Soil-Structure Interaction of Flexible Pipe Under Pressure

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This paper contains the results of a finite element analysis of the soil-structure interaction of a buried flexible pipe subjected to internal pressure. The analysis is performed using a fine mesh so that *(a)* an accurate representation is made of the highly variable soil stiffness in some installations, similar to that of a pipe with poorly supported haunches resting on a hard bedding, and *(b)* an accurate model is made of the pipe wall stresses in light of the rapid attenuation of local deformations and strains in flexible pipe buried in stiff soils. In addition, the nonlinear behavior of pressurized flexible pipe, including large deflections and the flexural stiffening effect of the pressure-induced membrane stresses, is modeled. A special purpose program, FLEXPIPE, is developed. The program uses Duncan's soil model and considers the step-by-step nature of construction. It is used to calculate the stresses and strains in a buried flexible pipe with improper haunch support and hard bedding. It is shown that significant flexural stresses are developed at the invert of the pipe after installation and that internal pressure magnifies the flexural stresses and strains thus developed.

In the early stages of the development of flexible pipe for underground installation, there was a general consensus that installation-induced pipe wall stresses and strains were small because the serviceability requirement limits the deflection of the installed pipe to a fraction of the total deflection that the flexible pipe can withstand without fracture. This consensus was contingent on a popular belief that pipe wall stresses are proportional to the deflection of the pipe. However, numerous failures of flexible pipe due to an obvious overstress at the invert of the pipe in some installations accompanied by the successful performance of the pipes in other installations demonstrated that pipe wall stresses and strains were not always small. Interestingly, the deflection of the pipe that failed was not necessarily large. It appeared that improper installation and compaction procedures may have led to high localized stresses and strains and resulted in the fracture of the pipe wall.

The state-of-the-art design formulas for predicting pipe wall stresses in flexible pipe are developed primarily for stiffer pipe and are based on the assumption that pipe wall stresses are proportional to deflections. New tools for analysis are required for the prediction of pipe wall stresses in flexible pipe, particularly when subjected to internal pressure. The sensitivity of flexible pipe to installation suggests the use of finite element soil-structure interaction methods. Although much work has been performed on the analysis of the behavior of buried conduits using soil-structure interaction models *(1–3)*, the behavior of buried pressurized pipe has been little explored with the aid of soil-structure models. Recently, two major efforts were made to introduce finite element soil-structure methodology to the analysis and design of flexible pipe. The American Concrete Pipe Association sponsored the development of a soil-pipe interaction design and analysis (SPIDA) program as a tool for the design of buried reinforced concrete pipe *(4, 5)*. Concurrently, Owens-Corning Fiberglas embarked on a program to investigate the behavior of buried flexible pipe, including the development of a finite element soil-structure interaction model *(6–8)*. Both of these programs are supposed to have the capability to predict pipe wall stresses in nonideal installation conditions and to handle internal pressure; however, no results have been published by which the adequacy of these programs for predicting the stresses and the strains in pressurized flexible pipe installed in less than ideal conditions can be judged.

In this paper is described a finite element, soil-structure interaction method for predicting the pipe wall stresses and strains in a buried flexible pipe subjected to internal pressure.

The behavior of a buried flexible pipe depends to a large extent on the uniformity of the soil stiffness provided by the bedding and the backfill for the pipe. Nonuniformity of support can result in local deformations and flexural stresses in the pipe wall, in addition to the pipe wall stresses that result from the ovaling deflection of the pipe. The local deformation and flexural stresses are developed as the stiffer soil inhibits the outward radial movement of the pipe wall while the softer soil permits it. For example, when a flexible pipe is installed on hard bedding with inadequate haunch support, significant local deformation (i.e., flattening and a high flexural stress at the invert) is to be expected.

When a buried flexible pipe with uniform soil support is subjected to internal pressure, membrane stresses will develop in the pipe wall and the pipe will expand radially. In addition, internal pressure will reround the pipe and thus alleviate much of the flexural stresses and strains developed in the pipe wall as a result of the ovaling deflection of the pipe. This behavior has little resemblance to the behavior of the same pipe when the soil support does not provide a uniform support for the pipe. When a flexible pipe with nonuniform soil support is subjected to internal pressure, the deformation of the pipe before pressurization results in a larger radius of curvature locally (e.g., at the invert of the pipe), which, in turn, causes higher membrane stresses there. In addition, the internal pressure causes radial movement of the pipe wall that if inhibited by the stiffer soil will increase the local flexural stresses. As the pipe becomes more flexible, the additional stresses that result from the nonuniformity of the soil support become more significant.

Deformations induced in a flexible pipe buried in relatively stiff backfill tend to attenuate rapidly. Using the analogy of a ring embedded in an elastic foundation (*9*, p. 157), the attenuation length for a half wavelength may be approximately expressed as $\pi[(EI/E'r^3)^{1/4}]$ where E is the pipe wall stiffness (lb-in.^{2}/in.), r is the pipe radius (in./in.), and E' is soil stiffness (psi); therefore deformations induced in a flexible pipe with a stiffness of 10 psi buried in moderately stiff backfill are expected to attenuate in about 30 degrees. (Pipe stiffness is defined as the stiffness of the

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pipe when subjected to two diametrically opposed loads similar to those imparted to the pipe in a parallel plate test, whereas pipe wall stiffness is the stiffness of the pipe wall when subjected to a bending moment.) Obviously the mesh size must be sufficiently fine to permit an accurate prediction of pipe wall stresses.

