A Dynamic Model of Urban Retail Location and Shopping Travel

NORBERT OPPENHEIM

ABSTRACT

The evolution of the distribution of urban retail locations and the resultant shopping travel over time is simulated. The dynamic model developed incorporates a standard gravity formulation for interzonal travel demand with a linear programming formulation for the determination of the incremental zonal retail spaces that maximize net aggregate profits from retail sales. From one time period to the next link travel costs are updated as a function of the current flow, and zonal retail space development costs are updated as a function of the level of sales performance of the zone. The results of the simulations are in some cases counterintuitive, and illustrate the varied impacts urban development and transportation polices have on the spatial patterns of shopping activity and travel. Further possible extensions of the model, as well as practical applications, are discussed in conclusion.

During the last few years, the urban systems and transportation research community has given increasing attention to the dynamic modeling approach--the development of models in which time is an explicit variable. The reasons for this emerging trend include the inability of the traditional, static models to represent urban evolutionary changes, such as the decentralization of spatial patterns of urban activity (residential, commercial, etc.), the agglomeration of individual retail facilities into large-sized centers, sudden growth or decline of individual urban zones, and other similar contemporary urban phenomena (1).

In the dynamic modeling approach the nature of changes in the levels of system variables between individual time points (e.g., rate of change, oscillations, and instabilities), is as significant as the state of the urban and transportation system at given time points (e.g., trip ends, and interzonal flows), in cross-sectional analysis, for example. In particular, the existence and nature of equilibria for the system can be investigated as steady states, for instance, as a continuous evolution, rather than as individual, isolated points.

Several dynamic models of urban structure have recently been developed. Among these models are those of Wilson et al. (2), Allen et al. (3), and Mirchandani (4). The formulation of the first two models was based on a "logistic" mechanism of individual zonal activity growth. The third model was based on a variant of the "exponential smoothing" method of time-series forecasting applied to the matrix of origin-destination flows. In all cases, the spatial interaction is described by a gravity formulation.

Wilson's model focuses on the retail sector. However, subsequent versions addressed the agricultural, residential, and commercial sectors [Clarke and Wilson (5), and Birkin et al. (6)], whereas Allen's and Mirchandani's are comprehensive models that integrate several urban activities including retail, residential, industrial, and recreational.

The regional version of Allen's model adequately replicated the evolution of an area in Belgium between 1947 and 1970. The other models were not empirically validated, partly because of the difficulty of obtaining appropriate longitudinal data for the numerous variables in the models. Nevertheless, the findings of the various hypothetical simulations conducted with these respective models showed that a large variety of spatial patterns and types of temporal evolution can result from various parameter values, thereby translating different developmental scenarios and policies.

In particular, oscillations, cycles, and other similar forms of unstable behavior were observed in zonal activity levels over time. "Catastrophic" or discontinuous changes can also take place for certain critical combinations of parameter values, while bifurcations in evolutionary path may occur for others (7). Furthermore, random fluctuations in the levels of key variables (externally induced changes in the system's state such as major transportation facilities construction) have the potential to fundamentally alter, and over a long term, to cause an evolution of the system.

MODEL FORMULATION

The present model focuses on urban retail activity, and the associated shopping travel, as an individual component of a comprehensive urban and transportation system model. Consequently, the other activities (principally residential), are assumed to be given exogenously as the output of other submodels. The formulation of the model is based on two basic assumptions concerning supply and demand for shopping activity.

First, in keeping with the standard approach to travel distribution modeling, the spatial pattern of interzonal shopping travel from a Residence Zone i to a Shopping Zone j is assumed to conform to a gravity pattern. The attractiveness of an individual shopping zone is assumed to be proportional to the size of the zone, as represented by the square footage of retail space A_j. The trip production of a given zone is assumed to be proportional to the level of residential population in the zone P_i, which is exogenously input into the model. The disutility of shopping travel is represented by the interzonal

Department of Civil Engineering and Institute for Transportation Systems, The City College, City University of New York, New York, N.Y. 10031.

travel time C_{ij}, which is itself a function of the link flow through the standard link performance function (8). The form of the spatial deterrence function is given as a negative expotential that conforms both to the entropy and the logit derivation of the gravity model (9). This is represented in Equations 1 and 2.

$$T_{ij}^t = \theta P_i^t A_j^t \exp(-\alpha C_{ij}^t)/\Sigma_j A_j^t \exp(-\alpha C_{ij}^t) \quad (1)$$

$$P_i^t = P_i^{t-1}(1.05 + 0.1Z_1) + Z_2 \quad (2)$$

Note that it is assumed, as a simplification, that there is only one link connecting any couple of zones, so that there is no trip assignment component in the model. The level of retail sales in a given individual zone S_j is made proportional to the number of trip ends in the zone by the average amount spent on a shopping trip λ and the number of such trips per person per time period θ. This is expressed in Equation 3.

$$S_j^t = \lambda \Sigma_i T_{ij}^t \quad (3)$$

Second, the incremental allocation of zonal retail spaces over each consecutive time period is assumed to be such that it maximizes the future net aggregate profit resulting from retail sales (Equation 4).

