Enhancement of Membrane Action for Analysis and Design of Box Culverts

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Current design procedures for cast-in-place reinforced concrete box culverts are based on the load factor design approach, as recommended by AASHTO, or on the working-stress method and the fundamental assumption of rigid culvert behavior. Also, the interaction between the soil cover and the structure is not considered beyond the computation of the soil load that is to be added to the other live and dead loads that affect the design. Recent developments in the understanding of the structural behavior of reinforced concrete combined with a modified formulation of this behavior may provide some ideas for improvement in the design of box culverts. Membrane (in-plane) forces are often present in reinforced concrete slabs as a result of boundary conditions and the geometry of slab deformation. Box culverts can be viewed as composed of slabs, and the restraints will be introduced by the joints and the surrounding soil backfill. These conditions will introduce in-plane forces, initially in compression and ultimately in tension, that are capable of enhancing the load-carrying capacity of the individual slabs. Such enhancement, in the domain of compressive membrane behavior, is associated with a certain amount of deflection that in many cases does not affect serviceability requirements. In the present study, one-, two-, and three-barrel culverts were analyzed using this approach, and the results can be used to demonstrate the modified behavior of such structures. It was found that the stiffness of the lateral restraint around the structure makes a significant contribution to structural capacity, that the membrane enhancement of the load is more than 50 percent larger compared with the yield line approach for the same culverts, and that this enhancement could be improved further by a relatively simple redesign of the culverts that would increase their load capacity by as much as 74 percent.

Design guidelines for cast-in-place reinforced concrete box culverts are provided by the AASHTO code (*1*) and similar codes that are based on linear elastic frame analysis of the box cross section combined with an assumed load distribution. The design parameters are obtained by employing an ultimate strength approach that eventually leads to rather stiff structural members. In these design considerations, soil-structure interaction is not considered, and the soil contribution is only to the loads that act on the box structure. The limited analytical capabilities incorporated in the design approach can be enhanced significantly by employing advanced numerical techniques such as the finite element approach, as discussed by Katona and Vittes (*2*). Another approach to the analysis and the design of box culverts is based on the theory of enhancement of membrane action in reinforced concrete slabs combined with the yield line method, as proposed by Fares and Krauthammer (*3*).

Previous studies of the behavior of reinforced concrete slabs and the significant contribution of membrane action to structural performance in the static domain have been adequately tested and documented in the last 20 years (*4*). Similar effects were also noticed for reinforced concrete slabs in the dynamic domain of behavior, and the analytical procedures that incorporated membrane action enhancement have led to accurate assessments of structural performance (*5*). Membrane action is the development of in-plane forces, due to geometric conditions at the slab supports, combined with the lateral and rotational support stiffnesses and possibly also with externally applied forces that are transmitted to the slab plane. The contribution of such action both in tension and in compression can be well beyond the load-carrying capacity that is based on the yield line theory (*6, 7*). Initially, at low central deflections, slabs behave according to the assumed one- or two-way slab behavior, but, as the central deflections increase, the compressive membrane action becomes an important mechanism that tends to peak when the central deflection is between 0.25 and 0.5 of the total depth of the slab. Beyond that point a steep decline in load capacity was noticed until the central deflection reached about 1.0 times the total depth of the slab, and at that point there is a transition into the tensile membrane domain where the resistance increases almost linearly with added central deflections. The loads are carried essentially by the steel reinforcement acting as a plastic tensile membrane with the concrete fully cracked through the entire slab depth.

This paper is intended to demonstrate how membrane action can be incorporated into the analysis, and eventually the design, of box culverts. The methodology of the approach is presented next, followed by several examples and recommendations for future development.

STRUCTURAL MECHANISMS

Two fundamental assumptions can be employed for evaluating the load-carrying capacity of reinforced concrete slabs, as extensively discussed in the literature (*4, 8, 9*). These methods consist of the yield line theory for reinforced concrete slabs based on the approach proposed by Johansen (*10, 11*) and the membrane approach that combines the concept of yield lines with the in-plane force enhancement, as discussed by Park and Gamble (*4*). At this time, only the yield line approach can be incorporated explicitly into design procedures (*8*), but the advantages of the membrane mechanism provide clear incentives for considering such contributions during slab analysis and design. Here, it is assumed that structural engineers are quite familiar with the yield line method, and therefore the membrane mechanism will be emphasized in the following discussion.

Nevertheless, it should be mentioned that the yield line theory is based on a variational approach, such as virtual work, in which the analyst assumes a collapse pattern for the structure (i.e., yield lines in the slab) and requires that the work done by the external loads over the deflected shape equal the work performed by the resisting moments over the corresponding rotations along the yield lines. The resulting equations lead to the evaluation of the limit load-carrying capacity of the slab.

