Effects of Capacity Constraints on Peak-Period Traffic Congestion

MOSHE BEN-AKIVA, ANDRE DE PALMA, AND PAVLOS KANAROGLOU

The model of Ben-Akiva, Cyna, and de Palma is extended to represent trip departure time and route choice decisions when total demand is elastic. The simple case treated has two parallel routes with travelers jointly selecting route and departure time. The delays are assumed to occur at bottlenecks of limited capacity (bridge, tunnel, etc.) and a simple queueing model is employed to determine waiting time as a function of queue length at the time of arrival at the end of the queue. The day-to-day adjustment of the distribution of traffic is derived from a dynamic Markovian model. Numerical simulation experiments are performed to demonstrate the possible changes in the pattern of peak-period congestion when capacity of a bottleneck is changed. The results demonstrate some of the interdependencies that may exist among different bottlenecks in a road network. It is shown, in particular, that, in the presence of elastic demand, congestion may persist even when capacity of a bottleneck is expanded to meet the highest level of existing traffic flows. This does not mean, however, that expanding the capacity of a bottleneck and thus diverting trips from other routes cannot be a successful strategy for reducing schedule delays and traffic congestion along other routes, if that is the objective of traffic management. In addition, it is shown that if the capacities of the bottlenecks remain constant on average, but fluctuate from day to day because of stochastic factors (such as weather conditions), average traffic delays tend to increase. The modeling approach presented in this paper can also be used for policy analyses such as finding the optimal capacity expansion, the optimal coarse toll or time-dependent toll, the impact of information in situations of stochastic capacity, and the impact of changing the characteristics of an alternative travel mode.

Traffic congestion occurs at critical bottlenecks on the network where large traffic volumes and limited roadway capacity cause queues to develop. A bottleneck may occur at a point where roadway capacity is reduced, such as a merge area, a bridge, a tollgate, or a tunnel. It is assumed that, as soon as the arrival flow at the bottleneck is larger than its capacity, a queue develops and the departure flow from the bottleneck is equal to its capacity. The limited resources available for the expansion of highway networks in dense urbanized areas are likely to cause further increases in levels of congestion. In this paper a model of peak-period traffic congestion is used to analyze the effects of capacity constraints. The model is applied to a simplified network to predict the lengths of the queues at different times. Simulation results of the model in a prototypical situation demonstrate the effects of changing capacity on the pattern of traffic congestion during a peak period.

The model assumes that a commuter may choose to avoid long queues by trading off the difference between actual and desired arrival times (termed schedule delay) against shorter travel time. For a discussion and empirical estimates of this trade-off, see, for example, Kraft and Wohl *(1)*, Cosslett *(2)*, Small *(3)*, Abkowitz *(4)*, and Hendrickson and Plank *(5)*. Equilibrium models of peak-period traffic congestion that incorporate this trade-off for a network with a single bottleneck facility were developed by Vickery *(6)*, Henderson *(7)*, Hendrickson and Kocur *(8)*, and Fargier *(9)*. A stochastic extension for this problem was developed by de Palma et al. *(10)*, and its dynamic version was analyzed by Ben-Akiva et al. *(11)*.

Although many useful results were obtained from these models, their assumption of inelastic total volume is often not satisfied. The presence of congestion after a significant increase in capacity is often attributed to diverted and induced demands. Additional travelers are attracted by the expanded facility and consequently the queues that were expected to vanish may persist *(12)*. To capture this effect the present authors use a model, recently developed by Ben-Akiva et al. *(13)*, that extends the previous work by employing an elastic demand function for the total number of road users. In the previous analyses, the total number of travelers crossing the bottleneck was fixed and only the choice of departure time was considered. However, travelers can also decide to travel or not, to choose among different destinations, to switch to alternative modes of travel, and to divert to alternative routes. A simple example would be the case of two parallel routes in which travelers are jointly selecting a route and a departure time. In this case, it is also useful to include the option of not traveling.

The numerical simulations that are presented in this paper are concerned with the case of two parallel roads, a high-capacity expressway and a shorter distance arterial. This example is used to demonstrate the effects of changes in capacity on the pattern of peak-period congestion.

The results presented in this paper could be generalized without any difficulty to more than two routes in parallel. In a companion paper, de Palma et al. *(14)* have considered the case of multiple origins, a single destination, and bottlenecks in series; this corresponds to an urban corridor situation. The simulation of general networks appears to be significantly more complex. Ben-Akiva *(15)* discusses some of the difficulties inherent in this generalization.

MODEL

Consider a network that consists of I parallel routes linking a single origin-destination pair. Let N be the number of potential travelers, each one of whom is faced with deciding whether to travel via one of the I routes. Given a decision to travel, the traveler selects a route ($i = 1, ..., I$) and a departure time (t) from the origin, $t \in [T_0, T_0 + T]$, where T_0 and $T_0 + T$ are the earliest and the latest possible departures from the origin, respectively.

It is assumed that individuals may alter their choices from day to day. The probability, $P^i(t,\omega)h$, that a given individual decides on day ω to use one of the I routes, to select route i from the I routes, and to depart from the origin during the time interval $[t, t + h[\in [T_0, T_0 + T]$ is obtained from a nested logit model. [See Ben-Akiva and Lerman *(16)* and Ben-Akiva et al. *(13)* for detailed presentations of the nested logit model and its application, respectively.] It

M. Ben-Akiva, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass. 02139. A. de Palma, Department of Economics, Université Libre de Bruxelles, B-1050 Brussels, Belgium. P. Kanaroglou, Department of Geography, McMaster University, Hamilton, Ontario L8S 4K1, Canada.