$$\Delta A_j^t = A_j^{t+1} - A_j^t \quad (4)$$

This profit is the difference between incremental retail sales and the annualized cost of developing and operating a retail center in the given zone (Equation 5).

$$\Delta S_j^t = S^{t+1} - S_j^t = f(\Delta A_1^t \ldots\ldots, \Delta A_j^t, \ldots\ldots \Delta A_n^t) \quad (5)$$

It is further assumed that incremental sales in a given zone can be predicted by multiplying incremental space by a growth factor, where the factor is equal to the ratio of incremental sales to incremental retail space in the previous period. Thus, the objective function is linearized as represented in Equation 6.

$$\text{Max}\Sigma_j (\Delta S_j^t - C_j^t \Delta A_j^t) \simeq \text{Max}\Sigma_j [(\Delta S_j^{t-1}/\Delta A_j^{t-1}) - C_j^t]\Delta A^t \quad (6)$$

Clearly this simplification is analytically convenient, and is probably closer to the manner in which retail space developers actually make their predictions of future sales activity than the analytically exact estimation, which is too intractable to be computationally practical.

In any case, the constraints on the incremental allocation of zonal retail spaces translate two sets of requirements. The first, at the individual zonal level, limits the retail space increase to the amount currently available in a zone given the ultimate zonal limits such as carrying capacities, and the amount of space used up so far in the zone. This results in Equation 7.

$$\Delta A_j^t \leq L_j - A_j^{t-1} \quad (7)$$

where j equals 1, 2, ... n. The second constraint is at the level of the entire urban area, and requires that the total amount of retail space developed be made consistent with the residential population increase in the given time period by a coefficient of proportionality, which represents a developmental intensity with the dimension of a per-capita square footage of retail facilities (Equation 8).

$$\Sigma_j \Delta A_j^t \leq \omega\, \Sigma_i \Delta P_i^t \quad (8)$$

Because the objective function, as well as all of the constraints, are linear in ΔA_j^t for the equations, the incremental zonal retail space allocation process can now be effected through the solution of a linear program represented by Equations 6, 7, and 8.

Equation 9 updates the unit costs of zonal retail space development (the cost coefficients in the Objective Function 9) as linear functions of the level of zonal retail sales relative to the level of sales at time t = 0.

$$C_j^t = C_j^o [1 + c(S_j^{t-1}/\bar{S}^{t-1})] \quad (9)$$

This translates the assumption that zonal rents, or land values, are directly affected by the financial performance of the zone. This equation in effect introduces the spatial dimension in the retail space allocation process. It also makes the factors of the evolution of the retail space system (and consequently that of the interzonal shopping travel), specific functions of time. Therefore, the model is inherently dynamic, and not simply a recursive one.

MODEL APPLICATION TO THE SIMULATION OF RETAIL SYSTEM EVOLUTION

The preceding model was programmed for execution by a microcomputer. The spatial system was a simple grid of 36 equally sized zones. The initial cost of travel between zones was represented by the travel time between zones under free flow conditions based on the euclidian distance between zone centroids. The number of time periods was set at 18, corresponding to 36 years. The preceding dimensions were dictated by the 128-K memory size of the computer.

The series of simulations consisted of a base case, together with various sensitivity analyses. The base case was represented by a population distribution at time t = 0, which was concentrated in the area's core (Figure 1). The respective retail space distributions at time t = 0 and t = 1 were also concentrated at the center of the area. Both t = 0 and t = 1 are required as initial conditions for the operation of the model because the coefficients of the objective function for any time period depend on the performance of the retail allocation space during the preceding period. Subsequently, the residential population growth was set at 5 percent