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The load versus central deflection curve of a uniformly loaded reinforced concrete slab with laterally restrained edges is shown in Figure 1. At early loading stages the behavior is the same as that described by the theories for one- or two-way slabs. However, as the load is increased and the center of the slab is pushed downward, the slab edges must rotate and move outward to accommodate the central displacement. Because the edges are fixed (or restrained by the adjoining walls) such rotation and outward motion cannot occur, which causes an in-plane compression in the slab that increases its load-carrying capacity. The slab reaches its enhanced ultimate load at B with an associated central deflection of about one-half its effective depth (*4*). Beyond B the slab exhibits a decrease in load-carrying capacity with an increasing central deflection. The stage between B and C marks the transition from compressive to tensile membrane behavior. For slabs with rigid boundaries, the central deflection of the slab at C has been found to approximately equal the slab thickness. Beyond C the slab carries the load by the reinforcement acting as a plastic tensile membrane. The slab continues to carry further load until at D the reinforcement fails.

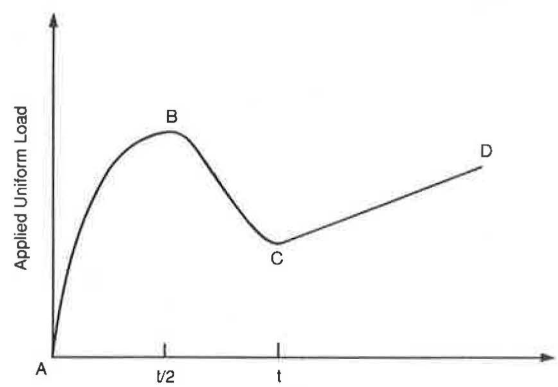

FIGURE 1 Load-deflection relationship for slab, after Park and Gamble (*4*).

The present approach is based on the model and the extensive discussion in Park and Gamble (*4*) in which a load-deflection relationship was derived for a reinforced concrete slab strip.

A fixed-end strip with developed plastic hinges is shown in Figure 2. The strip is initially of length L and is restrained against rotation and vertical translation at each end by the adjoining structural components. The ends are considered to be partly restrained against lateral displacement, and the outward lateral movement at each end is t. The strip in Figure 2 is considered to have symmetrically positioned plastic hinges. The portions of the strip between the critical sections (plastic hinges) are assumed to remain straight; it will be assumed that at each plastic hinge the tension steel has yielded, the compressive concrete has reached its uniaxial compressive strength, and the tensile strength of the concrete can be neglected. It also is assumed that the top steel at opposite supports has the same area per unit width, bottom steel is constant along the length of the strip, and top and bottom steel may be different. Because of the sensitivity of the theory to axial shortening, the axial strain (ε) will be assumed to have a constant value because the membrane force is constant along the length of the strip. The change in dimensions of end portion 1–2 due to ε and t is shown in Figure 3. It should be noted that ε is compatible with the introduction of in-plane forces to the slab and that such forces can exist only if the slab edges are effectively restrained by adjoining structural members, such as walls.

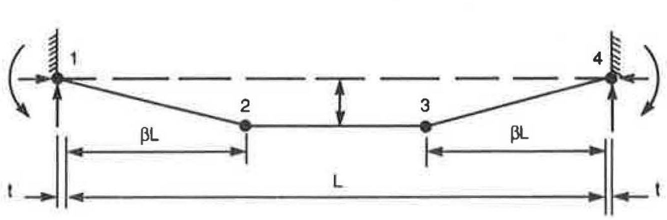

FIGURE 2 Deformation mechanism for slab strip, after Park and Gamble (*4*).

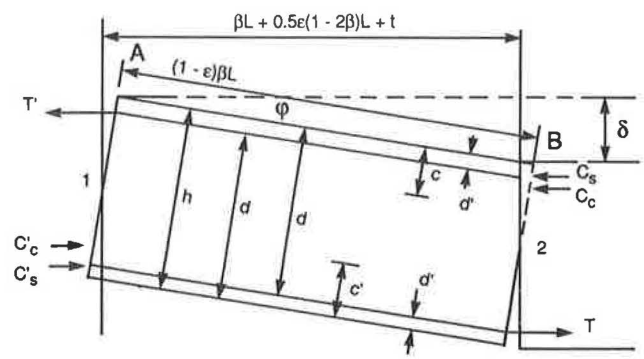

FIGURE 3 Free body diagram for strip segment, after Park and Gamble (*4*).