views a trip as the outcome of a three-stage decision process: choice of using one of the I routes, choice of leaving during the time interval $[t, t + h[$ conditional on the choice of making a trip, and choice of using route i conditional on the previous two choices. Following Ben-Akiva et al. *(13)*, the following product of conditional probabilities is obtained:

$$
\begin{aligned}
P^i(t,\omega)h = {} & \text{(Probability of a trip on day } \omega) \\
& \text{(Probability of a departure between } t \text{ and } t + h \\
& \text{given a trip on day } \omega) \\
& \text{(Probability of selecting route } i \text{ given a trip} \\
& \text{departure during the period } [t, t + h[\text{ on day } \omega)
\end{aligned} \tag{1a}
$$

Each probability is assumed to have the multinomial logit form with its own scale parameter; μ_1, μ_2, and μ_3 are the parameters of the route, departure time, and the travel or no travel choice probabilities, respectively. A scale parameter of a discrete choice model measures the degree of heterogeneity of preferences among individual decision makers in a market segment. The nested logit formulation for this choice probability can be expressed by

$$
\begin{aligned}
P^i(t,\omega)h = {} & \left(\exp[V^i(t,\omega)/\mu_1]/\exp[V^*(t,\omega)/\mu_1]\right) \\
& \left(\exp[V^*(t,\omega)/\mu_2]/\exp[V^*(*,\omega)/\mu_2]\right) \\
& \left(\exp[V^*(*,\omega)/\mu_3]/\{\exp[V^*(*,\omega)/\mu_3] \right. \\
& \left. + \exp[V_0/\mu_3]\}\right) \cdot h
\end{aligned} \tag{1b}
$$

where the following composite variables are used

$$V^*(t,\omega) = \mu_1 ln \sum_{j=1}^{I} \exp[V^j(t,\omega)/\mu_1] \tag{1c}$$

$$V^*(*,\omega) = \mu_2 ln \sum_{u=T_0}^{T_0+T} \exp[V^*(u,\omega)/\mu_2] \tag{1d}$$

and where an asterisk is used to indicate that a summation has been performed over the corresponding variable, $V^i(t,\omega)$ is the systematic utility of the choice described previously, and V_0 is the utility of not using one of the I routes (i.e., the null alternative). The composite variable defined in Equation 1c is the expected maximum utility from the choice among alternative routes. The variable defined in Equation 1d is the expected maximum utility from the choice among alternative trips (i.e., combinations of departure time period and route).

The utility function of a trip via route i departing from the origin at time t during day ω is assumed for simplicity to have the following linear form:

$$V^i(t,\omega) = d^i - \alpha tt^i(t,\omega) - SD^i(t,\omega) \tag{2}$$

where

d^i = a constant specific to route i,
$tt^i(t,\omega)$ = travel time from the origin to the destination on day ω for a departure at time t via route i,
$SD^i(t,\omega)$ = the disutility of schedule delay of a trip via route i departing at time t during day ω, and
α = a constant parameter that measures the marginal disutility of travel time.

The specification of schedule delay disutility assumes that the desired period of arrival at the destination is $[t^* - \Delta, t^* + \Delta]$ where t^* denotes the center of the desired arrival period and $\Delta \geq 0$ is a measure of arrival time flexibility. (Alternatively, Δ can be interpreted as a measure of desired arrival time variability among individuals.) The arrival time at the destination for a departure at time t for a trip via route i during day ω is given by

$$t_a^i(t,\omega) = t + tt^i(t,\omega) \tag{3}$$

Denote the departure times from the origin via route i during day ω for arrivals at the destination at times $t^* - \Delta$ and $t^* + \Delta$, respectively, by $\tilde{t}^i(\omega)$ and $\tilde{\tilde{t}}^i(\omega)$ to obtain

$$\tilde{t}^i(\omega) = t^* - \Delta - tt^i(\tilde{t}^i(\omega),\omega) \tag{4a}$$

$$\tilde{\tilde{t}}^i(\omega) = t^* + \Delta - tt^i(\tilde{\tilde{t}}^i(\omega),\omega) \tag{4b}$$

In other words, departures from the origin via route i on day ω during the period $[T_0, \tilde{t}^i(\omega)]$ result in early arrivals, and those during the period $[\tilde{\tilde{t}}^i(\omega), T_0 + T]$ result in late arrivals. The disutility of schedule delay is assumed to be piecewise linear and is specified as follows:

$$
SD^i(t,\omega) = \begin{cases} \beta\,[t^* - \Delta - t - tt^i(t,\omega) & \text{for } t \in [T_0, \tilde{t}^i(\omega)] \\ 0 & \text{for } t \in [\tilde{t}^i(\omega), \tilde{\tilde{t}}^i(\omega)] \\ \beta\gamma\,[t + tt^i(t,\omega) - t^* - \Delta] & \\ & \text{for } t \in [\tilde{\tilde{t}}^i(\omega), T_0 + T] \end{cases} \tag{5}
$$

where β and $\beta\gamma$ are constant marginal disutility parameters. β is therefore the disutility of 1-min early arrival and $\beta\gamma$ is the disutility of 1-min late arrival.

The delay on each route is assumed to occur at a single bottleneck facility, such as a bridge or a tollgate, with a fixed capacity of s^i. The road segments before and after the bottlenecks have fixed travel times. Queues may develop only at the entrances to the bottleneck facilities. The waiting time at the entrance to a bottleneck is determined by a deterministic queueing model: it is equal to the number of vehicles in the queue at the time of arrival at the bottleneck divided by the capacity. For more details, see Equations 2–4 in Ben-Akiva et al. *(11)*.