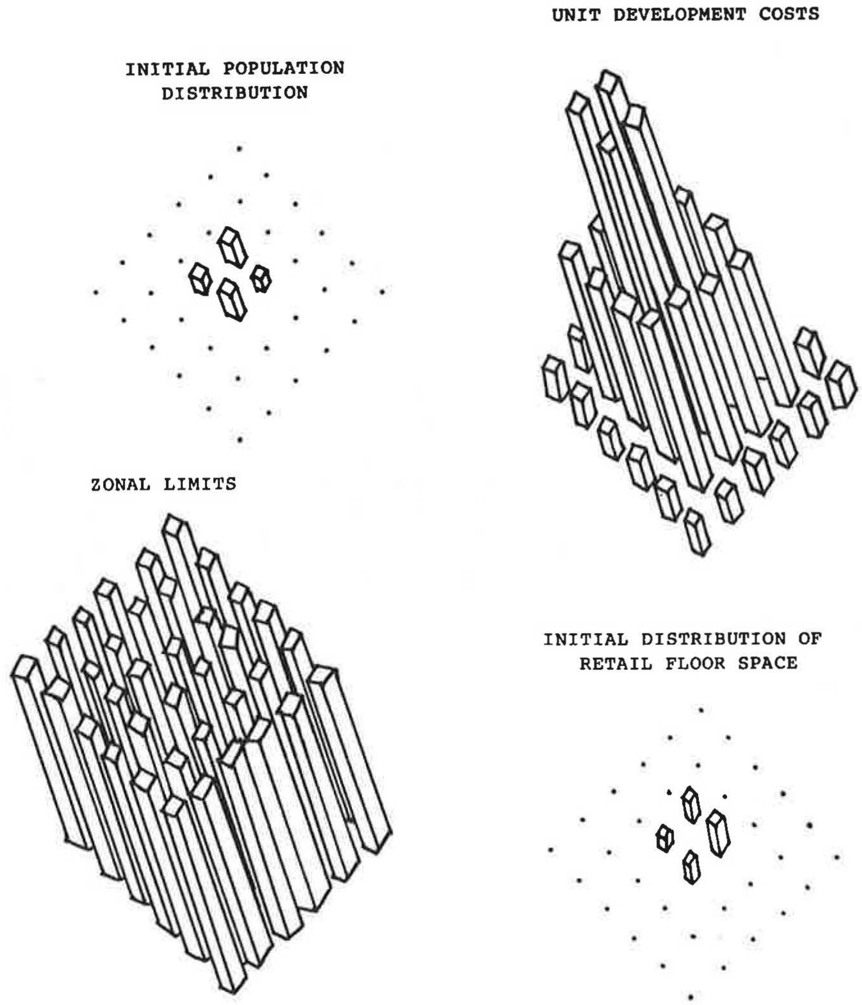

FIGURE 1 Initial conditions.

per period, plus random zonal differentials, as in Formula 2.

The initial zonal development costs were assumed to be highest in the inner ring of the four central zones and lowest on the area's periphery, with a medium level in the intermediate ring, and were roughly set equal to one-fifth the annual zonal sales revenue during the initial time period. Also, the limit on individual zonal retail space development was uniform for all zones, and set equal to 30 times the average zonal amount of retail space at time zero. Inflation was assumed to be nonexistent, or more precisely, it was assumed to affect both development costs and retail sales in the same manner so that all economic variables are in constant units over time.

The value of the parameter α in Equation 1, which measures the rate at which the level of interzonal shopping travel decreases with increasing distance (or which, in behavioral terms, represents the willingness of shoppers to travel long distances to retail facilities), was set at -0.2. This value results in a moderate distance decay effect, given the order of magnitude of the interzonal distances, ranging from 5 to 40 min.

The values of parameters a and b in Equation 10 represent the link performance function, and were set at the standard values for urban arterials of 0.15 for a, and 4 for b.

$$C^{t}_{ij} = C^{o}_{ij} \, [1 + a(T^{t-1}_{ij}/Cap_{ij})^{b}] \qquad (10)$$

Also, the value of coefficient c in Equation 9 was set at 1.0, thus implying an equality in the respective rates of increase in zonal sales, and in zonal cost of retail space development. The coefficient ω in Equation 8 was set at a value equal to the ratio between total population and total retail space at time t = 0, indicating a balanced retail development with respect to population growth.

Finally, coefficients θ in Equation 1, and λ in Equation 3, which in effect are scaling factors between the values of variables with different units of measurement, were both initially set at 1.0, thereby predetermining the order of magnitude of all the other variables.

RESEARCH FINDINGS

The results of the simulation of the base case just described are shown in Figures 2, 3, and 4. The evolution of the residential population distribution, which drives the evolution of the retail activity distribution, follows the expected pattern with residential levels increasing exponentially over time, and decreasing over space, away from the area's center. Concurrently, the distribution of zonal retail spaces shows that the location of retail development appears to alternate periodically over time between the area's core and its periphery.

Although absolute, the zonal retail spaces increase over time (as can be expected from the assumption that the area is in a growth mode), and the