Employing the principle of equilibrium for this system, Park and Gamble (*4*) showed that the resulting equation for the end portions 1–2 or 3–4 of the strip in Figures 2 and 3 is

$$\begin{aligned} m'_u + m_u - n_u\delta = {} & 0.85 f'_c\beta_1 h \ \{(h/2)(1-\beta_1/2) \\ & + (\delta/4)(\beta_1 - 3) + (\beta L^2/4\delta)(\beta_1 - 1) \\ & [\varepsilon + (2t/L)] + (\delta^2/8h)[2 - (\beta_1/2)] \\ & + (\beta L^2/4h)[1 - (\beta_1/2)][\varepsilon + (2t/L)] \\ & - (\beta_1 \beta^2 L^4/16h\delta^2)[\varepsilon + (2t/L)]^2\} \\ & - (1/3.4f'_c)(T' - T - C'_s + C_s)^2 \\ & + (C'_s + C_s)[(h/2) - d' - (\delta/2)] \\ & + (T' + T)[d - (h/2) + (\delta/2)] \end{aligned} \quad (1)$$

where

L = initial length of the strip;
t = outward lateral displacement of each boundary;
ε = axial strain in the strip due to elasticity, creep, and shrinkage;
f'_c = uniaxial compressive strength for concrete;
βL = location of the middle hinges;
h = thickness of the strip;
c, c' = neutral axis depth at Sections 1 and 2, respectively;
d = distance from top compression fiber of the concrete to the steel in tension;
d' = distance from compression steel to the outer fiber of concrete in compression;

T, T' = steel tensile forces acting on Sections 1 and 2;
C_s, C'_s = steel compressive forces acting on Sections 1 and 2, respectively;
C_c, C'_c = concrete compressive forces acting on Sections 1 and 2, respectively;
m_u, m'_u = positive and negative ultimate moment capacity for strip, respectively;
n_u = membrane force corresponding to all in-plane forces on the strip at midspan;
β = constant, less than 1, defining hinge position from support as βL;
δ = deflection of the middle part of the strip;
ϕ = inclination of portion 1–2;

and

$$\beta_1 = \begin{cases} 0.85 & \text{for } f'_c \leq 4{,}000 \text{ psi} \\ 0.85 - 0.05\,[(f'_c - 4000)/1000] \geq 0.65 & \text{for } f'_c > 4{,}000 \text{ psi} \end{cases} \quad (2)$$

It is also shown (*4*) that for a fixed-end reinforced concrete strip:

$$\varepsilon + 2t/L = \Big([(1/hE_c) + (2/LS)]\,\big[0.85f'_c\beta_1\,\{(h/2) - (\delta/4) - [(T' - T - C'_c + C_s)/1.7f'_c\,\beta_1]\} + C_s - T\big]\Big)/\{1 + 0.2125\,(f'_c\,\beta_1\,\beta\,L^2/\delta)\,[(1/hE_c) + (2/LS)]\} \quad (3)$$

where E_c is the elastic modulus for concrete and S is the lateral stiffness of the slab surroundings at each end in units of load per outward displacement of the support (i.e., support resistance function). The lateral stiffness is defined as a function of E_c and represents the primarily contributions of adjoining structural elements. The contribution of soil backfill to the lateral restraining of the slab may also be considered, but usually such contribution is rather limited. Furthermore, because the primary restraint is provided by structural components, the backfill effects can be ignored for the present model.

Employing the principle of virtual work for the strip under a uniform load (w) and a virtual rotation (Θ) for portions 1–2 or 3–4:

$$(wL/2)/(\Theta L/4) = (m'_u + m_u - n_u\,\delta)\,\Theta \quad (4)$$

from which

$$(wL^2/8) = m'_u + m_u - n_u\,\delta \quad (5)$$

The load on the slab (w) can be computed from Equation 1 after Equations 3 and 5 are introduced into it. It may be noticed that in Equation 1 w is a function of material properties of steel and concrete, geometry of the strip, and central deflection (δ). Therefore the load-carrying capacity (w) can be assessed as a function of the central deflection (δ) because all other parameters are known for a given slab.

The present study was limited to the compressive membrane range ($\delta < t$) because the peak load capacity is obtained in this range and because structural damage would be too severe in the tensile membrane domain, and, therefore, might not meet code deflection and cracking control requirements.

Furthermore, only uniformly distributed loading conditions were considered because the soil cover above the culverts transforms concentrated loads on the surface to distributed loads on the culvert, as recommended by AASHTO (*1*).

EFFECT OF SURROUND STIFFNESS

The model for membrane enhancement, as described by Equations 1 and 3, includes the parameter S, which was defined as the "surround" stiffness. The fundamental development of this model, as clearly discussed by Park and Gamble (*4*), is based on the assumption that such stiffness is provided by structural elements connected to the slab in question. To obtain an understanding of the magnitude of required surround restraint for achieving significant membrane action, S can be compared with the axial stiffness of a slab strip over each half span, S_b (where S_b is the load per unit shortening of the half span). When $S = S_b$, it was shown (*4*) that

$$S = (2h/L)\,E_c \quad (6)$$

where the parameters h, E_c, L, and S are as defined earlier. From this approximate model the magnitude of the restraint provided by the surround (i.e., the surround stiffness) can be assessed from the geometric and material properties for each case. For example, in the present case the information on the slabs is given in Table 1 from which it is found that for the top slab h = 11.5 in. and L = 144 in. When this information is introduced into Equation 6 it is found that S is practically equal to 0.16 E_c. Similarily, the bottom and side slabs will provide S = 0.167 E_c and S = 0.11 E_c, respectively. Therefore, on the basis of this brief discussion, it is clear that for the present structure it should be expected that $0.11\,E_c < S < 0.167\,E_c$, which, as will be shown later, produces significant membrane enhancement. Also, the reader should realize that such values of surround restraint (stiffness) are present in the structure without the addition of any special design features, and, if required, S can be enhanced by providing stiffer boundary conditions to the slabs. However, in this paper, the slabs will be analyzed under regular conditions to illustrate the existing membrane enhancement.