Certain installation conditions can result in highly localized soil reactions on the pipe. In addition, when such a pipe is pressurized, additional reaction forces can develop on the pipe and thus exacerbate the situation. The stresses in the pipe wall are significantly affected by the spread of the localized reaction. A hard bedding is expected to provide only a narrow reaction at the invert, and a soft bedding is expected to distribute the reaction over a wider region and thus relieve pipe wall stresses.

To model these phenomena in such a way that a reasonable estimate of pipe wall stresses can be obtained, a much finer mesh than those used by the existing finite element, soil-structure interaction programs is needed.

To properly simulate the effects of internal pressure acting on a flexible pipe, the nonlinear effects of the deformed configuration of the pipe and of the internal pressure must be taken into account. The nonlinear effects of the deformed configuration of the pipe and of the internal pressure are incorporated in the program by applying the internal pressure incrementally to the deformed geometry of the pipe. In addition, the stiffening effect of the membrane stress resultant developed as a result of the internal pressure on the flexural stiffness of the pipe is considered.

The program has been applied to a number of buried flexible pipes with and without internal pressure. The results indicate that in the absence of a soil support of uniform stiffness, large flexural strains can develop in a buried flexible pipe. The magnitude of such flexural strains cannot be predicted from the deflection of the pipe. For all practical purposes, they may be considered additional to the strains that would result when soil provides an approximately uniform support for the pipe. Internal pressure not only does not alleviate the flexural strains that result from the nonuniformity of soil support, it also exacerbates the situation by increasing the flexural strains at the invert of the pipe. Furthermore, local deformations of the pipe that result from the nonuniformity of the supporting soil stiffness and the consequent local changes in the radius of curvature of the deformed pipe give rise to higher pressure-induced membrane stresses.

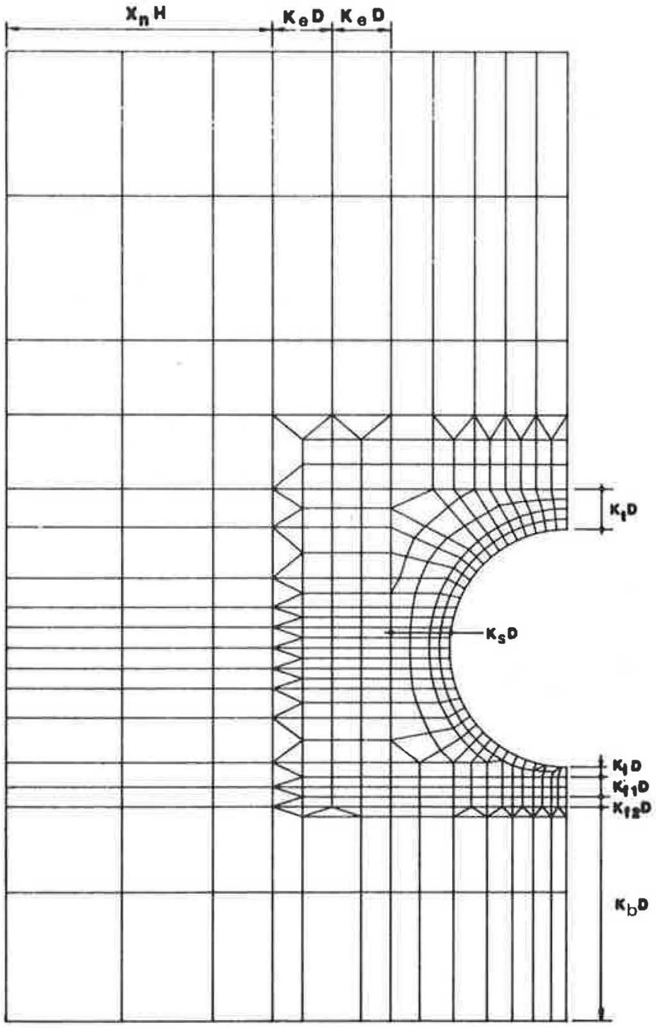
PIPE-SOIL INTERACTION MODEL

The mathematical model developed for the pipe-soil interaction is one in which the pipe and the surrounding soil are discretized in finite elements with specific load-deformation characteristics. The model has a mesh geometry that makes possible accurate analysis of flexible pipe in nonideal installations observed in the field that, in some instances, have led to the failure of the pipe wall. The model thus developed simulates the characteristics of the actual installation condition, of the soil, and of the flexible pipe and considers the step-by-step construction sequence of the pipe. The model as it stands now assumes a vertical plane of symmetry. It does not account for the method of compaction of the backfill at the time of its placement. Furthermore, it does not consider the soil compaction that may ensue after installation of the pipe such as the settlement that is expected from a collapsible backfill (e.g., an inadequately compacted sandy silt subjected to the simultaneous action of load and groundwater).

Finite Element Mesh

The mesh geometry adopted for the finite element model of pipe-soil interaction is shown in Figure 1. The general layout of the mesh is designed to analyze pipe installed in trench-type installations, considered to be the most common type of installation of flexible pipe. The mesh element size is fine enough that significant discretization errors are not expected in cases in which a buried flexible pipe is resting on a hard bedding and has loosely supported haunches. In addition, because the flexural stresses that result from highly localized soil reactions are significantly affected by the spread of the load, the mesh layout and fineness provide an adequate representation for predicting the actual pipe wall stresses for different bedding thicknesses including the installation condition in which the pipe is laid directly on a hard foundation.