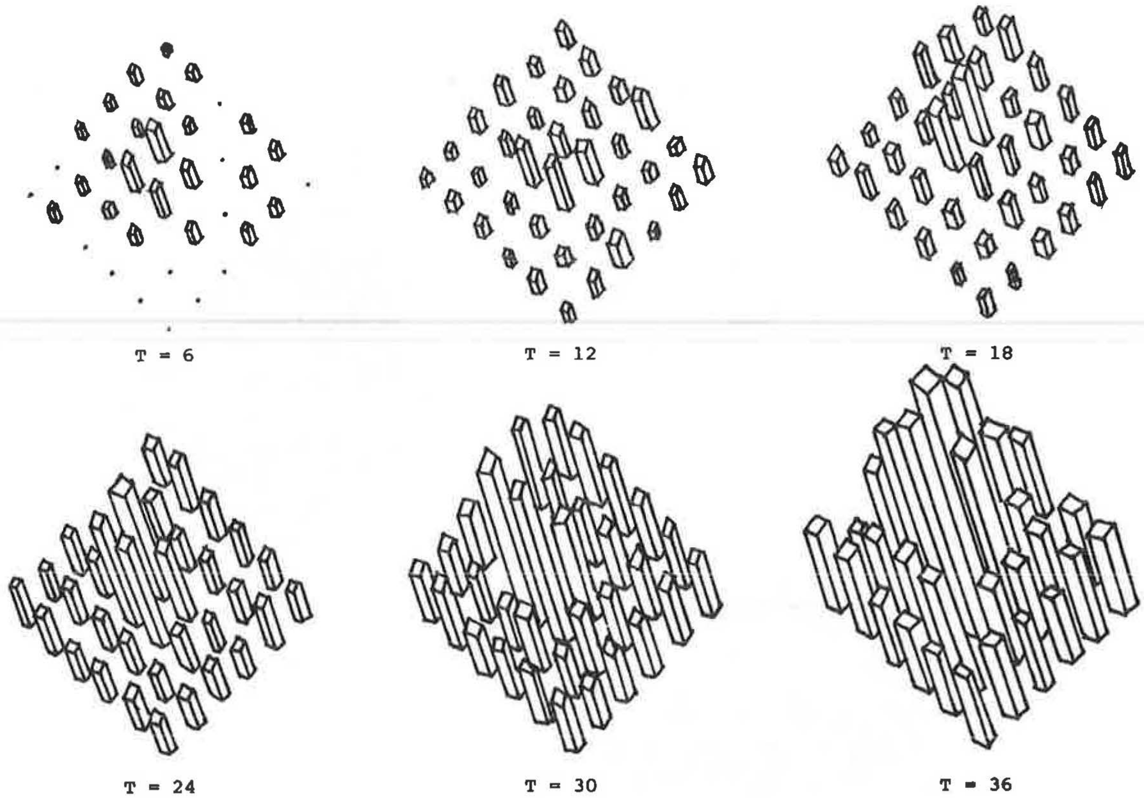

FIGURE 2 Evolution of the residential population distribution.

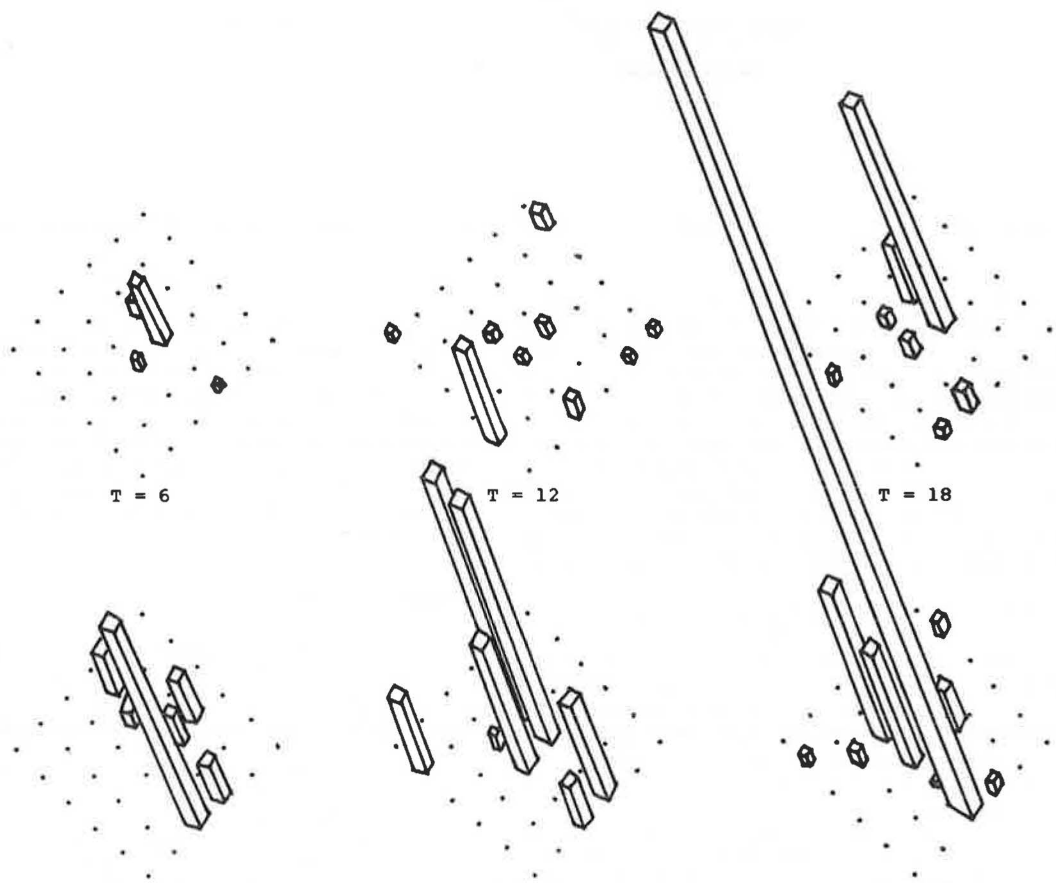

FIGURE 3 Evolution of the retail floor space distribution.