COMPUTER PROGRAM

The information and behavioral model, as outlined previously, was programmed in FORTRAN on the IBM 4341 of the Department of Civil and Mineral Engineering at the University of Minnesota. It should be noted that the program was written for eliminating long manual computations, but the present approach does not require a computer program if only simple assessments are needed. At present the engineering data are in standard units, and an SI version can be prepared without difficulty. Figure 4 is a flow diagram of the program, and the following comments are keyed to the corresponding letters in the diagram.

A. The length and thickness of the strip are read. Reinforcement areas of compressive and tensile steel in both sections of the strip are read. Diameter of bars parallel to short span and stirrup diameter are read. Concrete protective cover and material properties of concrete and steel are read. Finally, location of the middle hinge and concrete ultimate strain are read. All units must be compatible (i.e., pounds and inches are used).

B. The program computes the effective depth of compressive and tensile reinforcement at both ends of the section.

TABLE 1 DESIGN DATA FOR SINGLE CAST-IN-PLACE BOX CULVERT

INSIDE HEIGHT	12.00 FEET
INSIDE WIDTH	12.00 FEET
DEPTH OF FILL	2.00 FEET
UNIT WT. FILL	130.0 LBS/CU.FT.
LATERAL SIDE PRESS. COEFF.	
MAXIMUM	0.75
INSIDE FACE SLAB	0.16
OUTSIDE FACE SIDEWALL	0.16
REINF. YIELD STRENGTH	60000. P.S.I.
ULTIMATE CONCRETE STRESS	4000. P.S.I.
SEGMENT LENGTH (INTERIOR)	40. FEET
SKEW ANGLE	0.0 DEGREES
REINF. COVER	2.0 INCHES
BEDDING CONDITION CLASS - 1	
LOAD FACTOR	1.30
LIVE LOAD COEFF.	1.67
TOTAL CONCRETE QUANTITY	62.4 CUBIC YARDS
SOIL BEARING PRESSURE	2145. POUNDS PER SQUARE FOOT

REINFORCING BARS

BAR	NO.	SIZE	LENGTH	SPACING	LOCATION	WEIGHT
B 603	97	6	13'0"	5.0 IN.	ROOF BOTTOM LONG	1894.02
B 404	49	4	12'1"	10.0 IN.	ROOF TOP LONG	395.51
B 607	87	6	13'0"	5.5 IN.	FLOOR TOP LONG	1698.76
B 409	49	4	12'1"	10.0 IN.	FLOOR BOTTOM LONG	395.51
B 510	130	5	12'5"	7.5 IN.	WALL VERTICAL INSIDE	1683.58
B 512	194	5	6'4"	5.0 IN.	CORNER TOP	1281.50
B 613	150	6	6'6"	6.5 IN.	CORNER BOTTOM	1464.45
B 414	98	4	2'2"	10.0 IN.	WALL DOWELL	141.84
B 420	122	4	7'10"	8.0 IN.	WALL VERTICAL OUTSIDE	638.39
B 421	34	4	44'0"	18.0 IN.	FLOOR AND ROOF LONG.	999.33
B 422	34	4	44'0"	18.0 IN.	WALL LONG.	999.33

STIRRUPS

BAR	NO.	DIM "A"	LENGTH	SPACING	RANGE X	LONG. SPACING	LOCATION	WEIGHT
B 446	148	7.50"	16.50"	4.50'	4.50"	13.00"	BD	135.94

TOTAL REINFORCEMENT WEIGHT 11728.12

RESULTS OF MN/DOT CALCULATIONS

REQUIRED THICKNESS

TOP 11.50 INCH BOTTOM 12.00 INCH SIDE 8.00 INCH

C. The stress block parameter and the concrete modulus of elasticity are computed.

D. The program then computes the tensile and compressive forces and the ultimate moments at both ends of the section.

E. Computation of the Johansen's load is executed using the values of ultimate moments computed in D.

F. The program computes the coefficients of lateral restraint stiffness (K_1) defined as the ratio S/E_c, which are successively incremented, and S is then inserted into Equation 3.

G. The middle-span central deflection is incremented starting from zero and reaching one slab thickness.

H. The program then finds the membrane action load for every deflection step by using compressive membrane action and plastic theory.

I. The maximum membrane load for every lateral restraint stiffness specified and the corresponding deflection are found.

J. The program then computes the ratio of membrane load to Johansen load and the corresponding ratio of central deflection to the strip depth.

