16	-12.16	-15.58	-0.07	-0.07	-0.06	0.02	0.02	0.03	-0.07	-0.04	-0.03
30x116	-8.96	-11.82	-15.07	-0.06	-0.06	-0.05	0.02	0.02	0.03	-0.07	-0.04	-0.02
30x108	-8.70	-11.50	-14.67	-0.06	-0.05	-0.04	0.02	0.02	0.02	-0.06	-0.04	-0.02
30x99	-8.42	-11.15	-14.24	-0.05	-0.04	-0.03	0.02	0.02	0.02	-0.06	-0.03	-0.02
27x94	-8.16	-10.74	-13.63	-0.05	-0.04	-0.02	0.02	0.02	0.02	-0.05	-0.03	-0.01
27x84	-7.84	-10.35	-13.14	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.05	-0.02	-0.01
24x94	-7.93	-10.36	-13.04	-0.04	-0.03	-0.02	0.02	0.02	0.02	-0.05	-0.03	-0.01
24x84	-7.63	-9.99	-12.57	-0.04	-0.02	-0.01	0.02	0.02	0.02	-0.04	-0.02	-0.00
24x76	-7.37	-9.66	-12.16	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.02	0.00
24x68	-7.10	-9.31	-11.71	-0.02	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
24x62	-6.80	-8.92	-11.21	-0.02	0.00	0.02	0.01	0.02	0.02	-0.02	0.00	0.01
21x55	-6.55	-8.60	-10.78	-0.01	0.00	0.02	0.01	0.01	0.01	-0.01	0.00	0.02
21x68	-6.80	-8.83	-10.97	-0.02	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
21x62	-6.60	-8.56	-10.63	-0.01	0.00	0.02	0.01	0.02	0.02	-0.02	0.00	0.01
21x50	-6.08	-7.88	-9.73	0.00	0.01	0.03	0.01	0.01	0.01	-0.01	0.01	0.02
21x44	-5.84	-7.55	-9.29	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03
18x55	-5.97	-7.61	-9.28	-0.01	0.01	0.03	0.01	0.01	0.02	-0.01	0.01	0.02
18x50	-5.78	-7.37	-8.95	-0.00	0.01	0.03	0.01	0.01	0.01	0.00	0.02	0.03

TABLE 3 Thermal Coefficients k_1 to k_4 for $f'_c = 4$ ksi

Steel Section	k_1 for 2a =			k_2 for 2a =			k_3 for 2a =			k_4 for 2a =		
	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.
36x300	-14.18	-18.63	-23.63	-0.19	-0.22	-0.23	0.04	0.04	0.05	-0.13	-0.10	-0.07
36x280	-13.82	-18.16	-23.04	-0.18	-0.20	-0.22	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x260	-13.41	-17.62	-22.38	-0.17	-0.19	-0.20	0.03	0.04	0.05	-0.12	-0.09	-0.07
36x245	-13.10	-17.23	-21.89	-0.16	-0.18	-0.19	0.03	0.04	0.04	-0.12	-0.09	-0.06
36x230	-12.78	-16.82	-21.40	-0.15	-0.17	-0.17	0.03	0.04	0.04	-0.11	-0.08	-0.06
36x210	-12.11	-15.95	-20.32	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
36x194	-11.80	-15.58	-19.90	-0.12	-0.13	-0.13	0.03	0.03	0.04	-0.10	-0.07	-0.05
36x182	-11.54	-15.25	-19.50	-0.12	-0.12	-0.12	0.03	0.03	0.04	-0.09	-0.07	-0.05
36x170	-11.26	-14.91	-19.09	-0.11	-0.11	-0.11	0.03	0.03	0.03	-0.09	-0.06	-0.04
36x160	-11.00	-14.59	-18.70	-0.10	-0.10	-0.10	0.03	0.03	0.03	-0.09	-0.06	-0.04
36x150	-10.79	-14.31	-18.37	-0.09	-0.10	-0.09	0.03	0.03	0.03	-0.08	-0.06	-0.04
36x135	-10.29	-13.69	-17.60	-0.08	-0.08	-0.07	0.02	0.03	0.03	-0.07	-0.05	-0.03
33x221	-12.47	-16.37	-20.79	-0.14	-0.16	-0.16	0.03	0.04	0.04	-0.11	-0.08	-0.06
33x201	-12.02	-15.80	-20.09	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x141	-10.41	-13.78	-17.64	-0.09	-0.09	-0.08	0.03	0.03	0.03	-0.08	-0.05	-0.03
33x130	-10.08	-13.38	-17.15	-0.08	-0.07	-0.06	0.02	0.03	0.03	-0.07	-0.05	-0.03
33x118	-9.71	-12.92	-16.58	-0.07	-0.06	-0.05	0.02	0.03	0.03	-0.07	-0.04	-0.02
30x116	-9.49	-12.56	-16.03	-0.06	-0.06	-0.04	0.02	0.03	0.03	-0.06	-0.04	-0.02
30x108	-9.23	-12.22	-15.61	-0.06	-0.05	-0.03	0.02	0.02	0.03	-0.06	-0.03	-0.02
30x99	-8.94	-11.87	-15.17	-0.05	-0.04	-0.02	0.02	0.02	0.02	-0.05	-0.03	-0.01
27x94	-8.66	-11.42	-14.51	-0.04	-0.03	-0.02	0.02	0.02	0.02	-0.05	-0.02	-0.01
27x84	-8.33	-11.01	-13.99	-0.04	-0.02	-0.01	0.02	0.02	0.02	-0.04	-0.02	0.00
24x94	-8.41	-11.01	-13.86	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.05	-0.02	0.01
24x84	-8.10	-10.62	-13.37	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.02	0.00
24x76	-7.84	-10.28	-12.94	-0.03	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.00
24x68	-7.55	-9.91	-12.46	-0.02	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
24x62	-7.24	-9.51	-11.94	-0.01	0.00	0.02	0.01	0.02	0.02	-0.02	0.00	0.01
21x55	-6.98	-9.16	-11.48	-0.01	0.01	0.03	0.01	0.01	0.01	-0.01	0.01	0.02
21x68	-7.23	-9.38	-11.66	-0.02	0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
21x62	-7.02	-9.11	-11.29	-0.01	0.00	0.03	0.02	0.02	0.02	-0.02	0.00	0.02
21x50	-6.48	-8.39	-10.34	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03
21x44	-6.23	-8.04	-9.86	0.00	0.02	0.04	0.01	0.01	0.01	0.01	0.02	0.03
18x55	-6.34	-8.08	-9.83	0.00	0.01	0.04	0.01	0.01	0.02	0.00	0.01	0.03
18x50	-6.15	-7.82	-9.48	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03