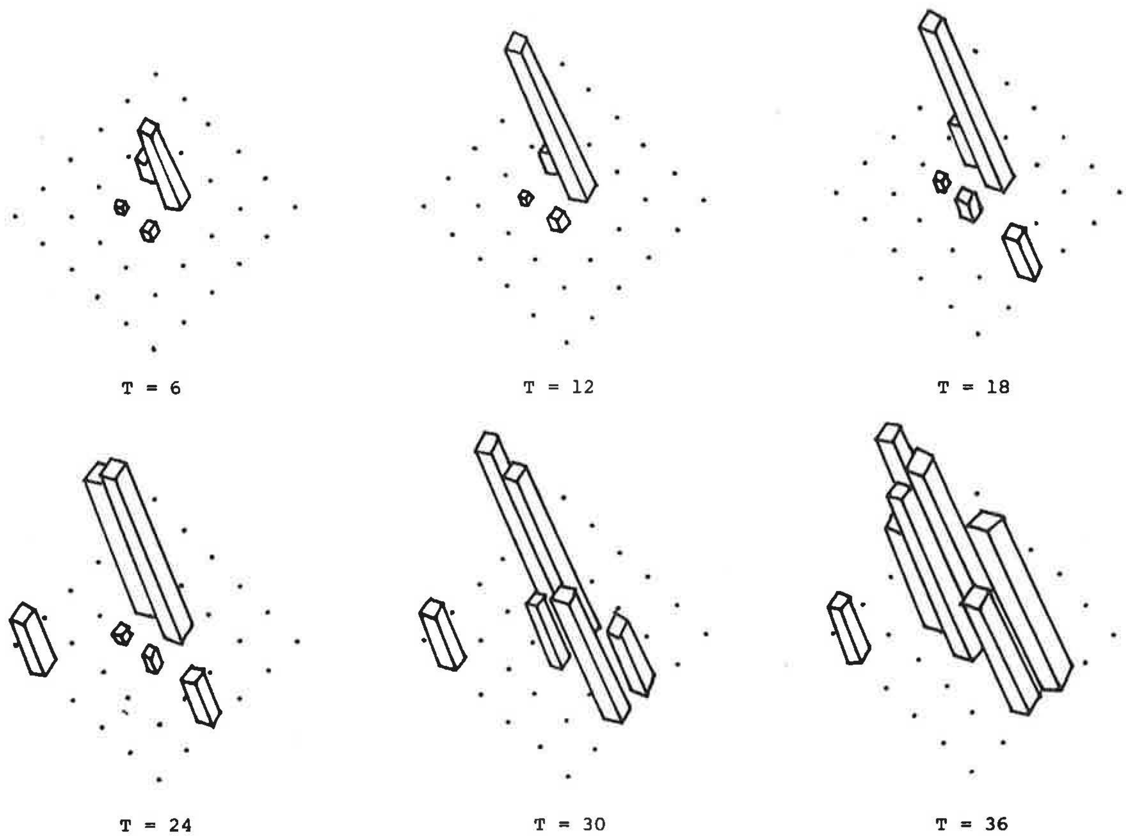

FIGURE 4 Evolution of the trip ends distribution.

level of retail activity (sales and shopping trip ends), in individual zones appears to oscillate. The characteristics of individual cycles in zonal activity appear to vary from zone to zone, as can be seen in Figure 5. For instance, Zone 19 shows a fairly steady cycle, whereas Zone 33 experiences an amplifying cycle. However, the period for most zones appears to be about 12 years (or six time periods).

It is worth noting that such oscillations in zonal activity levels have also been observed in all applications of the other dynamic models mentioned in the introductory section. However, oscillations in these models are expected as a standard feature of the output of systems of dynamic, nonlinear differential equations. In the present case, however, the dynamic component is provided not by a mechanistic law of evolution, but by an optimizing process of development.

In any case, such oscillations can be related to the multiple feedbacks between the zonal costs, activity levels, and the resulting zonal attractiveness to shoppers and developers. Specifically, when a zone's sales performance increases, its unit cost of retail development increases also. Consequently, the zone becomes less attractive to retail space developers. The zone may thus receive less development, in absolute terms, than other zones with lower levels of performance. In turn, this will decrease the attractiveness of the zone to potential shoppers, and consequently diminish its sales performance, until several periods later when the subsequent evolution of both the population and the retail activity spatial distribution might make the zone competitive again.

Concurrently with this unstable, cyclical behavior of zonal trips ends, the pattern of interzonal shopping travel flows also shows oscillations (Figure 6). For clarity only the six largest origin-destination flows are shown. The instability in zonal shopping trip ends is reflected in shifts in the type of interzonal travel pattern. Specifically, there appear to be two cycles of about 12 years each, starting with a primarily centripetal pattern at time $t = 6$, and becoming a purely centrifugal pattern at time $t = 12$, with a repeat from time $t = 18$ to time $t = 24$. At time $t = 30$, the major traffic flows are limited to the inner area, whereas at time $t = 36$, there is significant cross-town shopping traffic.