K. The output provide numerical as well as graphic results.

ANALYSIS AND DESIGN CONSIDERATIONS

The present study (*3*) concentrated on three one-, two-, and three-barrel box culverts for which design data were obtained from the Minnesota Department of Transportation. These structures were analyzed to derive the ratio between the membrane-enhanced load-carrying capacity and the Johansen load as a function of the central deflection (normalized by the slab total depth, δ/h). The computations were performed for each slab of the three culverts and as a function of the lateral restraint stiffness (represented by the parameter S in Equation 3), as presented elsewhere (*3*), and in general the analytical approach for those cases is the same as that discussed in this paper for a simpler case. Here, the approach will be illustrated on a one-barrel box culvert 40 ft long with a 12-ft x 12-ft opening. The wall thicknesses are 11.5 in. for the top slab, 12 in. for the bottom slab, and 8 in. for the side slab (Table 1). The analytical results are provided for the top, bottom, and wall slabs in Figures 5–7. From these figures it is noticed that the membrane enhancement effects and the restraint stiffness contributions are significant. Furthermore, the culvert can be redesigned by employing the present approach, as explained next.

The same top slab for the one-barrel culvert (Table 1) will be designed using the yield line theory. This example is chosen to demonstrate how the proposed approach can be incorporated in the design procedure. The panel carries a uniformly distributed service line load of 640 psf (based on a simplified rectangular loading for a 2-ft soil cover and a 16,000-lb wheel load).

$$w = 16{,}000/2 + [2 * 1.75\ (1 + 2 * 1.75)] = 646$$

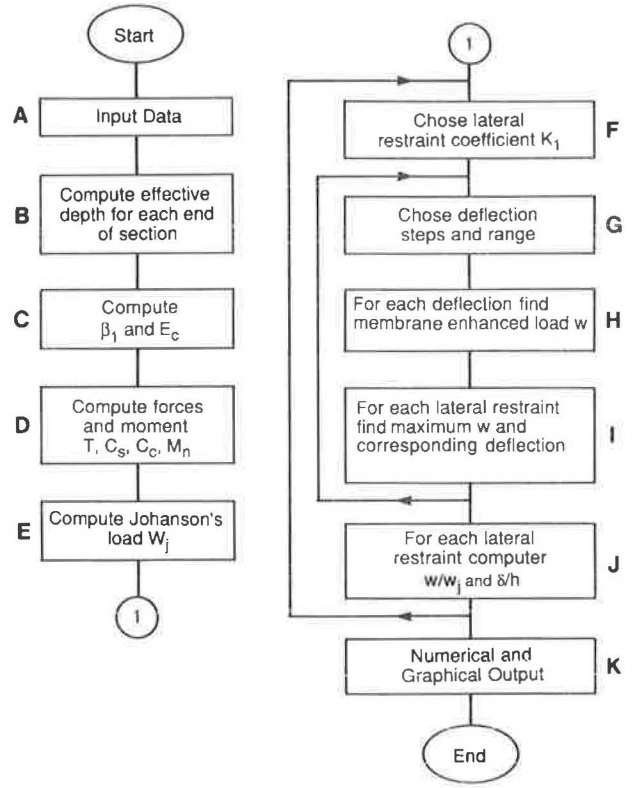

FIGURE 4 Computational approach.

In this case a 0 impact factor for 2 ft or more of fill was assumed. For this case it was assumed that w = 640 psf for purposes of illustration, but the designer may wish to perform more accurate computations.

The backfill dead load is 260 psf, and other conditions for concrete and steel are the same as for the data given in the actual design.

Select slab thickness of 11.5 in., the same as that employed in the existing structure.

Strength Requirement

Assuming that the concrete unit weight is 150 pcf, the service dead load is D = (11.5/12) 150 = 144 psf. Therefore the factored (ultimate) load according to AASHTO (*1*) is

$$w_u = \gamma\,[BdD + B_1\,(L + I) + B_e\,E] \quad (7)$$

where

γ = 1.3 for rigid culverts,
B_d = 1.0 for rigid culverts,
B_1 = 1.67 for rigid culverts,
B_e = 1.0 for rigid culverts,
D = 144 psf,
$L+I$ = 640 psf,
E = 2(130) = 260 psf, and
w_u = 1.3 {144 + 1.67 (640) + 260} = 1,915 psf.

The yield line pattern for the strip is a hinge at each end and in the center. The ultimate load is given by the following equation:

$$w_u = [24/(L_x)2]\,[1/3\,(L_y/L_x) - 1.0]\,[(L_y/L_x)\,(m'_{ux} + m_{ux}) + m'_{uy} + m_{uy}] \quad (8)$$

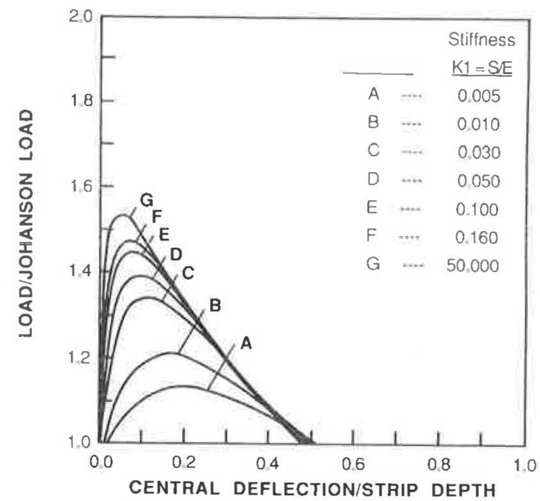

FIGURE 5 Analytical results for top slab.