The program allows the user to specify several mesh geometry parameters that will define the specific geometric profile of the trench that the user desires to analyze. The pipe may be placed



Default Values: $K_t = 0.1667$, $K_s = K_e = 0.25$, $K_b = 0.9$

$K_l = K_{f2} = 0.0417$, $K_{f1} = 0.0833$, $X_n = 0.6$

FIGURE 1 Fine mesh geometry of the finite element model of soil-structure interaction for flexible pressurized pipe, program FLEXPIPE.

either on native material or on backfill material the thickness of which can be varied. In addition, the mesh is adjustable through input parameters to facilitate analysis of pipe installed in different trench widths. Other parameters that can be specified define the pipe zone region immediately above the pipe and the overall mesh dimensions. The parameters that are not specified by the user will default to values defined in the program. The specific input parameters that the user can specify are shown in Figure 1. The mesh is designed in such a way that it can be deformed to any reasonable trench geometry while retaining good geometric proportions.

Characteristics of Pipe Element

The half circular arch of the pipe is discretized into 37 prismatic beam elements. The lengths of the beam elements range from a minimum that appears at the invert and subtends an angle of 2.5 degrees to a maximum that subtends an angle of 5 degrees away from the invert of the pipe. The 5-degree elements are approximately one-sixth of the attentuation interval of 30 degrees, calculated in the first section of this paper for a flexible pipe with a stiffness of 10 psi buried in a moderately stiff backfill. This element size is expected to yield sufficiently accurate results for pipe the stiffness of which is not much less than 10 psi based on the results of tests conducted on discretization of beams on elastic foundations. Obviously, the accuracy of analysis will deteriorate as pipe stiffness decreases below 10 psi. The beam elements are connected together so that full continuity of displacements and rotations is ensured. The material of the beam elements is linearly elastic. The stiffness matrix of the beam elements is modified incrementally, as construction progresses in steps and as pressure is incrementally applied, to incorporate *(a)* the effect of pipe deformation on the geometry of the pipe and *(b)* the stiffening effect of the axial force in the element, such as the tensile force that results from internal pressure, on the flexural stiffness of the elements. The first effect is modeled by modifying the coordinates of the joints by the magnitude of their displacements. The second effect is modeled by adding to the stiffness matrix of the beam elements a correction matrix described elsewhere (*10*, p. 262; *11*).

Characteristics of Soil Elements

The model employs two kinds of soil elements, an in situ element and a constructed soil element.

In situ soils are the preexisting natural soils in the foundation and the walls of the trench. In situ soils are undisturbed through installation and have been consolidated under the natural overburden. The in situ soil elements are assumed to be linear in the sense that their elastic constants do not vary with stress. This is believed to be a valid approximation because the final state of stress that results from pipe backfill is not expected to be significantly different from the natural state.

Constructed soils are soils used for bedding underneath the pipe and the backfill around and over the pipe. They are usually placed in layers and compacted mechanically or through the application of the subsequent layers. The constructed soil elements have a hyperbolic stress-strain relationship *(12)*. This relationship implies that the constructed soil stiffness decreases with increasing stress but increases with confining pressure. The properties of the constructed soil elements, which characterize the hyperbolic stress-strain relationship, are based on those proposed for several soil types in Duncan et al. *(13)*. Tables 1 and 2 give the soil properties used in the program. Note that the soil types studied in Duncan et al. *(13)* do not constitute the entire range of compactions that has been encountered in the field. In many cases, the constructed soil beneath the haunches of the pipe is much looser than the "loose" soil properties presented by Duncan et al. *(13)*. The elastic properties of the improperly compacted soils used for the installation of the pipe need to be established. This, however, must be performed separately for each soil type encountered in the field. Table 1 gives the properties of the soil used in the haunch areas of the flexible pipe with improper haunch support presented herein. In specific applications, these properties must be established on the basis of field observation and testing.

The constructed soil elements are elastic in the sense that loading and unloading are assumed to follow the same path on the stress-strain diagram. The soil elements are connected to each other and to the pipe elements so that full continuity of all of the components of displacement is ensured. In other words, no slip is permitted between the constructed soil elements and the pipe elements.

Loads

There are several sources of loads that the pipe-soil system may be subjected to, such as the weights of the soil, pipe, and water inside the pipe; internal pressure; the live load of a truck; and surcharge.

TABLE 1 SOIL PROPERTIES OF CONSTRUCTED SOIL (backfill)

Soil No.	Unified Classification	Compaction	γ_m (lb/ft^3)	K	n	R_f	K_b	m	C (psi)	ϕ (degrees)	$\Delta\phi$ (degrees)	k_0
1	GW, GP	H	145	550	0.4	0.7	150	0.2	0	42	8	1.5
2	SW, SP	M	135	300	0.4	0.7	75	0.2	0	35	5	1.0
3		L	125	100	0.4	0.9	25	0.2	0	30	2	0.5
4	SM	H	135	650	0.25	0.7	500	0	0	36	8	1.3
5		M	125	400	0.25	0.7	350	0	0	33	5	0.9
6		L	115	150	0.25	0.9	150	0	0	30	2	0.5
7	SM–SC	H	135	400	0.6	0.7	200	0.5	3.5	33	0	1.1
8		M	125	200	0.6	0.7	100	0.5	2.5	33	0	0.8
9		L	115	100	0.6	0.9	50	0.5	1.4	33	0	0.5
10	CL	H	130	250	0.45	0.7	150	0.2	2.8	30	0	0.9
11		M	120	150	0.45	0.7	100	0.2	1.8	30	0	0.6
12		L	110	50	0.45	0.9	50	0.2	0.6	30	0	0.3
19	(haunch voids)	VL	10	5	0.25	0.9	0.2	0.0	0.0	20	0.2	0.2