The implications of these oscillations in traffic volumes and origin-destination patterns are multiple, not only from the point of view of the operation of the retail facilities, but also for the operation of the transportation network. In particular, the fluctuations in level of trip ends in individual zones over time are detrimental to operational efficiency because they essentially imply underutilization of facilities at certain times, regardless of whether such facilities are retail stores, parking lots, or urban streets. Conversely, congestion will prevail at other times.

In the case of the retail sector, this problem may be alleviated through such temporary measures as adding or deleting personnel, short-term reconversion to other commercial activities, or other such business practices. Concerning the transportation network, such adaptations to fluctuations in travel demand may be more difficult to achieve, particularly given the relatively short period of the demand cycles, as described earlier. In any case, they will have significant impacts on traffic operations and control.

The next step in the analysis was to investigate the sensitivity of the foregoing results to changes in the urban conditions, as represented in the parameter values for the model. The first feature to be so varied was the value of the parameter α in

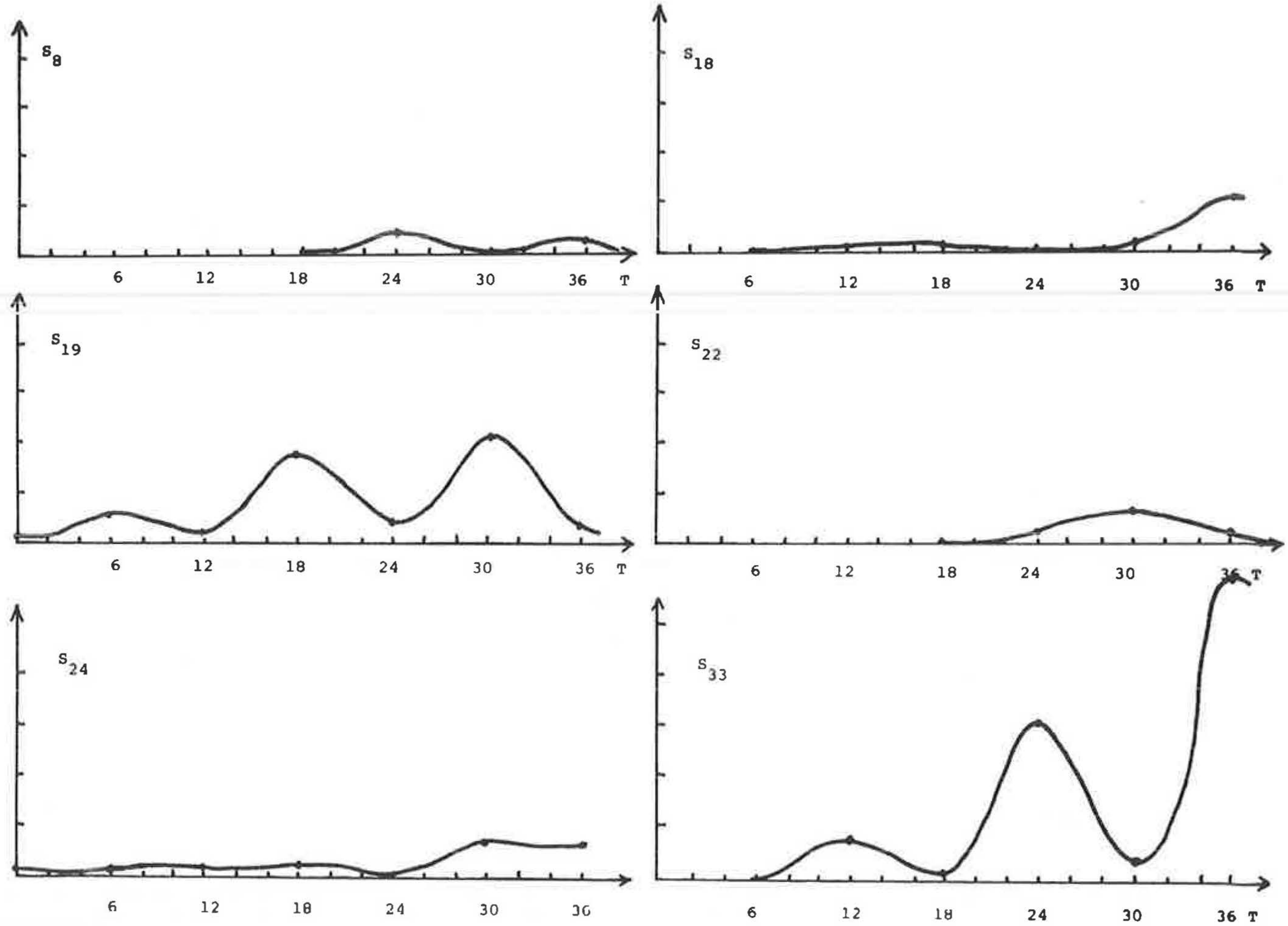

FIGURE 5 Evolution of trip ends levels in selected individual zones.