To simulate this model, the time period $[T_0, T_0 + T]$ is divided into equal time intervals of length h. Define $R^i(t,\omega)$ to be the number of users choosing the departure time interval $[t, t + h]$ and route i. The parameter h could be interpreted as a measure of the ability of individuals to discriminate among alternative departure times. This view is supported by Mahmassani et al. *(17)* who developed an experimental procedure to study the choice of departure time and "found that the participants adjusted their departure times by multiples of 5 minutes, with a minimum adjustment interval of 5 minutes." Moreover, various values of h have been explored, and it has been found that, if h is small enough (on the order of 5 to 10 min), the results are extremely stable. In the following, $R^i(t,\omega)$ will denote the departure rate per unit of time that is equal to $R^i(t,\omega)/h$.

Following de Palma and Lefèvre *(18)* and Ben-Akiva et al. *(11)*, it is assumed that the day-to-day adjustment process used by individuals to revise their behavior can be modeled using the following set of difference equations:

$$R^i(t,\omega + 1) = R^i(t,\omega) + R\,[NP^i(t,\omega)h - R^i(t,\omega)] \tag{6}$$

where R is a constant rate at which individuals switch their choices or the probability that a randomly chosen individual will review his travel decision on a given day. Note that $[NP^i(t,\omega)h - R^i(t,\omega)]$ is

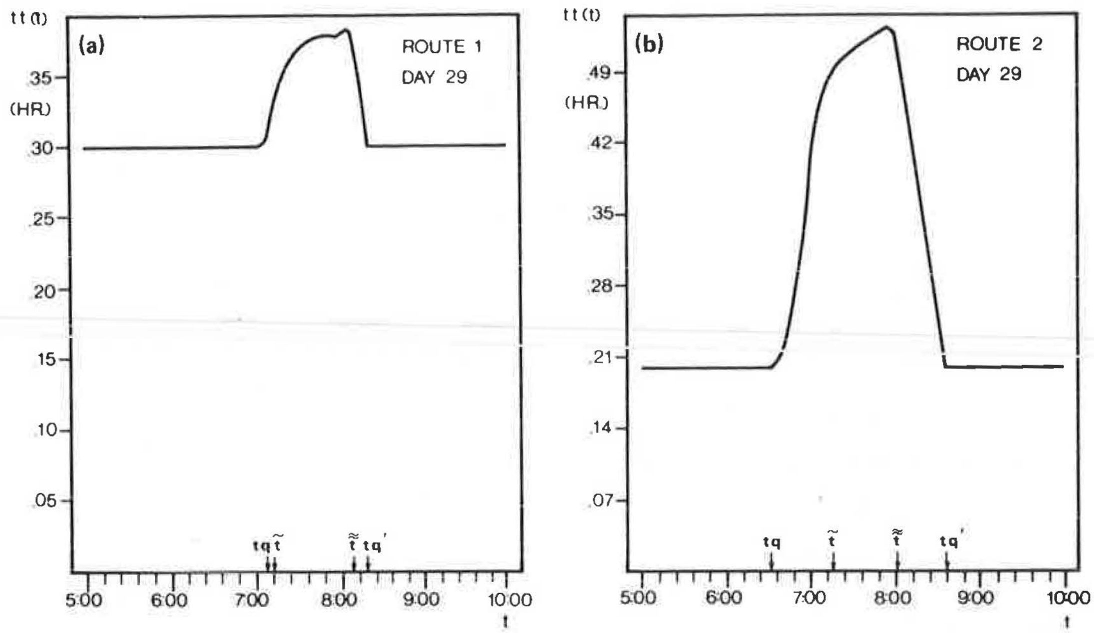

FIGURE 2 Base case stationary distributions of travel times for Route 1 and Route 2 (s^1 = 8,000 vph and s^2 = 3,000 vph).

runs with different parameters, it appears that these abrupt changes are caused neither by computational problems nor by the nature of the dynamic model.

In an earlier theoretical analysis of the shape of the distribution of departure and travel times, de Palma et al. *(10)* have shown that

1. The departure rate increases (decreases) exponentially for $t < t_q$ ($t > t_{q'}$);
2. When congestion is low, the distribution of departure times tends to be flat for $\tilde{t} < t < \tilde{\tilde{t}}$, for a small value of μ_2; and
3. The distribution of travel times can also be derived for the deterministic limit: it is linear for $t < \tilde{t}$ and $t > \tilde{\tilde{t}}$ and constant for $\tilde{t} < t < \tilde{\tilde{t}}$ *(25)*.

Analysis of Changes in Bottleneck Capacities

The following two changes in the capacities of the two bottlenecks are considered:

1. In the base case Route 2 is highly congested and the maximum arrival rate at this bottleneck reaches approximately 4,900 vph. As an attempt to eliminate the congestion on Route 2, its capacity is increased from 3,000 to 5,000 vph.
2. Route 1 in the base case represents a major expressway that carries almost two-thirds of the traffic in the network under study. Considered is the situation in which this highway needs to undergo major reconstruction; during the construction period the maximum peak-period capacity of this highway decreases from 8,000 to 6,000 vph (i.e., an effective loss of one lane).

The first situation is analyzed under two assumptions about the total demand: elastic total demand using the parameters of the base case and inelastic total demand with the parameters of the base case except that the total volume is constrained to be 21,698 vehicles, as in the stationary state of the base case (Table 1).

The inelastic total demand assumption is an approximation that may be more acceptable for the second situation in which the change in the capacity of Route 1 is due to road repair. Because of the temporary nature of the higher level of congestion, drivers are likely to adjust routes and departure times and maintain the same overall travel pattern in terms of origins, destinations, and modes of travel. The demand will always be elastic except when specified otherwise.

The stationary distributions for these two capacity changes are summarized in Table 1. Figure 3 shows the stationary departure rate distributions in the first situation. The distributions under the elastic and inelastic total demand assumption are quite similar, and there are no qualitative differences between the stationary distributions for the two demand assumptions. Higher capacity on Route 2 or lower capacity on Route 1 results in major shifts of traffic from Route 1 to Route 2. In the case of higher capacity on Route 2, congestion does not vanish from Route 2 even with inelastic total demand. There is a significant shift from Route 1 to Route 2 that actually eliminates congestion on Route 1. This increased capacity has substantial user benefits because the delays are significantly shorter.