Place the minimum steel in the y-direction because the load is carried effectively in the x-direction between the supported edges (where x is the direction from support to support, and y is along the culvert axis).

Use No. 4 bars with 2 in. of cover; therefore,

d = 11.5 − 2.00 − 0.25 − 0.375 = 8.875 in. in x-direction and

d = 11.5 − 2.00 − 0.50 − 0.375 − 0.25 = 8.375 in. in y-direction.

The amount of steel permitted is 0.0018 of the gross section, giving A_s = 0.0018(11.5) = 0.0207 in.²/in. width that requires No. 4 bars on 0.20/0.0207 = 9.66 in., say 9 in. on center. Therefore A_s = 0.022 in.²/in.

Place minimum steel in the y-direction at the bottom of the slab and compute the allowable moment capacity.

$$m_u = \phi\,A_s f_y\,(d - 0.59 A_s f_y\,/f'_c) \quad (9)$$

The moment is

$$m_{uy} = 0.9\,(0.022)\,(60{,}000)\,[8.375 - (0.59)\,(0.022)\,(60{,}000/4{,}000)] = 9{,}718 \text{ ft-lb/ft width.}$$

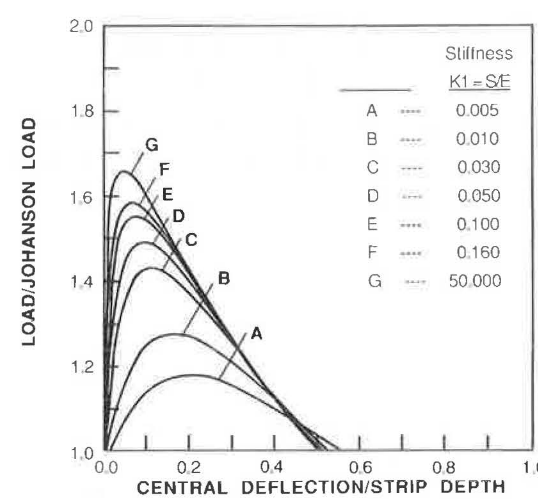

FIGURE 6 Analytical results for bottom slab.

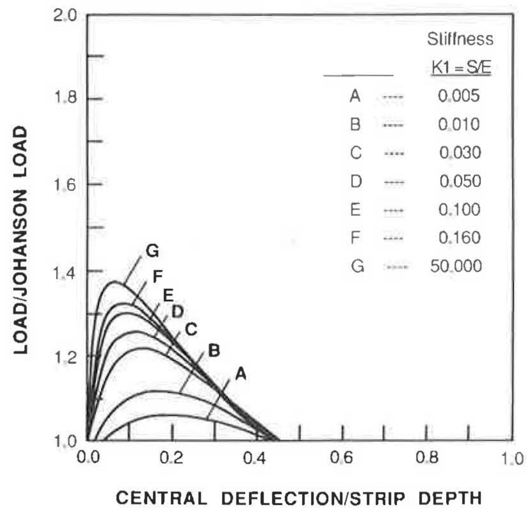

FIGURE 7 Analytical results for wall slab.

Certain assumptions about the induced moments in the x- and y-directions can be made in order to derive the required design parameters, and such assumptions can be adjusted for specific cases on the basis of better information about site and serviceability conditions. Here, such assumptions were made to enable the authors to continue with the example by introducing appropriate values into Equation 7, as follows.

If the x-direction positive moment is 1.5 times larger than the y-direction positive moment, and the x-direction negative moment is 1.5 times larger than the x-direction positive moment, then

m_{ux} = 1.5(9,718) = 14,577 ft-lb/ft width,

m'_{ux} = 1.5(14,577) = 21,865 ft-lb/ft width, and

$m_{ux} + m'_{ux}$ = 14,577 + 21,865 = 36,442 ft-lb/ft,

which is larger than $w_u L^2/8 = 1{,}915(12)^2/8 = 34{,}470$ ft-lb/ft.

Therefore, from Equation 7

$$w_u = \{24/(12^2\,[3(40/12) - 1.0])\}\ \{9{,}178 + (40/12)\,(36{,}442)\} = 2{,}419 \text{ psf},$$

which is larger than 1,915 psf. Therefore the design meets its ultimate load requirement.