TABLE 2 SOIL PROPERTIES OF PREEXISTING SOIL (in situ)

Soil No.	State	γ_m (lb/ft^3)	K	n	R_f	K_b	m	C (psi)	ϕ (degree)	$\Delta\phi$ (degree)	k_0
Coarse-Grained Soils											
13	A	145	680	0	0	22,000	0	0	0	0	1.0
14	B	130	408	0	0	453	0	0	0	0	1.0
15	C	115	136	0	0	76	0	0	0	0	1.0
Fine-Grained Soils											
16	D	125	408	0	0	340	0	0	0	0	1.0
17	E	117	238	0	0	393	0	0	0	0	1.0
18	F	110	68	0	0	2,200	0	0	0	0	1.0

The dead weights are applied with each construction layer. When all layers have been placed, the pipe is subjected to the weight of the water and internal pressure. The internal pressure is applied incrementally; the number of increments is dependent on the extent of nonlinearity expected in the behavior of the pressurized pipe. A larger number of pressure increments are expected for a pipe with an installation-induced deformation configuration that results in a significant change in the local radius of curvature of the pipe before pressurization. This condition occurs when, for example, a flexible pipe has an improper haunch support and is installed directly on hard bedding.

Construction Sequence

The model considers the step-by-step sequence of construction. It starts with the assemblage of the in situ soil elements and in each step adds a layer of the constructed soil. The pipe may be placed either on in situ soil or on constructed backfill the thickness of which may be specified by the user. After the pipe is placed, five additional layers of backfill are placed in sequence until the backfill just covers the crown of the pipe. The number of layers applied after this is dependent on the height of the fill that the user specified. Above the pipe zone regions, layers are constructed that have a thickness of approximately one-half the pipe diameter. In each construction step, a soil layer is applied by adding new nodes and elements to the previously constructed finite element model. The weight of the soil layer increases the confining pressure of the previous soil elements. The increase in the confining pressure increases the stiffness of the constructed soil elements. (At any stage of construction, the top layer of soil is not confined and therefore initially lacks stiffness; care must be exercised not to unduly load the flexible pipe by the weight of the soil elements that lack the necessary stiffness to support the pipe, particularly when the construction reaches pipe haunches.)

At each step of construction, the pipe-soil system is subjected to a two-iteration analysis. In the first iteration the solution before the application of the load increment is used to approximate the states of stress and deformation of the pipe-soil system after the application of the load increment. The results of the first iteration are then used to estimate the average states of stress and deformation of the pipe-soil system during the application of the load increment, which are in turn used to modify the stiffness matrix and the geometry of the deformed pipe-soil system. This iterative procedure has been shown to give results that are sufficiently accurate for all practical purposes when load increments are not very big. However, for problems for which experience does not justify the two-iteration analysis, a larger number of iterations is unavoidable.

Pressurization

When the last construction layer is applied, the pipe is subjected to the weight of the water and the pressure increments. At each pressure increment, the pipe-soil system is subjected to two-iteration analysis. In the first iteration, the state of stress and deformation of the pipe-soil system at the end of the pressure increment is approximated. The results are then used to calculate the average stresses and deformations in that load increment. These average values are used to modify the soil and the pipe stiffnesses that are in turn used to calculate the state of stress and strain in the pipe-soil system at the end of the pressure increment.

As the flexible pipe is subjected to internal pressure, the pipe expands and moves against the surrounding soil. This increases the radial load on the soil elements around the pipe. In addition, with increasing pressure, a larger share of the weight of the cover will be picked up by the pipe, thus reducing the confining stress on some soil elements near the springline. The program ignores any inelastic behavior of the soil resulting from such a reduction of the confining stress and assumes that the loading and unloading paths are coincident. The effects of this approximation on the accuracy of the results obtained have not been evaluated.

Note that the geometric nonlinearity of the pipe, resulting from large deformation and from the effect of the axial force in the elements on the flexural stiffness of the pipe elements, and the nonlinearity of the stress-strain relationship of the soil render the problem highly nonlinear and sensitive to pressure increment size.

PROGRAM

The computer program FLEXPIPE is developed for the finite element analysis of soil-structure interaction of a flexible pipe subjected to internal pressure. The soil model is a variant of the Nonlinear Soil-Structure Interaction Program (NLSSIP), a general-purpose finite element soil-structure interaction program developed originally at the University of California at Berkeley and modified at the University of Massachusetts at Amherst for application to SPIDA *(4)*. It generates the mesh geometry and construction sequence internally from a limited input. It computes the stiffness of the pipe elements considering large displacement and the nonlinear effects of the axial force in the element on its flexural stiffness. The results are printed as deformations and stress resultants in the pipe wall. The input to this program consists of the following parameters:

- Trench geometry. Specifications for trench geometry include height of fill, trench width, bedding thickness, height of pipe zone region, and overall mesh dimensions.

- Pipe parameters. These include pipe diameter, wall thickness, material density, and elastic modulus.
- Construction sequence. The construction sequence is by and large internally specified. The user may only specify whether the pipe is to be placed on in situ material or on constructed backfill.
- Soil properties. The soil properties are built into the program, but the user may choose to define his own. To facilitate the assignment of soil properties to each element, the finite elements are grouped into zones that are typical of present engineering and construction practices. The user simply assigns a number that represents the desired soil properties to each zone. In addition, he may override soil properties for any element in any given zone and specify the properties he desires for that particular element.
- Pressure. The user may specify the magnitude of pressure and the number of increments in which the pressure is to be applied.
- Additional loads. The user may specify additional loadings such as AASHTO truck loading and surface surcharge loads.