Note that the importance of the temporal distribution of the demand is demonstrated by the fact that the percentage change in average delay is significantly greater than the percentage change in total volume. In the case of larger capacity on Route 2, there are no important differences between the elastic and the inelastic total demand assumptions (Table 1).

The comparisons with the base case in Figures 3a and 3b demonstrate that increasing capacity eliminates congestion on Route 1, shortens the length of the congestion period (by 25 percent) on Route 2, and decreases the average and maximum delays. It also results in a significant shift toward later departure times and a large increase [decrease] in the maximum of the departure rate distribution for Route 2 [1] because of the shift from Route 1 to Route 2. Figures 3a and 3b provide a clear demonstration of how added capacity causes an increase in traffic volume, a shift from one route to another, and a shift in the temporal distributions.

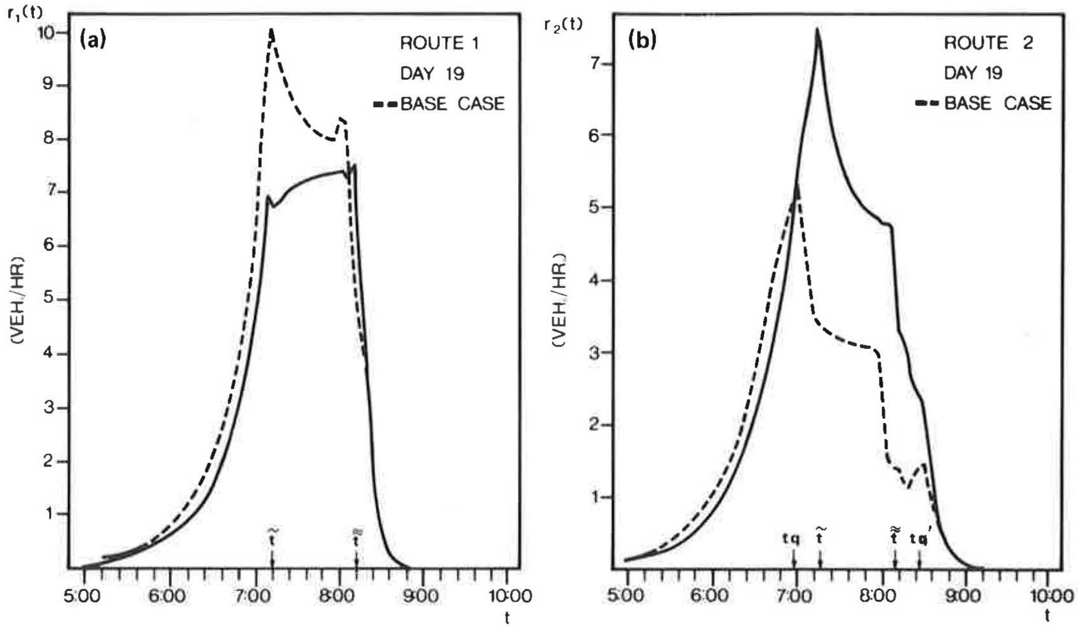

FIGURE 3 Stationary distributions of departure times for Route 1 and Route 2 (s^1 = 8,000 vph and s^2 = 5,000 vph).

The comparisons with the base case for the situation of reduced capacity on Route 1 are shown in Figure 4. In this case, the shift from Route 1 to Route 2 is less significant and the major change is a shift on both routes toward earlier departure times. There is also a smaller increase in late departures. The maximum departure rates have increased and shifted to an earlier time and the durations of the congestion periods on both routes have increased significantly. Thus the major effect of closing one lane on Route 1 is a shift of traffic from the congested on-time arrival period on Route 1 to early arrival periods on Routes 1 and 2.

In a situation of drastic change in the capacities of the bottlenecks in a highway network it is important to predict the transient adjustments of the volumes in addition to the new equilibrium state. Major reductions in capacities often occur for short periods of time when a highway section is being repaired and the dynamics of the traffic are of direct interest. For permanent changes in capacity such as the construction of an additional lane, it is also useful to study the length of the adjustment period.

The predicted dynamic evolutions of the traffic flows and delays toward their new stationary states starting from the stationary state of the base case are shown in Figure 5. The rate of convergence to a stationary state is dependent on the value of the review rate. For a high value, a convergence to a stationary state is not guaranteed. Simulation experiments consistently show that for small values of