TABLE 4 Thermal Coefficients k_1 to k_4 for $f'_c = 5$ ksi

Steel Section	k_1 for 2a =			k_2 for 2a =			k_3 for 2a =			k_4 for 2a =		
	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.
36x300	-14.62	-19.21	-24.37	-0.19	-0.22	-0.23	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x280	-14.25	-18.72	-23.78	-0.18	-0.20	-0.22	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x260	-13.82	-18.18	-23.11	-0.17	-0.19	-0.20	0.04	0.04	0.05	-0.12	-0.09	-0.06
36x245	-13.51	-17.77	-22.61	-0.16	-0.18	-0.18	0.03	0.04	0.05	-0.11	-0.08	-0.06
36x230	-13.18	-17.36	-22.10	-0.15	-0.17	-0.17	0.03	0.04	0.04	-0.11	-0.08	-0.06
36x210	-12.49	-16.47	-21.01	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
36x194	-12.18	-16.10	-20.58	-0.12	-0.13	-0.13	0.03	0.04	0.03	-0.10	-0.07	-0.05
36x182	-11.91	-15.76	-20.18	-0.12	-0.12	-0.12	0.03	0.04	0.03	-0.09	-0.06	-0.04
36x170	-11.63	-15.41	-19.76	-0.11	-0.11	-0.11	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x160	-11.37	-15.09	-19.36	-0.10	-0.10	-0.10	0.03	0.03	0.03	-0.08	-0.06	-0.04
36x150	-11.15	-14.81	-19.02	-0.09	-0.10	-0.09	0.03	0.03	0.03	-0.08	-0.05	-0.03
36x135	-10.64	-14.18	-18.25	-0.08	-0.08	-0.07	0.02	0.03	0.03	-0.07	-0.05	-0.03
33x221	-12.86	-16.90	-21.47	-0.14	-0.16	-0.16	0.03	0.04	0.04	-0.11	-0.08	-0.05
33x201	-12.40	-16.32	-20.76	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x141	-10.76	-14.26	-18.27	-0.09	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.05	-0.03
33x130	-10.43	-13.85	-17.77	-0.08	-0.07	-0.06	0.02	0.03	0.03	-0.07	-0.04	-0.03
33x118	-10.05	-13.38	-17.19	-0.07	-0.06	-0.05	0.02	0.03	0.03	-0.06	-0.04	-0.02
30x116	-9.82	-13.00	-16.61	-0.06	-0.06	-0.04	0.02	0.03	0.03	-0.06	-0.04	-0.02
30x108	-9.55	-12.66	-16.19	-0.06	-0.05	-0.03	0.02	0.02	0.03	-0.05	-0.03	-0.01
30x99	-9.26	-12.30	-15.74	-0.05	-0.04	-0.02	0.02	0.02	0.02	-0.05	-0.03	-0.01
27x94	-8.97	-11.84	-15.04	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.05	-0.02	-0.01
27x84	-8.63	-11.42	-14.51	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.02	0.00
24x94	-8.70	-11.40	-14.36	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.04	-0.02	0.00
24x84	-8.39	-11.00	-13.86	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.01	0.00
24x76	-8.12	-10.65	-13.41	-0.03	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
24x68	-7.83	-10.28	-12.92	-0.02	0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
24x62	-7.51	-9.86	-12.38	-0.01	0.00	0.03	0.01	0.02	0.02	-0.01	0.00	0.02
21x55	-7.25	-9.51	-11.90	-0.01	0.01	0.03	0.01	0.01	0.01	-0.01	0.01	0.02
21x68	-7.49	-9.72	-12.07	-0.02	0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
21x62	-7.28	-9.44	-11.69	-0.01	0.01	0.03	0.02	0.02	0.02	-0.01	0.01	0.02
21x50	-6.72	-8.70	-10.71	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03
21x44	-6.46	-8.33	-10.21	0.01	0.02	0.04	0.01	0.01	0.01	0.01	0.02	0.03
18x55	-6.57	-8.37	-10.16	0.00	0.02	0.04	0.01	0.02	0.02	0.00	0.02	0.03
18x50	-6.37	-8.10	-9.80	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03