The output consists of the following items:

- Summary of installation data, pipe properties, deflections, thrusts, moments, shear stresses, and arching factors;
- Soil properties;
- Finite element node geometry, connectivity, and construction layer information; and
- Incremental node forces, deformations, axial forces, shears, moments, and normal and tangential soil pressures on the pipe elements.

The user may specify as much or as little output as is desired. In addition, the user may specify at what construction layers the detailed information should be printed. In addition, a summary table of the pipe deformations and the stress resultants and strains in the pipe wall are printed at the end of the analysis for all construction layers.

At present, the program is privately maintained on a CDC Cyber 176 computer at the Control Data Corporation Eastern Cybernet Service Center, 6006 Executive Boulevard, Rockville, Maryland.

RESULTS OF ANALYSIS

The program was used to investigate the effects of inadequate haunch support and hard bedding on the stresses and the strains induced in flexible pipe subjected to internal pressure. For this purpose, a flexible pipe with a pipe stiffness of 10 psi was selected for analysis. The pipe diameter is assumed to be 36 in. with a thickness of 0.4 in. and a modulus of elasticity of 1.63×10^6 psi. The pipe is installed with 6 ft of cover and is subjected to 100 psi of internal pressure after installation.

Installation is in a narrow trench, only a foot wider than the pipe, with vertical walls. The in situ soil of the walls of the side trenches and the foundation is a moderately dense coarse-grained soil. The backfill adjacent to the pipe is a moderately dense loose gravel and sand mixture with silt fines, and the cover over the pipe is loose sand. The haunches are supported by extremely loose soil with a very low bulk modulus. The pipe is placed directly on the hard coarse-grained native soil supported only by the 2.5-degree element at the invert of the pipe. Figure 2 shows the soil types and construction layers used in this analysis. The soil properties for each soil number shown are those given in Tables 1 and 2.

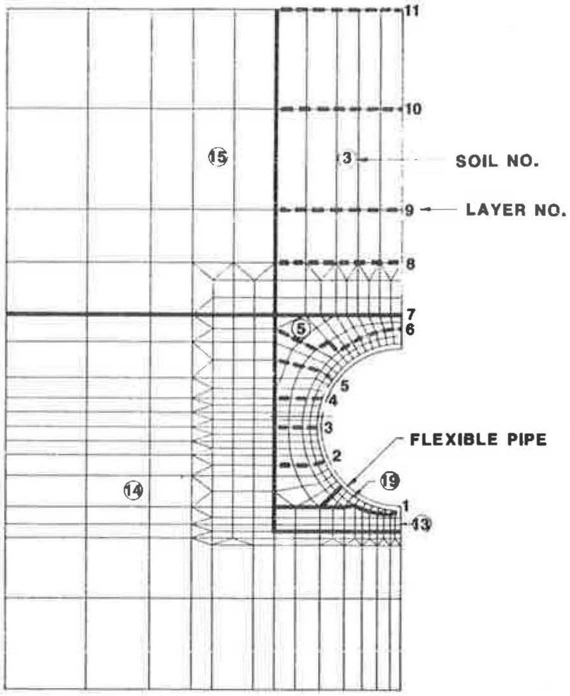


FIGURE 2 Mesh geometry, soil types, and construction layers used in analysis of buried flexible pipe with poorly supported haunches and hard bedding.

The results of the analysis are shown in Figures 3 and 4. Figure 3 shows the deformed configuration of the pipe before and after pressurization. The bending moments, or equivalently the flexural strains, before and after pressurization are shown in Figure 4.

In addition, a second analysis in which the same flexible pipe was installed properly was performed. In this analysis, a 4-in.-thick bedding was provided underneath the pipe and the pipe

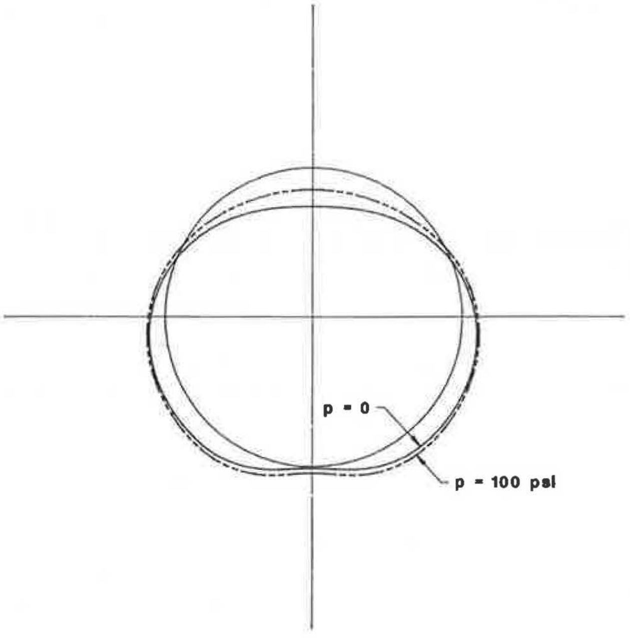


FIGURE 3 Deformed shape of a buried flexible pipe with improper haunch support and hard bedding before and after pressurization.