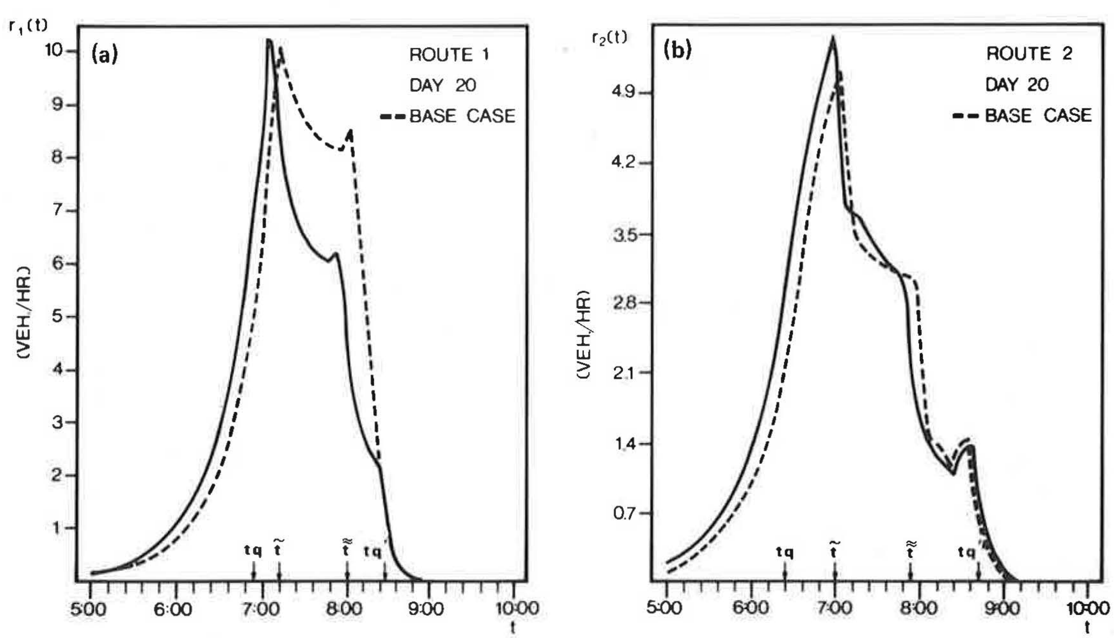

FIGURE 4 Stationary distributions of departure times for Route 1 and Route 2 (s^1 = 6,000 vph and s^2 = 3,000 vph).

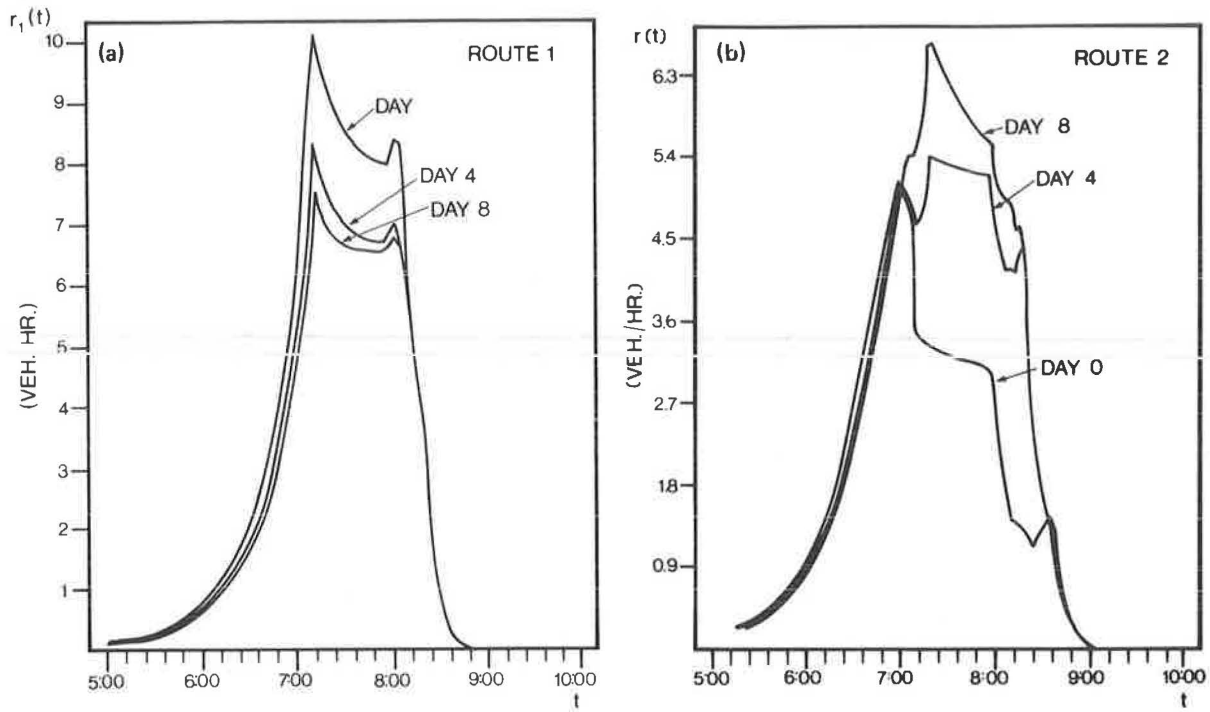

FIGURE 5 Transient distributions of departure times for Route 1 and Route 2 (s^1 = 8,000 vph and s^2 = 3,000 vph).

the review rate convergence occurs toward a unique stationary state. A value of 0.1 was selected for the simulations presented in this paper because it leads to stable stationary states after a period of 2 to 3 weeks, which is reasonable for these types of traffic adjustments. As also suggested by other simulations not presented here, the dynamic evolutions of the departure rate distributions appear to have two time scales. The first period corresponds to the major shifts among the alternative routes. During the second period significant adjustments occur in the departure time distributions while the total volume on each route remains stable. [A more detailed presentation of the dynamic evolutions in the temporal distributions of traffic flows is given elsewhere *(25)*.]

The on-time arrival period on Route 1 is shortened by only 6 min, from 58 to 52 min, and begins 15 min earlier. This result could be interpreted by noting that the length of the on-time arrival period for a deterministic choice model is equal to 2Δ, which is equal to 1 hr in the simulations *(27)*.

Adding or Closing a Route

Next is considered a change in the number of routes available between the origin and the destination (Figure 6 and Table 2). Considered first is the case in which there is only one route. In this case, the congestion level, measured by average delay, is twice its value in the base case. However, the length of the congestion period increases by only 50 percent. Because of the higher level of congestion, the volume of vehicles and the consumer surplus decrease substantially.