TABLE 5 Thermal Coefficients k_1 to k_4 for $f'_c = 6$ ksi

Steel Section	k_1 for 2a =			k_2 for 2a =			k_3 for 2a =			k_4 for 2a =		
	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.
36x300	-14.84	-19.50	-24.76	-0.19	-0.22	-0.23	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x280	-14.46	-19.01	-24.15	-0.18	-0.20	-0.22	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x260	-14.04	-18.46	-23.48	-0.17	-0.19	-0.20	0.04	0.04	0.05	-0.11	-0.08	-0.06
36x245	-13.72	-18.06	-22.98	-0.16	-0.18	-0.18	0.04	0.04	0.05	-0.11	-0.08	-0.06
36x230	-13.39	-17.64	-22.47	-0.15	-0.17	-0.17	0.03	0.04	0.04	-0.11	-0.08	-0.06
36x210	-12.69	-16.74	-21.36	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
36x194	-12.37	-16.36	-20.93	-0.12	-0.13	-0.13	0.03	0.04	0.04	-0.09	-0.07	-0.04
36x182	-12.10	-16.03	-20.53	-0.12	-0.12	-0.12	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x170	-11.82	-15.67	-20.10	-0.11	-0.11	-0.11	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x160	-11.56	-15.34	-19.70	-0.10	-0.10	-0.10	0.03	0.03	0.03	-0.08	-0.05	-0.04
36x150	-11.33	-15.07	-19.36	-0.09	-0.09	-0.09	0.03	0.03	0.03	-0.08	-0.05	-0.03
36x135	-10.82	-14.43	-18.58	-0.08	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.04	-0.03
33x221	-13.06	-17.17	-21.82	-0.14	-0.16	-0.16	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x201	-12.60	-16.58	-21.11	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x141	-10.94	-14.51	-18.59	-0.08	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.05	-0.03
33x130	-10.61	-14.10	-18.09	-0.08	-0.07	-0.06	0.02	0.03	0.03	-0.07	-0.04	-0.02
33x118	-10.23	-13.62	-17.51	-0.07	-0.06	-0.05	0.02	0.03	0.03	-0.06	-0.04	-0.02
30x116	-9.99	-13.24	-16.91	-0.06	-0.05	-0.04	0.02	0.03	0.03	-0.06	-0.03	-0.02
30x108	-9.72	-12.89	-16.49	-0.05	-0.05	-0.03	0.02	0.02	0.03	-0.05	-0.03	-0.01
30x99	-9.43	-12.53	-16.03	-0.05	-0.04	-0.02	0.02	0.02	0.02	-0.05	-0.02	-0.01
27x94	-9.12	-12.05	-15.32	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.04	-0.02	-0.01
27x84	-8.79	-11.63	-14.77	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.02	0.00
24x94	-8.85	-11.60	-14.62	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.04	-0.02	0.00
24x84	-8.54	-11.20	-14.11	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.03	-0.01	0.00
24x76	-8.27	-10.85	-13.65	-0.02	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
24x68	-7.97	-10.47	-13.15	-0.02	0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
24x62	-7.65	-10.05	-12.61	-0.01	0.01	0.03	0.01	0.02	0.02	-0.01	0.00	0.02
21x55	-7.39	-9.69	-12.12	-0.01	0.01	0.03	0.01	0.01	0.01	-0.01	0.01	0.02
21x68	-7.63	-9.90	-12.29	-0.01	0.00	0.03	0.02	0.02	0.02	-0.02	0.00	0.02
21x62	-7.41	-9.61	-11.90	-0.01	0.01	0.03	0.02	0.02	0.02	-0.01	0.01	0.02
21x50	-6.85	-8.86	-10.89	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03
21x44	-6.58	-8.49	-10.38	0.01	0.02	0.05	0.01	0.01	0.01	0.01	0.02	0.03
18x55	-6.68	-8.51	-10.33	0.00	0.02	0.04	0.01	0.02	0.02	0.00	0.02	0.03
18x50	-6.48	-8.24	-9.96	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03