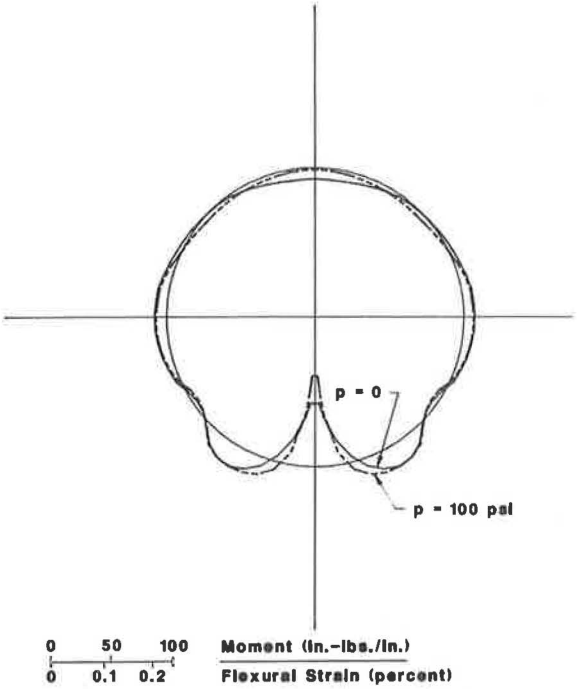


FIGURE 4 Bending moment, or flexural strain, of a buried flexible pipe with improper haunch support and hard bedding before and after pressurization.

haunches were supported by the backfill material with the same compaction as the sidefills. The soil types and construction layers used in this analysis are shown in Figure 5. Note that the pipe in this case is laid on a soil layer modeled as a region subtending a 10-degree angle. The resulting deformations of the pipe and the moments (or strains) in the pipe wall are shown in Figures 6 and 7.

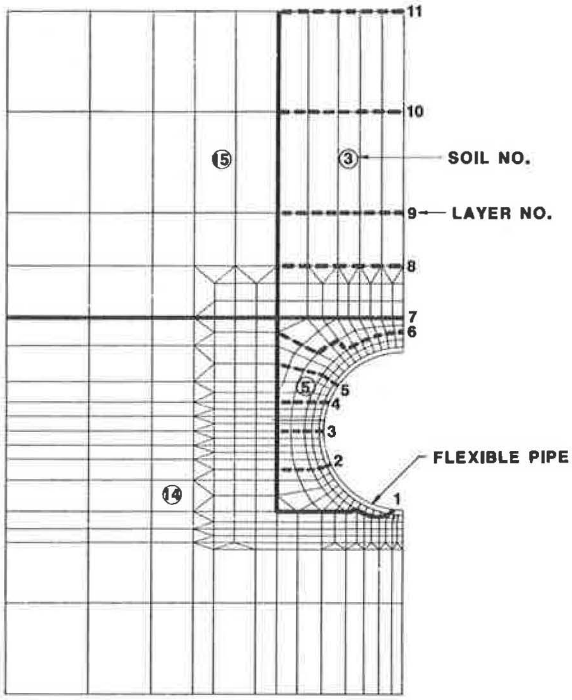


FIGURE 5 Mesh geometry, soil types, and construction layers used in analysis of buried flexible pipe with haunches and invert properly supported.

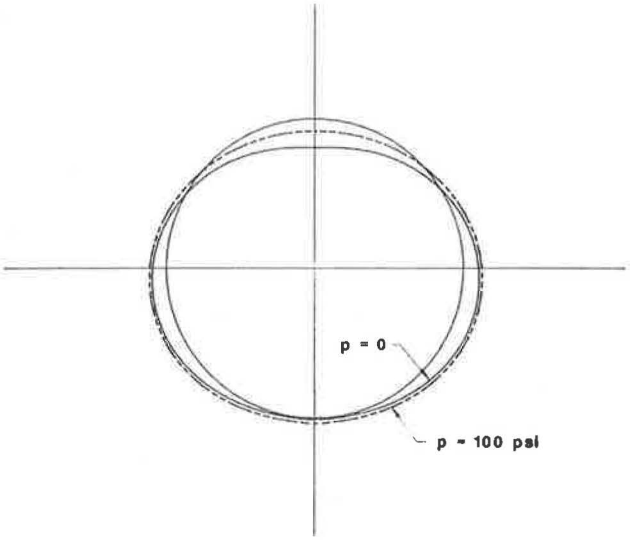


FIGURE 6 Deformed shape of a buried flexible pipe with proper haunch support and bedding before and after pressurization.

Table 3 gives a summary of the results of the analyses performed on the flexible pipe with inadequate haunch support laid on hard bedding and for the same pipe installed on a 4-in. bedding with adequate haunch support. The results are presented for the empty pipe immediately after installation and for the water-filled pipe before and after pressurization.

The following statements, based on the review of the numerical results obtained, can be made:

1. A comparison of the flexural strains at the invert of a flexible pipe with improperly supported haunches laid on hard bedding and

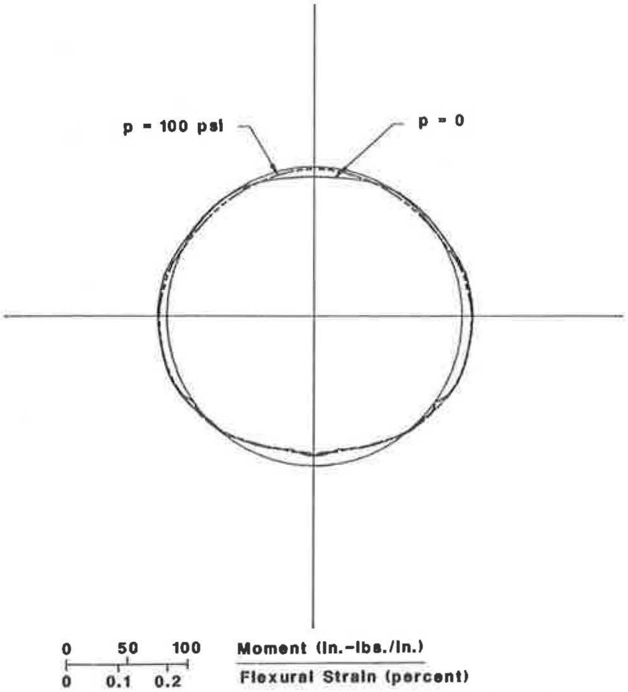


FIGURE 7 Bending moment, or flexural strain, of a buried flexible pipe with proper haunch support and bedding before and after pressurization.