Second, a situation is considered in which the base case network is augmented by a third route that has a capacity of 3,000 vph and the same length as Route 1. The number of road users increases slightly (by 3 percent). The main effect is a shift from Route 1 to Route 3 (31 percent) and to a lesser extent from Route 2 to Route 3 (12 percent). Thus Route 1 is no longer congested. The level of congestion decreases on Route 2 and increases on Route 3. The average delay is approximately two times greater on Route 2 than on Route 3 because Route 2 is shorter than Route 3. The average total travel time is 0.300 hr on Route 1, 0.308 hr on Route 2, but 0.356 hr on Route 3, which implies that even on average the travel times on alternative routes are not equal. The distribution of departure time at the stationary state for Route 3 is shown in Figure 7. The distribution for Route 1 is typical of a noncongested situation *(10)*. The shape of the distributions for Routes 2 and 3 is typical of a congested situation: compare Figure 7 with the base case for Route 2 (Figure 1).

Stochastic Capacity

All previous analyses have assumed that the capacities of the bottlenecks are fixed and do not vary from day to day. However, observation of traffic flow conditions on major highways that are saturated during peak periods shows that there exist day-to-day variations in capacity in the range of ±10 percent *(28)*. These deviations may be attributed to weather conditions, the mix of vehicles in the traffic stream, accidents, roadside interruptions, and other uncontrollable stochastic events that affect the maximal flow on a congested highway. The larger maximal flows that are observed could be attributed to a homogeneous traffic stream and ideal weather conditions.

Simulation results of an extension of the model in which the capacity of a bottleneck on any given day is a random variable (which is not known to the drivers when they plan their trips) follow. Let $\overline{s}^i(\omega)$ be the capacity of route i on day ω and express it as

$$\overline{s}^i(\omega) = s^i(1 + \varepsilon_\omega), \qquad i = 1,2,\ldots,I \qquad (7)$$

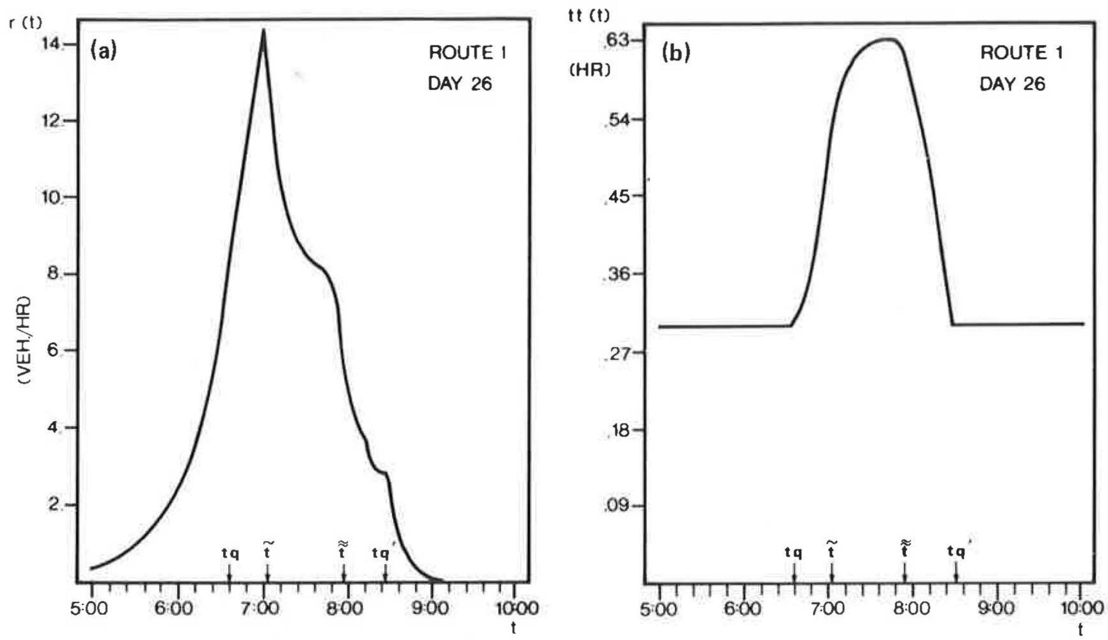

FIGURE 6 Stationary distributions of departure times and travel times for a single route (s^1 = 8,000 vph).

TABLE 2 SUMMARY OF THE STATIONARY DISTRIBUTIONS FOR CLOSING OR ADDING A ROUTE

	One Route s^1 = 8,000 vph	Three Routes s^1 = 8,000 vph, s^2 = 3,000 vph; s^3 = 3,000 vph
Total volume (vehicles)	19,654	22,343
Average delay (hr)	0.169	0.048
Average consumer surplus (utility)	4.713	6.726
Volume on Route 1 (vehicles)	19,654	9,560
Volume on Route 2 (vehicles)		6,878
Volume on Route 3 (vehicles)		5,905
Average delay on Route 1 (hr)	0.169	0.000
Average delay on Route 2 (hr)		0.108
Average delay on Route 3 (hr)		0.056
Maximum delay on Route 1 (hr)	0.331	0.000
Maximum delay on Route 2 (hr)		0.201
Maximum delay on Route 3 (hr)		0.107

where ε_ω is a uniformly distributed random variable in a range $[-m, +m]$. It is assumed that the values of ε_ω are the same for all routes and are independent from one day to another. Equation 7 implies therefore that the average capacity remains equal to s^i.

The obvious question to be addressed is: Starting from the base case and allowing the capacity to fluctuate according to Equation 7, how do traffic conditions fluctuate from day to day and what is the effect on the average flows and delays? A reduced capacity on a given day may cause travelers to shift to later or earlier departure times on the following day and possibly to less congested routes thus reducing the overall level of congestion.

On the other hand, it is expected that travel times will increase because of the convexity of the travel time function *(11)*. Finally, although on average the capacity is equal in the deterministic and the stochastic simulations, the average traffic conditions (even over a large number of days) will not be identical. The results are shown in Figure 8.