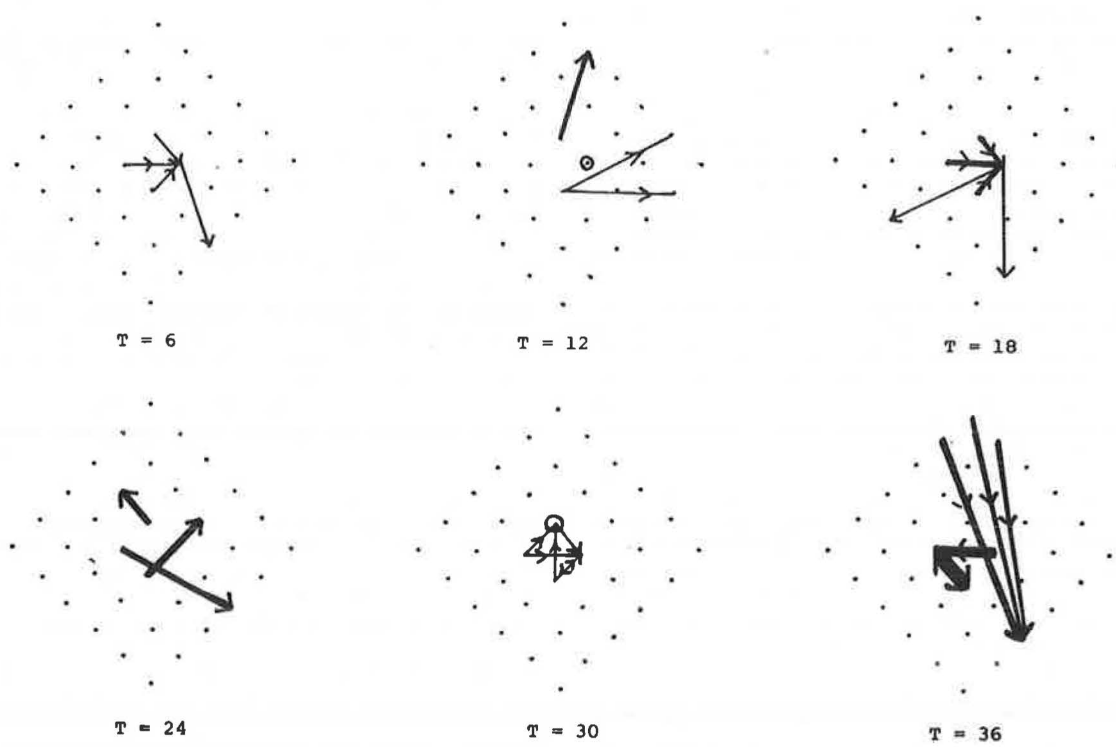

FIGURE 6 Evolution of the interzonal flows pattern.

Equation 1. This value was set at one-half its previous value of 0.2, translating a decrease in the spatial deterrence effect of distance on shopping travel, or equivalently, an increase in the willingness of shoppers to travel to distant shopping centers.

The results showed that in the first half of the simulation period, there were fewer dominant retail centers than before, but more of them in the second half of the period. Also, the average dispersion among retail centers remained about the same, although the pattern of interzonal shopping flows was changed, especially in the latter part of the period.

Another investigation was concerned with how increased levels of competition between both suppliers and consumers of retail activity affect the preceding results. The unit zonal development costs were thus made a steeper function of the zonal sales performance, while the travel time on individual links was made a faster rising function of the traffic flow. These changes can also be interpreted as increasing congestion levels in both the supply and demand sides. In numerical terms, coefficient c in Equation 9 was doubled, and the values of a and b in Equation 10 were set at 0.5 and 4.5, respectively.

The resulting evolution of the zonal and interzonal activity pattern showed that intrazonal shopping travel was prevalent in this case. Also, a more differentiated retail activity distribution, in the form of a dominant retail zone, together with more numerous secondary centers, emerged in the latter part of the period. In general, the development of the zonal retail spaces appeared to be somewhat more homogenous than those mentioned earlier.

The influence of the type of population evolution on retail system evolution was also investigated. Several patterns of population growth, including linear, and exponential, as well as different growth rates were used. Although the results are difficult to categorize in the form of simple, clear relationships, both zonal trips ends patterns and interzonal trip patterns were significantly affected.

Spatial irregularities in both the overall residential distribution and the residential evolution of individual zones are transmitted to the retail activity and shopping trip patterns in the form of greater temporal instabilities in their evolution. Furthermore, the characteristics of the initial residential population distribution had significant impacts throughout the period on both the type of trip ends, or interzonal trip patterns at given time points, and their characteristics of change over time.

CONCLUSION

An unlimited number of scenarios, beyond those discussed here, can be simulated in the same fashion, resulting in a wide variety of retail system evolutions. The interactions between the variables in the system are complex. Nevertheless, it is clear that some parameter values may lead to desirable configurations (e.g., a dispersed system of retail centers, or a radial pattern of shopping travel). Others result in detrimental configurations (e.g., center area congestion or rapid oscillations in retail sales).