TABLE 6 Thermal Coefficients k_1 to k_4 for $f'_c = 7$ ksi

Steel Section	k_1 for 2a =			k_2 for 2a =			k_3 for 2a =			k_4 for 2a =		
	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.
36x300	-14.89	-19.57	-24.85	-0.19	-0.22	-0.23	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x280	-14.51	-19.08	-24.24	-0.18	-0.20	-0.22	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x260	-14.09	-18.53	-23.56	-0.17	-0.19	-0.20	0.04	0.04	0.05	-0.11	-0.08	-0.06
36x245	-13.77	-18.12	-23.06	-0.16	-0.18	-0.18	0.04	0.04	0.05	-0.11	-0.08	-0.06
36x230	-13.44	-17.70	-22.55	-0.15	-0.17	-0.17	0.03	0.04	0.04	-0.11	-0.08	-0.06
36x210	-12.74	-16.80	-21.44	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
36x194	-12.42	-16.43	-20.02	-0.12	-0.13	-0.13	0.03	0.04	0.04	-0.09	-0.07	-0.04
36x182	-12.15	-16.09	-20.61	-0.12	-0.12	-0.12	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x170	-11.86	-15.74	-20.18	-0.11	-0.11	-0.11	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x160	-11.60	-15.40	-19.78	-0.10	-0.10	-0.10	0.03	0.03	0.03	-0.08	-0.05	-0.04
36x150	-11.38	-15.13	-19.44	-0.09	-0.09	-0.09	0.03	0.03	0.03	-0.08	-0.05	-0.03
36x135	-10.87	-14.49	-18.66	-0.08	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.04	-0.03
33x221	-13.11	-17.23	-21.91	-0.14	-0.16	-0.16	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x201	-12.64	-16.64	-21.19	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x141	-10.98	-14.57	-18.67	-0.08	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.05	-0.03
33x130	-10.65	-14.15	-18.16	-0.08	-0.07	-0.06	0.03	0.03	0.03	-0.07	-0.04	-0.02
33x118	-10.27	-13.68	-17.58	-0.07	-0.06	-0.05	0.02	0.03	0.03	-0.06	-0.04	-0.02
30x116	-10.03	-13.29	-16.98	-0.06	-0.05	-0.04	0.02	0.03	0.03	-0.06	-0.03	-0.02
30x108	-9.75	-12.95	-16.55	-0.05	-0.05	-0.03	0.02	0.02	0.03	-0.05	-0.03	-0.01
30x99	-9.46	-12.58	-16.10	-0.05	-0.04	-0.02	0.02	0.02	0.02	-0.05	-0.02	-0.01
27x94	-9.16	-12.10	-15.38	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.04	-0.02	-0.01
27x84	-8.83	-11.68	-14.84	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.01	0.00
24x94	-8.89	-11.65	-14.68	-0.04	-0.03	0.00	0.02	0.02	0.02	-0.04	-0.02	0.00
24x84	-8.57	-11.25	-14.16	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.03	-0.01	0.00
24x76	-8.30	-10.89	-13.71	-0.02	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
24x68	-8.01	-10.51	-13.21	-0.02	-0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
24x62	-7.68	-10.09	-12.66	-0.01	0.01	0.03	0.01	0.02	0.02	-0.01	0.00	0.02
21x55	-7.42	-9.73	-12.17	-0.01	0.01	0.03	0.01	0.01	0.01	-0.01	0.01	0.02
21x68	-7.66	-9.94	-12.34	-0.01	0.00	0.03	0.02	0.02	0.02	-0.02	0.00	0.02
21x62	-7.44	-9.65	-11.95	-0.01	0.01	0.03	0.02	0.02	0.02	-0.01	0.01	0.02
21x50	-6.88	-8.89	-10.94	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03
21x44	-6.61	-8.52	-10.42	0.01	0.02	0.05	0.01	0.01	0.01	0.01	0.02	0.03
18x55	-6.71	-8.55	-10.37	0.00	0.02	0.04	0.01	0.02	0.02	0.00	0.02	0.03
18x50	-6.51	-8.27	-10.00	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03