Figure 8 shows that average delay is quite sensitive to variation of m from 0 to 5 percent. Beyond this range of variations, the system appears to absorb better the stochastic variations in capacity. It is worth noting that the sensitivity of average and maximum delay to m is not the same. Maximum delay is approximately constant for $m < 0.05$ whereas average delay varies significantly in this range. An implication of these results is that, in order to have the same consumer surplus level, the capacity that is used in a deterministic model should be smaller than the average capacity. A mean preserving capacity distribution lowers the efficiency of the system as its range of fluctuations increases.

The vertical lines in Figure 8 indicate the range of day-to-day fluctuations in travel time. The ability of a network to absorb unpredictable fluctuations should be taken into consideration as well as its performance under optimal conditions. It may be preferable to have a road with stable day-to-day performance instead of a road that has larger maximum capacity but that is less reliable. A similar conclusion was reached by Kahn et al. *(29)* for a mode choice model.

The way individuals build their expectations is critical in a stochastic capacity model. Here it is assumed that the expected travel time for day ω is equal to the travel time experienced on day ω - 1. In future research, more complex hypotheses should be tested. Little experimental evidence is available on this aspect of driver behavior. Moreover, little is known in general about such adjustment processes. There exist, however, some situations in which the road user may have better expectations. For example, if capacity level is a function of weather conditions, the value of capacity on day ω - 1 and on day ω will be correlated. This, however, does not necessarily mean that road users will be better off. Arnott et al. *(30)* have studied a simplified version of the model presented here and have shown that this could be the case. More information provided to the road users, which makes the system more predictable, may thus lower the utility level for the

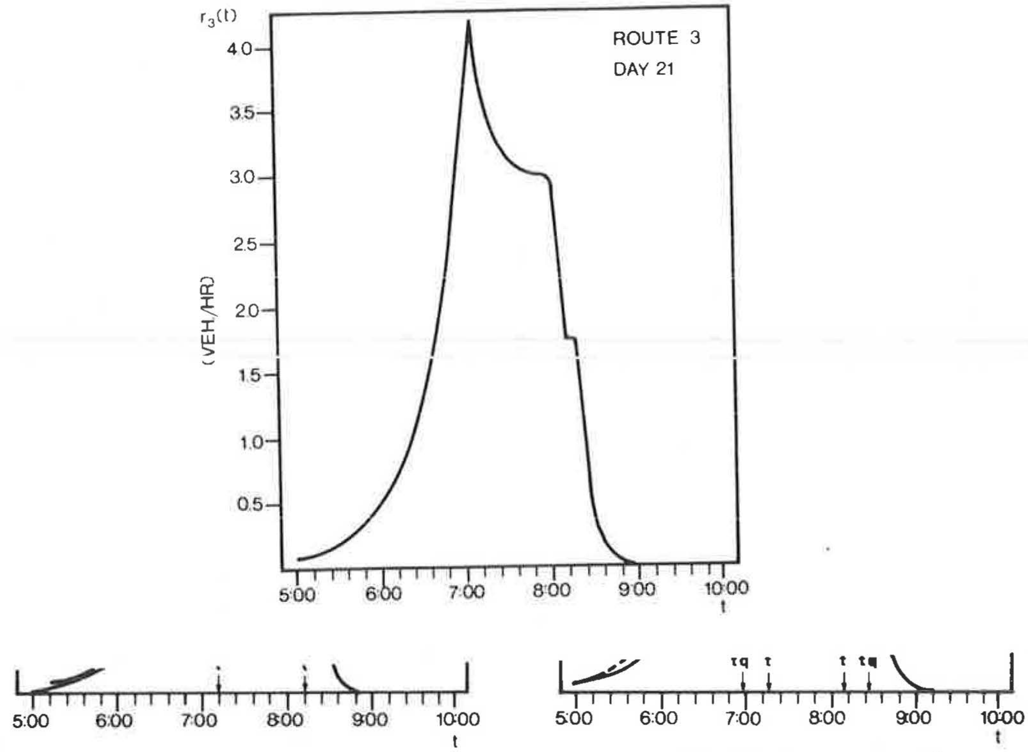

FIGURE 7 Stationary distribution of departure times in a network with three routes (s^1 = 8,000 vph, s^2 = 3,000 vph, and s^3 = 3,000 vph).

road users. This issue (which has a strong practical interest) is an important topic for future research.

CONCLUSION

The major limitations of this approach follow.

- The simplified time adjustment process, which excludes learning behavior. The mechanisms by which individuals process the information attained in their past experience should be investigated further on both theoretical and experimental grounds. This may lead to more realistic dynamic adjustment processes than those that have been considered so far. This extension will also explore the impact of detours and road construction.
- The linear specification of the utility function. It was chosen only because there does not appear to be general agreement on how to generalize the linear specification. The specification of the utility function could be changed quite easily in the simulation program that was developed.
- The simplified network that has been considered so far [see the discussion in Ben-Akiva *(15)*].
- The study of a homogeneous population without explicit treatment of taste variations. Moore et al. *(31)* found, for example, that "older workers and those living at great distances from the workplace tend to arrive earlier." This corresponds to smaller values of β but the same value of $\beta\gamma$ (Equation 5). They also found that households constrained by the presence of a working spouse and young children have less flexibility to alter arrival times with flextime. This corresponds to a smaller value of Δ (Equation 5).

The results obtained nevertheless demonstrate that the model is able to explain, at least qualitatively, the experimental properties of departure time choice situations. It is believed that the results will not be significantly different under slightly different hypotheses.

The simulation experiments have replicated important phenomena in the response of traffic flows to changes in roadway capacities. It was shown how changing the capacity of one bottleneck affects traffic conditions in a parallel facility. It was also shown that the capacity of a bottleneck may be expanded to meet the highest existing traffic flow without eliminating congestion. It is thought that the model, even with its limitations, should be able to effectively analyze simple networks.