For positive reinforcement

$$14{,}577 = 0.9A_s\,(60{,}000)\,[8.875 - 0.59A_s\,(60{,}000/4{,}000)].$$

Solving the quadratic equation for A_s, yields

A_s = 0.0314 in.2/in.

Using No. 4 bars on 0.20/0.0314 = 6.37 in., say 6 in., on centers. Then A_s = 0.0333 in.2/in. The resulting moment is

$$m_{ux} = 0.9\,(0.0333)\,(60{,}000)\,[8.875 - 0.59\,(0.0333)\,(60{,}000/4{,}000)] = 15{,}429 \text{ ft-lb/ft}.$$

For negative reinforcement

A_s = 0.0483 in.2/in.

Using No. 5 bar on 0.31/0.0483 = 6.4 in. Select 6 in. on centers; therefore, A_s = 0.0517 in.2/in. The resulting negative moment is

m'_{ux} = 23,499 ft-lb/ft.

Serviceability Check for Cracking

The elastic theory distribution of moments for a fixed-edge strip carrying a service load of (640 + 260) = 900 psf gives maximum x-direction moments.

Negative moment = $w_u L^2/12 = 900(12^2)/12 = 10{,}800$ ft-lb/ft

and

Positive moment = $w_u L^2/24 = 900(12^2)/24 = 5{,}400$ ft-lb/ft.

The maximum steel stress found from $f_s = M/jdA_s$ is

Top steel in x-direction $f_s = 10{,}800/[0.0517(12 - 5.5/3)] = 20.5$ ksi

and, from ACI 318–83 Section 10.6.4;

$z = 20.5[2.7(32.25)]^{1/3} = 91$ kips/in. and

Bottom steel in x-direction $f_s = 10.3$ ksi,

which is less than 20.5 ksi; therefore the design is adequate for both interior and exterior exposure according to AASHTO 8.16.8 (*1*) or ACI 318–83, Section 10.6.4 (*8*).

The new design for the one-barrel culvert was also analyzed by the present program, and the results for the top slab are shown in Figure 8. A comparison of the two designs for this culvert is provided in Table 2 from which it can be seen that the second design provides a higher load-carrying capacity, primarily because of the different reinforcement arrangement. It is important to notice in Table 2 that for $S/E_c = 0.16$ (i.e., the surround stiffness is 0.16 of the concrete elastic modulus) the membrane contributes an additional 47 percent to the original case and 66 percent to the redesigned case. Also, a significant increase in the surround stiffness will not provide much higher capacities, as is clearly indi-

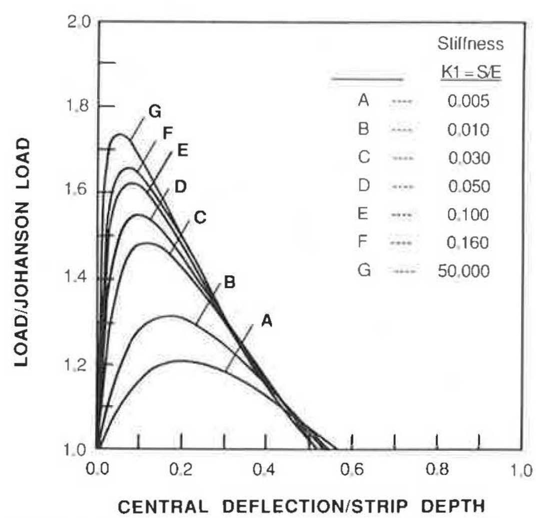

FIGURE 8 Analytical results for redesigned top slab.

TABLE 2 COMPARISON OF ELASTIC DESIGN AND DESIGN BY MEMBRANE APPROACH FOR THE TOP SLAB IN THE SINGLE-BARREL CULVERT

$K1$	Thickness, $H1$ (in.)	Span Length, L (in.)	Johansen Load Wj (lb/in.2)	Membrane Action Load, W (lb/in.2)	Strip Central Deflection, δ (in.)	Membrane Load, Johansen Load (W/Wj)	Central Deflection Thickness ($\delta/H1$)
Elastic Design							
0.005	11.5	144	24.98	28.20	2.30	1.29	0.20
0.16	11.5	144	24.98	36.80	0.81	1.47	0.07
50.0	11.5	144	24.98	38.34	0.56	1.54	0.05
Membrane Design							
0.005	11.5	144	20.98	25.34	2.30	1.21	0.20
0.16	11.5	144	20.98	34.86	0.81	1.66	0.07
50.00	11.5	144	20.98	36.42	0.56	1.74	0.05

cated in Table 2 and shown in Figures 5–8. When the surround stiffness is increased from 0.16 to 50.0, the load capacity enhancement increases only from 1.66 to 1.74. The ratio S/E_c was chosen as a parameter for representing possible restraint conditions on in-plane slab motion. Here it is important to note that such restraint is actually not provided by the soil backfill; the end conditions of the slab are a major contributor (i.e., the connection and culvert walls have a major effect on the in-plane forces in the slab). As a result, a ratio of 0.16 is not extraordinarily high when all contributions to these restraining conditions are considered.