TABLE 7 Thermal Coefficients k_1 to k_4 for f'_c = 8 ksi

Steel Section	k_1 for 2a =			k_2 for 2a =			k_3 for 2a =			k_4 for 2a =		
	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.	7 in.	8 in.	9 in.
36x300	-14.79	-19.43	-24.67	-0.19	-0.22	-0.23	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x280	-14.41	-18.95	-24.07	-0.18	-0.20	-0.22	0.04	0.04	0.05	-0.12	-0.09	-0.07
36x260	-13.99	-18.40	-23.39	-0.17	-0.19	-0.20	0.04	0.04	0.05	-0.12	-0.08	-0.06
36x245	-13.67	-17.99	-22.89	-0.16	-0.18	-0.18	0.03	0.04	0.05	-0.11	-0.08	-0.06
36x230	-13.34	-17.57	-22.38	-0.15	-0.17	-0.17	0.03	0.04	0.04	-0.11	-0.08	-0.06
36x210	-12.65	-16.68	-21.28	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
36x194	-12.33	-16.30	-20.85	-0.12	-0.13	-0.13	0.03	0.04	0.04	-0.09	-0.07	-0.05
36x182	-12.06	-15.97	-20.45	-0.12	-0.12	-0.12	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x170	-11.78	-15.61	-20.02	-0.11	-0.11	-0.11	0.03	0.03	0.04	-0.09	-0.06	-0.04
36x160	-11.51	-15.28	-19.62	-0.10	-0.10	-0.10	0.03	0.03	0.03	-0.08	-0.06	-0.04
36x150	-11.29	-15.01	-19.28	-0.09	-0.09	-0.09	0.03	0.03	0.03	-0.08	-0.05	-0.03
36x135	-10.78	-14.37	-18.50	-0.08	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.04	-0.03
33x221	-13.01	-17.11	-21.74	-0.14	-0.16	-0.16	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x201	-12.55	-16.52	-21.03	-0.13	-0.14	-0.14	0.03	0.04	0.04	-0.10	-0.07	-0.05
33x141	-10.89	-14.45	-18.52	-0.09	-0.08	-0.07	0.03	0.03	0.03	-0.07	-0.05	-0.03
33x130	-10.56	-14.04	-18.01	-0.08	-0.07	-0.06	0.02	0.03	0.03	-0.07	-0.04	-0.02
33x118	-10.19	-13.57	-17.43	-0.07	-0.06	-0.05	0.02	0.03	0.03	-0.06	-0.04	-0.02
30x116	-9.95	-13.18	-16.84	-0.06	-0.05	-0.04	0.02	0.03	0.03	-0.06	-0.03	-0.02
30x108	-9.68	-12.84	-16.42	-0.05	-0.05	-0.03	0.02	0.02	0.03	-0.05	-0.03	-0.01
30x99	-9.39	-12.47	-15.96	-0.05	-0.04	-0.02	0.02	0.02	0.02	-0.05	-0.02	-0.01
27x94	-9.09	-12.00	-15.25	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.04	-0.02	-0.01
27x84	-8.75	-11.58	-14.71	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.04	-0.02	0.00
24x94	-8.82	-11.56	-14.56	-0.04	-0.03	-0.01	0.02	0.02	0.02	-0.04	-0.02	0.00
24x84	-8.50	-11.15	-14.05	-0.03	-0.02	0.00	0.02	0.02	0.02	-0.03	-0.01	0.00
24x76	-8.23	-10.80	-13.59	-0.02	-0.01	0.01	0.02	0.02	0.02	-0.03	-0.01	0.01
24x68	-7.94	-10.42	-13.10	-0.02	-0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
24x62	-7.62	-10.00	-12.55	-0.01	0.01	0.03	0.01	0.02	0.02	-0.01	0.00	0.02
21x55	-7.35	-9.65	-12.07	-0.01	0.01	0.03	0.01	0.01	0.01	-0.01	0.01	0.02
21x68	-7.59	-9.86	-12.24	-0.01	0.00	0.02	0.02	0.02	0.02	-0.02	0.00	0.01
21x62	-7.38	-9.57	-11.85	-0.01	0.01	0.03	0.02	0.02	0.02	-0.01	0.01	0.02
21x50	-6.82	-8.82	-10.85	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03
21x44	-6.56	-8.45	-10.34	0.01	0.02	0.05	0.01	0.01	0.01	0.01	0.02	0.03
18x55	-6.66	-8.48	-10.29	0.00	0.02	0.04	0.01	0.02	0.02	0.00	0.02	0.03
18x50	-6.46	-8.21	-9.92	0.00	0.02	0.04	0.01	0.01	0.01	0.00	0.02	0.03

The thermal stresses given by Equations 17 and 18 are for simply supported composite bridges. In the case of continuous composite bridges the resultant thermal stresses are obtained by superimposing the thermal stresses given by Equations 17 and 18 and the stresses induced by the presence of the intermediate supports; these latter stresses are derived from the redundant moments resulting from continuity, as shown in Figure 2. This will be illustrated in the design example given later.