TABLE 3 SUMMARY OF RESULTS OF FINITE ELEMENT SOIL-STRUCTURE INTERACTION ANALYSIS FOR FLEXIBLE PIPE IN PROPER AND IMPROPER INSTALLATIONS

	Hard Bedding and Poor Haunch Support			4-in. Bedding and Supported Haunches		
Item	After Installation	Before Internal Pressure	After Internal Pressure	After Installation	Before Internal Pressure	After Internal Pressure
Arching factor	0.44	0.36	0.56	0.55	0.50	0.77
Vertical deflection (in.)	-0.29	-0.30	-0.13	-0.22	-0.23	0.06
Horizontal deflection (in.)	0.23	0.23	0.27	0.21	0.21	0.26
Approximate invert reaction (lb/in.)	27	35	148	–	–	–
Bending moment (in.-lb/in.)						
At crown	8.7	8.9	1.3	8.1	8.3	1.8
At springline	-9.6	-9.7	-8.8	-8.3	-8.3	-7.1
At invert	44.1	52.8	76.5	8.4	8.5	10.6
Flexural strain (%) at invert	0.101	0.121	0.176	0.019	0.020	0.024
Membrane strain (%) at invert	-0.003	-0.001	0.267	-0.004	-0.003	0.266
Combined strain (%) at invert	0.104	0.122	0.443	0.023	0.023	0.290
Nominal hoop strain (%)	–	–	0.28	–	–	0.28

of the same pipe properly installed suggests that the deformation configuration of the installed pipe, and consequently the resulting installation-induced strains in the pipe wall, may be visualized as the sum of two distinct configurations, an ovaling deformation that would have been obtained if the pipe had been installed on a soft bedding with haunches adequately supported (Figure 6) and a pear-shaped configuration resulting from the localized soil reaction at the invert of the buried pipe, such as the shape corresponding to the difference or deformations shown in Figures 3 and 6. The magnitude of the deformation that results from the pear-shaped component is roughly proportional to the magnitude of the reaction at the invert of the pipe and, therefore, depends on several factors such as the size of the haunch area with inadequate support, the stiffness of the bedding material, the arching factor (i.e., the fraction of the total weight of cover supported by the pipe), and the depth of cover.

This result suggests a generalization that the deformed configuration of any pipe with less-than-ideal installation condition may be considered as the sum of an oval-shaped configuration, which would be obtained if the invert were properly supported, and a pear-shaped configuration resulting from the deviation of the installation from the ideal condition (i.e., the localized soil reaction at the invert) (Figure 8).

Because the oval configuration contributes more to pipe deflection and the pear-shaped configuration to moment, the strain cannot be defined uniquely in terms of the deflection unless a unique mix of the two configurations is assumed. Molin's formula for strain *(14)*, used by most standards including those of AWWA, expresses the flexural strain in the pipe wall as $6(\Delta/d)(t/d)$, where Δ is the deflection, t is the thickness, and d is the diameter of the pipe. For the pipe analyzed herein, this formula yields a value for strain of less than 0.06 percent, underestimating it by almost an order of magnitude.

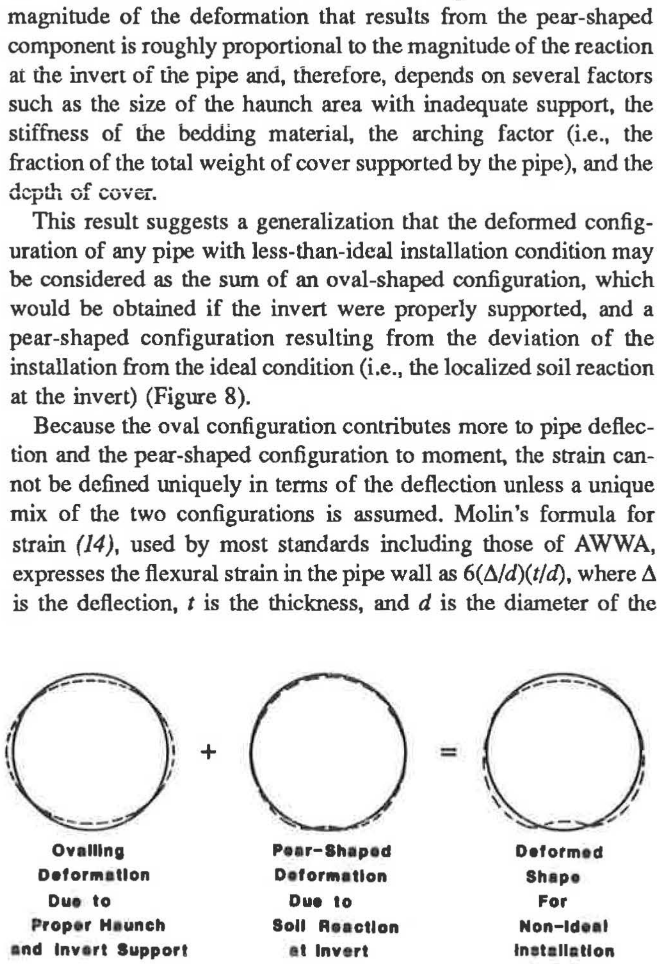


FIGURE 8 Schematics.