CONTROL OF CRACKING AND DEFLECTION

Structural serviceability is an important issue that needs to be addressed by the analyst and the designer. For cracking control (*1, 8*), it was shown in the previous example that the modified design will assure that cracking will not become a problem. In the present case $z = 91$ kips/in., which is well below the limits (*1, 8*), and therefore the design is acceptable. Nevertheless, the analyst should perform similar checks for other cases to verify that they meet code requirements.

It can be seen from Figures 5–8 that the deflections associated with the peak membrane capacity are approximately in the range $0.05 < \delta/h < 0.2$ depending on the surround restraint. For the top slab $h = 11.5$ in., which corresponds to a central deflection range of 0.57 in. $< \delta <$ 2.3 in. Because the span length is 144 in. these deflections lead to $0.004 < \delta/L < 0.016$. These values can be compared with recommended deflection control criteria (*1, 8*) as follows: $L/180 = 0.8$ in. leads to δ/h values of about 0.07 for the top slab, 0.067 for the bottom slab, and 0.1 for the side slab. These values correspond to E/E_c ratios of about 0.16 for these slabs, as can be obtained from Figures 5–8. From the previous discussion on surround restraint (stiffness) it should be clear that these values are in line with existing conditions for the structure.

Another important comment that should be made is that under normal service conditions a structure is not loaded to its ultimate capacity. Therefore the anticipated normal deflections should be lower than those computed for peak resistance, and under such conditions there is no doubt that neither deflections nor cracking are serious problems when membrane action is considered.

CONCLUSIONS AND RECOMMENDATIONS

A modified analytical approach to the assessment of the behavior and design of cast-in-place reinforced concrete culverts was presented in this paper. This approach is based on the enhancement of the structural capacity of reinforced concrete slabs by the effects of in-plane forces that are provided through the jamming of the slab edges by adjoining structural elements. The membrane action mechanism was observed experimentally and is well documented (*4*).

The analytical method previously described can be employed for the analysis of cast-in-place reinforced concrete box culverts subjected to various loading conditions and encourages the user to employ engineering judgment before computer analyses. On the basis of information presented in the literature (*4*) and the results obtained in the present study (*3*), the following conclusions are drawn:

1. The present approach is simple for application to the design and behavioral assessment of cast-in-place reinforced concrete culverts. The consideration of membrane action enhancement provide good estimates of ultimate capacity and associated deflections.
2. Ultimate load is not reached at a sharp peak in the load-deflection curve, and structural capacity does not differ significantly from the ultimate value over a small range of deflections (Figures 5–8). Therefore the exact determination of the deflection at ultimate load may be unnecessary.
3. Strips with small L/h ratios are less sensitive to outward displacements at their ends than are strips with large L/h ratios. It is also evident that the surround stiffness (i.e., the restraint by adjoining structural elements) need not be enormous to achieve membrane action that is close in value to that for an infinitely rigid surround. Furthermore, the added load capacity on the order of about 10 percent does not justify the expense of providing a very stiff surround (i.e., $S/E_c = 50$ instead of 0.16, as shown in Figures 5–8).
4. The actual deflection cannot always be assumed to be accurate at maximum load. This is because the plastic theory curves do not hold for small deflections when the slab is acting elastically, or at greater deflections when the slab is acting elastically and partly plastically before the yield line pattern has fully formed. However,

this should not be a major problem if the slab is assessed to perform close to Point B in Figure 1.

5. The load-central deflection curves of Figures 5–8 for the strips tend to indicate the attainment of the maximum load at small central deflections. However, it should be noted that in the strip the whole of the positive moment yield section has the same deflection as the strip center. Thus strips (and one-way slabs) will reach ultimate loads at a smaller central deflection. Also, calculation of the ultimate load using the δ/h value of 0.15 may at first sight appear to be a crude approximation for these cases, but it can provide a practical estimate of loads and deflections. As briefly mentioned previously, the load-deflection curve is fairly flat near the ultimate load and the load is near ultimate load over a good range of deflections for stiff surrounds.

6. The use of compressive membrane action allows the designer to reduce the amount of reinforcement to less than that required by Johansen's yield line theory. For economical use of compressive membrane action, it is anticipated that the resulting reduction in the steel content of the slabs should be greater than the extra reinforcement that could be placed in the supports.

The results that were obtained by the present method demonstrate its effectiveness in evaluating the performance of the strips under consideration; however, further studies are needed to refine the approach and to evaluate it against experimental data. Also, the present approach should be reevaluated for complete slabs, rather than strips, and this should be performed by a combined experimental analytical study. The strength and safety of structures to be assessed by this approach need to be adjusted in light of similar requirements for other transportation structures (*1*). Furthermore, it is recommended that explicit contributions of the soil backfill in terms of surround stiffness be derived and a tool for optimal design of buried culverts be thus attained.

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