DISCUSSION

The adopted maximum and minimum temperature differentials (ΔT) for both summer and winter, given in Table 1, are based on shade air temperatures found from maps of isotherms for the particular construction site considered. These temperature differentials were chosen to represent the most unfavorable conditions expected, as proposed by Zuk (13) and confirmed by Emanuel and Hulsey (1). The suggested minimum temperature differentials in summer and winter of -7.5°F compare favorably with those given by the British Standards BS 5400 (16) and with the values calculated by Emanuel and Hulsey (1) using a finite element analysis. Although the maximum temperature differential of 40°F for summer is higher than the 25°F given in BS 5400 (16), it is in good agreement with the value obtained by Emanuel and Hulsey (1).

Both German specifications DIN 1072 and DIN 1078 (17) have suggested a linear temperature distribution through the section depth from the top of the concrete deck slab to the soffit of the steel beam; however, because the diffusivity of the steel is much higher than that of the concrete, such a temperature distribution is unrealistic compared with the one adopted herein (14). The use of Tables 2 through 7 for the constants k_1 to k_4 in estimating the thermal stresses will be demonstrated by the following worked example.

Worked Example

It is necessary to estimate the thermal stresses induced in a two-span continuous concrete composite bridge with the following characteristics:

- Length of span = 80 ft;
- Effective concrete deck slab width (w) = 96 in.;
- Concrete deck slab thickness (2a) = 8 in.;
- f'_c = 8 ksi;
- f_y for WF 36x245 steel beam of grade A36 = 36 ksi;
- $E_s = 20 \times 10^3$ ksi;
- $E_c = 3.9 \times 10^3$ ksi (private communication from D.W. Pfeifer, Portland Cement Association, May 1970);
- m = 7.44;
- Poisson's ratio of concrete (ν) = 0.2; and
- $\alpha_c = 5.5 \times 10^{-6}$/°F.

With reference to Figure 1(c), T_o = 60°F and T_b = 85°F maximum in summer and -22°F minimum in winter.

Solution

There are four cases of temperature variations through the depth of the section that have to be considered. However, for brevity only Case (iv), shown in Figure 3(a), will be given in detail here; the same procedure will apply to the other three cases. From geometry of the composite section, A_c = 768 in.^{2}; I_c = 4,096 in.^{4}; A_s = 72.1 in.^{2}; I_s = 16,100 in.^{4}; and moment of inertia of transformed composite section (I_t) = 37,283 in.^{4}. Using Table 7

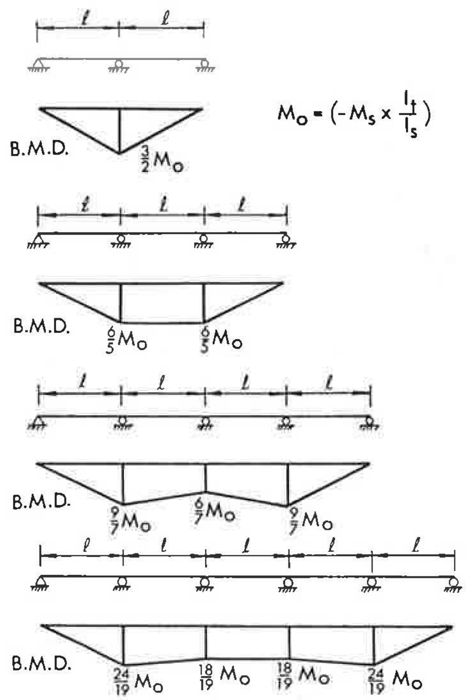

FIGURE 2 Redundant bending moments in continuous beams caused by temperature change.

for f'_c = 8 ksi, (2a) = 8 in., yields k_1 = -17.99 kip-in., k_2 = -0.18 kip-in., k_3 = 0.04 kip., and k_4 = -0.08. Thus, from Equations 11-13,

$$Q = (40)(-17.99) - (85 - 60)(-0.18) = -715.10 \text{ kip-in.}$$

$$F = (85 - 60)(0.04) - (-715.10)(-0.08) = -56.21 \text{ kips}$$

$$M_s = (-56.21)(18.04) + (-715.10) = -1{,}729.13 \text{ kip-in.}$$

From Equations 17 and 18, the thermal stresses induced (with the intermediate pier support assumed removed) are

$$\sigma_{xc}\begin{Bmatrix}\text{top}\\ \text{bottom}\end{Bmatrix} = (-56.21/768) - (-715.10/4096)\begin{Bmatrix}-4\\ +4\end{Bmatrix} + [(-56.21)(4)/4096]\begin{Bmatrix}-4\\ +4\end{Bmatrix} = \begin{matrix}-0.55 \text{ ksi}\\ 0.41 \text{ ksi}\end{matrix}$$

$$\sigma_{xs}\begin{Bmatrix}\text{top}\\ \text{bottom}\end{Bmatrix} = (56.21/72.1) + [(-56.21)(18.04)/16100]\begin{Bmatrix}-18.04\\ +18.04\end{Bmatrix} + (-715.10/16100)\begin{Bmatrix}-18.04\\ +18.04\end{Bmatrix} = \begin{matrix}2.72 \text{ ksi}\\ -1.16 \text{ ksi}\end{matrix}$$