2. High flexural strains can develop at the invert of a flexible pipe installed with improper haunch support and on hard bedding. For the problem analyzed, the resulting flexural strain before pressurization is between one-third and one-half of the nominal strain in the pipe wall resulting from internal pressure (pr/Et). Such high flexural strains are not present when the pipe is installed on a softer bedding with proper haunch support.

Comparison of the flexural strains developed in the flexible pipe installed with inadequate haunch support and on hard bedding with those developed in the same pipe installed properly demonstrates the supreme importance of proper installation to the performance of buried flexible pipe. In other words, design of flexible pipe in accordance with the requirements of the AWWA specification is contingent on adequate assurance of the quality of installation.

3. The calculated arching factors after installation and before pressurization indicate that only a fraction of the total weight of the column of the soil above the pipe is supported by the pipe. With time and under the action of the fluctuating groundwater level, the arching factor will increase and more of the weight of the cover will rest on the pipe. This is expected to increase the soil reaction at the invert of the pipe laid on hard bedding with poor haunch support and to increase the flexural strain at the invert of the pipe significantly. For pipe with proper haunch and invert support, the increase in arching factor is expected to have a much smaller effect on pipe wall strains.

4. Internal pressure affects the two components of the deformation in different ways. Its effect on the ovaling deformation component is to reround the pipe. However, the flexural strains in the pipe wall will not decrease as the pipe rerounds even for the flexible pipe installed with proper haunch and invert support because of the nonuniformity of the support that the soil provides for the pipe in the radial direction. The effect of the internal pressure on the pear-shaped deformation component, which results from the improper haunch support and hard bedding, is totally different. As the pressure-induced radial expansion of the pipe is inhibited by the hard invert while permitted into the loose haunch support areas, the pipe will flatten and the flexural strains will increase at the invert. The data in Table 3 indicate that the flexural strain at the invert of the pipe is increased significantly when the pipe is pressurized to 100 psi.

5. In the cases analyzed, the rerounding of the flexible pipe with internal pressure was not accompanied by an unloading of the sidefills; therefore, the assumption of elastic behavior of soil used in the program is not expected to entail large errors.

6. The quality control procedure for the installation of flexible pipe must include steps to ensure that the compaction of the backfill supporting the haunches of the pipe is adequate and that the bedding is not very hard. Note that the deflections of the pipe analyzed are quite small, not exceeding 1 percent of the diameter, and the sidefills have adequate compaction, but the strain in the pipe wall is extremely high. In other words, the quality of installation cannot be ensured by checking pipe deflection and sidefill compaction.

7. In the examples presented herein, the flattening at the invert of the pipe is not large enough to give rise to higher pressure-induced membrane stresses. This is not always the case and higher membrane stresses can result when the invert flattens in a more pronounced way *(15)*. Obviously, a larger arching factor or a lower pipe stiffness could result in larger flattening of the invert.

CONCLUSIONS

The analysis of buried flexible pipe subjected to internal pressure is performed by a finite element soil-structure interaction program. The program has a fine mesh geometry to enable accurate modeling of the variability of soil stiffness around the pipe and of the rapid attenuation of the flexural stresses of flexible pipe installed in stiff backfills. In addition, the program considers the nonlinear behavior of the pipe resulting from large deflection and from the flexural stiffening effect of the pressure-induced membrane stresses. The program provides for the hyperbolic stress-strain relationship of the soil and for the step-by-step nature of construction. Using the program, the stresses and the strains for a buried flexible pipe subjected to internal pressure with improper haunch support and hard bedding have been calculated and compared with those calculated for the same pipe installed properly.

The results show that the deformed configuration of a flexible pipe in less-than-ideal installation conditions may be visualized as the sum of two components. One component is an ovaling configuration, such as that which results from installation with proper haunch and invert supports, and the other is a pear-shaped configuration, which results from the deviation of installation from the ideal configuration, caused by a localized soil reaction at the invert of the buried pipe. Molin's formula, used in most of the present standards, gives a fixed ratio of strain to deflection in terms of pipe deflection. It may be considered to be based on a fixed mix of these two configurations and, therefore, may not be valid for general application (i.e., improper installation cases). High flexural strains can develop in a flexible pipe laid on hard bedding with improper haunch support, although the deflection of the pipe is small. The results demonstrate the supreme importance of proper installation to the performance of buried flexible pipe. In addition, the quality of an installation for a flexible pipe cannot be ensured by checking the deflection of the pipe and the compaction of sidefills. Steps must be taken to ensure that the haunch support areas are well compacted and that a proper bedding is provided.

Internal pressure is shown to reround the buried flexible pipe, but, even when the haunches and the invert are properly supported, it may not alleviate the flexural stresses in the pipe wall. In the case of a flexible pipe with an improper haunch support and a hard bedding, internal pressure increases the installation-induced flexural stresses significantly.

The program cannot at the present time simulate the time-dependent behavior of the soil and the resulting higher stresses in the pipe wall that are expected to occur with time. Therefore the results of the finite element analysis of soil-structure interaction should be reviewed in light of the expected increase in the arching factor.

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