Among the macroscopic factors of urban retail development and shopping travel that can thus be analyzed are: (a) the type of population growth or decline (e.g., residential densities, monocentric versus polycentric residential patterns); (b) basic economic activity distribution (e.g., inner versus peripheral, industrial decline); (c) zoning restrictions (e.g., developmental limits, land use policy); and (d) transportation network characteristics (e.g., layout, capacity, parking supply, and so on). In particular, the impacts of varying levels of congestion on the spatial structure of shopping travel have been investigated (10).

Among the microscopic factors of the retail system development are the behavior of the shopper (e.g., preferences for large retail stores, willingness to travel far, and so on); and behavior of the retail developer (e.g., willingness for taking risk).

One of the challenges of explorations of the type presented here is to categorize the results of such investigations in terms of clear, concise relationships between system parameters, and types of retail activity and travel system evolution.

The preliminary results illustrate a fundamental aspect of dynamic spatial analysis. Contradictory to the traditional standpoint, the evolutionary characteristics here are equally as significant as the cross-sectional information generated by the static, equilibrium-type urban structure models (magnitude and distribution of urban activities at given time points). This analysis highlights such dynamic features as oscillations in zonal trip level and interzonal trip pattern, evolutionary differences between individual zones, rates of growth or decline, long-term versus short-term characteristics, steady states, and so on. These dynamic impacts are as important to the evaluation of urban developmental policies as the equilibrium impacts predicted by static models. The importance of dynamic models lies in the fact that they alone can represent such impacts.

In conclusion, it might be appropriate to review briefly possible approaches for improving the described model. First, the dependency of the retail space allocation process on the other urban activities should be incorporated because all activities compete for the same limited urban space. This amounts to expanding the model by linking it with other models for residential (here assumed to be given exogenously), economic, or other activities, and incorporating the feedbacks between their respective variables (11). For instance, the amount of land available in a given zone would depend on the level of the other activities in the zone (relating to residential or office density). Also the cost of retail space development might be an increasing function of the percentage of remaining space suitable for development.

Another potential improvement in the present formulation would consist of the inclusion of additional economic variables, principally the zonal price of goods. This zonal characteristic should be important for most shoppers in their evaluation of zonal attractiveness. In turn, its value could be made a decreasing function of the level of zonal sales, reflecting volume discounts, and an increasing function of cost of retail development, reflecting the transmission of such costs to the consumer. These additional feedbacks among individual model variables might potentially give rise to other types of retail system evolution.

Another main area of development for the present model would be the allowance for a decline in zonal retail space. This might, for instance, be achieved by stipulating that the retail space for any zone with sales performance less than a specified level (e.g., a minimum rate of return), would be reduced by a given fraction (e.g., a failure rate).

Other refinements might in time be desirable, such as taking into account both multiple stops and multipurpose shopping trips, or nonhome-based trips, and other such complex aspects of urban travel behavior. Most important, the empirical validation of the results will have to be undertaken before dynamic models such as this can be used in practical situations. This task is subsequently dependent on the development of adequate longitudinal data bases, and the operationalization of a suitable calibration methodology (10).

REFERENCES

1. N. Oppenheim. A Critical Survey of Current Developments in Urban and Regional Modeling. In Advances in Urban Systems Modeling. (B. Hutchinson and M. Batty, eds.), North Holland, Amsterdam, The Netherlands, 1986.
2. A.G. Wilson. Some New Sources of Instability and Oscillation in Dynamic Models of Shopping Centers and Other Urban Structures. Sistemi Urbani, Vol. 3, 1981, pp. 391-401.
3. P.M. Allen et al. Models of Urban Settlement and Structure as Dynamic, Self-Organizing Systems. Report DOT/RSPA/DPA-10/6. U.S. Department of Transportation, 1981.
4. P. Mirchandani et al. A Dynamic Model of the Urban Process. Report DOT-RD-82016. U.S. Department of Transportation, 1981.
5. M. Clarke and A.G. Wilson. The Dynamic of Urban Structure. Croom-Helm, London, England, 1982.
6. M. Birkin, M. Clarke, and A.G. Wilson. Interacting Fields: Comprehensive Models for the Dynamic Analysis of Urban Spatial Structure. Working Paper 385. School of Geography, University of Leeds, England, 1984.
7. A.G. Wilson and M. Clarke. Some Applications of Catastrophe Theory to Urban Retailing Structures. In Papers in Regional Science, Pion, London, England, 1979, pp. 5-27.
8. Y. Scheffi. Urban Transportation Networks. Prentice Hall, Englewood Cliffs, N.J., 1985.
9. A. Anas. Discrete Choice Theory, Information Theory and The Multinomial Logit and Gravity Models. Transportation Research, B, Vol. 17, No. 1, 1983, pp. 13-23.
10. N. Oppenheim. Dynamic Models of Urban and Regional Systems: Some Conceptual and Methodological Issues. In Mathematical Modeling in Science and Technology, (Avula et al. eds.), Pergamon Press, Elmsford, N.Y., 1984.
11. N. Oppenheim. Applied Models in Urban and Regional Analysis. Prentice Hall, Englewood Cliffs, N.J., 1980.