Because of the thermal strains and the presence of the intermediate pier support, the redundant moment at the pier support for a two-span continuous bridge is (Figure 2)

$$M_R = (3/2)(-M_s)(I_t/I_s) \qquad (19)$$

or

$$M_R = (3/2)(1729.13)(37283/16100) = 6{,}006.26 \text{ kip-in.}$$

The resulting stresses from $\sigma = (M/I)y$ are

$$\sigma_{xc}\begin{Bmatrix}\text{top}\\ \text{bottom}\end{Bmatrix} = [6{,}006.26/37283(7.44)]\begin{Bmatrix}-13.06\\ -5.06\end{Bmatrix} = \begin{matrix}-0.28 \text{ ksi}\\ -0.11 \text{ ksi}\end{matrix}$$

$$\sigma_{xs}\begin{Bmatrix}\text{top}\\ \text{bottom}\end{Bmatrix} = (6{,}006.26/37283)\begin{Bmatrix}-5.06\\ 31.02\end{Bmatrix} = \begin{matrix}-0.82 \text{ ksi}\\ +5.00 \text{ ksi}\end{matrix}$$

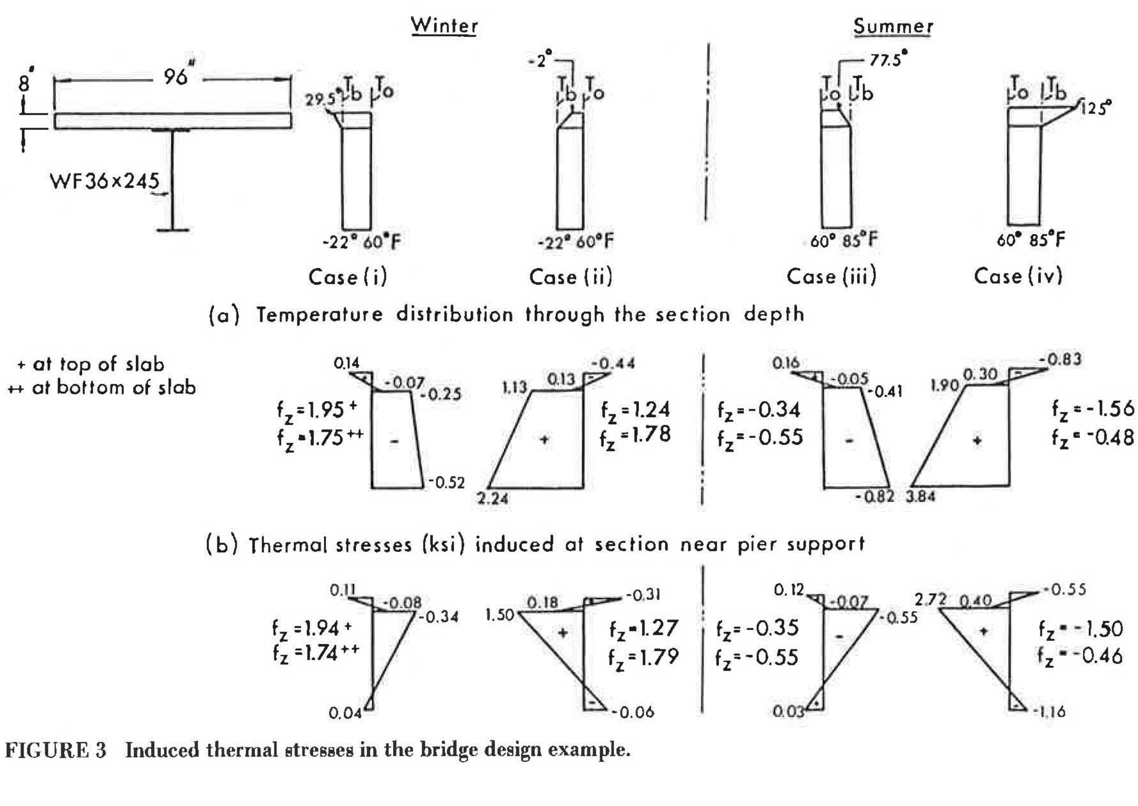

FIGURE 3 Induced thermal stresses in the bridge design example.