For example, the simulation model employed in this analysis can also be used to analyze a variety of other policy measures aimed at reducing peak-period congestion. In particular, it is useful for comparisons of the effectiveness of low-capital policy options such as variable work hours and peak-load pricing with capital-intensive capacity expansions. Additional simulation results reported in Ben-Akiva et al. *(13)* replicated the phenomenon of shifting peaks that occurs when peak-period tolls are established.

Finally, further work on this modeling approach should include a detailed validation test with data from before and after an actual change in a transportation network. Attention should also be given to the theoretical properties of the stationary state of the model and the stability of its dynamic evolution. Further extensions could be directed to capturing differences among market segments with different travel behavior preferences and origin-destination patterns.

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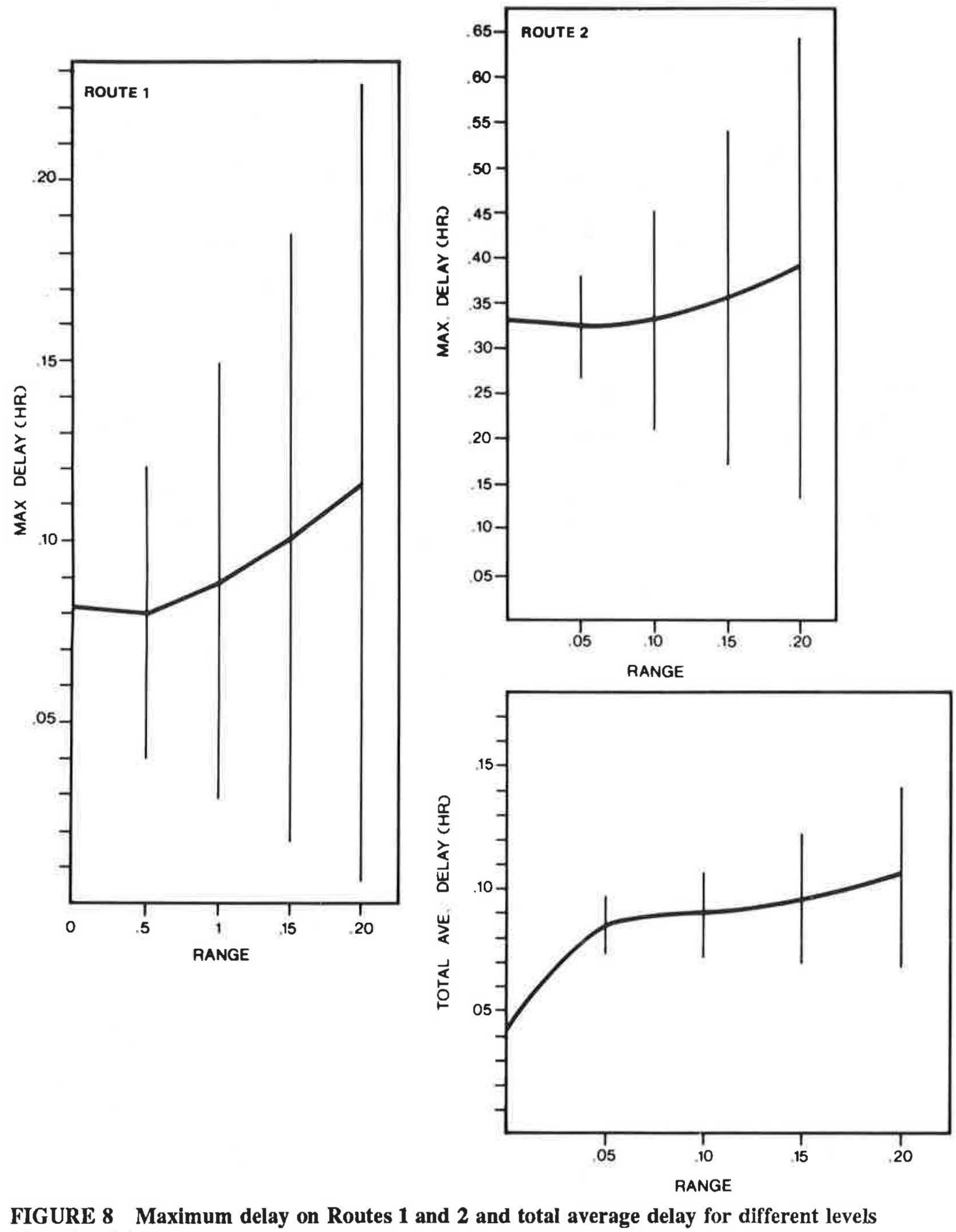

FIGURE 8 Maximum delay on Routes 1 and 2 and total average delay for different levels of stochasticity.

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APPENDIX

Base Values for Network Characteristics

	Free-flow travel time from origin to destination for Route 1 [Route 2]	0.3 hr [0.2 hr]
$s^1 [s^2]$	Capacity of Route 1 [Route 2]	8,000 vph^{-1} [3,000 vph^{-1}]

Base Values for Demand Parameters

α	Marginal disutility of travel time	6.40 hr^{-1}
β	Marginal disutility of early arrivals	3.90 hr^{-1}
β_γ	Marginal disutility of late arrivals	15.2 hr^{-1}
μ_1	Heterogeneity factor of travel decision	1.0
μ_2	Heterogeneity factor of departure time choice	2.0
μ_3	Heterogeneity factor of route choice	3.0
R	Individual review rate	0.10 day^{-1}
Δ	Work start time flexibility	0.5 hr
t^*	Center of desired period of arrival	8.00 a.m.
V_0	Utility of the null alternative	0
N	Number of potentential travelers	25,000
T_0	Time of earliest possible departure	5.00 a.m.
T	Daily study period	5 hr