Thus the resultant thermal stresses at a section near the pier support become

$$\sigma_{xc} = \begin{cases} \text{top} & -0.55 - 0.28 = -0.83 \text{ ksi} \\ \text{bottom} & 0.41 - 0.11 = 0.30 \text{ ksi} \end{cases}$$

$$\sigma_{xs} = \begin{cases} \text{top} & 2.72 - 0.82 = 1.90 \text{ ksi} \\ \text{bottom} & -1.16 + 5.00 = 3.84 \text{ ksi} \end{cases}$$

The thermal stresses in the concrete deck slab in the transverse direction are calculated from Equation 16 as

$$\sigma_{zc} = \begin{cases} \text{top} & (0.2)(-0.83) - (5.5)(10^{-6})(3.9)(10^{3})(125 - 60) = -1.56 \text{ ksi} \\ \text{bottom} & (0.2)(0.30) - (5.5)(10^{-6})(3.9)(10^{3})(85 - 60) = -0.48 \text{ ksi} \end{cases}$$

The induced thermal stresses for the other three cases, (i) to (iii), are presented in Figure 3(b). The thermal stresses at sections near the outer supports are shown in Figure 3(c). These results indicate that significant tensile thermal stresses can develop in the concrete deck slab in the longitudinal direction not only at the top of the concrete deck slab but also at its bottom. Furthermore, the transverse stresses in the deck slab, which result from thermal variations, can be quite large and are likely to cause severe longitudinal cracking of the concrete deck unless adequately reinforced.

SUMMARY AND CONCLUSIONS

A simple method of analysis is presented for estimating the thermal stresses in composite concrete deck slab-on-steel beam bridges; the method can be applied to both simple spans and continuous spans. The results are based on a realistic temperature distribution through the depth of the bridge section and on accepted maximum and minimum temperature differentials for summer and winter. From the results given it can be concluded that

- The magnitude of the longitudinal thermal stresses in composite bridges can be significant compared with stresses due to dead and live loads.
- The thermal stresses induced in the concrete deck slab in the transverse direction can exceed the tension capacity of the concrete by several fold; therefore additional steel reinforcements are necessary in the transverse direction to guard against the formation of longitudinal cracks in the concrete deck.

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APPENDIX

The explicit expressions for the thermal coefficients k_1, k_2, k_3, and k_4 are as follows:

$$k_1 = [AC_2/(AR - KB)]$$

$$k_2 = [KC_1/(AR - KB)]$$

$$k_3 = (C_1/A)$$

$$k_4 = B/A$$

where

$C_1 = -[(1 + \nu)\alpha_c + \alpha_s]$,

$C_2 = -(1 + \nu)\alpha_c wI_sE_sE_c a^2$,

$A = (1/E_sA_s) + (d_1^2/I_sE_s) + [2(1 - \nu^2)/waE_c]$,

$B = (d_1/E_sI_s) - [1.5(1 - \nu^2)/a^2wE_c]$,

$K = 2wd_1 a^3 E_c - 3(1 - \nu^2) aI_sE_s$, and

$R = 2wa^3 E_c + 3(1 - \nu^2) I_sE_s$.

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Multi-Increment Cost-Allocation Methodology for Bridges

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ABSTRACT

In this paper is presented an overview of a bridge cost-allocation procedure that uses data from bridges built in the state of Indiana between 1980 and 1983. The framework of the present analysis was based on the incremental concept with modification at various steps in the allocation process such that a larger number of cost increments were obtained. A technique is discussed for obtaining a large number of cost increments in order to render the economies-of-scale problem associated with the incremental methodology insignificant. The quantitative correlation between study vehicle classification and AASHTO vehicles was based on the relative effect of both axle loading and axle spacing of each vehicle on continuous-span bridges. The cost responsibility of each vehicle class was determined first on the basis of its structural and geometric requirements and then on the relative frequency with which it uses the bridges. A discussion of the approach employed in the distribution of the bridge construction, bridge replacement, and bridge rehabilitation cost is presented.

There are several methods of allocating bridge costs in the literature. However, the most commonly used and widely accepted methodology for structural cost-allocation analysis is classical incremental analysis (1-6). Although the concept of classical incremental analysis for bridge cost allocation is well documented, the approach for developing the various bridge cost functions in the allocation process varies among studies. The bridge cost-allocation process discussed in this paper is also based on the classical incremental framework but with modification at various steps. Presented herein is a multi-increment cost-allocation process based on data from bridges built in Indiana between 1980 and 1983.

VEHICULAR CLASSIFICATION AND DESIGN LOADING

Vehicles that use the Indiana highway system were categorized into 14 basic classifications (Table 1).

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Each class was further divided into various weight groups. For the purpose of incremental analysis, with the exception of the live loads, all loads and forces were handled in the same manner as the original bridge. The live loads that represented the weight of the moving traffic were modified to reflect a range of different types of vehicles. According to AASHTO bridge specifications (7), traffic-related loadings can be represented by standard trucks or by equivalent lane loads. The trucks specified are designated with an H prefix followed by a number indicating the total weight of the trucks in tons for the two-axle trucks or with an HS prefix followed by a number indicating the weight of the tractor in tons for the tractor-trailer combinations. The AASHTO bridge specification provides only five classes of design loading, namely, HS 20, HS 15, H 20, H 15, and H 10. Other loadings required for the present analysis can be obtained by proportionally changing the weights of the designated trucks (7). The modified AASHTO live loadings and the corresponding lane loadings used in the present study are shown in Figure 1. The cost functions of each bridge element were